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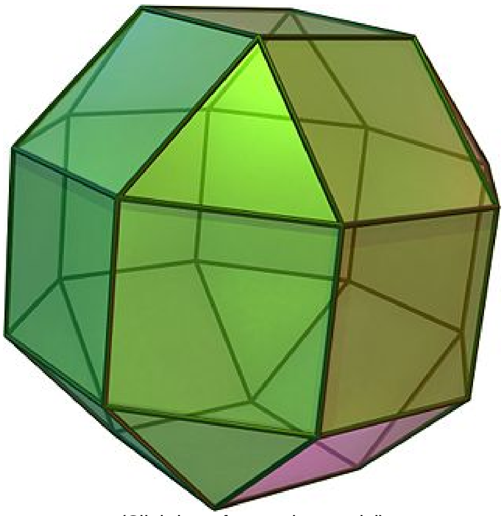
Rhombicuboctahedron

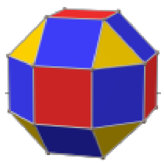
In geometry, the **rhombicuboctahedron**, or **small rhombicuboctahedron**, is a polyhedron with eight triangular, six square, and twelve rectangular faces. There are 24 identical vertices, with one triangle, one square, and two rectangles meeting at each one. If all the rectangles are themselves square (equivalently, all the edges are the same length, ensuring the triangles are equilateral), it is an Archimedean solid. The polyhedron has octahedral symmetry, like the cube and octahedron. Its dual is called the deltoidal icositetrahedron or trapezoidal icositetrahedron, although its faces are not really true trapezoids.

Rhombicuboctahedron

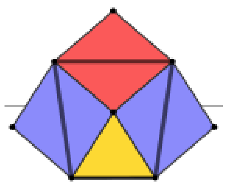


(Click here for rotating model)

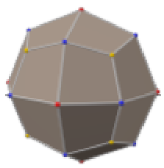
Type	Archimedean solid Uniform polyhedron
Elements	$F = 26$, $E = 48$, $V = 24$ ($\chi = 2$)
Faces by sides	8{3}+(6+12){4}
Conway notation	eC or aaC aaaT
Schläfli symbols	rr{4,3} or $r\begin{Bmatrix} 4 \\ 3 \end{Bmatrix}$ $t_{0,2}\{4,3\}$
Wythoff symbol	3 4 | 2
Coxeter diagram	
Symmetry group	O_h, B_3, [4,3], (*432), order 48
Rotation group	O, $[4,3]^+$, (432), order 24
Dihedral angle	3-4: 144°44′08″ (144.74°) 4-4: 135°
References	U_{10}, C_{22}, W_{13}
Properties	Semiregular convex



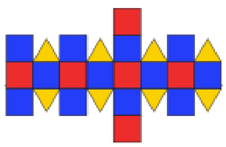
Colored faces



3.4.4.4
(Vertex figure)

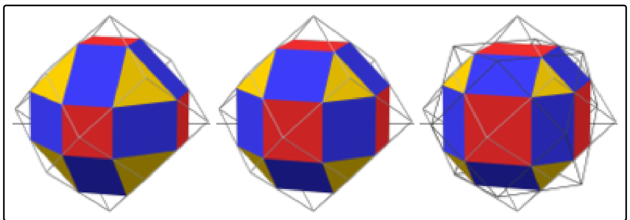


Deltoidal icositetrahedron
(dual polyhedron)



Net

Names

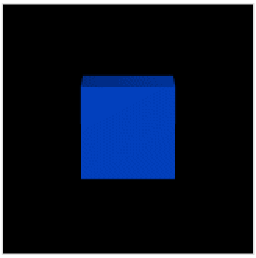


Johannes Kepler in Harmonices Mundi (1618) named this polyhedron a *rhombicuboctahedron*, being short for *truncated cuboctahedral rhombus*, with *cuboctahedral rhombus* being his name for a rhombic dodecahedron.[1] There are different truncations of a rhombic dodecahedron into a topological rhombicuboctahedron: Prominently its rectification (left), the one that creates the uniform solid (center), and the rectification of the dual cuboctahedron (right), which is the core of the dual compound.

