# Competitive programming Notebook •



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# Sumário

1	$\mathbf{D}\mathbf{s}$		2	
	1.1	sparse-table	2	
	1.2	DSU	2	
	1.3	prefix-sum-array	2	
	1.4	delta-encoding	2	
	1.5	Segtree	2	
2	Graph			
	2.1	Dijkstra	3	
	2.2	Bipartite	3	
	2.3	BFS	4	
	2.4	Kruskal	4	
	2.5	BellmanFord	4	
	2.6	DFS	5	
	2.7	MCBM	5	
	2.8	Bridge	5	
	2.9	Warshall	5	
	2.10	CycleDetection	5	
_			_	
3		orithm	6	
	3.1	merge-sort	6	
	3.2	bsearch-iterative	6	
	3.3	counting-inversions	6	
	3.4	kadane	6	
4	Math 7			
	4.1	floor-log	7	
	4.2	fast-exponentiation	7	
	4.3	matrix-exponentiation	7	
۲	D		7	
5	<b>Dp</b> 5.1	knangaak	7	
	$\frac{5.1}{5.2}$	knapsack		
	$\frac{5.2}{5.3}$	LCS	8	
		coin-change	8	
	5.4	unbouded-knapsack	8	
6	Stri	O .	8	
	6.1	General	8	
		Manacher	8	
	6.3	Z-function	6	
	6.4	Trie	6	
	6.5	AllSubPalindromes	Ĉ	
	6.6	Kmp	8	
7	Geometry 9			
	7.1	2D	ç	
			1	

Ds

1

#### if(r[a] == r[b]){ 17 p[a] = b;18 r[b]++; 1.1 sparse-table 19 qtd[b]+=qtd[a]; }else if(r[a] > r[b]){ 1 // Sparse-Table p[b] = a; 21 2 // O(log n) 22 qtd[a]+=qtd[b]; 3 const int logn = 22; // max log lelsef 2.3 p[a] = b; 24 5 int logv[MAX]; 25 qtd[b]+=qtd[a]; 6 // Pre comp log values } 26 7 void make\_log(){ 27 } logv[1] = 0;for(int i = 2; i <= MAX; i++)</pre> 9 29 // Initializing values in main() logv[i] = logv[i/2]+1;30 for(int i = 1; i <= n; i++) p[i]=i;</pre> 11 } 1.3 prefix-sum-array 13 struct Sparse { vector < vector < int > > st; 14 1 // Preffix sum 1D 15 2 // O(n) Sparse(vector<int>& v) { 16 3 int v[MAXN]: 17 int n = v.size(); 4 int psum[MAXN]; st.assign(n, vector<int>(logn, 0)); 18 // Unitary values st[i][0] = v[i, i+2^0] = v[ $^5$ 19 6 int create\_psum(){ int acc = 0;for(int i = 0; i < n; i++){</pre> 20 for(int i = 0; i < v.size(); i++){</pre> st[i][0] = v[i]; 21 acc+=v[i]; psum[i] = acc; // Constructing Sparse Table in O(log n) 23 11 for(int k = 1; k < logn; k++){</pre> 24 12 } for(int i = 0; i < n; i++){</pre> 13 if(i + (1 << k)-1 >= n)14 int query(int 1, int r){ continue; 27 return 1 == 0 ? psum[r] : psum[r]-psum[1-1]; 15 int prox = i + (1 << (k-1));</pre> st[i][k] = min(st[i][k-1], st[prox][k 16 } -1]); 1.4 delta-encoding 30 } } 31 1 // Delta encoding 2 // O (n) 33 34 int f(int a, int b){ 4 for(int i = 0; i < queries; i++){</pre> // Can be: min, max, gcd int 1, r, x; // f must have idempotent property 36 cin >> 1 >> r >> x; return min(a, b); delta[1]+=x; 38 // Queries in O(1)delta[r+1]-=x; 9 } 40 int query(int 1, int r){ int size = r-l+1; 10 int acc = 0; 41 int k = logv[size]; 11 for(int i = 0; i < v.size(); i++){</pre> // cat jump for queries in O(1)acc+=delta[i]; 12 43 int res = f(st[l][k], st[r - ((1 << k)-1)][k 13</pre> v[i]+=acc; 1): return res; 45 1.5 Segtree } 46 47 }; 1 // Segtree MAX 1.2 DSU 2 // O(log n) operations 1 // Disjoint union set 4 // DESCRIPTION: $_{5}$ // sti: id do nodo que estamos na segment tree 2 // Operation ~ O(1) 3 int r[MAXN]; 6 // stl: limite inferior do intervalo que aquele nodo 4 vector < int > qtd(MAXN, 1); representa(inclusivo) 7 // str: limite superior do intervalo que aquele nodo representa(inclusivo) 6 int get(int x) { return p[x] = (p[x] == x ? x : get(p[x])); $_{8}$ // l : limite inferior do intervalo que queremos fazer a consulta 9 // r : limite superior do intervalo que queremos void unite(int a, int b){ fazer a consulta 11 a = get(a); 10 // i : indice do vetor que queremos atualizar b = get(b); 11 // amm: novo valor daquele indice no vetor if(a == b) return; 13 class SegTree{ 14 vector < int > st; 1.5 14

