Cosmology with galaxy cluster abundance

An unbinned likelihood with Super Sample Covariance

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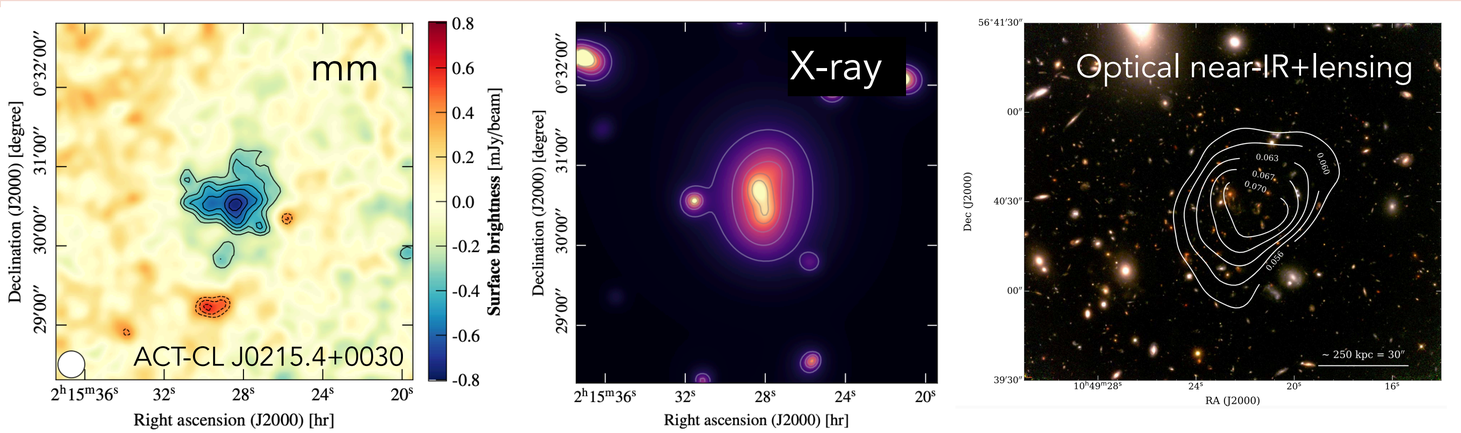
Journées Rubin LSST-France 12/2023, CC-IN2P3

Galaxy clusters



Kéruzoré et al. (2020)

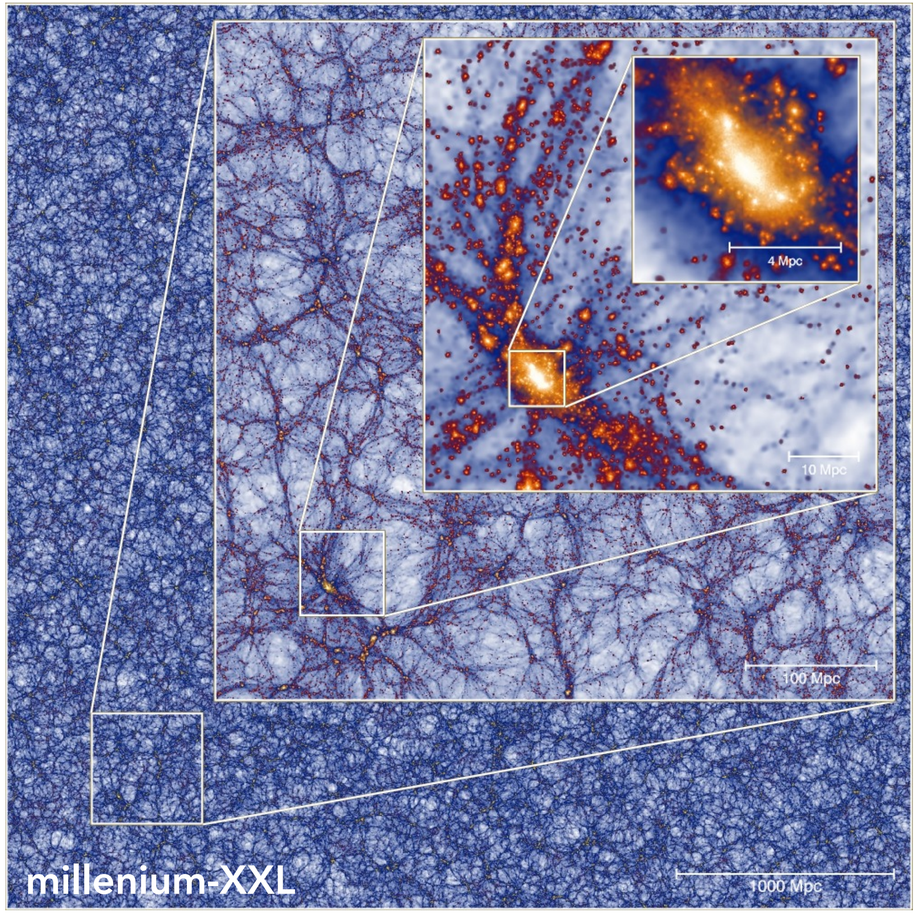
Finner et al. (2020)