It can also be called an *expanded* or *cantellated* cube or octahedron, from truncation operations on either uniform polyhedron.

Since its inclusion in Wings 3D as an "octotoad"[2] this unofficial moniker is spreading.

Geometric relations



The rhombicuboctahedron can be seen as either an expanded cube (the blue faces) or an expanded octahedron (the red faces).

There are distortions of the rhombicuboctahedron that, while some of the faces are not regular polygons, are still vertex-uniform. Some of these can be made by taking a cube or octahedron and cutting off the edges, then trimming the corners, so the resulting polyhedron has six square and twelve rectangular faces. These have octahedral symmetry and form a continuous series between the cube and the octahedron, analogous to the distortions of the rhombicosidodecahedron or the tetrahedral distortions of the cuboctahedron. However, the rhombicuboctahedron also has a second set of distortions with six rectangular and sixteen trapezoidal faces, which do not have octahedral symmetry but rather T_h symmetry, so they are invariant under the same rotations as the tetrahedron but different reflections.

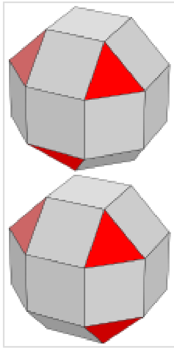
The lines along which a Rubik's Cube can be turned are, projected onto a sphere, similar, topologically identical, to a rhombicuboctahedron's edges. In fact, variants using the Rubik's Cube mechanism have been produced which closely resemble the rhombicuboctahedron.[3][4]

The rhombicuboctahedron is used in three uniform space-filling tessellations: the cantellated cubic honeycomb, the runcitruncated cubic honeycomb, and the runcinated alternated cubic honeycomb.

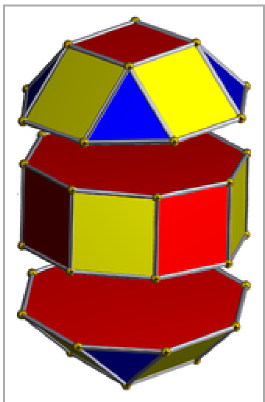
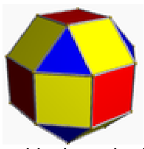
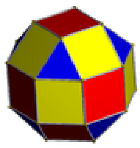
Dissection

The rhombicuboctahedron can be dissected into two square cupolae and a central octagonal prism. A rotation of one cupola by 45 degrees creates the *pseudorhombicuboctahedron*. Both of these polyhedra have the same vertex figure: 3.4.4.4.

There are three pairs of parallel planes that each intersect the rhombicuboctahedron in a regular octagon. The rhombicuboctahedron may be divided along any of these to obtain an octagonal prism with regular faces and two additional polyhedra called square cupolae, which count among the Johnson solids; it is thus an *elongated square orthobicupola*. These pieces can be reassembled to give a new solid called the elongated square gyrobicupola or *pseudorhombicuboctahedron*, with the symmetry of a square antiprism. In this the vertices are all locally the same as those of a rhombicuboctahedron, with one triangle and three squares meeting at each one, but are not all identical with respect to the entire polyhedron, since some are closer to the symmetry axis than others.



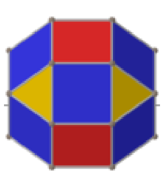
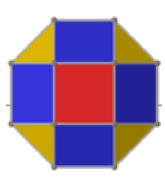
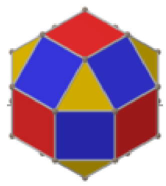
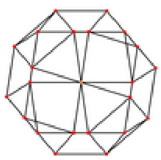
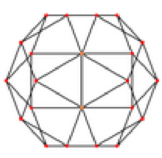
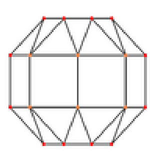
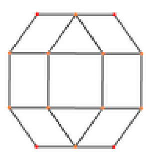
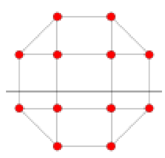
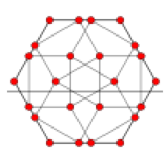
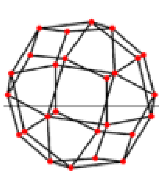
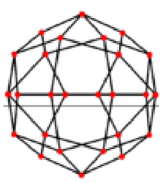
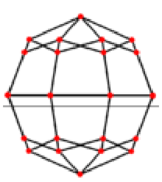
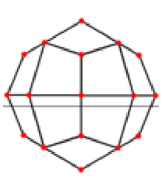
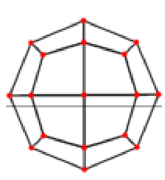
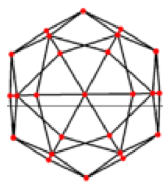
The triangles are staggered in a pseudorhombicuboctahedron (top) but aligned in a rhombicuboctahedron (bottom)