16

18

32

37

41

49

50

54

79

80

82

```
vector < int > lazv:
1.5
                                                           84
16
      vector < bool > has;
                                                           85
                                                                  public:
                                                                       SegTree(int n): st(4*n, 0){size=n;}
      int size;
                                                           86
                                                                       int query(int 1, int r){return query(0,0,size
                                                           87
       int el_neutro = -(1e9 + 7);
                                                                      void update(int i, int amm){update(0,0,size
20
                                                           88
       int f(int a, int b){
                                                                  -1, i, amm);}
           return max(a,b);
                                                                      void update_range(int 1, int r, int amm){
                                                           89
                                                                  update_range(0,0,size-1,l,r,amm);}
23
                                                           90 };
24
      void propagate(int sti, int stl, int str){
25
                                                           91
           if(has[sti]){
                                                           92 // In main()
               st[sti] = lazy[sti]*(str-stl+1);
                                                           93
               if(stl!=str){
                                                           94 SegTree st(v.size());
                   lazy[sti*2+1] = lazy[sti];
29
                                                           95
                   lazy[sti*2+2] = lazy[sti];
                                                           96 for(int i = 0; i < n; i++) {
30
                                                           97
                                                                  st.update(i, v[i]);
                                                           98 }
                   has[sti*2+1] = true:
                   has[sti*2+2] = true;
               }
34
                                                                   Graph
               has[sti] = false;
           }
                                                              2.1
      }
                                                                    Dijkstra
      int query(int sti, int stl, int str, int l, int r
39
                                                            1 // Dijkstra
                                                            2 // O(V + E log E)
           if(str < l || stl > r) return el_neutro;
40
                                                            3 #define INF 1e9+10
                                                            4 vector<pair<int, int>> adj[MAXN];
           if(stl >= 1 && str <= r)</pre>
                                                            5 vector <int> dist;
               return st[sti];
43
                                                            6 vector < bool > visited;
44
                                                            7 priority_queue <pair <int,int>> q;
           // intervalo parcialmente incluido em 1-r
45
           int mid = (stl+str)/2;
46
                                                            9 void Dijkstra(int n, int start){
                                                                  for(int i = 0; i <= n; i++){
           return f(query(2*sti+1, stl, mid, l, r),
48
                                                                      dist.push_back(INF);
                                                           11
       query(2*sti+2, mid+1, str, 1, r));
                                                                       visited.push_back(false);
                                                           13
                                                                  dist[start] = 0;
       void update(int sti, int stl, int str, int i, int ^{14}
                                                                  q.push(make_pair(0, start));
        amm){
                                                                  while (!q.empty()) {
                                                           16
           if(stl == i && str == i){
                                                           17
                                                                       int a = q.top().second; q.pop();
               st[sti] += amm;
                                                                       if(visited[a]) continue;
                                                           1.8
               return;
                                                           19
                                                                       visited[a] = true;
           }
5.5
                                                           20
                                                                       for(auto u : adj[a]){
                                                                          int b = u.first, w = u.second;
                                                           21
           if(stl > i || str < i) return;</pre>
                                                                           if(dist[a]+w < dist[b]){</pre>
                                                           22
                                                                               dist[b] = dist[a]+w;
                                                           23
           int mid = (stl+str)/2;
                                                                               q.push({-dist[b], b});
60
                                                                           }
                                                           25
           // Processo de atualização dos nos filhos
                                                                      }
                                                           26
           update(sti*2+1, stl, mid, i, amm);
62
                                                           27
                                                                  }
           update(sti*2+2, mid+1, str, i, amm);
63
                                                           28 }
           st[sti] = f(st[sti*2+1], st[sti*2+2]);
                                                                   Bipartite
                                                              2.2
66
67
      void update_range(int sti, int stl, int str, int 1 // Checking if graph is Bipartite
68
      1, int r, int amm){
                                                            2 // O(V+E)
           if(stl >= 1 && str <= r){</pre>
               lazy[sti] = amm;
                                                            4 const int MAXN { 100010 };
               has[sti] = true;
                                                            5 vector < vector < int > > g(MAXN);
               propagate(sti, stl, str);
                                                            6 vector < int > color (MAXN);
               return:
           }
74
                                                            8 bool bfs(int s){
                                                                 const int NONE=0, B=1, W=2;
                                                            9
           if(stl > r || str < 1) return;</pre>
76
                                                                  queue < int > q;
                                                           10
                                                                  q.push(s);
           int mid = (stl+str)/2;
                                                                  color[s]=B;
                                                           12
           update_range(sti*2+1, stl, mid, l, r, amm);
           update_range(sti*2+2, mid+1, str, 1, r, amm); 14
                                                                  while(!q.empty()){
81
                                                                      auto u = q.front(); q.pop();
           st[sti] = f(st[sti*2+1],st[sti*2+2]);
                                                           1.6
83
      }
                                                                      for(auto v : g[u]){
                                                           17
```

```
if(color[v] == NONE){
1.8
19
                    color[v]=3-color[u];
20
                    q.push(v);
               }else if(color[v]==color[u]){
2.1
                   return false;
23
           }
2.5
           return true;
26
       }
27
28 }
30 bool is_bipartite(int n){
      for (int u = 1; u <= n; u++)
           if (color[u] == NONE && !bfs(u))
33
34
               return false;
3.5
       return true;
37 }
  2.3 BFS
1 // BFS
2 // O(V+E)
4 const int MAXN { 100010 };
6 vector < vector < int > > g(MAXN);
7 vector < bool > visited(MAXN);
8 vector < int > dist(MAXN, oo);
9 queue < int > q;
void bfs(int s){
      q.push(s);
12
       dist[s] = 0;
13
      visited[s] = true;
14
15
16
      while(!q.empty()){
          int u = q.front(); q.pop();
1.7
           for(auto v : g[u]){
19
               if(not visited[v]){
20
                    dist[v] = dist[u]+1;
21
                    visited[v] = true;
22
                    q.push(v);
23
               }
24
           }
      }
26
27 }
  2.4 Kruskal
1 // Minimum Spanning tree
2 // w/ DSU structure
4 struct edge {
      int a, b, w;
      bool operator < (edge const& other) {</pre>
       return w < other.w;</pre>
7
    }
9 };
11 /* ---- DSU Structure ----*/
12 int get(int x) {
13
   return p[x] = (p[x] == x ? x : get(p[x]));
14 }
16 void unite(int a, int b){
    a = get(a);
    b = get(b);
18
```

