CS F320 – FOUNDATIONS OF DATA SCIENCE

ASSIGNMENT 2A – Implementing PCA from Scratch

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Importing the Libraries

In [1]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import math
import random
```

⏳ Loading the Dataset

In [2]:

```
df = pd.read_csv("audi.csv")
df.head()
```

Out[2]:

	model	year	price	transmission	mileage	fuelType	tax	mpg	engineSize
0	A1	2017	12500	Manual	15735	Petrol	150	55.4	1.4
1	A6	2016	16500	Automatic	36203	Diesel	20	64.2	2.0
2	A1	2016	11000	Manual	29946	Petrol	30	55.4	1.4
3	A4	2017	16800	Automatic	25952	Diesel	145	67.3	2.0
4	A3	2019	17300	Manual	1998	Petrol	145	49.6	1.0

In [3]:

```
df.shape
```

Out[3]:

```
(10668, 9)
```

In [4]:

```
# checking categorical variables
df.select_dtypes(exclude=['number']).head()
```

Out[4]:

	model	transmission	fuelType
0	A1	Manual	Petrol
1	A6	Automatic	Diesel
2	A1	Manual	Petrol
3	A4	Automatic	Diesel
4	A3	Manual	Petrol

Dropping Categorical Variables

In [5]:

```
# Drop all categorical columns
df = df.select_dtypes(exclude='object')
df
```

Out[5]:

	year	price	mileage	tax	mpg	engineSize
0	2017	12500	15735	150	55.4	1.4
1	2016	16500	36203	20	64.2	2.0
2	2016	11000	29946	30	55.4	1.4
3	2017	16800	25952	145	67.3	2.0
4	2019	17300	1998	145	49.6	1.0
...	...	...	...	...	...	...
10663	2020	16999	4018	145	49.6	1.0
10664	2020	16999	1978	150	49.6	1.0
10665	2020	17199	609	150	49.6	1.0
10666	2017	19499	8646	150	47.9	1.4
10667	2016	15999	11855	150	47.9	1.4

10668 rows × 6 columns

Excluding the target variable

In [6]:

```
target_var = df["price"]
```

In [7]:

```
df = df.drop(columns='price')
```

1. Data Understanding and Representation

In [8]:

```
print("Number of records in the given dataset are: ",len(df))
print("Number of features in the given dataset are: ",len(df.columns))
```

```
Number of records in the given dataset are:  10668
Number of features in the given dataset are:  5
```

In [9]:

```
# representing data in matrix format, each row representing a car, columns re
feature_matrix = df.values
```

In [10]:

```
print(feature_matrix)
```

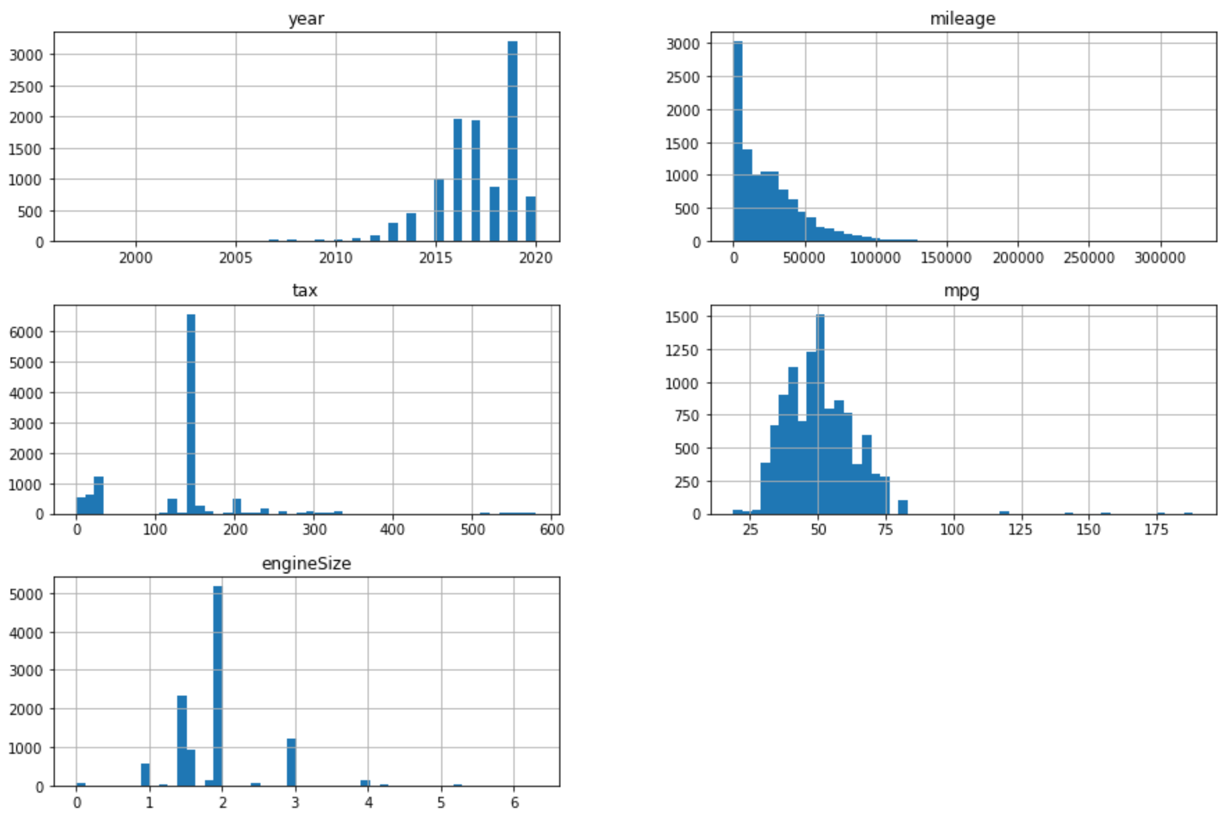
```
[[2.0170e+03 1.5735e+04 1.5000e+02 5.5400e+01 1.4000e+00]
 [2.0160e+03 3.6203e+04 2.0000e+01 6.4200e+01 2.0000e+00]
 [2.0160e+03 2.9946e+04 3.0000e+01 5.5400e+01 1.4000e+00]
 ...
 [2.0200e+03 6.0900e+02 1.5000e+02 4.9600e+01 1.0000e+00]
 [2.0170e+03 8.6460e+03 1.5000e+02 4.7900e+01 1.4000e+00]
 [2.0160e+03 1.1855e+04 1.5000e+02 4.7900e+01 1.4000e+00]]
```

🔬Preprocess and perform exploratory data analysis of the dataset obtained

📊 Plotting Histograms

In [11]:

```
df.hist(bins=50,figsize=(15,10))
plt.show()
```



In [12]:

```
# understanding the given data
df.describe()
```

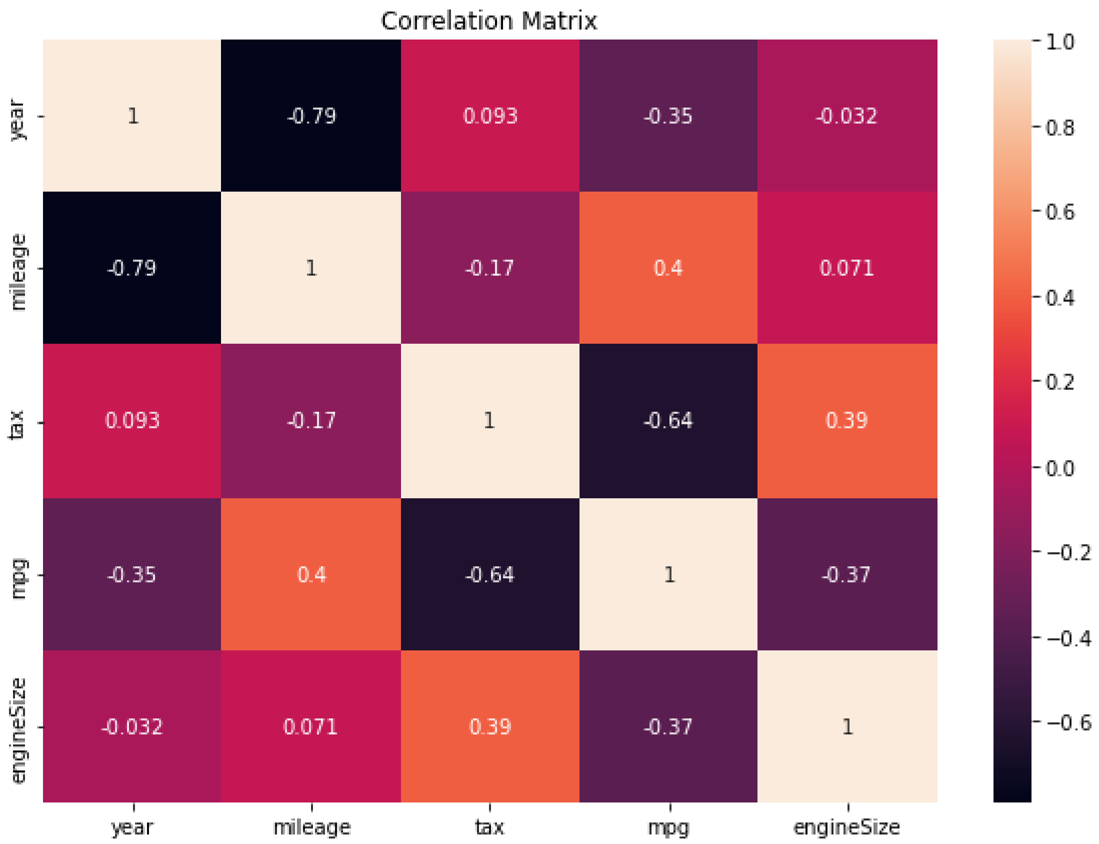
Out[12]:

	year	mileage	tax	mpg	engineSize
count	10668.000000	10668.000000	10668.000000	10668.000000	10668.000000
mean	2017.100675	24827.244001	126.011436	50.770022	1.930709
std	2.167494	23505.257205	67.170294	12.949782	0.602957
min	1997.000000	1.000000	0.000000	18.900000	0.000000
25%	2016.000000	5968.750000	125.000000	40.900000	1.500000
50%	2017.000000	19000.000000	145.000000	49.600000	2.000000
75%	2019.000000	36464.500000	145.000000	58.900000	2.000000
max	2020.000000	323000.000000	580.000000	188.300000	6.300000

Correlation Matrix

In [13]:

```
correlation = df.corr()
plt.subplots(figsize=(10,7))
heatmap = sns.heatmap(correlation,annot=True)
heatmap.set(title='Correlation Matrix')
plt.show()
```

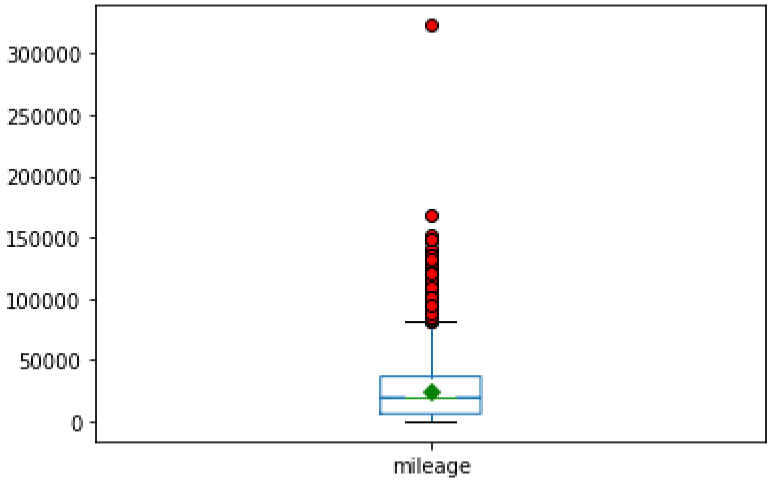


Plotting Box Plots

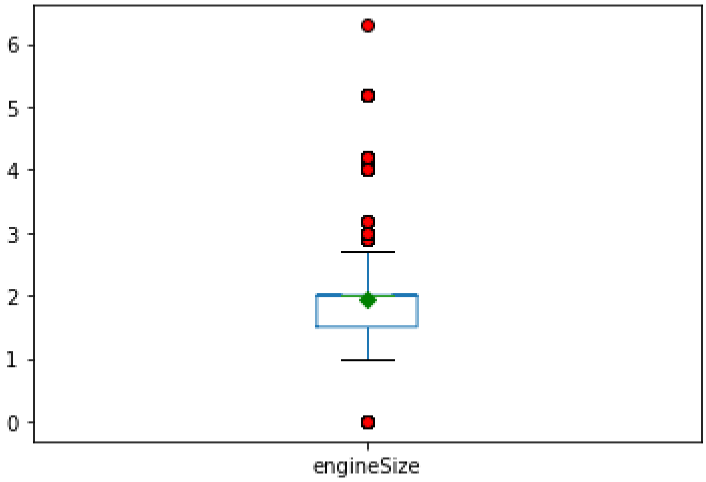
In [14]:

```
def plot_boxplot(dataframe,feature):
    red_circle = dict(markerfacecolor='red', marker='o')
    mean_shape = dict(markerfacecolor='green',marker='D',markeredgecolor='gre
    dataframe.boxplot(column=[feature],flierprops = red_circle,showmeans=True
    plt.grid(False)
    plt.show()
```

In [15]:
```
# red circles will be the outliers
plot_boxplot(df,"mileage")
```

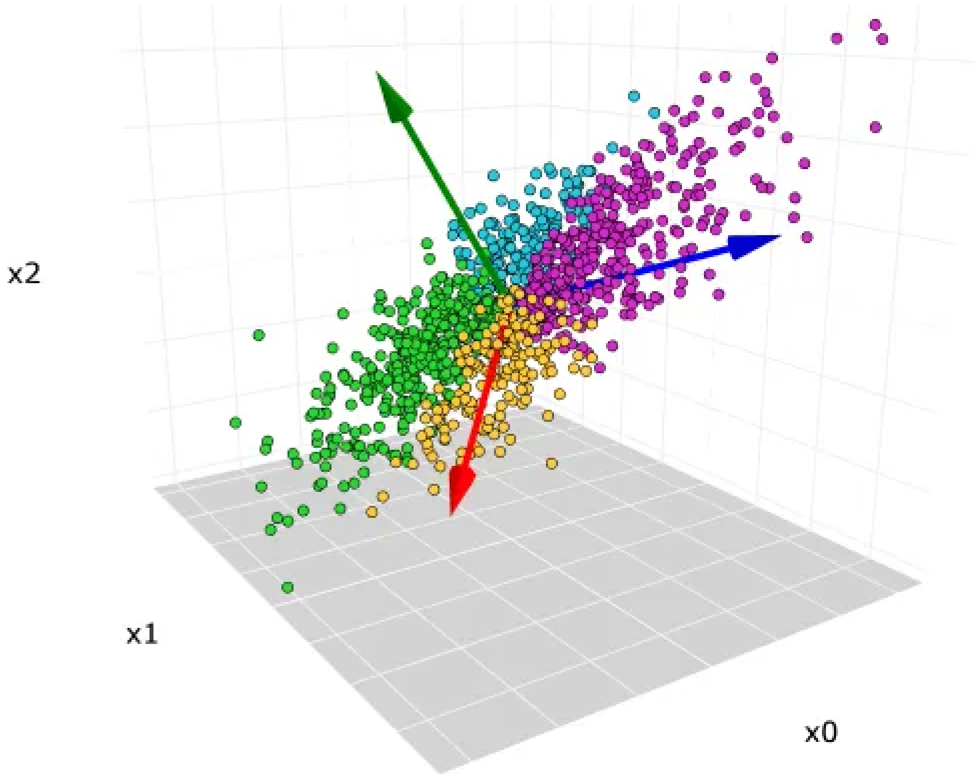


In [16]:
```
# red circles are the outliers
plot_boxplot(df,"engineSize")
```



2. Implementing PCA using Covariance Matrices

- It is used to reduce the dimensionality of dataset by transforming a large set into a lower dimensional set that still contains most of the information of the large dataset
- Principal component analysis (PCA) is a technique that transforms a dataset of many features into principal components that "summarize" the variance that underlies the data
- PCA finds a new set of dimensions such that all dimensions are orthogonal and hence linearly independent and ranked according to variance of data along them
- Eigen vectors point in direction of maximum variance among data, and eigen value gives the importance of that eigen vector



- First the dataset is centered by subtracting means from the feature values
- Means are calculated by

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

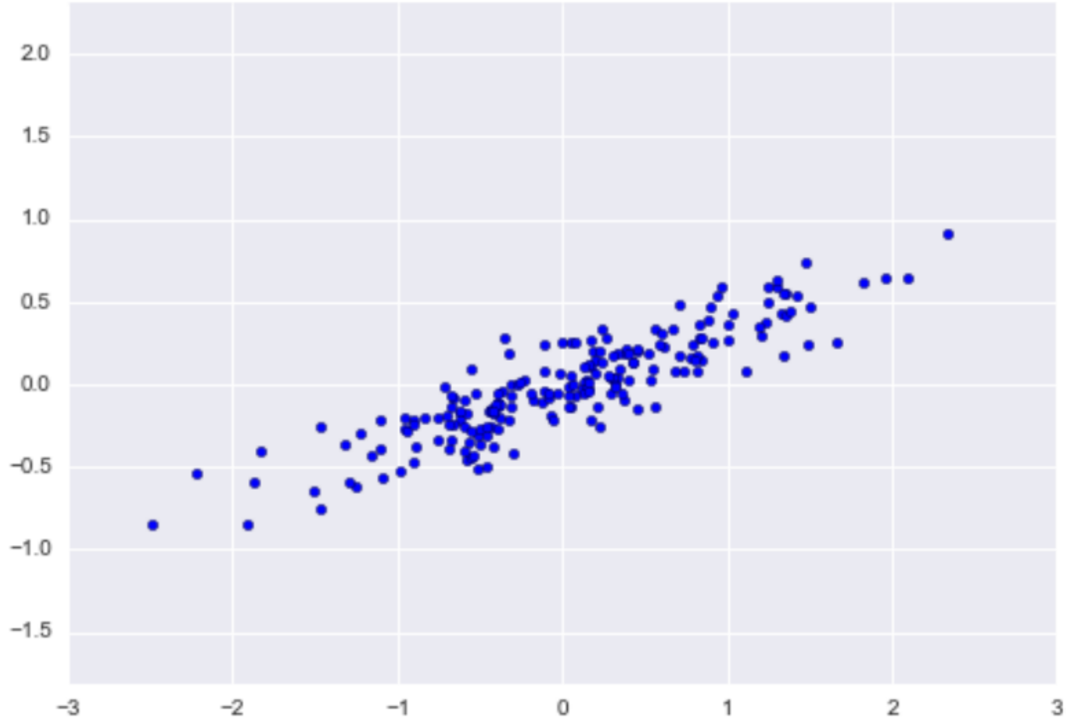
- Centered features are then compted as

$$x_i = x_i - \mu$$

- Let x_1, x_2... x_n be be N training examples, each having D features.
- Mean of N training examples is given by $\bar{x}$, which can be computed as

$$\bar{x} = \frac{1}{N}\sum_{n=1}^{N} x_{n1} + \frac{1}{N}\sum_{n=1}^{N} x_{n2} \ldots + \frac{1}{N}\sum_{n=1}^{N} x_{nd}$$

- Now suppose we have a graph with 2-dimensional points as follows:



- Our motive is to bring down the 2D points to 1D by projecting on a vector. We need to project the points on a 1D vector such that the variance between data points is maximum.

- We need to compute unit vector such that the variance is as maximum as possible.

- We do some mathematical computations as follows:

$$cos\theta = \frac{OA}{OB}$$

$$\bar{u} cdot \bar{x_n} = (||u||)(||x||)cos\theta = (||u||)(OB)cos\theta = (||u||)(OA)$$

- The above equation gives us the below result

$$OA = \frac{\bar{u} \cdot \bar{x_n}}{||u||}$$

- We take projection on unit vector $||u||$ = 1

- Our final result is as follows:

$$OA = \bar{u} \cdot \bar{x_n}$$

- The mean of the projected points is given by

$$\frac{1}{N} \sum_{n=1}^{N} \bar{u} \cdot \bar{x_n} = \bar{u} \cdot \sum_{n=1}^{N} \frac{x_n}{N} = \bar{u} \cdot \bar{x}$$

- Here, $\bar{x}$ is the mean of training points in their dimension

- We then compute variance as

$$Variance = \frac{1}{N}\sum_{n=1}^{N}(\bar{u} \cdot \bar{x_n} - \bar{u} \cdot \bar{x})^2$$

- We then compute $\bar{u}$ which maximizes variance as much as possible such that $||u|| = 1$
- Consider x_n and $\bar{x}$ to be matrices of dx1 size represented as follows

$$\bar{x_n} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_d \end{bmatrix}$$

$$\bar{x} = \begin{bmatrix} \bar{x_1} \\ \bar{x_2} \\ \bar{x_3} \\ \vdots \\ \bar{x_d} \end{bmatrix}$$

- Consider $\bar{u}$ to be a 1xd matrix represented by

$$\bar{u} = [u_1, u_2 \ldots u_d]$$

- We then observe that we need to maximize the following expression

$$max[\frac{1}{N}\sum_{n=1}^{N}(\bar{u} \cdot (\bar{x_n} - \bar{x}))(\bar{u} \cdot (\bar{x_n} - \bar{x})^T]$$

- While trying to maximize the above expression by expanding the same, we get

$$max[\frac{1}{N}\sum_{n=1}^{N}(\bar{u}(x_n - \bar{x})(x_n - \bar{x})^T\bar{u}^T)]$$

- The above expression in turn becomes

$$max[\bar{u}\frac{1}{N}[\sum_{n=1}^{N}(x_n - \bar{x})(x_n - \bar{x})^T]\bar{u}^T]$$

- The above expression simplifies to

$$max[\bar{u}S\bar{u}^T]$$

$$||u|| = 1$$

- Here, S is called covariance matrix
- Principal Component Analysis (PCA) gives linear combination of these features to get matured features
- We then try to convert the above constraint optimization problem to an unconstrained optimization problem, as follows:

$$E(u, \lambda) = max[\bar{u}S\bar{u}^T + \frac{\lambda}{2}(1 - \bar{u}\bar{u}^T)]$$

- Taking derivation with respect to $\bar{u}$ and λ and setting it to 0, we get final answer to be

$$\bar{u}S\bar{u}^T = \lambda$$

- λ is called the eigen value found from the equation

$$|A - \lambda I| = 0$$

- Let u_1, u_2,...u_d be the eigen vectors, and λ_1, λ_2,...λ_d be the eigen values, A is a dxd square matrix, we get

$$A\gamma = \lambda\gamma$$

$$Au_1 = \lambda_1 u_1$$

$$Au_2 = \lambda_2 u_2$$

- Any of d $\bar{u}$ values are feasible solutions, we need to find optimal solution from the following set of equations

$$Su_1 = \lambda_1 u_1$$

$$Su_2 = \lambda_2 u_2$$

$$.$$

$$.$$

$$.$$

$$Su_d = \lambda_d u_d$$

- The above set of equations simplifies to

$$u_1 S u_1^T = \lambda_1$$

$$u_2 S u_2^T = \lambda_2$$

$$.$$

$$.$$

$$.$$

$$u_d S u_d^T = \lambda_d$$

- For instance, if we project all points on eigen vector u_1 then variance comes out to be λ_1

- λ_1, λ_2, ..., λ_d are variances after projecting values/points on eigen vectors u_1, u_2,....,u_d. We need to find that eigen vector which has maximum variance, or simply, maximum λ.

- For instance, consider the first eigen vector to be of the form

$$u_1 = \begin{bmatrix} \bar{u_{11}} \\ \bar{u_{12}} \\ \bar{u_{13}} \\ \vdots \\ \bar{u_{1d}} \end{bmatrix}$$

- Transformed point is

$$u_{11}x_{11} + u_{12}x_{12} + \ldots + u_{1d}x_{1d}$$

- Transformation of a point from multidimensional space (d-dimensional in this case) to a uni-dimensional space is a linear transformation (where multiples are componenents of eigen vectors in PCA)

In [17]:
```
# calculating mean of each feature in the dataset
feature_means = df.mean()
feature_means
```

Out[17]:
```
year              2017.100675
mileage          24827.244001
tax                126.011436
mpg                 50.770022
engineSize           1.930709
dtype: float64
```

In [18]:
```
# Centering the dataset by subtracting the mean from each feature.
centered_features = df - feature_means
```

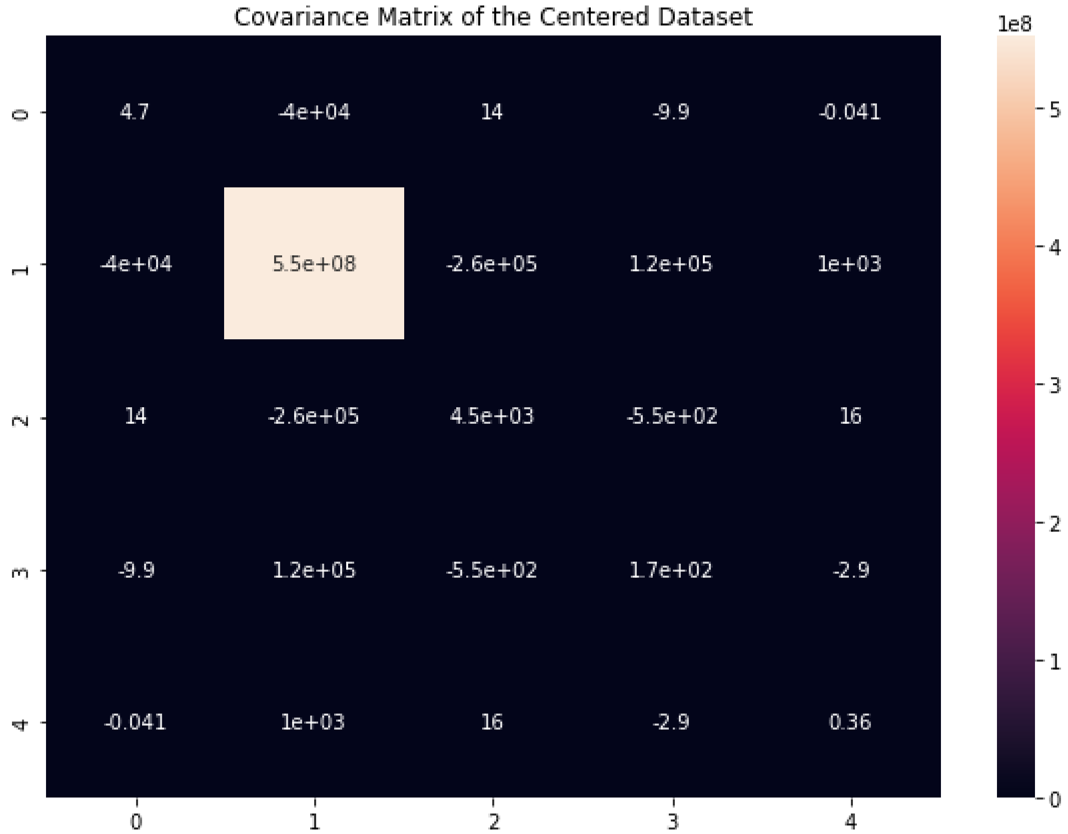
In [19]:
```
centered_features.head()
```

Out[19]:

	year	mileage	tax	mpg	engineSize
0	-0.100675	-9092.244001	23.988564	4.629978	-0.530709
1	-1.100675	11375.755999	-106.011436	13.429978	0.069291
2	-1.100675	5118.755999	-96.011436	4.629978	-0.530709
3	-0.100675	1124.755999	18.988564	16.529978	0.069291
4	1.899325	-22829.244001	18.988564	-1.170022	-0.930709

Covariance matrix of the centered dataset

In [20]:
```
# covariance matrix of centered feature values
covariance_matrix = np.cov(centered_features,rowvar=False)
plt.subplots(figsize=(10,7))
heatmap = sns.heatmap(covariance_matrix,annot=True)
heatmap.set(title='Covariance Matrix of the Centered Dataset')
plt.show()
```



3. Eigenvalue Eigenvector Equation

For a square matrix A, if $\mathbf{v}$ is an eigenvector and λ is the corresponding eigenvalue, the eigenvalue-eigenvector equation is given by

$$A\mathbf{v} = \lambda\mathbf{v}$$

In [21]:

```
# finding eigenvalues and eigenvectors
eigenvalues, eigenvectors = np.linalg.eig(covariance_matrix)

# transpose eigenvector
eigenvectors = eigenvectors.T

# will give indexes according to eigen values, sorted in decreasing order
idx = np.argsort(eigenvalues)[::-1]
eigenvalues = eigenvalues[idx]
eigenvectors = eigenvectors[idx]
```

4. Solving for Principal Components

In [22]:

```
print(eigenvalues)
```

```
[5.52497271e+08 4.44392581e+03 8.44121646e+01 1.72584583e+00
 2.82457928e-01]
```

In [23]:

```
print(eigenvectors)
```

```
[[ 7.28176631e-05 -9.99999860e-01  4.75940888e-04 -2.17675259e-04
  -1.81384120e-06]
 [-1.22350540e-03  4.97635688e-04  9.93414801e-01 -1.14504431e-01
   3.74489446e-03]
```

```
 [-2.10100343e-02 -1.63185503e-04  1.14495419e-01  9.93103163e-01
  -1.39806367e-02]
 [ 9.99593344e-01  6.98862164e-05  3.58032275e-03  2.10014069e-02
   1.89542395e-02]
 [-1.92411557e-02 -7.28558351e-06 -2.18799844e-03  1.39189142e-02
   9.99715587e-01]]
```

In [24]:

```
# for finding how much variance does each principal component capture
explained_variance = eigenvalues / np.sum(eigenvalues)
```

In [25]:

```
print(explained_variance)
```

```
[9.99991800e-01 8.04327841e-06 1.52781700e-07 3.12369269e-09
 5.11234403e-10]
```

In [26]:

```
# slicing first k eigenvectors
# let k = 5
k = 5
k_principal_components = eigenvectors[:k]
```

In [27]:

```
print(k_principal_components)
```

```
[[ 7.28176631e-05 -9.99999860e-01  4.75940888e-04 -2.17675259e-04
  -1.81384120e-06]
 [-1.22350540e-03  4.97635688e-04  9.93414801e-01 -1.14504431e-01
   3.74489446e-03]
 [-2.10100343e-02 -1.63185503e-04  1.14495419e-01  9.93103163e-01
  -1.39806367e-02]
 [ 9.99593344e-01  6.98862164e-05  3.58032275e-03  2.10014069e-02
   1.89542395e-02]
 [-1.92411557e-02 -7.28558351e-06 -2.18799844e-03  1.39189142e-02
   9.99715587e-01]]
```

In [28]:

```
k_principal_components_eigenvalues = eigenvalues[:k]
```

5. Sequential Variance Increase

In [29]:

```
# total variance covered by principal components
total_variance = np.sum(k_principal_components_eigenvalues)
```

In [30]:

```
print(total_variance)
```

```
552501800.8994668
```

In [31]:

```
# new features after applying PCA
pca_df = np.dot(centered_features,k_principal_components.T)
pca_df = pd.DataFrame(pca_df)
pca_df.head()
```

Out[31]:

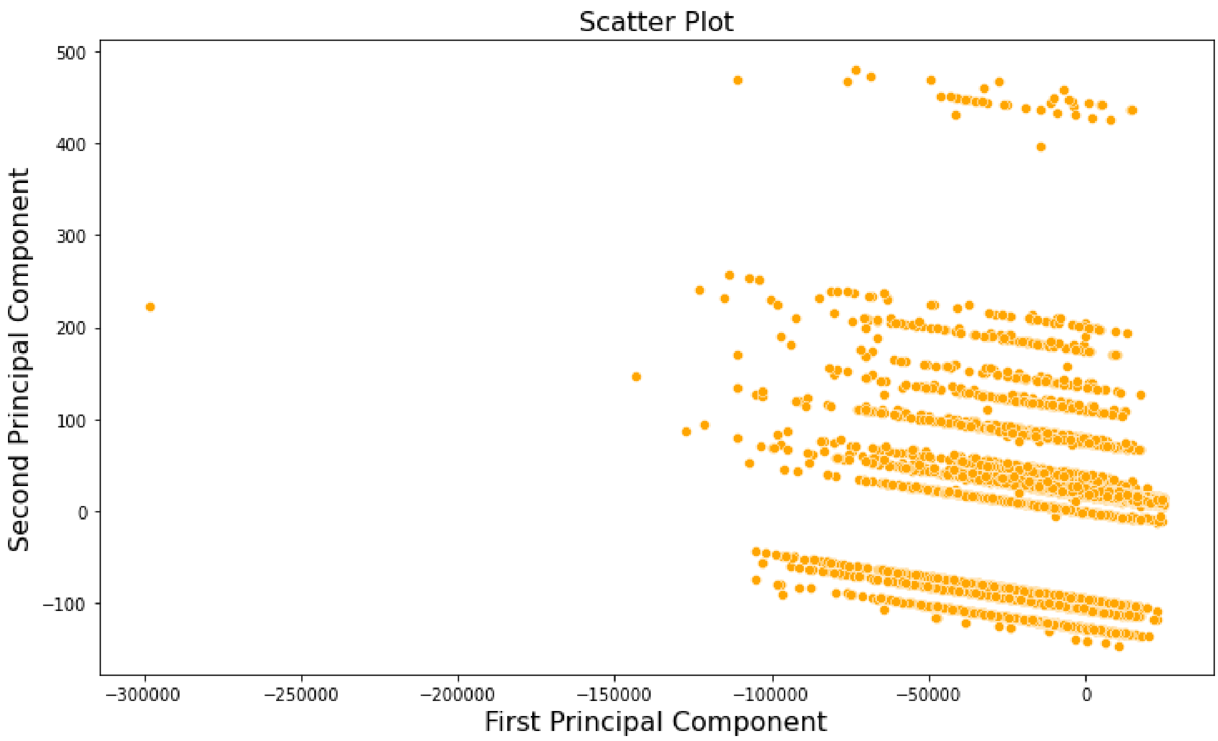
	0	1	2	3	4
0	9092.253134	18.773952	8.837883	-0.562993	-0.450421
1	-11375.807870	-101.188533	-0.634673	-0.401412	0.426454

	0	1	2	3	4
2	-5118.802067	-93.362700	-7.199586	-0.999072	-0.272155
3	-1124.750410	17.530866	18.407679	0.394422	0.251547
4	22829.250246	7.631037	4.710659	0.328875	-0.858497

Plot showing spread of data along first 2 Principal Components

In [32]:

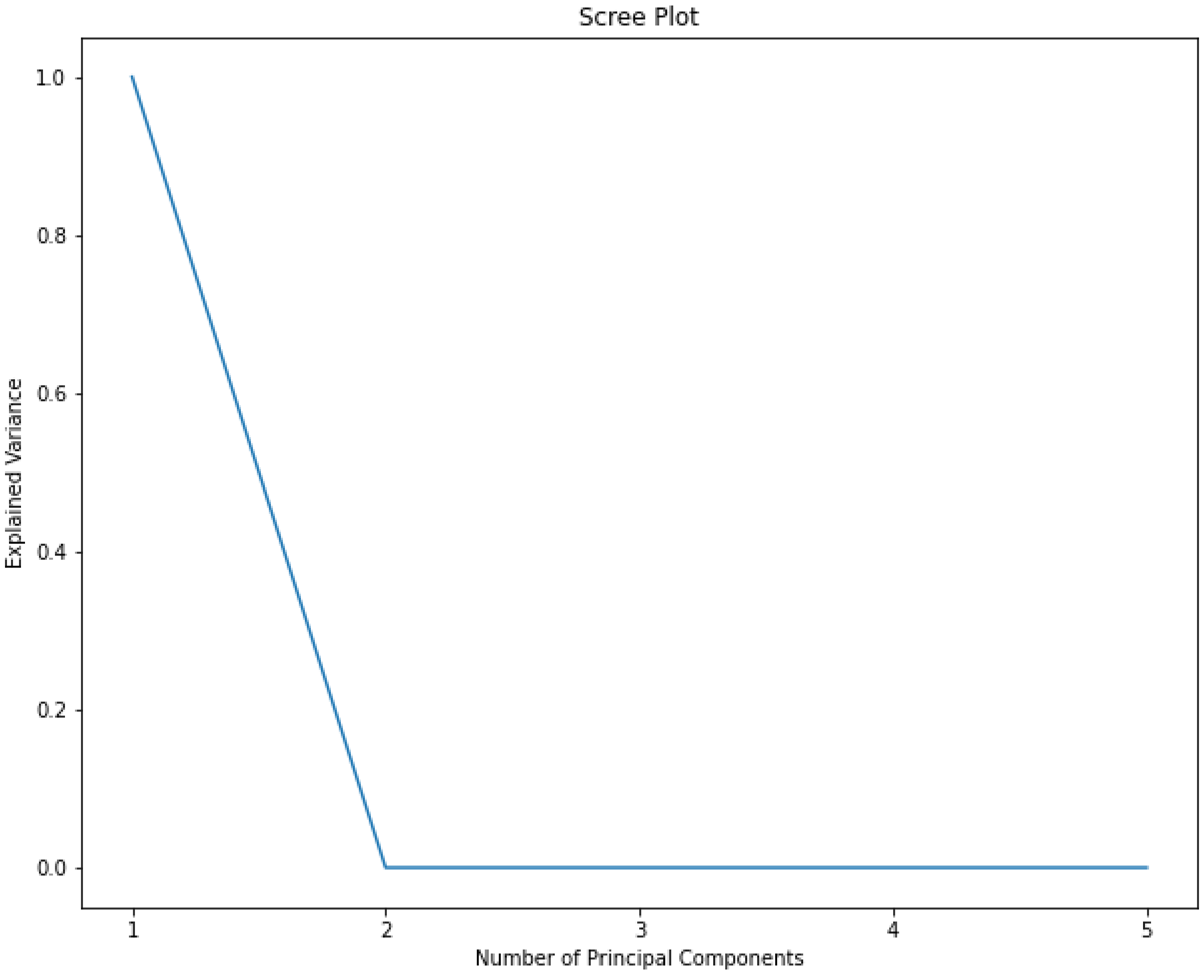
```
plt.figure(figsize=(12,7))
sns.scatterplot(data=pca_df,x=0,y=1,color='orange')
plt.title("Scatter Plot",fontsize=16)
plt.xlabel('First Principal Component',fontsize=16)
plt.ylabel('Second Principal Component',fontsize=16)
plt.show()
```



Plot showing variance captured by each Principal Component

In [33]:

```
num_components = len(explained_variance)
components = np.arange(1, num_components + 1)
plt.figure(figsize=(10, 8))
plt.plot(components, explained_variance)
plt.xlabel('Number of Principal Components')
plt.ylabel('Explained Variance')
plt.title('Scree Plot')
plt.xticks(components)
plt.show()
```



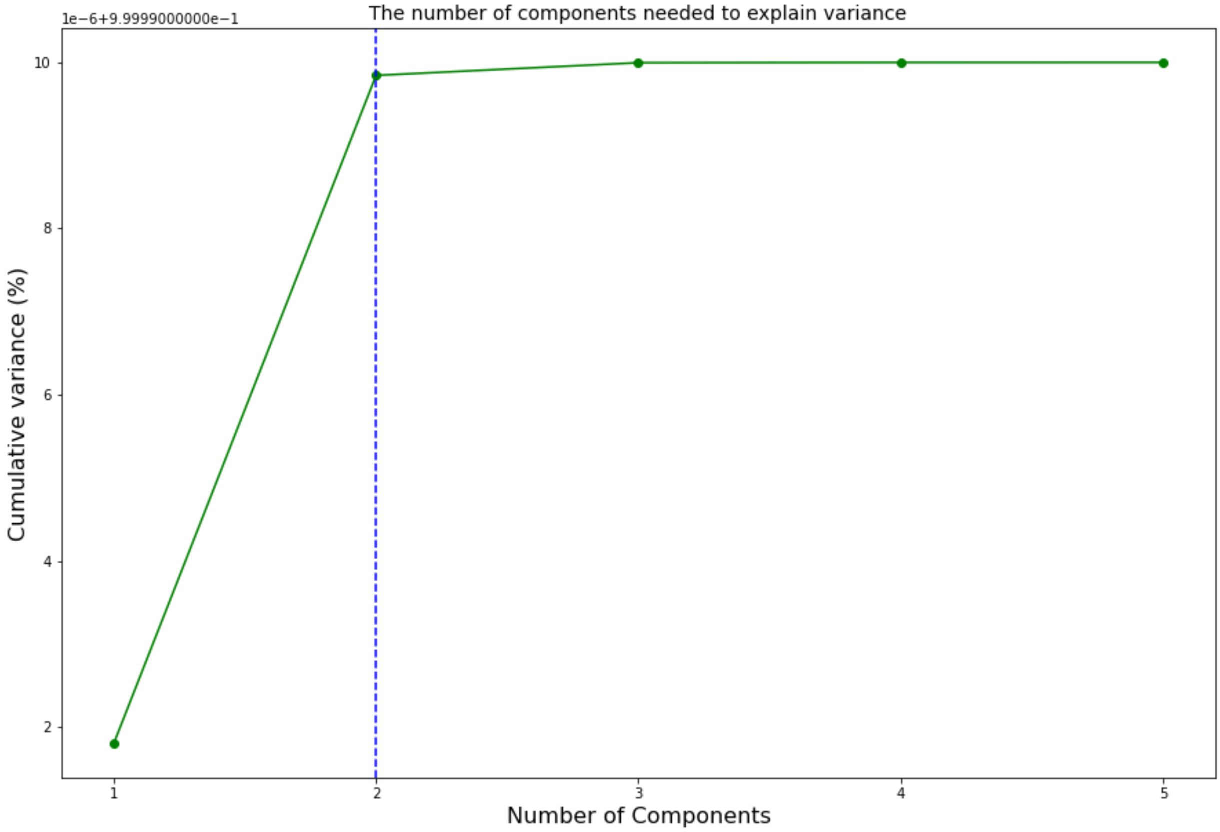
In [34]:

```
# finding cumulative variance captured by principal components
y_var = np.cumsum(explained_variance)

plt.figure(figsize=(15,10))
plt.plot(components, y_var, marker='o', linestyle='-', color='green')
plt.title('The number of components needed to explain variance',fontsize=14)
plt.xlabel('Number of Components',fontsize=16)
plt.ylabel('Cumulative variance (%)',fontsize=16)
plt.xticks(components)