	 Rhombicuboctahedron
	 Pseudorhombicuboctahedron

Orthogonal projections

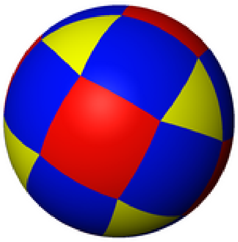
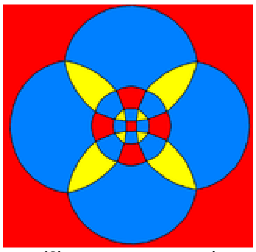
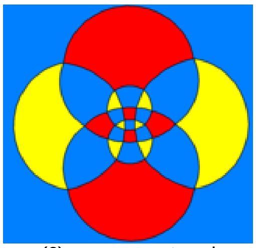
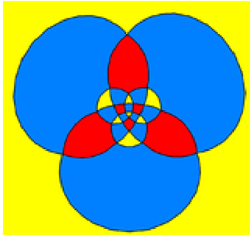
The *rhombicuboctahedron* has six special orthogonal projections, centered, on a vertex, on two types of edges, and three types of faces: triangles, and two squares. The last two correspond to the B_2 and A_2 Coxeter planes.

Orthogonal projections

Centered by	Vertex	Edge 3-4	Edge 4-4	Face Square-1	Face Square-2	Face Triangle
Solid						
Wireframe						
Projective symmetry	[2]	[2]	[2]	[2]	[4]	[6]
Dual						

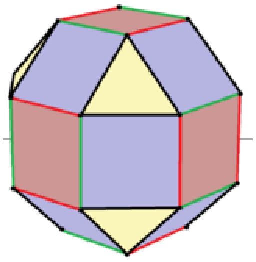
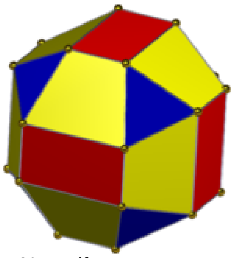
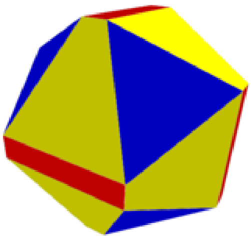
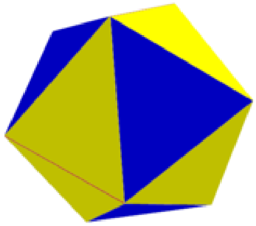
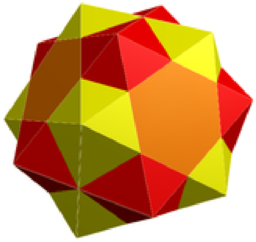
Spherical tiling

The rhombicuboctahedron can also be represented as a spherical tiling, and projected onto the plane via a stereographic projection. This projection is conformal, preserving angles but not areas or lengths. Straight lines on the sphere are projected as circular arcs on the plane.

	 (6) square-centered	 (6) square-centered	 (8) triangle-centered
Orthogonal projection	**Stereographic projections**		

Pyritohedral symmetry

A half symmetry form of the rhombicuboctahedron, , exists with pyritohedral symmetry, $[4,3^+]$, (3*2) as Coxeter diagram , Schläfli symbol $s_2\{3,4\}$, and can be called a *cantic snub octahedron*. This form can be visualized by alternatingly coloring the edges of the 6 squares. These squares can then be distorted into rectangles, while the 8 triangles remain equilateral. The 12 diagonal square faces will become isosceles trapezoids. In the limit, the rectangles can be reduced to edges, and the trapezoids become triangles, and an icosahedron is formed, by a *snub octahedron* construction, , $s\{3,4\}$. (The compound of two icosahedra is constructed from both alternated positions.)

