2. La variable continua: Unos problemillas

Ozner Leyva A01742377

Parte 1

$$Cx^2 \qquad 0 \le x \le 2$$

$$\int_0^2 Cx^2 = 1$$

$$c\int_0^2 x^2 = 1$$

$$c(8/3) = 1$$

$$C = \frac{1}{\frac{8}{3}} = \frac{3}{8}$$

$$P[0 < x < 1]$$

$$\int_0^1 \frac{3}{8}x^2$$

$$\frac{3}{8}\int_0^1 x^2 = \frac{3}{8}\left[\frac{x^3}{3}\right]\Big|_0^1$$

$$= \frac{1^3}{3}\cdot\frac{3}{8} = \frac{1}{8}$$

Parte 2

a) $$k\int_1^\infty \frac{1}{x^4} = k\int_1^\infty x^{-4} = k\,\frac{x^{-3}}{-3}\Big|_1^\infty = k\,\frac{1}{3}$$

$$k\left(\frac{1}{3}\right) = 1 \rightarrow k = \frac{1}{\left(\frac{1}{3}\right)} = 3$$

b) $$E[x] = \int_1^\infty x\cdot f(x)\,dx = \int_1^\infty x\cdot\frac{k}{x^4}\,dx = \int_1^\infty x\cdot\frac{3}{x^4}\,dx$$

$$= 3\int_1^\infty x\cdot\frac{1}{x^4} = 3\int\frac{x}{x^4} = 3\int_1^\infty\frac{1}{x^3} = 3\int_1^\infty x^{-3}$$

$$= 3\,\frac{x^{-2}}{-2}\Big|_1^\infty = 3\,\frac{0^{-2}}{-2} - 3\,\frac{1^{-2}}{-2} = 0 - \left[-\frac{3}{2}\right] = \frac{3}{2}$$

Varianza

$$E[x^2] = \int_1^{\infty} x^2 \cdot \frac{3}{x^4}\, dx = 3\int_1^{\infty} x^2 \cdot \frac{1}{x^4}\, dx = 3\int_1^{\infty} \frac{x^2}{x^4}\, dx$$

$$= 3\int_1^{\infty} \frac{1}{x^2}\, dx = 3\int_1^{\infty} x^{-2}\, dx = 3\left.\frac{x^{-1}}{-1}\right|_1^{\infty} = -3x^{-1}\Big|_1^{\infty} = 3$$

$$Var(x) = E[x^2] - (E[x])^2 = 3 - \left(\frac{3}{2}\right)^2 = 3 - \left(\frac{9}{4}\right)$$

$$= \frac{3}{4}$$

c)

$$P(x>2) = \int_2^{\infty} \frac{3}{x^4}\, dx = 3\int_2^{\infty} \frac{1}{x^4}\, dx = 3\int_2^{\infty} x^{-4}\, dx$$

$$= 3\left.\frac{x^{-3}}{-3}\right|_2^{\infty} = 0 - \left[-\frac{1}{8}\right] = \frac{1}{8}$$

$$P(x<2) = 1 - \frac{1}{8} = \frac{7}{8}$$

$$P(X \leq x) = \int_1^{x} \frac{3}{x^4} = 3\int_1^{x} \frac{1}{x^4}\, dx = 3\int_1^{x} x^{-4}$$

$$= 3 \cdot \left.\frac{x^{-3}}{-3}\right|_1^{x} = 3 \cdot \frac{x^{-3}}{-3} - \left[3\left(\frac{1^{-3}}{-3}\right)\right]$$

$$= \frac{-3x^{-3}}{3} - [-1] = -x^{-3} + 1$$