# line showing number of principal components required to capture most of the
plt.axvline(x=2.00, color='blue', linestyle='--')

plt.show()
```



- We can see that the complete variance is captured by the first 2 principal components and there is very insignificant sequential increase in the variance as we consider more principal components

Standardized Dataset

In [35]:
```
df_standardized = (df-df.mean())/(df.std())
df_standardized
```

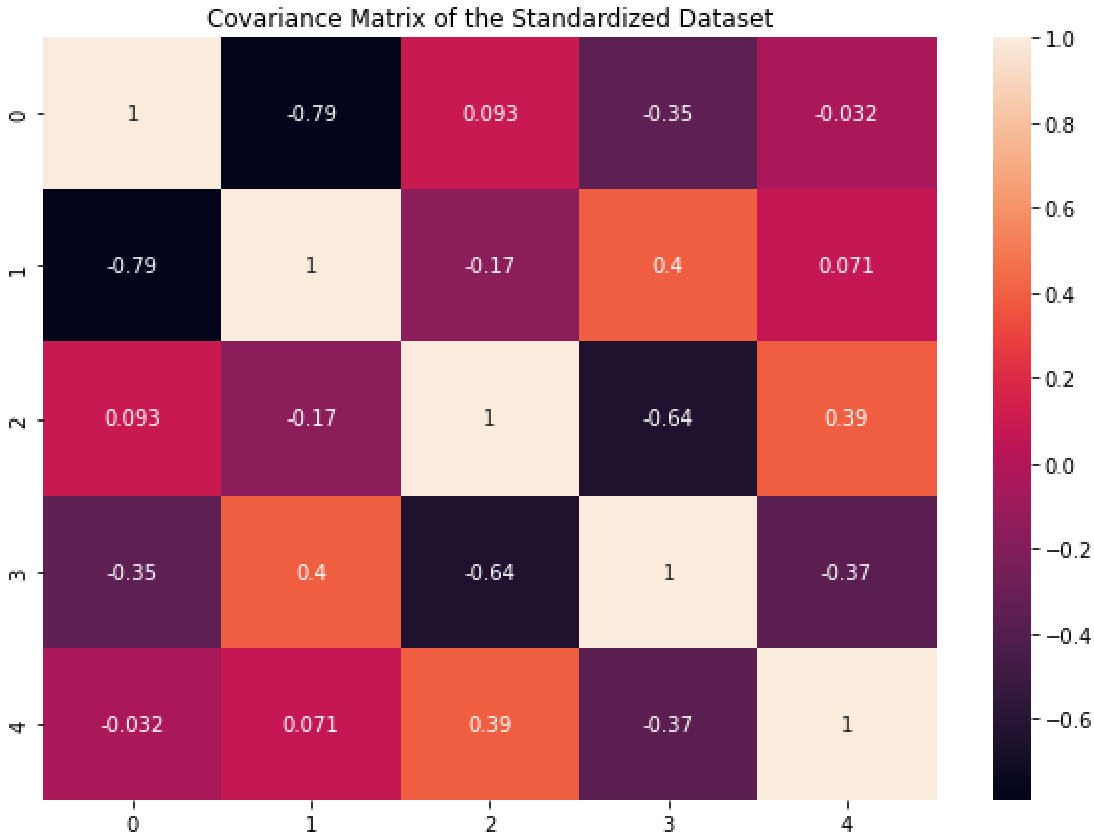
Out[35]:

	year	mileage	tax	mpg	engineSize
0	-0.046448	-0.386817	0.357131	0.357533	-0.880177
1	-0.507810	0.483966	-1.578249	1.037081	0.114919
2	-0.507810	0.217771	-1.429373	0.357533	-0.880177
3	-0.046448	0.047851	0.282693	1.276468	0.114919
4	0.876277	-0.971240	0.282693	-0.090351	-1.543575
...	...	...	...	...	...
10663	1.337639	-0.885302	0.282693	-0.090351	-1.543575
10664	1.337639	-0.972091	0.357131	-0.090351	-1.543575
10665	1.337639	-1.030333	0.357131	-0.090351	-1.543575
10666	-0.046448	-0.688410	0.357131	-0.221627	-0.880177
10667	-0.507810	-0.551887	0.357131	-0.221627	-0.880177

10668 rows × 5 columns

In [36]:

```
# covariance matrix of centered feature values
covariance_matrix = np.cov(df_standardized,rowvar=False)
plt.subplots(figsize=(10,7))
heatmap = sns.heatmap(covariance_matrix,annot=True)
heatmap.set(title='Covariance Matrix of the Standardized Dataset')
plt.show()
```



In [37]:

```
# finding eigenvalues and eigenvectors
eigenvalues, eigenvectors = np.linalg.eig(covariance_matrix)

# transpose eigenvector
eigenvectors = eigenvectors.T

# will give indexes according to eigen values, sorted in decreasing order
idx = np.argsort(eigenvalues)[::-1]
eigenvalues = eigenvalues[idx]
eigenvectors = eigenvectors[idx]
```

4. Solving for Principal Components

In [38]:

```
print(eigenvalues)
```

```
[2.31297823 1.54968025 0.62002561 0.31552269 0.20179322]
```

In [39]:

```
print(eigenvectors)
```

```
[[-0.46373634  0.48575669 -0.43356476  0.5485194  -0.24522869]
 [ 0.48249022 -0.46716498 -0.43702648  0.22908421 -0.55271001]
 [-0.26369364  0.0732536   0.49512711 -0.24802587 -0.7864044 ]
 [-0.10806712  0.17892982 -0.60322362 -0.76491314 -0.08564328]
 [ 0.68624992  0.7130325   0.10954391 -0.00616002 -0.09277865]]
```

In [40]:
```
# for finding how much variance does each principal component capture
explained_variance = eigenvalues / np.sum(eigenvalues)
```

In [41]:
```
print(explained_variance)
```

```
[0.46259565 0.30993605 0.12400512 0.06310454 0.04035864]
```

In [42]:
```
# slicing first k eigenvectors
# let k = 5
k = 5
k_principal_components = eigenvectors[:k]
```

In [43]:
```
print(k_principal_components)
```

```
[[-0.46373634  0.48575669 -0.43356476  0.5485194  -0.24522869]
 [ 0.48249022 -0.46716498 -0.43702648  0.22908421 -0.55271001]
 [-0.26369364  0.0732536   0.49512711 -0.24802587 -0.7864044 ]
 [-0.10806712  0.17892982 -0.60322362 -0.76491314 -0.08564328]
 [ 0.68624992  0.7130325   0.10954391 -0.00616002 -0.09277865]]
```

In [44]:
```
k_principal_components_eigenvalues = eigenvalues[:k]
```

5. Sequential Variance Increase

In [45]:
```
# total variance covered by principal components
total_variance = np.sum(k_principal_components_eigenvalues)
```

In [46]:
```
print(total_variance)
```