19

```
if(r[a] == r[b]) r[a]++;
2.0
21
    if(r[a] > r[b]) p[b] = a;
22
    else p[a] = b;
23 }
25 // Initializing values in main()
26 for(int i = 1; i <= n; i++) p[i]=i;
27
28 /* -----*/
30 vector<edge> edges, result;
31 int total_weight=0;
32
33 void mst(){
34
      sort(edges.begin(), edges.end());
35
36
      for(auto e : edges){
3.7
          if(get(e.a) != get(e.b)){
39
              unite(e.a, e.b);
               result.pb(e);
40
               total_weight+=e.w;
41
          }
42
      }
43
44 }
```

### 2.5 BellmanFord

```
1 // Bellman Ford - Min distance
3 // O(V*E)
4 // Min dist from a start node
5 // Can be aplied to negative weights
7 using edge = tuple<int, int, int>;
9 vector<int> bellman_ford(int s, int N, const vector<</pre>
       edge > & edges) {
       const int oo { 1000000010 };
11
       vector < int > dist(N + 1, oo);
       dist[s] = 0;
13
14
       for (int i = 1; i <= N - 1; i++)</pre>
15
16
           for (auto [u, v, w] : edges)
               if (dist[u] < oo and dist[v] > dist[u] +
17
       w) {
                    dist[v] = dist[u] + w;
1.8
19
                    // pred[v]=u to find path
20
21
       return dist;
23 }
24
25 // Identifying negative Cycle
26 bool has_negative_cycle(int s, int N, const vector <
       edge > & edges) {
       const int oo { 1000000010 };
27
       vector < int > dist(N + 1, oo);
29
       dist[s] = 0;
30
31
       for (int i = 1; i <= N - 1; i++)</pre>
32
33
           for (auto [u, v, w] : edges)
               if (dist[u] < oo and dist[v] > dist[u] +
34
                    dist[v] = dist[u] + w;
3.5
36
       // If after all rounds, exists a better answer -
37
       Negative cycle found
       for (auto [u, v, w] : edges)
           if (dist[u] < oo and dist[v] > dist[u] + w)
39
               return true;
40
```

4.1

```
if (dfs_low[v] > dfs_num[u])
42
      return false;
                                                           19
                                                                              bridges.emplace_back(u, v);
43
                                                           2.0
                                                           2.1
                                                                          dfs_low[u] = min(dfs_low[u], dfs_low[v]);
  2.6 DFS
                                                           22
                                                                      } else if (v != p)
                                                                          dfs_low[u] = min(dfs_low[u], dfs_num[v]);
                                                           23
                                                           24 }
1 // DFS
                                                           2.5
_{2} // 0(n+m)
                                                           26 vector<edge> bridges(int n){
3 vector < vector < int > > graph(MAX_NODES);
                                                           27
4 vector < bool > visited(MAX_NODES);
                                                                  memset(dfs_num, 0, (n + 1)*sizeof(int));
                                                           28
                                                                  memset(dfs_low, 0, (n + 1)*sizeof(int));
                                                           29
6 void dfs(int s){
                                                           3.0
      if(visited[s]) return;
                                                                  vector<edge> bridges;
                                                           31
      visited[s] = true;
                                                           32
9
      for(auto v : graph[s]){
                                                                  for (int u = 1, next = 1; u <= n; ++u)
                                                           33
          dfs(v);
10
                                                           34
                                                                      if (not dfs_num[u])
                                                                          dfs_bridge(u, u, next, bridges);
                                                           3.5
12 }
                                                                  return bridges;
                                                           3.7
  2.7 MCBM
                                                           38 }
_{1} // Augmenting Path Algorithm for Max Cardinality
                                                             2.9
                                                                   Warshall
      Bipartite Matching
2 // O(V*E)
                                                           1 // Floyd - Warshall
                                                           2 // O(n^3)
4 // Algorithm to find maximum matches between to set
                                                           3 #define INF 1e9+10
5 // of nodes (bipartite graph)
                                                           5 int adj[MAXN][MAXN];
7 vector < int > match . visited:
                                                           6 int distances[MAXN][MAXN];
9 int aug(int u){
                                                           8 void Warshall(int n, int start){
      if(visited[u]) return 0;
10
                                                                  for (int i = 1; i <= n; i++) {</pre>
                                                           9
       visited[u]=1;
                                                                      for (int j = 1; j <= n; j++) {</pre>
                                                           10
                                                                          if (i == j) distances[i][j] = 0;
                                                           11
13
       for(auto v : g[u]){
                                                                          else if (adj[i][j]) distances[i][j] = adj
          if (match[v] == -1 | | aug(match[v])) {
1.4
                                                                  [i][j];
               match[v]=u;
15
                                                                          else distances[i][j] = INF;
               return 1;
16
                                                                      }
           }
                                                           14
17
      }
                                                           1.5
18
                                                                  for (int z = 1; z <= n; z++) {
19
      return 0;
                                                                      for (int i = 1; i <= n; i++) {</pre>
20 }
                                                                          for (int j = 1; j <= n; j++) {
                                                           18
                                                                              distances[i][j] = min(distances[i][j
                                                           19
22 // Inside Main()
                                                                  ], distances[i][z] + distances[z][j]);
_{23} // Good to try - left v: [0,n-1], right: [n, m-1]
                                                                          }
24 int MCBM = 0;
25 match.assign(V, -1); // V = all vertices(left+right)
                                                           21
26 for(int i = 0; i < n; i++){ // n = size of left set
27
      visited.assign(n, 0);
      MCBM+=aug(i);
                                                                     CycleDetection
29 }
       Bridge
  2.8
                                                           1 // Existency of Cycle in a Graph
                                                           3 // 1. Better to use when path is important
1 // Algorithm to get bridges in a graph
                                                           4 // O(V+E)
3 using edge = pair<int, int>;
                                                           5 const int MAXN { 100010 };
                                                           6 vector < int > visited(MAXN, 0);
5 const int MAX { 100010 };
                                                           7 vector < vector < int > > g(MAXN);
6 int dfs_num[MAX], dfs_low[MAX];
7 vector < vector < int > > adj;
                                                           9 bool dfs_cycle(int u){
                                                                 if(visited[u]) return false;
                                                           10
9 void dfs_bridge(int u, int p, int& next, vector<edge 11
                                                                  visited[u] = true;
      >& bridges){
                                                           12
                                                           13
      dfs_low[u] = dfs_num[u] = next++;
                                                           14
                                                                  for(auto v : g[u]){
                                                                      if(visited[v] && v != u) return true;
                                                           15
12
      for (auto v : adj[u])
                                                                      if(dfs_cycle(v)) return true;
                                                           16
          if (not dfs_num[v]) {
14
                                                           1.7
                                                                  return false;
                                                           18
               dfs_bridge(v, u, next, bridges);
                                                           19 ]
16
```