Pyritohedral symmetry variations				
 Uniform geometry	 Nonuniform geometry	 Nonuniform geometry	 In the limit, an icosahedron snub octahedron, , from one of two positions.	 Compound of two icosahedra from both alternated positions.

Algebraic properties

Cartesian coordinates

Cartesian coordinates for the vertices of a rhombicuboctahedron centred at the origin, with edge length 2 units, are all the even permutations of

$$(\pm 1, \pm 1, \pm(1 + \sqrt{2})).$$

If the original rhombicuboctahedron has unit edge length, its dual strombic icositetrahedron has edge lengths

$$\frac{2}{7}\sqrt{10-\sqrt{2}} \quad \text{and} \quad \sqrt{4-2\sqrt{2}}.$$

Area and volume

The area A and the volume V of the rhombicuboctahedron of edge length a are:

$$\begin{aligned} A &= \left(18+2\sqrt{3}\right)a^2 &&\approx 21.464\,1016a^2 \\ V &= \frac{12+10\sqrt{2}}{3}a^3 &&\approx 8.714\,045\,21a^3. \end{aligned}$$

Close-packing density

The optimal packing fraction of rhombicuboctahedra is given by

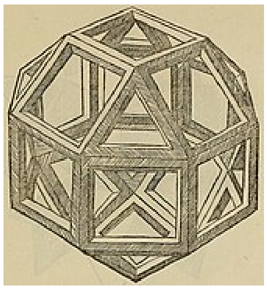
$$\eta = \tfrac{4}{3}\left(4\sqrt{2}-5\right).$$

It was noticed that this optimal value is obtained in a Bravais lattice by de Graaf (2011). Since the rhombicuboctahedron is contained in a rhombic dodecahedron whose inscribed sphere is identical to its own inscribed sphere, the value of the optimal packing fraction is a corollary of the Kepler conjecture: it can be achieved by putting a rhombicuboctahedron in each cell of the rhombic dodecahedral honeycomb, and it cannot be surpassed, since otherwise the optimal packing density of spheres could be surpassed by putting a sphere in each rhombicuboctahedron of the hypothetical packing which surpasses it.

In the arts

The 1495 *Portrait of Luca Pacioli*, traditionally attributed to Jacopo de' Barbari, includes a glass rhombicuboctahedron half-filled with water, which may have been painted by Leonardo da Vinci.[6] The first printed version of the rhombicuboctahedron was by Leonardo and appeared in Pacioli's *Divina proportione* (1509).

Portrait of Luca Pacioli (c. 1495)[5]



Leonardo da Vinci's illustration in *Divina proportione* (1509)

A spherical 180° × 360° panorama can be projected onto any polyhedron; but the rhombicuboctahedron provides a good enough approximation of a sphere while being easy to build. This type of projection, called *Philosphere*, is possible from some panorama assembly software. It consists of two images that are printed separately and cut with scissors while leaving some flaps for assembly with glue.[7]

Objects

The Freescape games *Driller* and *Dark Side* both had a game map in the form of a rhombicuboctahedron.

The "Hurry-Scurry Galaxy" and "Sea Slide Galaxy" in the videogame *Super Mario Galaxy* have planets in the similar shape of a rhombicuboctahedron.

Sonic the Hedgehog 3's Icecap Zone features pillars topped with rhombicuboctahedra.

