Applied Mathematics for Games Reflective Report - Jacob Costen (23025180)

Summary and Instructions

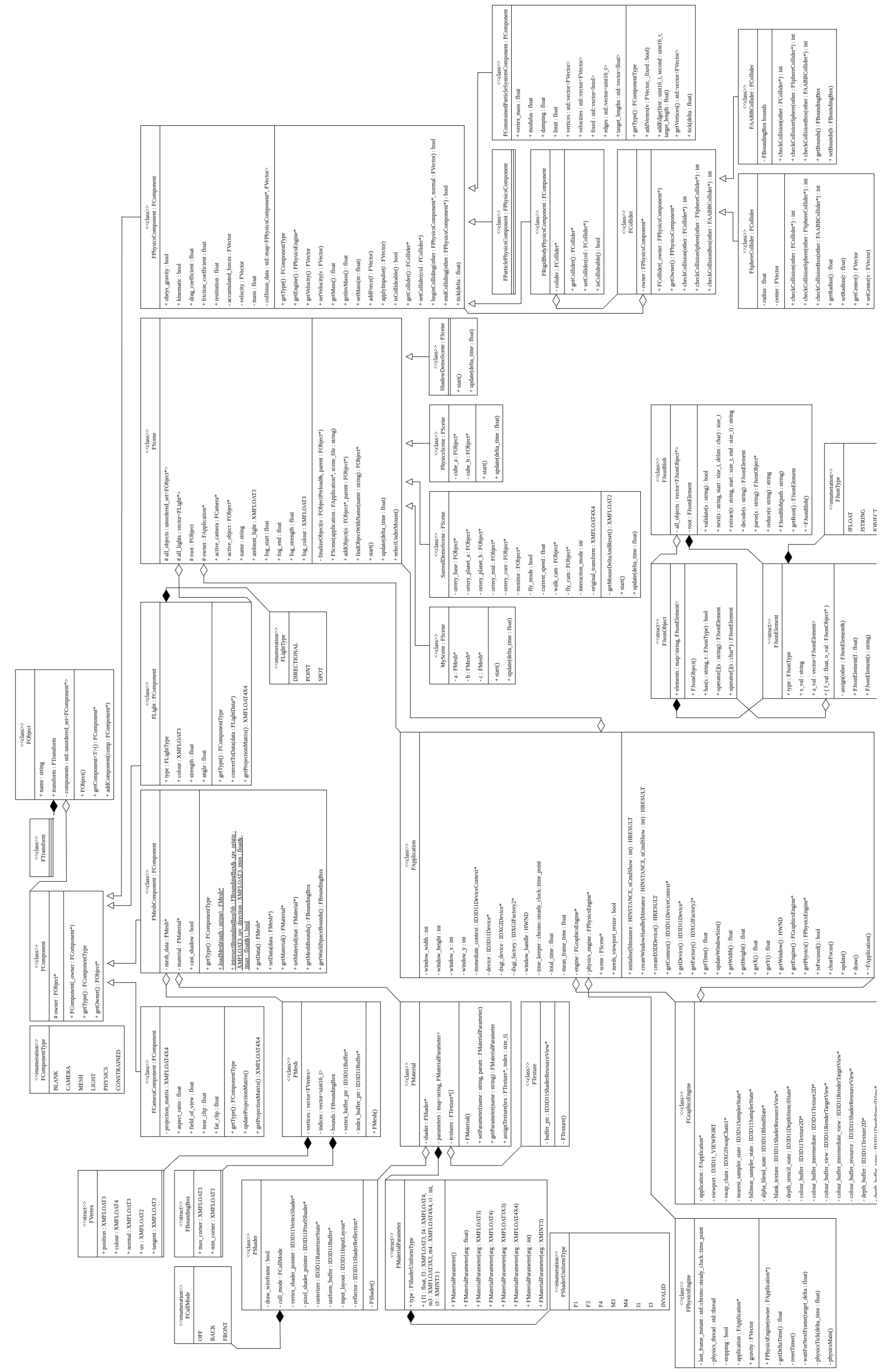
The artefact showcases various areas of development of a physics engine:

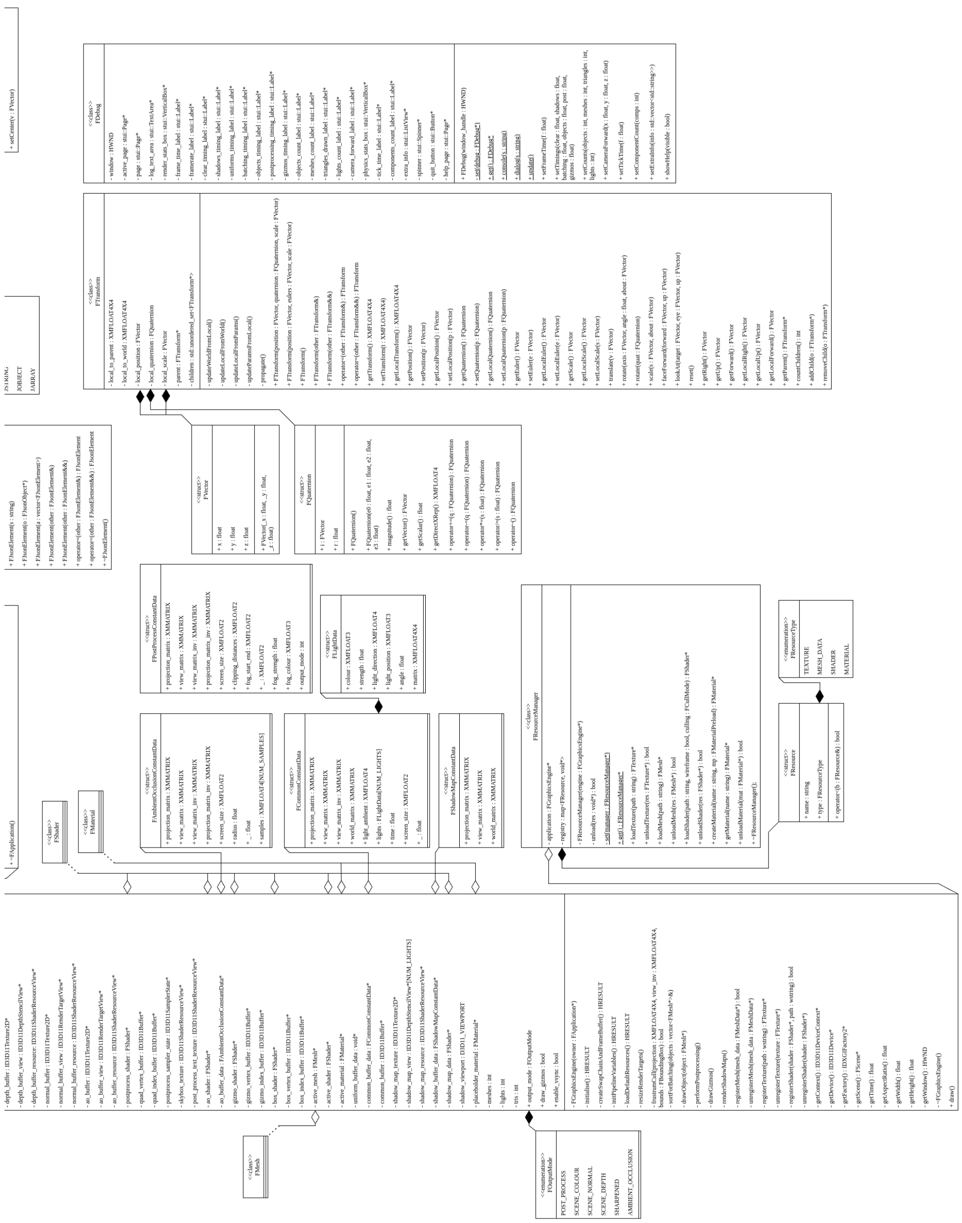
- Semi-implicit Euler integration for physics modelling, resulting in slightly higher stability than explicit Euler
- Multi-threaded physics processing, where physics is processed on a separate background thread to rendering, allowing physics to maintain a very consistent fixed time step of 60fps.
- Point mass physics: mass, force accumulation, velocity, acceleration, gravity, drag, and friction
- Rigid body physics: collision detection and resolution between combinations of spherical and axis-aligned bounding box (AABB) colliders, including restitution
- Constrained particle physics using spring forces between particles and configurable spring length and modulus of elasticity
  In addition, the artefact implements an scene tree, objects, and components, all of which can be deserialised from a scene file in the form of JSON (using a custom JSON parser developed for the project). The artefact also includes debugging tools: a system for selecting objects in the scene, gizmos to show the origins and orientations of each object, and a bounding box to highlight selection; also a detailed debug window using my own text user interface library STUI (which is header only, immediate mode, and very lightweight, making it perfect for this application) to display a terminal-based overview of the frame timing, details of the currently selected objects, and physics events like collisions.

To operate the framework, launch the project and move the debug window out of the way. Focus the graphical window, and observe the control scheme below to navigate:
WASD - move around (hold shift to move faster)
E/Q - move up/down
Arrow keys - look around
Left click - select object under mouse
J/L - apply force to selected object, on X axis
I/K - apply force to selected object, on Y axis
U/O - apply force to selected object, on Z axis
B - reset selected object velocity
N - toggle gravity for selected object
M - toggle kinematic (affected by collisions) for selected object
You can also view these by focusing the debug window and pressing H. The debug window can be used to view various statistics about the scene, including the timing of various rendering operations, the

length of the physics tick, the number of objects, meshes, physics components, etc, and a log of events, specifically physics collision events.