1.8

2.0

```
21 bool has_cycle(int n){
22
       visited.reset();
23
       for(int u = 1; u <= n; u++)</pre>
24
           if(!visited[u] && dfs(u))
               return true;
26
       return false:
28
29 }
_{
m 31} // 2. Better when only detect cycle is important
32 // Only for undirected graphs
_{33} // When E>=V, a cycle exists
35 void dfs(int u, function < void(int) > process){
       if (visited[u])
36
37
           return;
38
       visited[u] = true;
40
      process(u);
41
42
       for (auto v : adj[u])
43
           dfs(v, process);
45
46
47 bool has_cycle(int N) {
       visited.reset();
48
49
       for (int u = 1; u <= N; ++u)</pre>
5.0
           if (not visited[u])
51
52
                vector < int > cs;
53
                size_t edges = 0;
55
                dfs(u, [&](int u) {
                    cs.push_back(u);
5.7
58
                    for (const auto& v : adj[u])
                         edges += (visited[v] ? 0 : 1);
               });
                if (edges >= cs.size()) return true;
64
           }
65
66
       return false;
67 }
```

# 3 Algorithm

#### 3.1 merge-sort

```
1 // Merge Sort
2 // O(n log n)
3 void merge_sort(vector<int>& v){
       if(v.size() == 1) return;
       vector < int > 1, r;
       for(int i = 0; i < v.size()/2; i++)</pre>
           l.push_back(v[i]);
       for(int i = v.size()/2; i < v.size(); i++)</pre>
1.0
           r.push_back(v[i]);
       merge_sort(1);
13
14
       merge_sort(r);
15
       l.push_back(INF);
16
      r.push_back(INF);
       int inil = 0, inir = 0;
19
20
```

#### 3.2 bsearch-iterative

```
1 // Binary search in iterative questions
2 // O(log n)
3 bool query(int mid, int x){
       cout << mid << endl;</pre>
       cout.flush();
       int ans;
       cin >> ans;
9
       return ans == x;
10 }
11
12 int solve(int x){
       int 1 = 1, r = n;
13
14
       int res = -1;
15
       while(1 <= r){
           int mid = (1+r)/2;
1.7
           if(query(mid, x)){
18
               res = mid;
19
               l = mid+1;
20
           }else{
21
               r = m-1:
22
23
       }
2.4
25
       return res;
26
```

#### 3.3 counting-inversions

```
1 // Counting inversions in Array
2 // O(n log n)
3 int merge_sort(vector<int>& v){
       if(v.size() == 1) return 0;
       vector < int > 1, r;
       for(int i = 0; i < v.size()/2; i++)</pre>
           1.push_back(v[i]);
9
       for(int i = v.size()/2; i < v.size(); i++)</pre>
10
           r.push_back(v[i]);
       int ans = 0;
1.3
       ans += merge_sort(1);
       ans += merge_sort(r);
14
15
       l.push_back(1e9);
16
17
       r.push_back(1e9);
18
19
       int inil = 0, inir = 0;
20
       for(int i = 0; i < v.size(); i++){</pre>
21
           if(l[inil] <= r[inir]) v[i] = l[inil++];</pre>
           else{
23
                v[i] = r[inir++];
24
                ans+=1.size()-inil-1;
25
26
27
28
29
       return ans;
30 }
```

#### 3.4 kadane

```
1 // Maximum possible sum in Array
2 // O(n)
3 int array[MAXN];
4
5 int kadane(){
6    int sum = 0, best = 0;
7    for(int i = 0; i < n; i++){
8        sum = max(array[i], sum+array[i]);
9        best = max(sum, best);
10    }
11
12    return best;
13 }</pre>
```

#### 4 Math

### 4.1 floor-log

```
1 // Find floor(log(x))
2 // 0(n)
3 int logv[MAXN];
4 void make_log(){
5     logv[1] = 0;
6     for(int i = 2; i <= MAXN; i++)
7     logv[i] = logv[i/2]+1;
8 }</pre>
```

#### 4.2 fast-exponentiation

```
1 // Fast Exponentiation
2 // O(log n)
3 ll fexp(ll b, ll e) {
4     if(e == 0) {
5         return 1;
6     }
7     ll resp = fexp(b, e/2)%MOD;
8     resp = (resp*resp)%MOD;
9     if(e%2) resp = (b*resp)%MOD;
10
11     return resp;
12 }
```