During the Rubik's Cube craze of the 1980s, at least two twisty puzzles sold had the form of a rhombicuboctahedron (the mechanism was similar to that of a Rubik's Cube).[3][4]

Sundial (1596)

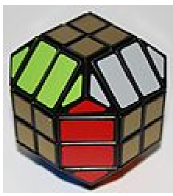
Sundial

Street lamp in Mainz

Die with 18 labelled faces

Cabela's shooting target

Rubik's Snake

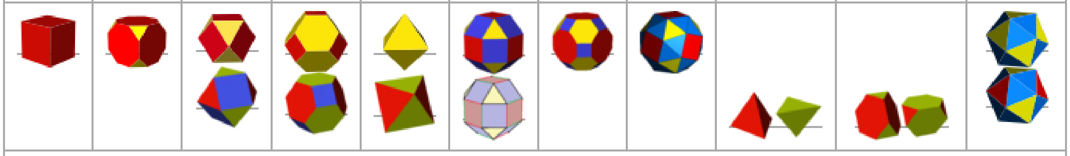
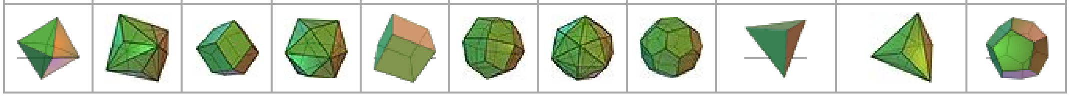


Rubik's Cube variation

Pyrite crystal

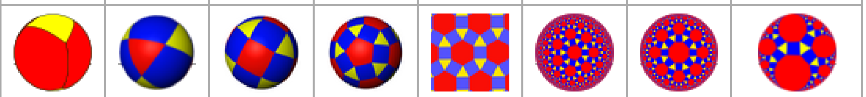
Related polyhedra

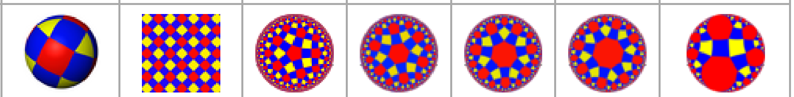
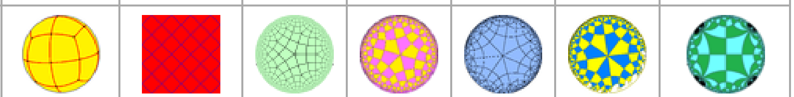
The rhombicuboctahedron is one of a family of uniform polyhedra related to the cube and regular octahedron.

Uniform octahedral polyhedra										
Symmetry: [4,3], (*432)							$[4,3]^+$ (432)	$[1^+,4,3]$ = [3,3] (*332)		$[3^+,4]$ (3*2)
{4,3}	t{4,3}	r{4,3} r$\{3^{1,1}\}$	t{3,4} t$\{3^{1,1}\}$	{3,4} $\{3^{1,1}\}$	rr{4,3} $s_2\{3,4\}$	tr{4,3}	sr{4,3}	h{4,3} {3,3}	$h_2\{4,3\}$ t{3,3}	s{3,4} s$\{3^{1,1}\}$
										
Duals to uniform polyhedra										
$V4^3$	$V3.8^2$	$V(3.4)^2$	$V4.6^2$	$V3^4$	$V3.4^3$	V4.6.8	$V3^4.4$	$V3^3$	$V3.6^2$	$V3^5$
										

Symmetry mutations

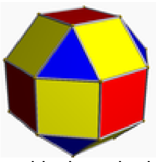
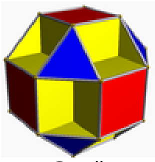
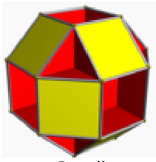
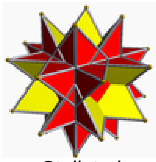
This polyhedron is topologically related as a part of sequence of cantellated polyhedra with vertex figure (3.4.*n*.4), and continues as tilings of the hyperbolic plane. These vertex-transitive figures have (**n*32) reflectional symmetry.