```
4.999999999999998
```

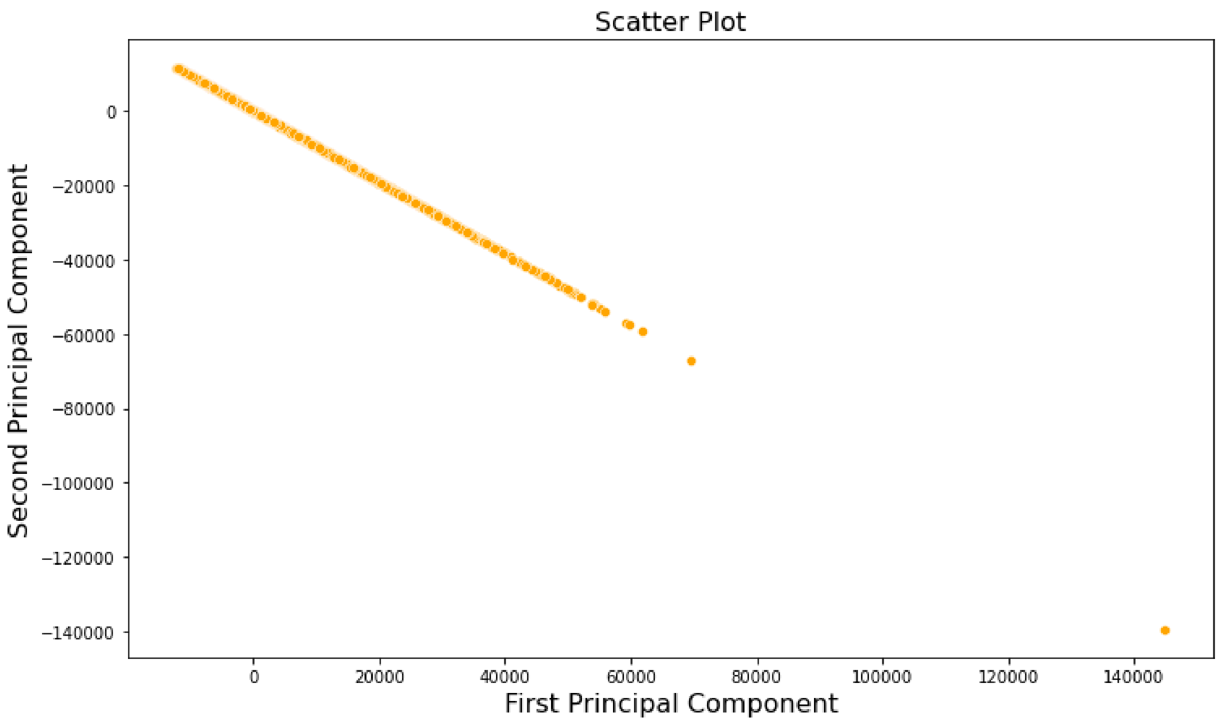
In [47]:
```
# new features after applying PCA
pca_df = np.dot(centered_features,k_principal_components.T)
pca_df = pd.DataFrame(pca_df)
pca_df.head()
```

Out[47]:

	0	1	2	3	4
0	-4424.302480	4238.399752	-654.866693	-1644.829238	-6480.486066
1	5579.672435	-5265.517780	777.730738	2089.250806	8098.826369
2	2531.277346	-2348.521084	326.988690	970.437316	3638.587313
3	547.221687	-530.045234	87.666376	177.158985	803.890325
4	-11098.985097	10657.887583	-1662.401338	-4095.517419	-16274.515937

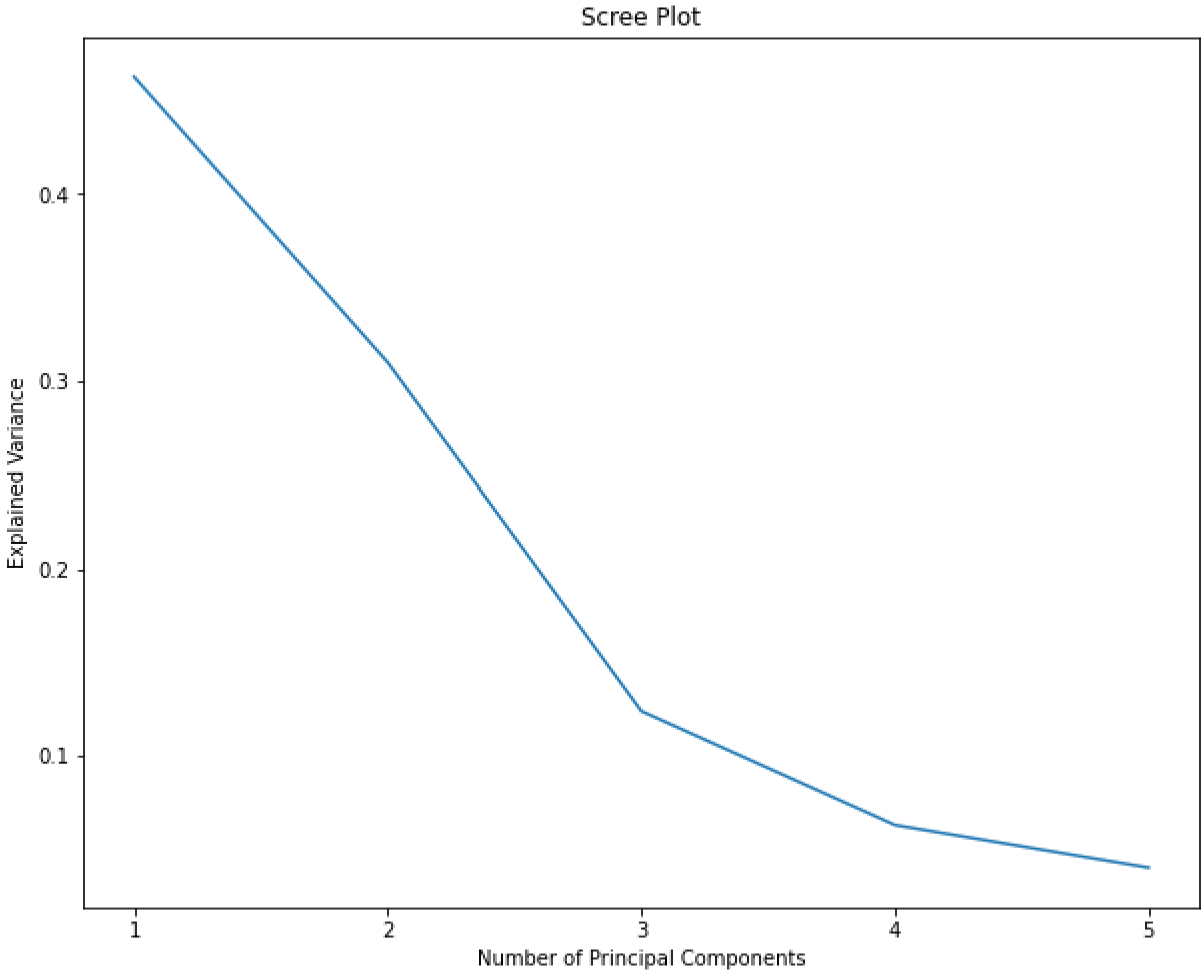
Plot showing spread of data along first 2 Principal Components

In [48]:
```
plt.figure(figsize=(12,7))
sns.scatterplot(data=pca_df,x=0,y=1,color='orange')
plt.title("Scatter Plot",fontsize=16)
plt.xlabel('First Principal Component',fontsize=16)
plt.ylabel('Second Principal Component',fontsize=16)
plt.show()
```



Plot showing variance captured by each Principal Component

In [49]:
```
num_components = len(explained_variance)
components = np.arange(1, num_components + 1)
plt.figure(figsize=(10, 8))
plt.plot(components, explained_variance)
plt.xlabel('Number of Principal Components')
plt.ylabel('Explained Variance')
plt.title('Scree Plot')
plt.xticks(components)
plt.show()
```



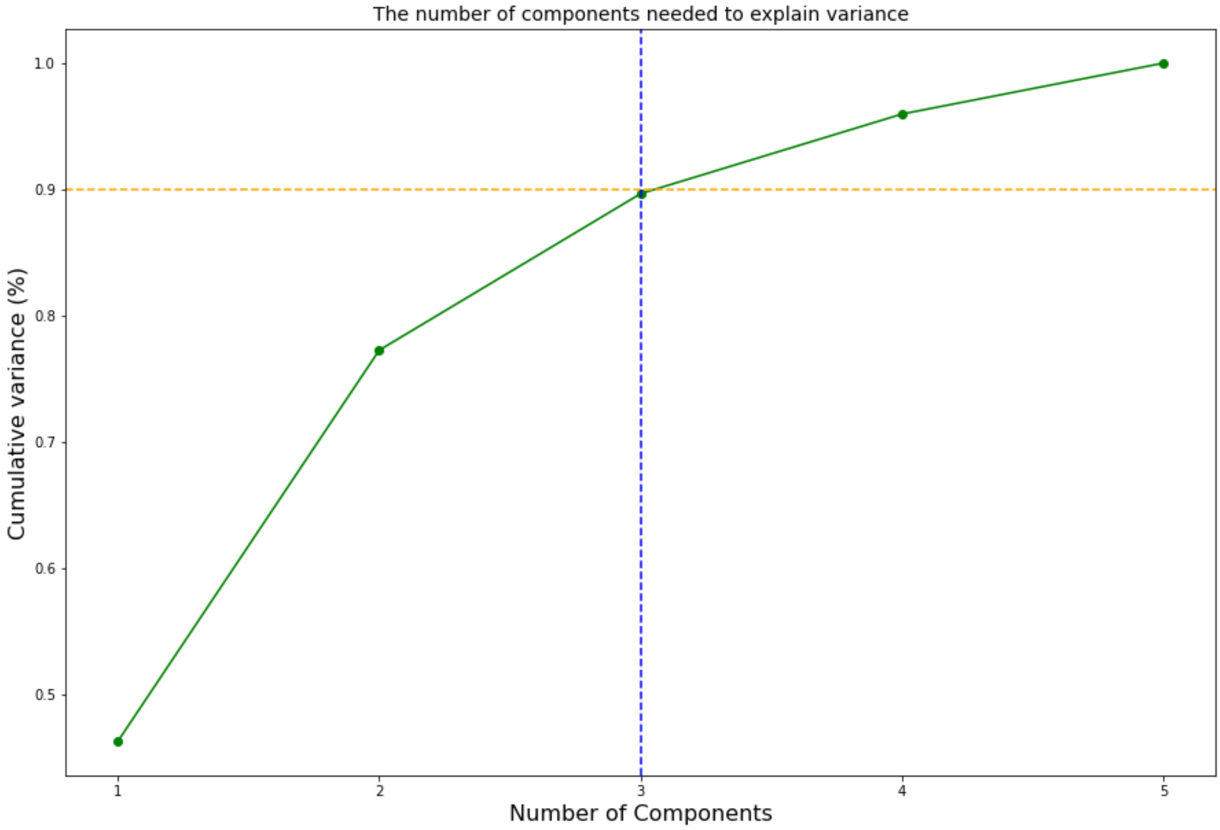
In [50]:

```
# finding cumulative variance captured by principal components
y_var = np.cumsum(explained_variance)

plt.figure(figsize=(15,10))
plt.plot(components, y_var, marker='o', linestyle='-', color='green')
plt.title('The number of components needed to explain variance',fontsize=14)
plt.xlabel('Number of Components',fontsize=16)
plt.ylabel('Cumulative variance (%)',fontsize=16)
plt.axhline(y=0.9,color='orange',linestyle='--')
plt.xticks(components)

# line showing number of principal components required to capture most of the
plt.axvline(x=3.00, color='blue', linestyle='--')

plt.show()
```



- We can see that the complete variance is captured by the first 3 principal components and there is very insignificant sequential increase in the variance as we consider more principal components

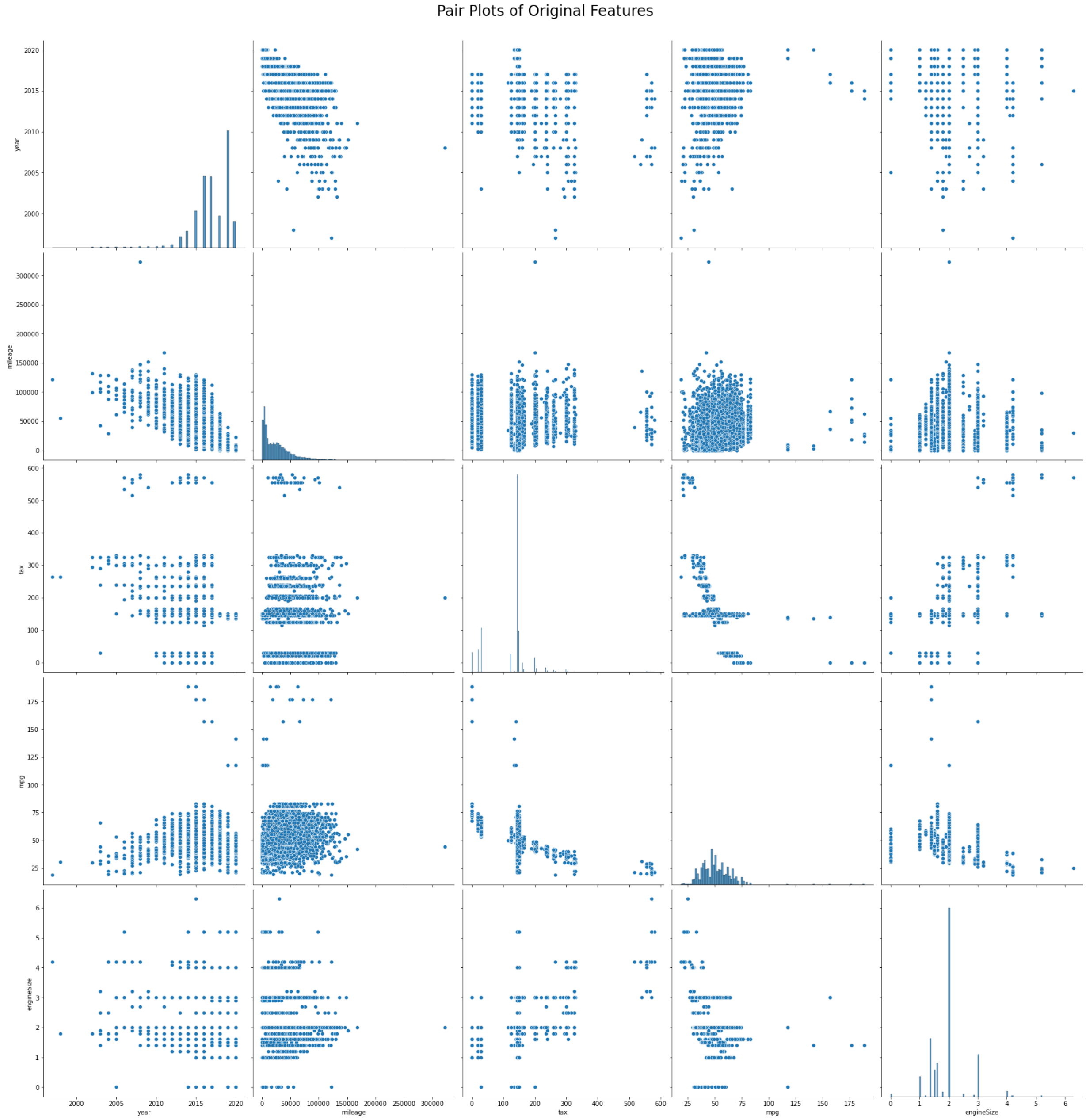
6. Visualization using Pair Plots

In [51]:

```
# pair plot of original features
original_pair_plot = sns.pairplot(pd.DataFrame(df,columns=df.columns),height=

# Set the title of the figur
original_pair_plot.fig.suptitle('Pair Plots of Original Features', y=1.02, fo

# Display the pair plots
plt.show()
```



Projecting Principal Components on Original Dataset

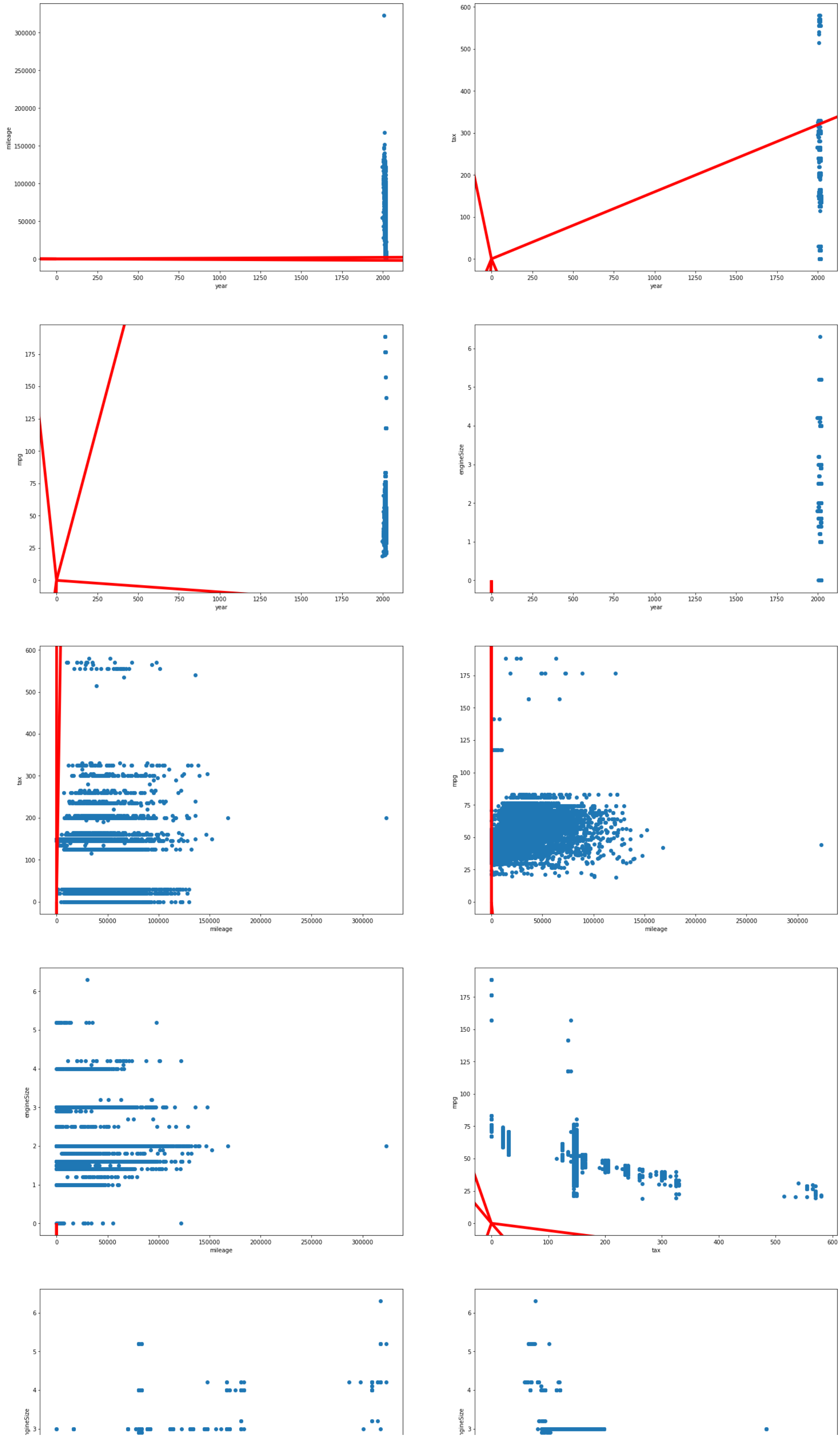
- Projecting principal components onto these pair plots and visualizing them as vectors, as we have 5 principal components and each of these 5 vectors have 5 components, so for projecting principal components onto these plots, we will take ith and jth component of every eigenvector for plotting it on the pair plot for ith and jth feature in original dataset

In [52]:

```
# projecting principal components onto these plots and visualizing them as ve
fig, ax = plt.subplots(nrows=5, ncols=2, figsize=(25,50))

# for going on every feature in original dataset
row, col = 0,0
for i in range(0,5):
    for j in range(i+1,5):
        # this will plot scatter plot for all points for ith and jth feature
        ax[row,col].scatter(df.iloc[:,i],df.iloc[:,j],label=f'original')
        ax[row,col].set_xlabel(df.columns[i])
```

```
        ax[row,col].set_ylabel(df.columns[j])
        # plotting each and every eigenvector
        for vec in eigenvectors:
            # plotting ith and jth component of each eigenvector
            ax[row,col].quiver(0, 0, vec[i], vec[j], angles='xy', scale_units
        col += 1
        if col > 1:
            col = 0
            row += 1
plt.show()
```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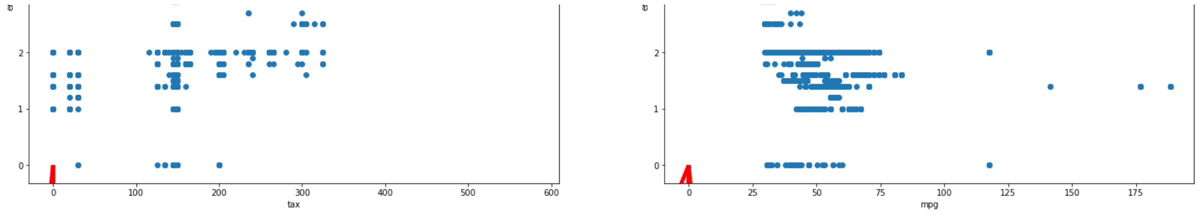


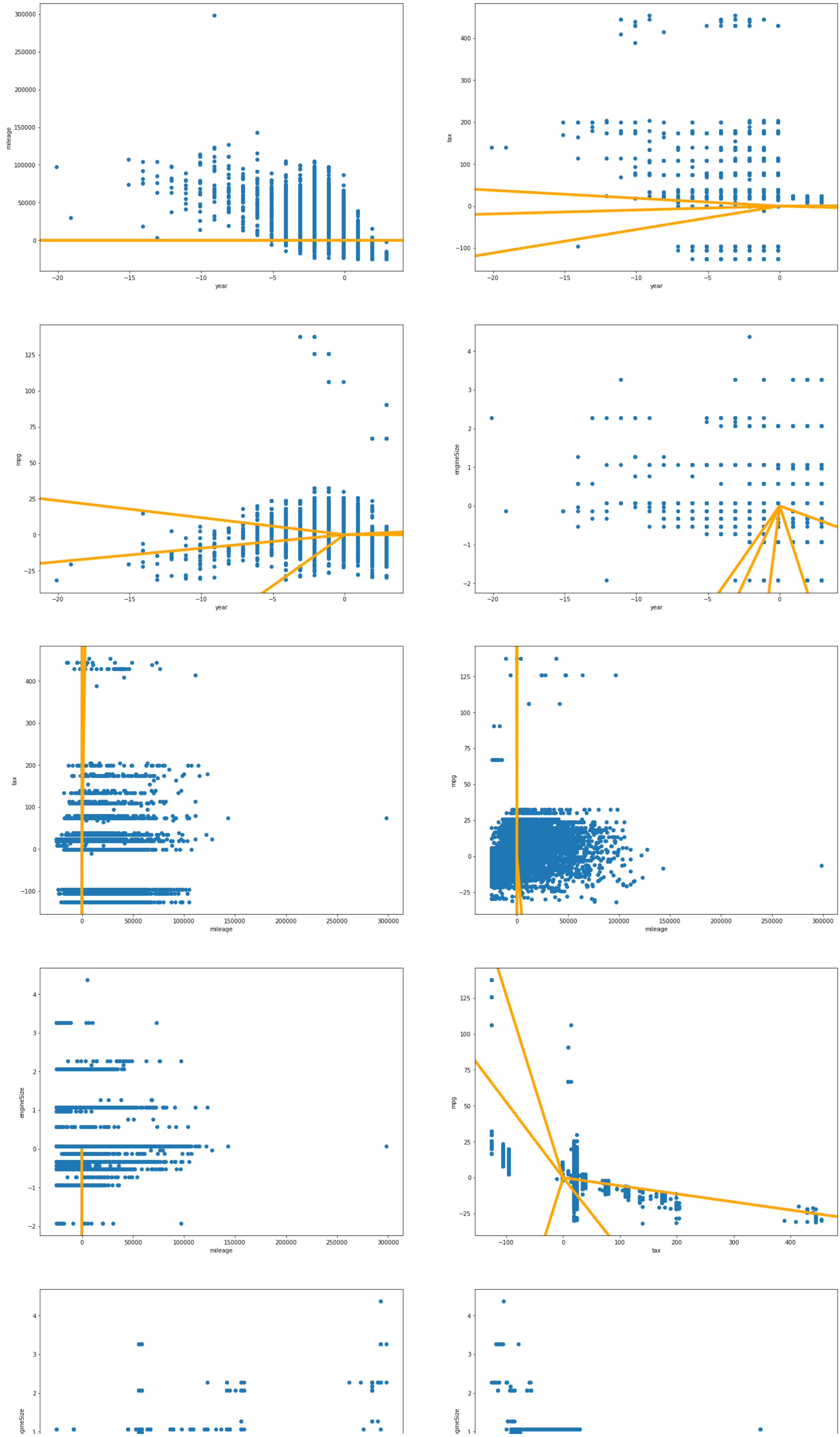
Projecting Principal Components on Centered Dataset

- We can observe that points on scatter plot are around projected principal components, because the data is centered around the mean.

In [53]:

```
# projecting principal components onto these plots and visualizing them as ve
fig, ax = plt.subplots(nrows=5, ncols=2, figsize=(25,50))