### 4.3 matrix-exponentiation

```
1 // Matrix Exponentiation
2 // O(log n)
3 #define ll long long int
4 #define vl vector<ll>
5 struct Matrix {
      vector < vl> m;
      int r, c;
      Matrix(vector<vl> mat) {
9
1.0
          m = mat;
          r = mat.size();
          c = mat[0].size();
12
14
      Matrix(int row, int col, bool ident=false) {
15
         r = row; c = col;
16
          m = vector < vl > (r, vl(c, 0));
1.7
           if(ident)
              for(int i = 0; i < min(r, c); i++)</pre>
19
                   m[i][i] = 1;
20
21
      Matrix operator*(const Matrix &o) const {
       assert(c == o.r); // garantir que da pra
24
      multiplicar
         vector<vl> res(r, vl(o.c, 0));
25
26
```

```
for(int i = 0; i < r; i++)</pre>
2.7
28
               for (int j = 0; j < o.c; j++)
                    for(int k = 0; k < c; k++)
29
                        res[i][j] = (res[i][j] + m[i][k]*
30
       o.m[k][j]) % 100000007;
31
32
           return Matrix(res);
3.3
34
35
       void printMatrix(){
           for(int i = 0; i < r; i++)</pre>
36
37
               for(int j = 0; j < c; j++)
                   cout << m[i][j] << " \n"[j == (c-1)];
38
39
40 };
41
42 Matrix fexp(Matrix b, ll e, int n) {
       if(e == 0) return Matrix(n, n, true); //
43
       identidade
       Matrix res = fexp(b, e/2LL, n);
44
       res = (res * res);
45
       if(e\%2) res = (res * b);
46
47
       return res;
49 }
50
51 // Fibonacci Example O (log n)
52 /* Fibonacci
53
      | 1 1 | * | Fn | = | Fn + 1 |
       | 1 0 | | Fn - 1 | | Fn |
5.4
55
       Generic
5.6
       |a1 a2 ... an| ** K * |Fn-1| = |Fk+n-1|
5.7
58
       |1 0 ... 0|
                                |Fn-2| |Fk+n-2|
       0 1 0 ... 0
                               Fn -3
                                         Fk+n-3
59
60
       0 0 0 ...1 0
                              |F0 | |Fk
6.1
62 */
63
64 int main() {
65
       11 n;
       cin >> n; // Fibonacci(n)
66
67
68
       if(n == 0) {
           cout << 0 << end1;
69
70
           return 0;
72
       vector < vl> m = {{1LL, 1LL}, {1LL, 0LL}};
7.3
74
       vector < vl> b = {{1LL}}, {0LL}};
7.5
       Matrix mat = Matrix(m);
76
       Matrix base = Matrix(b);
77
7.8
7.9
       mat = fexp(mat, n-1, 2);
80
       mat = mat*base:
81
       cout << mat.m[0][0] << endl;</pre>
82
83
84
8.5
       return 0:
        Dp
   5.1 knapsack
```

```
1 // Knapsack problem
2 // O(n.w)
3 int valor[MAXN], peso[MAXN], memo[MAXN];
4
5 ll solve(int i, int w){ // Recursive version
```

```
if(i <= 0 || w <= 0) return 0;</pre>
       if(memo[i][w] != -1) return memo[i][w];
       11 pegar=-1e9;
       if(peso[i] <= w){</pre>
           pegar = solve(i-1,w-peso[i])+valor[i];
11
12
1.3
       11 naopegar = solve(i-1,w);
14
15
       memo[i][w] = max(pegar, naopegar);
16
17
18
       return memo[i][w];
19 }
20
21 int dp[MAXN][MAXN], valor[MAXN], peso[MAXN];
22 int solve(int n, w){ // Iterative version
23 // n objects | max weight
       for(int i = 0; i <= n; i++)</pre>
           for(int j = 0; j <= w; j++)</pre>
25
               dp[i][j] = 0;
26
27
       for(int i = 0; i <= n; i++){</pre>
28
           for(int j = 0; j <= w; j++){
                if(i == 0 || j == 0) return dp[i][j];
3.0
                else if(peso[i-1] <= j)</pre>
3.1
                    dp[i][j] = max(dp[i-1][j-peso[i-1]]+
32
       valor[i-1],dp[i-1][j]);
33
                    dp[i][j] = dp[i-1][j];
3.4
           }
       }
36
37
       return dp[n][w];
38 }
39
40 int val[MAX], wt[MAX], dp[MAX]; // Optimization for
       space
41 int solve(int n, int W){
       for(int i=0; i < n; i++)</pre>
42
           for(int j=W; j>=wt[i]; j--)
43
44
                dp[j] = max(dp[j],dp[j-wt[i]]+val[i]);
4.5
       return dp[W];
46 }
```

#### 5.2 LCS

```
1 // LCS maior subs comum
2 // ** usar s[1 - n]
3 #define MAXN 1010
5 int s1[MAXN], s2[MAXN], tab[MAXN][MAXN];
7 int lcs(int a, int b){
      if(a == 0 || b == 0) return tab[a][b] = 0;
9
10
      if(tab[a][b] != -1) return tab[a][b];
11
      if(s1[a] == s2[b]) return lcs(a-1,b-1)+1;
13
14
      return tab[a][b] = max(lcs(a-1, b), lcs(a, b-1)); 15
15
16 }
```

#### 5.3 coin-change

```
1 // You have n coins {c1, ..., cn}
2 // Find min quantity of coins to sum K
3 // O(n.c)
4 int dp(int acc) { // Recursive version
5     if(acc < 0) return oo;
6     if(acc == 0) return 0;
7</pre>
```

```
if (memo[acc] != -1) return memo[acc];
8
9
       int best = oo:
10
       for(auto c : coins){
           best = min(best, dp(acc-c)+1);
13
14
1.5
       return memo[acc] = best;
16
17 }
18
19 int dp(){ // Iterative version
       memo[0] = 0
20
       for(int i = 1; i <= n; i++){</pre>
21
22
           memo[i] = oo;
           for(auto c : coins){
23
24
               if(i-c >= 0)
                    memo[i] = min(memo[i], memo[i-c]+1);
2.5
26
       }
27
28 }
```

#### 5.4 unbouded-knapsack

```
1 // Knapsack (unlimited objects)
2 // O(n.w)
4 int w, n;
5 int c[MAXN], v[MAXN], dp[MAXN];
7 int unbounded_knapsack(){
      for(int i=0;i<=w;i++)</pre>
9
          for(int j=0;j<n;j++)
10
               if(c[j] <= i)
                   dp[i] = max(dp[i], dp[i-c[j]] + v[j])
12
1.3
      return dp[w];
14
15 }
```