UML





FTransform
FDebug
FVector
FQuaternion
FPostProcessConstantData
FAmbientOcclusionConstantData
FCommonConstantData
FLightData
FShadowMapConstantData
FResourceManager
FResource
FResourceType
FShader
FMaterial
FMesh
FOutputMode

Successful Feature

The framework successfully implements non-oriented rigid bodies. This supports both collision detection and resolution for collisions between combinations of AABBs (axis-aligned bounding boxes) and spheres.

The design of this system is such that colliders are separated from the rigid body itself (i.e. any collider type can be assigned to a rigid body), and a single collider corresponds to a single physics object, a model which is common in libraries such as PhysX (Gregory (2019))[1]. This fact may be a limitation, since a common trick in games is to build an object's collider out of multiple simple primitives (e.g. two spheres), rather than building a custom (potentially convex) mesh collider, which would is almost always more computationally expensive (Ericson (2005))[2]
Collision detection and resolution were implemented appropriate ways: sphere-sphere collision is trivial to solve using the relative positions and combined radii as described by Ericson (2005)[2]; sphere-AABB collision is also relatively simple by computing a nearest-point-on-box. AABB-AABB is more complex as it involves determining the collision normal based on the axis with minimum interpenetration (i.e. to estimate which axis it would be easiest to push the boxes apart along), an approach developed specifically for the artefact. The use of interpenetration testing in all three cases means that collision resolution is quite stable.

The rigid body system, implemented in `FRigidBodyPhysicsComponent`, inherits behaviour from its parent `FPhysicsComponent` class which handles force accumulation, acceleration, gravity, drag, and friction automatically. This simplifies the implementation of the rigid body collision system.

As noted above, in addition to course materials, understanding of the rigid body system was supplemented using Gregory (2019)[1] and Ericson (2005)[2], but the Real-Time Rendering website by Haines (2025)[4] which provides links to further resources for implementing intersection detection between various geometric primitives was also used heavily.

A system for tracking currently colliding objects was also implemented. Instead of simply returning whether a collision occurred, collider testing functions return an integer, which represents whether objects began colliding, continued colliding, or finished colliding. In addition, each collider contains a `std::map` of the other colliders it is currently intersecting, and the collision normal. In a more advanced form of this feature, the key part of the map would also store the position at which the collision occurred. This feature was implemented with reference to Unity's `OnCollisionEnter`, `OnCollisionExit`, and `OnCollisionStay` methods (Unity (2024))[3], which are very useful for implementing gameplay features on collision (such as sound of visual effects). In the case of the framework, this also allows the `FRigidBodyPhysicsComponent` to compute the frictional forces affecting it due to ongoing collisions.

Another useful feature of the rigid body system implementation is support for kinematic bodies. Any physics component can be marked as kinematic, in which case it ignores forces entirely, as well as interpenetration response. This makes it easy to 'freeze' and object with respect to physics simulation (although it's velocity is still respected), while maintaining the ability for that object to interact with other objects. This ability is included in reference to similar options for rigid bodies in Unreal Engine (Epic Games (2024))[5] and Unity. In the framework, this feature is used to create an immovable floor object.

The key feature which is missing from the rigid body system is support for rotations and oriented bounding boxes (OBBs) in collisions. Implementation of these would require handling of angular velocity, optimally using quaternions to express rotations (which eliminates the problem of Gimbal lock, and eliminates confusion of axis ordering) as described by Goldman (2011)[6], and development of a new set of collision detection/resolution tests between OBBs and sphere colliders using the separating axis theorem as described by Ericson (2005)[2].

Feature Requiring Improvement

The framework includes a component for simulating constrained spring physics, but which was not implemented particularly effectively. The spring model allows the user to alter the modulus of elasticity, the damping coefficient, and the minimum and maximum length factor of the model, as well as being able to define the vertices (point masses) and edges (spring constraints) between them. In theory this provides a simple framework from which to build reasonably complex rope or even cloth systems.

A demonstration scene with a simple double-pendulum-like setup was created to test the feature, which is essentially a simple rope. Small sections of cloth-like configurations (with structure and shear springs as described by Bourg and Bywalec (2013)[8]) were also tested, but were far too unstable to meaningfully demonstrate the feature.

The key problem with the spring physics system is that it suffers very heavily from computational inaccuracy and poor stability in integration. Points affected by multiple forces tend to jitter violently unless heavy damping is applied, and springs may oscillate where otherwise they would come to rest in equilibrium. This is fundamentally due to the integration method used being a semi-implicit Euler method, which while being simple to implement and efficient in this case, leads to an unstable simulation, as backed up by the findings of Galachenko, et al. (2024)[7] who found similar results with another numerical integration problem.