# for going on every feature in original dataset
row, col = 0,0
for i in range(0,5):
    for j in range(i+1,5):
        # this will plot scatter plot for all points for ith and jth feature
        ax[row,col].scatter(centered_features.iloc[:,i],centered_features.ilo
        ax[row,col].set_xlabel(df.columns[i])
        ax[row,col].set_ylabel(df.columns[j])
        # plotting each and every eigenvector
        for vec in eigenvectors:
            # plotting ith and jth component of each eigenvector
            ax[row,col].quiver(0, 0, vec[i], vec[j], angles='xy', scale_units
        col += 1
        if col > 1:
            col = 0
            row += 1
plt.show()
```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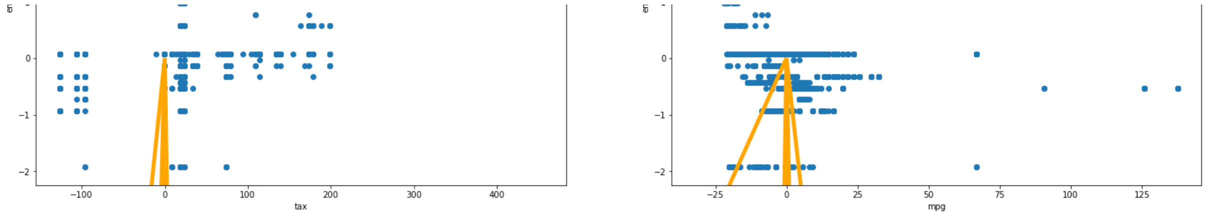


Projecting Principal Components on Standardized Dataset

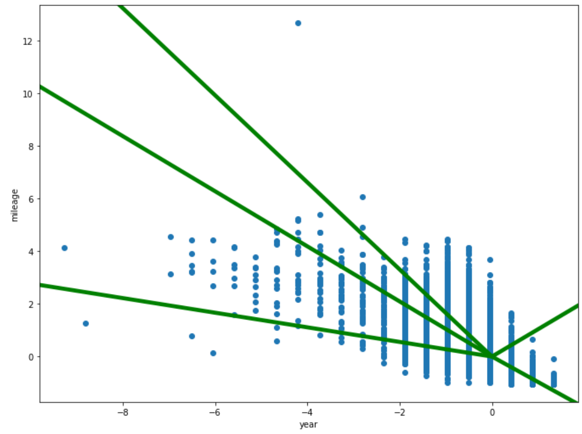
- We can see that points on scatter plot are around the vectors, as here we have properly scaled dataset

In [54]:

```
df_std = (df-df.mean())/df.std()

# projecting principal components onto these plots and visualizing them as ve
fig, ax = plt.subplots(nrows=5, ncols=2, figsize=(25,50))

# for going on every feature in original dataset
row, col = 0,0
for i in range(0,5):
    for j in range(i+1,5):
        # this will plot scatter plot for all points for ith and jth feature
        ax[row,col].scatter(df_std.iloc[:,i],df_std.iloc[:,j],label=f'origina
        ax[row,col].set_xlabel(df.columns[i])
        ax[row,col].set_ylabel(df.columns[j])
        # plotting each and every eigenvector
        for vec in eigenvectors:
            # plotting ith and jth component of each eigenvector
            ax[row,col].quiver(0, 0, vec[i], vec[j], angles='xy', scale_units
        col += 1
        if col > 1:
            col = 0
            row += 1
plt.show()
```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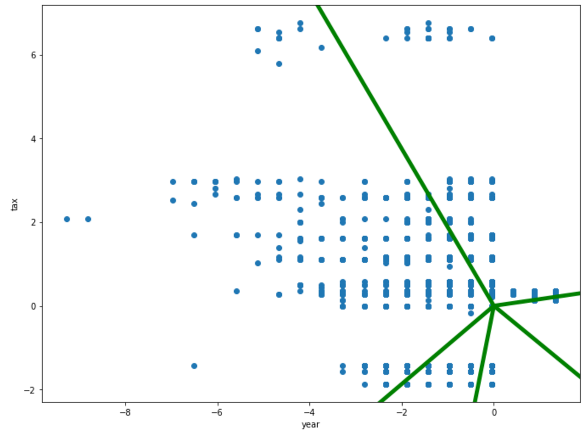
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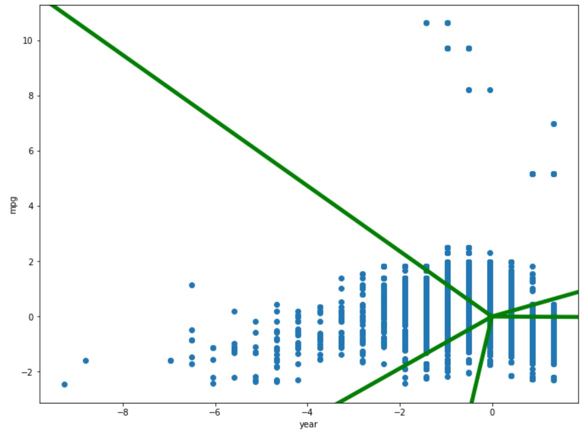
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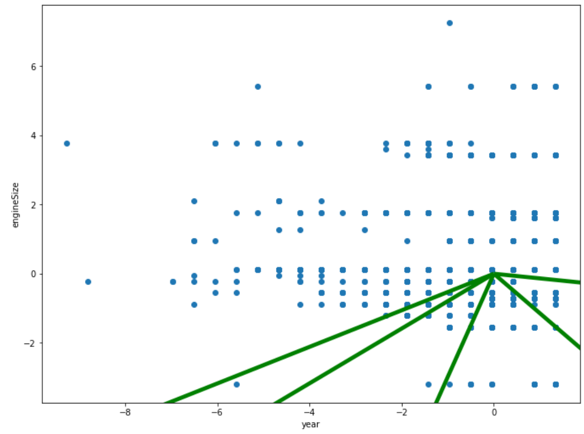
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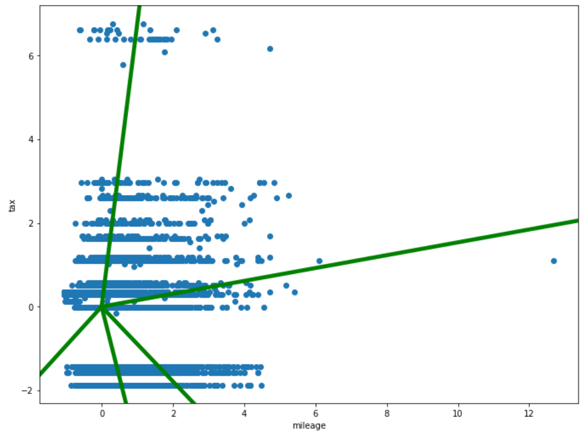
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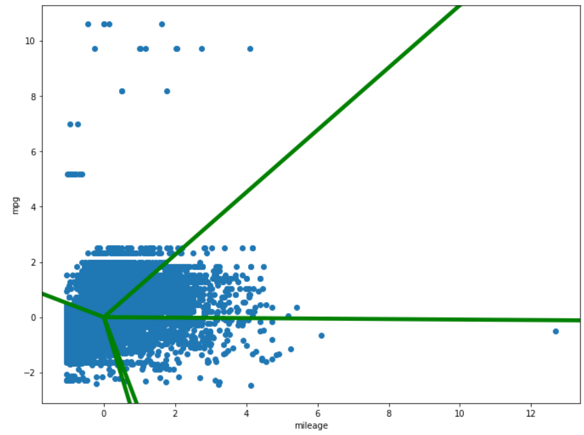
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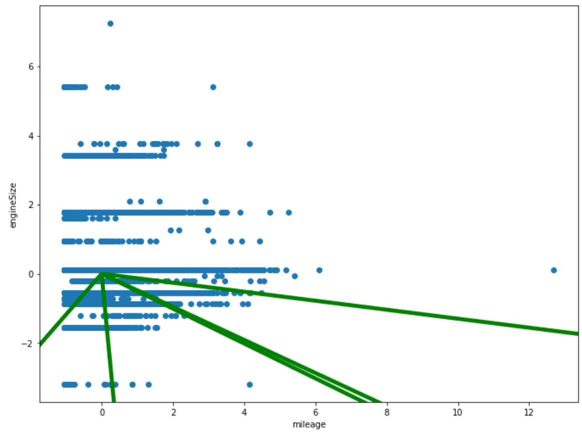
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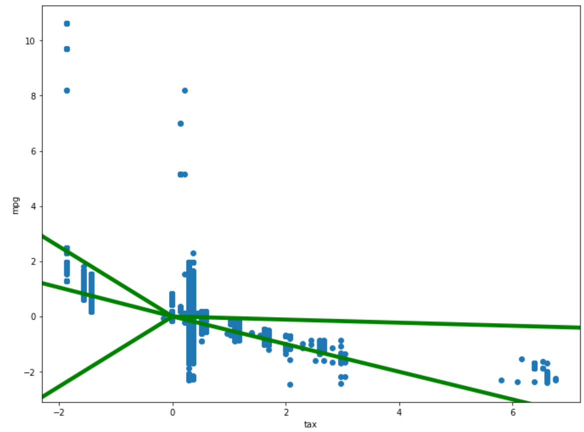
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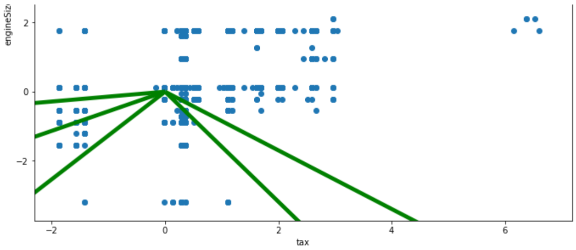

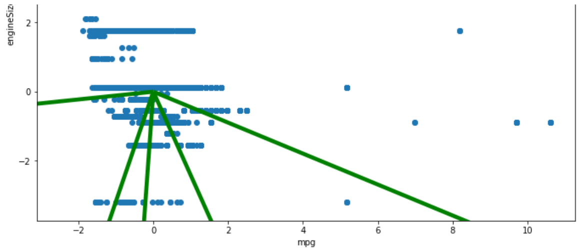

7. Conclusion and Interpretation

- We can see that maximum variance is captured by first two principal components, which helps us in reducing dimension of dataset from 5 features to 2 features while retaining most of the information.
- PCA is a dimensionality reduction technique, and dimensionality reduction is the process of reducing the number of features in a dataset while retaining as much information as possible. This can be done to reduce the complexity of a model, improve the performance of a learning algorithm, or make it easier to visualize the data.
- PCA converts a set of correlated features in the high dimensional space into a series of uncorrelated features in the low dimensional space. These uncorrelated features are also called principal components.
- We can see that points on scatter plot are around the vectors when we take projections of principal components for a standardized dataset, as here we have properly scaled dataset, unlike when we took projection on original dataset.