### 6 String

#### 6.1 General

```
1 // General functions to manipulate strings
2
3 // find function
4 int i = str.find("aa");
5 i = pos ou -1
6
7 // find multiples strings
8 while(i!=string::npos){
9         i = str.find("aa", i);
10 }
11
12 // replace function
13 str.replace(index, (int)size_of_erased, "content");
14 "paablo".replace(1, 2, "a"); // = Pablo
15
16 // string concatenation
17 string a = "pabl"
18 a+="0" or a+='0' or a.pb('o')
```

#### 6.2 Manacher

```
1 // Manacher Algorithm
2 // O(n)
3
4 // Find all sub palindromes in a string
5 // d1 = Odd palin, d2 = Even palin
```

```
if(i+z[i]-1 > R)
                                                           1.8
vector<int> manacher(string &s, vector<int> &d1,
                                                           19
                                                                          L = i;
                                                                          R = i + z[i] - 1;
      vector < int > &d2) {
                                                           2.0
      int n = s.size();
                                                           2.1
      for(int i = 0, l = 0, r = -1; i < n; i++) {
                                                                  }
          int k = (i > r) ? 1 : min(d1[l + r - i], r - 23
                                                                  z[0]=n:
10
                                                                  return z;
          while(0 <= i - k && i + k < n && s[i - k] == 25 }
      s[i + k]) {
                                                                    Trie
                                                             6.4
              k++;
          }
13
14
           d1[i] = k - -;
                                                             6.5
                                                                    AllSubPalindromes
          if(i + k > r) {
1.5
              l = i - k;
16
               r = i + k;
1.7
                                                           1 // Function to find all Sub palindromes
          }
18
                                                           2 // O(n*n)
19
      }
                                                            4 string s; // n = s.size();
20
      for(int i = 0, l = 0, r = -1; i < n; i++) {
                                                            5 vector < vector < bool > is_pal(n, vector < bool > (n, true)
          int k = (i > r) ? 0 : min(d2[1 + r - i + 1],
22
      r - i + 1);
          while(0 <= i - k - 1 && i + k < n && s[i - k
                                                           7 // formando todos os subpalindromos
      -1] == s[i + k]) {
                                                            8 forne(k, 1, n-1)
              k++;
                                                                 forne(i, 0, n-k-1)
          }
                                                                      is_pal[i][i+k] = (s[i]==s[i+k] && is_pal[i
25
          d2[i] = k--;
26
                                                                  +1][i+k-1]);
           if(i + k > r) {
                                                             6.6 Kmp
              1 = i - k - 1;
28
               r = i + k;
          }
3.0
      }
31
                                                                   Geometry
32
      // special vector to construct query by interval
33
                                                             7.1
                                                                    2D
      vector \langle int \rangle res(2*n-1);
      for (int i = 0; i < n; i++) res[2*i] = 2*d1[i]-1;</pre>
35
      for (int i = 0; i < n-1; i++) res[2*i+1] = 2*d2[i_1 // 2D structures template]
      +17:
37
      return res;
                                                            3 // Code from - Github: Tiagosf00/Competitive-
38
                                                                 Programming !!
39 }
                                                            4 // Writer: Tiago de Souza Fernandes
40
41 struct palindrome {
                                                            6 #define EPS 1e-6
      vector < int > res;
42
                                                            7 #define PI acos(-1)
43
                                                            8 #define vp vector<point>
      palindrome(const & s): res(manacher(s)){}
44
45
                                                           10 // typedef int cod;
      // Query if [i..j] is palindrome
46
                                                           11 // bool eq(cod a, cod b){ return (a==b); }
      bool is_palindrome(int i, int j){
                                                           12 typedef ld cod;
          return res[i+j] >= j-i+1;
48
                                                           13 bool eq(cod a, cod b){ return abs(a - b) <= EPS; }</pre>
49
                                                           14
50 }
                                                           15 struct point{
                                                                 cod x, y;
  6.3 Z-function
                                                                  int id;
                                                           17
                                                                 point(cod x=0, cod y=0): x(x), y(y){}
                                                           18
1 // Z-function
                                                           19
2 // O(n)
                                                           20
                                                                  point operator+(const point &o) const{
4 // Return array z(n) that each value z[i] tells the
                                                                      return {x+o.x, y+o.y};
                                                           22
5 // longest subsequence from i that is prefix of
      string s.
                                                           24
                                                                  point operator - (const point &o) const{
                                                                      return {x-o.x, y-o.y};
                                                           25
7 // Pattern Matching = z-func(s1$s2) acha s1 em s2.
                                                           26
                                                                  point operator*(cod t) const{
                                                           27
9 vector<ll> z_algo(const string &s){
                                                           28
                                                                     return {x*t, y*t};
      ll n = s.size();
1.0
                                                           29
      11 L = 0, R = 0;
                                                                  point operator/(cod t) const{
                                                           30
      vector<11> z(n, 0);
12
                                                           3.1
                                                                      return {x/t, y/t};
      for(11 i = 1; i < n; i++){</pre>
                                                           32
13
          if(i <= R)</pre>
                                                                  cod operator*(const point &o) const{ // dot
                                                           33
              z[i] = min(z[i-L], R - i + 1);
                                                                      return x * o.x + y * o.y;
1.5
                                                           34
           while (z[i]+i < n \text{ and } s[z[i]+i] == s[z[i]
                                                           35
      ])
                                                                  cod operator^(const point &o) const{ // cross
                                                           36
               z[i]++;
                                                                      return x * o.y - y * o.x;
                                                           37
```

```
return c/len:
38
       bool operator<(const point &o) const{</pre>
                                                            108 }
           if(!eq(x, o.x)) return x < o.x;</pre>
40
                                                            109
           return y < o.y;</pre>
                                                            110 point forca_mod(point p, ld m){
41
                                                                   ld cm = norm(p);
       bool operator == (const point &o) const{
                                                                   if(cm<EPS) return point();</pre>
43
                                                            112
           return eq(x, o.x) and eq(y, o.y);
                                                                   return point(p.x*m/cm,p.y*m/cm);
44
                                                            113
                                                            114 }