The problem arises from the fact that a spring may be under a certain amount of tension, and thus produce a certain force to resolve that. However, without sufficiently small timesteps, the spring will overshoot its contraction due to the applied force and by the time the next timestep occurs, it will be far enough in compression that it must be stretched again with a similar force. This insufficient granularity of simulation leads to the jittering and instability observed both in the framework and in other studies covering numerical integration of systems of related equations.

In an attempt to improve stability, and as an additional feature, the ability to clamp the length of the spring was added. This allows the user to specify a fraction of the rest length of the spring at which the spring cannot become shorter, and this restriction was implemented by altering the force applied to points during solving to prevent (or attempt to prevent) the connecting edge from becoming shorter than or longer than . However, testing showed that if this clamping mechanism was activated too frequently (particularly on successive frames), or if the upper and lower limits were too close together, the simulation would actually become much less stable and jittering of points would be much worse.

One element of this feature which was implemented successfully was pinning: this allows select vertices to be fixed in place, and spring forces will only be applied to the other end of each spring, which is a useful feature present in many applications supporting cloth simulations, for example Blender (2025)[9] which actually allows users to specify a degree to which each vertex is pinned, giving a greater degree of control than is present in the framework.

The solution to improve the quality and stability of the simulation would be to use a more effective integration method. The best choice would be a Runge-Kutta technique, specifically RK4 which is commonly used in videogame physics engines due to its high stability and consistent convergence to a solution, balanced with not being too computationally expensive as illustrated by Hauth (2003)[10]. Runge-Kutta represents a Taylor series expansion of the expression defining acceleration at different points along the timeline between the beginning and end of the timestep, which are then used to calculate an estimated overall gradient between the start and end, hence giving a more accurate result. Runge-Kutta is also open to further improvement in terms of speed and efficiency, as shown by Wang, Hu, and Zhuang (2009)[11] who produced a 6-iteration (rather than 4) Runge-Kutta method which supports dynamic timesteps (in fact, the wider timesteps possible with Runge-Kutta are what allows it to be more less computationally expensive) and solves a test scenario in barely half the time of RK4. The one downside to RK4 is that it tends to lose energy compared to semi-implicit integration (Fielder (2004))[12], which could become a problem in this spring physics application, where an additional damping force is also applied.

It would almost certainly be best to implement this new integration method using DirectX's math functions, rather than the framework's custom `FVector` implementation. This will allow the more advanced integration to take advantage of DirectX's use of SIMD (single instruction multiple data) instructions.

Bibliography

[1]: Gregory, J. (2019) *Game Engine Architecture.* 3rd edn. Boca Raton, FL: CRC Press, Taylor & Francis Group.
[2]: Ericson, C. (2005) *Real-time collision detection.* Amsterdam: Elsevier.
[3]: Technologies, Unity (2024) 'Collider', *Unity.* Available at: https://docs.unity3d.com/6000.0/Documentation/ScriptReference/Collider.html (Accessed: 12 February 2025).
[4]: Haines, E. (2025) 'Ray Tracing Resources', *Real-Time Rendering.* Available at: https://www.realtimerendering.com/intersections.html (Accessed: 12 February 2025).
[5]: Games, Epic (2024) 'Physics bodies reference', *Unreal Engine Documentation.* Available at: https://dev.epicgames.com/documentation/en-us/unreal-engine/physics-bodies-reference-for-unreal-engine (Accessed: 12 February 2025).
[6]: Goldman, R. (2011) ‘Understanding quaternions’, *Graphical Models*, 73(2), pp. 21–49. doi:10.1016/j.gmod.2010.10.004.
[7]: Galchenko, M. *et al.* (2024) ‘Semi-implicit numerical integration of Boundary Value Problems’, *Mathematics*, 12(23), p. 3849. doi:10.3390/math12233849.
[8]: Bourg, D.M. and Bywalec, B. (2013) *Physics for game developers.* 2nd edn. Sebastopol, Calif: O’Reilly Media.
[9]: Blender (2025) 'Shape', *Blender 4.3 Manual.* Available at: https://docs.blender.org/manual/en/latest/physics/cloth/settings/shape.html (Accessed: 12 February 2025).
[10]: Hauth, M., (2003) 'Numerical techniques for cloth simulation' *SIGGRAPH 'Clothing Simulation and Animation'*
[11]: Wang, J., Hu, X. and Zhuang, Y. (2009) ‘The dynamic cloth simulation performance analysis based on the improved spring-mass model’, *2009 International Conference on Wireless Networks and Information Systems*, pp. 282–285. doi:10.1109/wnis.2009.41.
[12]: Fielder, G. (2004) 'Integration basics', *Gaffer On Games.* Available at: https://gafferongames.com/post/integration_basics/ (Accessed: 12 February 2025).