PPI calibration challenge

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1 Approximate Markov-chain methods / Linear dynamical systems approximation

In this approach I tried to approximate the dynamics of the agent-based system in terms of linear dynamical systems which would mean that the final outcomes could be derived as a sequence of matrix operations which would be extremely fast. However, the accuracy of the solution could decrease due to the approximations.

In the following equations I will try to follow the notation of the paper as closely as possible. With $\mathbf{1}$ I will denote the vector of ones.

$$q = \frac{T - I_0}{max(T - I_0)} \tag{1}$$

$$P = \frac{B}{\mathbf{1}^T q} q \tag{2}$$

$$F = \frac{\alpha}{\mathbf{1}^T \alpha} \tag{3}$$

$$C = P \left(\frac{1}{2}\mathbf{1} + \frac{F}{4} \right) \tag{4}$$

where I use the following local approximation[1]: $\frac{1}{1+e^{-x}} \approx \frac{1}{2} + \frac{x}{4}$

$$z = \beta \circ C + \bar{C} \tag{5}$$

where $\bar{C}$ is the mean and $\circ$ is the elementwise, Hadamard-product operator.

$$\gamma = z \left(\frac{1}{2} + \frac{\gamma A}{4} \right) \tag{6}$$

from which I derive γ using numerical optimization procedures, bounding its valid interval to be $\gamma_i \in [0, 1] \quad \forall i$. Then the final equation is given by:

$$I = \tau \cdot \alpha \Gamma \tag{7}$$

[1] Based on the Taylor-expansion.

where Γ is a diagonal matrix.

$$\Gamma = \begin{bmatrix} \gamma_1 & & \\ & \ddots & \\ & & \gamma_n \end{bmatrix}$$