45
46
                                                            115
47 };
                                                            116
                                                            117 ///////////
48
49 ld norm(point a){ // Modulo
                                                            118 // Line
                                                            119 ///////////
5.0
       return sqrt(a*a);
51 }
                                                            120
52 bool nulo(point a){
                                                            121 struct line{
                                                                   point p1, p2;
       return (eq(a.x, 0) and eq(a.y, 0));
53
                                                            122
                                                                    cod a, b, c; // ax+by+c = 0;
54 }
                                                                   // y-y1 = ((y2-y1)/(x2-x1))(x-x1)
5.5
                                                            124
int ccw(point a, point b, point e){ //-1=dir; 0=
                                                                   line(point p1=0, point p2=0): p1(p1), p2(p2){
       collinear; 1=esq;
                                                            126
                                                                        a = p1.y-p2.y;
       cod tmp = (b-a)^(e-a); // from a to b
                                                                        b = p2.x - p1.x;
       return (tmp > EPS) - (tmp < -EPS);</pre>
                                                            128
                                                                        c = -(a*p1.x + b*p1.y);
       // if int: tira comentario
59
                                                            129
       // if(tmp==0) return 0;
                                                                   line(cod a=0, cod b=0, cod c=0): a(a), b(b), c(c)
       // if(tmp>0) return 1;
6.1
       // return -1;
                                                                        // Gera os pontos p1 p2 dados os coeficientes
62
                                                            131
63 }
                                                                        // isso aqui eh horrivel mas quebra um galho
                                                                   kkkkkk
64 point rotccw(point p, ld a){
       // a = PI*a/180; // graus
                                                                        if(b==0){
       return point((p.x*cos(a)-p.y*sin(a)), (p.y*cos(a)134
                                                                            p1 = point(1, -c/a);
66
       +p.x*sin(a)));
                                                                            p1 = point(0, -c/a);
67 }
                                                                        }else{
68 point rot90cw(point a) { return point(a.y, -a.x); }; 137
                                                                            p1 = point(1, (-c-a*1)/b);
69 point rot90ccw(point a) { return point(-a.y, a.x); };138
                                                                            p2 = point(0, -c/b);
                                                                        }
70
71 ld proj(point a, point b){ // a sobre b
       return a*b/norm(b):
7.2
                                                            141
73 }
                                                            142
                                                                    cod eval(point p){
                                                                        return a*p.x+b*p.y+c;
74 ld angle(point a, point b){ // em radianos
                                                            143
       ld ang = a*b / norm(a) / norm(b);
75
                                                            144
76
       return acos(max(min(ang, (ld)1), (ld)-1));
                                                                   bool inside(point p){
77 }
                                                            146
                                                                        return eq(eval(p), 0);
78 ld angle_vec(point v){
                                                            147
79
       // return 180/PI*atan2(v.x, v.y); // graus
                                                            148
                                                                   point normal(){
                                                                       return point(a, b);
       return atan2(v.x, v.y);
80
                                                            149
81 }
82 ld order_angle(point a, point b){ // from a to b ccw 151
       (a in front of b)
                                                                    bool inside_seg(point p){
       ld aux = angle(a,b)*180/PI;
83
                                                                       return (inside(p) and
       return ((a^b) <=0 ? aux:360-aux);</pre>
                                                                                min(p1.x, p2.x) \le p.x and p.x \le max(p1.
84
                                                            154
85 }
                                                                    x, p2.x) and
86 bool angle_less(point a1, point b1, point a2, point 155
                                                                                min(p1.y, p2.y) \le p.y  and p.y \le max(p1.y)
       b2){ // ang(a1,b1) <= ang(a2,b2)
                                                                   y, p2.y));
       point p1((a1*b1), abs((a1^b1)));
87
                                                            156
       point p2((a2*b2), abs((a2^b2)));
                                                            157
88
89
       return (p1^p2) <= 0;
                                                            158 };
90 }
                                                            159
                                                            vp inter_line(line 11, line 12){
91
                                                                   ld det = l1.a*l2.b - l1.b*l2.a;
92 ld area(vp &p){ // (points sorted)
                                                            161
                                                                    if(det==0) return {};
93
       ld ret = 0;
94
       for(int i=2;i<(int)p.size();i++)</pre>
                                                                   ld x = (l1.b*l2.c - l1.c*l2.b)/det;
           ret += (p[i]-p[0])^(p[i-1]-p[0]);
                                                                   1d y = (11.c*12.a - 11.a*12.c)/det;
95
                                                            164
       return abs(ret/2);
                                                                   return {point(x, y)};
96
97 }
                                                            166 }
98 ld areaT(point &a, point &b, point &c){
       return abs((b-a)^(c-a))/2.0;
                                                            168 point inter_seg(line 11, line 12){
99
100 }
                                                                   point ans = inter_line(11, 12);
                                                            169
101
                                                                    if(ans.x==INF or !11.inside_seg(ans) or !12.
                                                            170
102 point center(vp &A){
                                                                   inside_seg(ans))
       point c = point();
                                                                        return point(INF, INF);
103
       int len = A.size();
                                                                    return ans;
104
                                                            172
       for (int i = 0; i < len; i ++)</pre>
                                                            173
105
106
            c=c+A[i];
                                                            174
```

```
175 ld dseg(point p, point a, point b){ // point - seg
                                                                   ld m2 = norm(p1 - p3);
                                                           244
176
       if(((p-a)*(b-a)) < EPS) return norm(p-a);
                                                           245
                                                                   1d m3 = norm(p1 - p2);
       if(((p-b)*(a-b)) < EPS) return norm(p-b);
                                                                   point c = (p1*m1+p2*m2+p3*m3)*(1/(m1+m2+m3));
177
                                                           246
       return abs((p-a)^(b-a))/norm(b-a);
178
                                                           247
                                                                   1d s = 0.5*(m1+m2+m3);
179 }
                                                           248
                                                                   1d r = sqrt(s*(s-m1)*(s-m2)*(s-m3))/s;