**n*32 symmetry mutation of expanded tilings: 3.4.*n*.4								
Symmetry *n32 [n,3]	Spherical				Euclid.	Compact hyperb.		Paracomp.
	*232 [2,3]	*332 [3,3]	*432 [4,3]	*532 [5,3]	*632 [6,3]	*732 [7,3]	*832 [8,3]...	*∞32 [∞,3]
Figure								
Config.	3.4.2.4	3.4.3.4	3.4.4.4	3.4.5.4	3.4.6.4	3.4.7.4	3.4.8.4	3.4.∞.4

**n*42 symmetry mutation of expanded tilings: *n*.4.4.4							
Symmetry [n,4], (*n42)	Spherical	Euclidean	Compact hyperbolic				Paracomp.
	*342 [3,4]	*442 [4,4]	*542 [5,4]	*642 [6,4]	*742 [7,4]	*842 [8,4]	*∞42 [∞,4]
Expanded figures							
Config.	3.4.4.4	4.4.4.4	5.4.4.4	6.4.4.4	7.4.4.4	8.4.4.4	∞.4.4.4
Rhombic figures config.	 V3.4.4.4	V4.4.4.4	V5.4.4.4	V6.4.4.4	V7.4.4.4	V8.4.4.4	V∞.4.4.4

Vertex arrangement

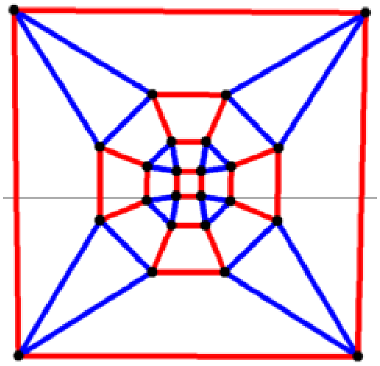
It shares its vertex arrangement with three nonconvex uniform polyhedra: the stellated truncated hexahedron, the small rhombihexahedron (having the triangular faces and six square faces in common), and the small cubicuboctahedron (having twelve square faces in common).

 Rhombicuboctahedron	 Small cubicuboctahedron	 Small rhombihexahedron	 Stellated truncated hexahedron

Rhombicuboctahedral graph

The **rhombicuboctahedral graph** is the graph of vertices and edges of the rhombicuboctahedron. It has 24 vertices and 48 edges, and is a quartic Archimedean graph.[8]

Rhombicuboctahedral graph



4-fold symmetry

Vertices	24
Edges	48
Automorphisms	48
Properties	Quartic graph, Hamiltonian, regular

Table of graphs and parameters

See also

- Compound of five rhombicuboctahedra
- Cube
- Cuboctahedron
- Nonconvex great rhombicuboctahedron
- Truncated rhombicuboctahedron
- Elongated square gyrobicupola
- Moravian star
- Octahedron
- Rhombicosidodecahedron
- Rubik's Snake – puzzle that can form a Rhombicuboctahedron "ball"
- National Library of Belarus – its architectural main component has the shape of a rhombicuboctahedron.
- Truncated cuboctahedron (great rhombicuboctahedron)

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External links

- Weisstein, Eric W., "Rhombicuboctahedron (https://mathworld.wolfram.com/SmallRhombicuboctahedron.html)" ("Archimedean solid (http://mathworld.wolfram.com/ArchimedeanSolid.html)") at *MathWorld*.
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- The Uniform Polyhedra (http://www.mathconsult.ch/showroom/unipoly/)
- Virtual Reality Polyhedra (http://www.georgehart.com/virtual-polyhedra/vp.html) The Encyclopedia of Polyhedra
- Editable printable net of a rhombicuboctahedron with interactive 3D view (http://www.dr-mikes-math-games-for-kids.com/polyhedral-nets.html?net=D9tM7OPpTeb5fdRIwHgyGnGQuG8Er0hjmb3YtRZaHYQAAns6eVm2BFtWBcy9fTY8oFG11608c05WEHBOIwUgR9mWjE9aHectfR7dboSNgfU8YAeliUFlMaH63asg2zGgf6foy68Hxg4VxIsKuxmBG9yNlpLAO1A9euZZPJ7&name=Rhombicuboctahedron#applet)
- *Rhombicuboctahedron Star (http://demonstrations.wolfram.com/RhombicuboctahedronStar/)* by Sándor Kabai, Wolfram Demonstrations Project.
- Rhombicuboctahedron: paper strips for plaiting (http://www.hbmeyer.de/flechten/rhku/indexeng.htm)