[1] Maggie Miller and Patrick Naylor, *Non-orientable trisections*, arXiv:2010.07433.

We prove an analogue of a classical theorem of Laudenbach-Poénaru for non-orientable 4-dimensional 1-handlebodies, which has several applications. This shows that homotopic diffeomorphisms of $\#^k S^1 \tilde{\times} S^2$ are isotopic, and in particular that the diffeotopy group of $S^1 \tilde{\times} S^2$ is $\mathbb{Z}_2 \oplus \mathbb{Z}_2$, reproving a theorem of Kim-Raymond. Building on work of Rubinstein-Tillmann, we define trisection diagrams for closed non-orientable 4-manifolds, and extend much of the theory of trisections from the orientable case, including relative trisections, and bridge trisections for knotted surfaces in 4-manifolds.

[2] Gabriel Islambouli and Patrick Naylor, *Multisections of 4-manifolds*, arXiv:2010.03057. Submitted.

We define and study *multisections* of closed 4-manifolds, which are an analogue of trisections, but with more than three 1-handlebody pieces. This provides some additional flexibility, and as a result, we are able to give infinitely many examples of exotic 4-manifolds with the same multisection genus. We are able to explicitly describe the cork twist of the Mazur manifold as an operation on a subsection of a multisection, which has interesting diagrammatic consequences. We also show that the analogue of Waldhausen's theorem fails for 4-sections (using exotic Mazur manifolds of Hayden-Mark-Piccirillo).

[3] Patrick Naylor and Hannah Schwartz, *Gluck twisting roll spun knots*, arXiv:2009.05703. Submitted.

The *Gluck twist* of a embedded 2-sphere in the 4-sphere is a surgery operation producing a homotopy 4-sphere. It remains an open question (since its introduction in the 1960s) whether all Gluck twists are standard, i.e., are diffeomorphic to S^4. Various families of knotted 2-spheres arising from spinning constructions are known to have standard Gluck twists, but roll spun knots are a well known family that have resisted usual methods of attack.

In this paper, we study the Gluck twist of the m-twist n-roll spin of a knot K, and show that if K has unknotting number equal to one, then this homotopy 4-sphere is standard. We take a completely different approach from existing literature, and relate regular homotopies of these 2-spheres to logarithmic transformations along unknotted tori in S^4, which are known to be standard by a theorem of Montesinos. The proof does not obviously extend to knots with higher unknotting number, which is particularly interesting. We also show that an infinite family of Gompf's twisted doubles arising from his infinite order cork construction are standard, generalizing a result of Akbulut.

[4] Patrick Naylor, *Trisection diagrams and twists of 4-manifolds*, arXiv:1906.0149. Submitted.

In this paper, we give a new proof of a theorem of Katanaga, Saeki, Teragaito, and Yamada, which relates the Gluck twist to the *Price twist* (a surgery along an embedded $\mathbb{RP}^2$). We use trisections, as well as recent results of Gay-Meier and Kim-Miller on trisection diagrams of Gluck twists and relative trisections of complements of surfaces in 4-manifolds. In particular, we show that such Gluck and Price twists are not only diffeomorphic, but always admit equivalent trisections up to stabilization. After setting up the appropriate trisections, the proof is entirely diagrammatic.

[5] Robert Craigen, Colin Desmarais, Ted Eaton, and Patrick Naylor, *Negacylic weighing matrices*. Submitted.

In this paper, we survey *negacyclic weighing matrices*, which are similar to circulant matrices; each row is obtained from the previous by cyclically permuting the entries, but with a sign change. These matrices have combinatorial properties related to Hadamard and weighing matrices. We give some recursive constructions, and report the results of a comprehensive examination of existence in small orders. This paper was the result of a summer undergraduate research project.

[6] Ahmet Beyaz, Patrick Naylor, Sinem Onaran, and Doug Park, *From automorphisms of Riemann surfaces to smooth 4-manifolds*, Math Res. Lett. 27(3), 629-645, 2020.

In this paper, we give a general branched covering method which, given an suitable collection of automorphisms of a Riemann surface, produces a surface bundle over a surface with positive signature. This has applications to the minimal genus problem, i.e., finding the smallest natural number b for which there exists a surface bundle over a surface with base genus b, and with fixed (positive) signature and fiber genus. The construction also has applications to the symplectic geography problem: with these surface bundles as input, we are able to use a theorem of Ahkmedov-Park to improve existing bounds (on p and q) for when when the symmetric billinear form $pE_8 \oplus qH$ is the intersection form of infinitely many pairwise non-diffeomorphic simply connected irreducible symplectic *and* non-symplectic 4-manifolds.

[7] Adam Clay, Colin Desmarais, and Patrick Naylor, *Testing bi-orderability of knot groups*, Canad. Math. Bull. 59(3), 472-482, 2016.

For the purposes of this paper, a group is *bi-orderable* if it admits an ordering which is compatible with both left and right multiplication. Knot groups are always left-orderable, but are not bi-orderable in general. Using a theorem of Chiswell-Glass-Wilson on 1-relator groups, we are able to completely determine bi-orderability for all twist knot groups, and many 2-bridge knot groups. Using the *KnotInfo* table and SnapPy, we are also able to determine bi-orderability for many knot groups admitting a 2-generator 1-relator presentation, among all knots with 12 or fewer crossings.