                                                                   return circle(c, r);
180
                                                           249
181 ld dline(point p, line l){ // point - line
                                                           250
       return abs(1.eval(p))/sqrt(1.a*1.a + 1.b*1.b);
182
183 }
                                                           252 circle circumcircle(point a, point b, point c) {
                                                                   circle ans;
184
                                                           253
185 line mediatrix(point a, point b){
                                                                   point u = point((b-a).y, -(b-a).x);
                                                           254
186
       point d = (b-a)*2;
                                                                   point v = point((c-a).y, -(c-a).x);
                                                                   point n = (c-b)*0.5;
187
       return line(d.x, d.y, a*a - b*b);
                                                                   1d t = (u^n)/(v^u);
188
                                                           257
                                                                   ans.c = ((a+c)*0.5) + (v*t);
189
                                                           258
190 line perpendicular(line 1, point p){ // passes
                                                           259
                                                                   ans.r = norm(ans.c-a);
       through p
                                                           260
                                                                   return ans;
       return line(l.b, -l.a, -l.b*p.x + l.a*p.y);
                                                           261
191
                                                           262
                                                           263 vp inter_circle_line(circle C, line L){
193
                                                                   point ab = L.p2 - L.p1, p = L.p1 + ab * ((C.c-L.
                                                           264
195 ///////////
                                                                   p1)*(ab) / (ab*ab));
196 // Circle //
                                                                   ld s = (L.p2-L.p1)^(C.c-L.p1), h2 = C.r*C.r - s*s
197 //////////
                                                                   / (ab*ab);
                                                                   if (h2 < 0) return {};</pre>
198
199 struct circle{
                                                                   if (h2 == 0) return {p};
                                                           267
                                                                   point h = (ab/norm(ab)) * sqrt(h2);
200
       point c; cod r;
                                                           268
       circle() : c(0, 0), r(0){}
                                                                   return {p - h, p + h};
201
                                                           269
       circle(const point o) : c(o), r(0){}
                                                           270
       circle(const point a, const point b){
203
           c = (a+b)/2;
                                                           272 vp inter_circle(circle C1, circle C2){
204
           r = norm(a-c);
                                                                   if(C1.c == C2.c) { assert(C1.r != C2.r); return
205
                                                                   {}; }
206
       circle(const point a, const point b, const point 274
                                                                   point vec = C2.c - C1.c;
                                                                   1d d2 = vec*vec, sum = C1.r+C2.r, dif = C1.r-C2.r
           c = inter_line(mediatrix(a, b), mediatrix(b,
       cc));
                                                                   1d p = (d2 + C1.r*C1.r - C2.r*C2.r)/(d2*2), h2 =
           r = norm(a-c);
                                                                   C1.r*C1.r - p*p*d2;
209
                                                                   if (sum*sum < d2 or dif*dif > d2) return {};
210
                                                           277
       bool inside(const point &a) const{
                                                                   point mid = C1.c + vec*p, per = point(-vec.y, vec
211
                                                           278
212
           return norm(a - c) <= r;</pre>
                                                                   .x) * sqrt(max((1d)0, h2) / d2);
                                                                   if(eq(per.x, 0) and eq(per.y, 0)) return {mid};
213
                                                           279
       pair < point , point > getTangentPoint(point p) {
                                                           280
                                                                   return {mid + per, mid - per};
214
215
           1d d1 = norm(p-c), theta = asin(r/d1);
                                                           281 }
           point p1 = rotccw(c-p,-theta);
216
                                                              7.2 ConvexHull
           point p2 = rotccw(c-p,theta);
217
           p1 = p1*(sqrt(d1*d1-r*r)/d1)+p;
218
           p2 = p2*(sqrt(d1*d1-r*r)/d1)+p;
                                                            1 // Convex Hull
220
            return {p1,p2};
                                                            2 // Algorithm: Monotone Chain
221
                                                             3 // Complexity: O(n) + ordenacao O(nlogn)
222 };
223
                                                             5 // Regra mao direita p2->p1 (dedao p cima ? esq : dir
224 // minimum circle cover O(n) amortizado
                                                                   || colinear)
225 circle min_circle_cover(vector<point> v){
       random_shuffle(v.begin(), v.end());
226
                                                             7 int esq(point p1, point p2, point p3){
       circle ans;
                                                                   cod cross = (p2-p1)^{(p3-p1)};
       int n = v.size();
                                                                   if(cross == 0) return 0;
       for(int i=0;i<n;i++) if(!ans.inside(v[i])){</pre>
229
                                                                   else if(cross > 0) return 1;
                                                            10
230
           ans = circle(v[i]);
                                                                   return -1;
231
            for(int j=0;j<i;j++) if(!ans.inside(v[j])){</pre>
                                                            12 }
                ans = circle(v[i], v[j]);
                                                            13
                for(int k=0;k<j;k++) if(!ans.inside(v[k]) 14 vector<point> convex_hull(vector<point> p) {
233
       ) {
                                                                   sort(p.begin(), p.end());
                                                            15
                    ans = circle(v[i], v[j], v[k]);
234
                                                            16
235
                }
                                                                   vector <point > L, U;
           }
236
                                                            18
237
                                                            19
                                                                   // Lower Hull
       return ans;
238
                                                            20
                                                                   for(auto pp : p){
239
                                                                       while (L.size() \geq 2 && esq(L[L.size()-2], L.
240
                                                                   back(), pp) == -1)
241
                                                                           L.pop_back();
242 circle incircle( point p1, point p2, point p3){
                                                            23
                                                                       L.pb(pp);
       ld m1=norm(p2-p3);
                                                            24
```

```
}
25
                                            32
     reverse(all(p));
26
                                            33
     // Upper Hull
                                                 L.pop_back();
27
                                            34
    for(auto pp : p){
                                            35
                                                 L.insert(L.end(), U.begin(), U.end()-1);
28
                                                 return L;
           U.pop_back();
31
        U.pb(pp);
```