Properties

- Massive bound systems $M > 10^{14}\ M_{\odot}$
- Recent objects, last steps of structure formation, formed by accretion of matter
- Laboratory for baryon physics (galaxies + ICM $\sim 20\,\%$) and dark matter $\sim 80\,\%$

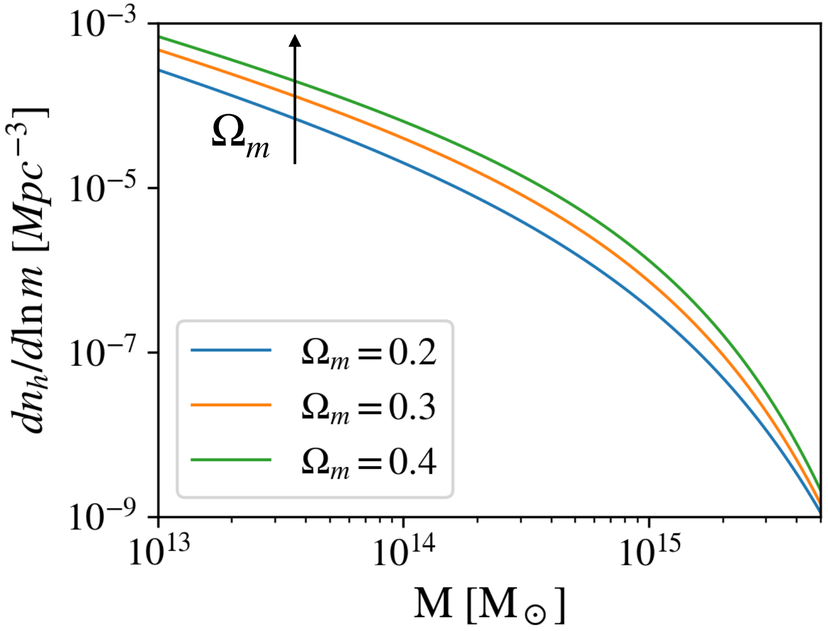
Cosmological probes

- Lie at the intersections of the cosmic web filaments
- Tracers of the matter distribution in the Universe

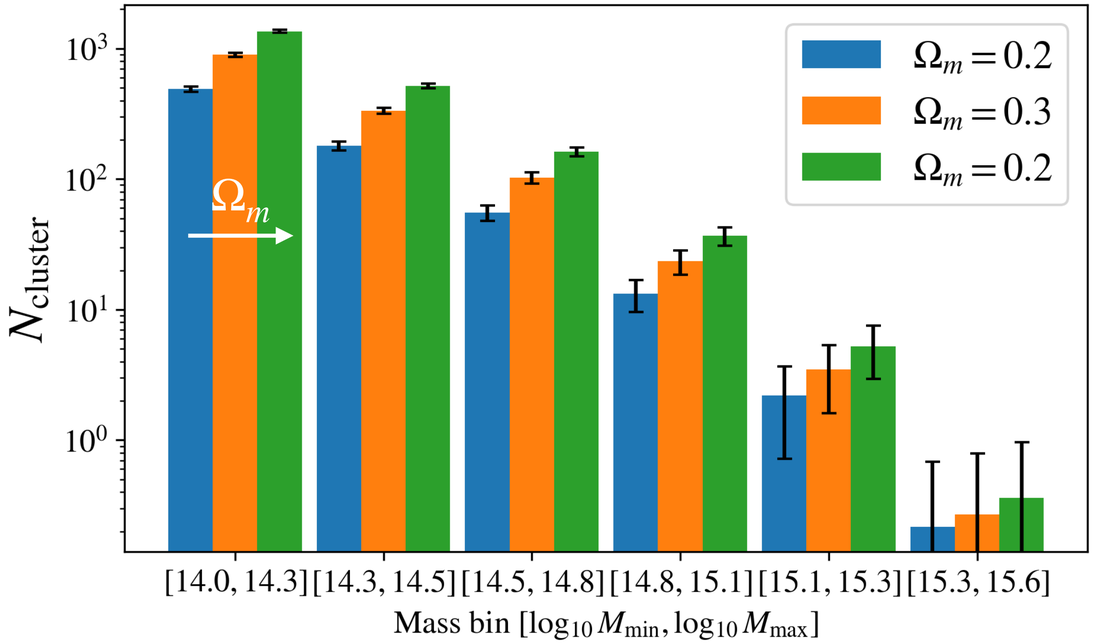


Cosmology with cluster counts

Change Ω_m ⟶ Change halo mass function + distances



Number of clusters
$0.28 < z < 0.36$



The abundance of galaxy clusters

- Privileged probe for structure formation and geometry

$$\frac{\partial^2 N_{\rm th}^{\rm halo}}{\partial m \partial z} \propto \frac{dn(m,z)}{dm} \frac{d^2V(z)}{dz d\Omega}$$

Future cluster surveys

- Detection of $\sim 10^4 - 10^5$ clusters (LSST, *Euclid*)
- Large statistical power
- Requires robust modeling of observable
- In this talk, we focus on the likelihood

Abundance statistics

Poisson process

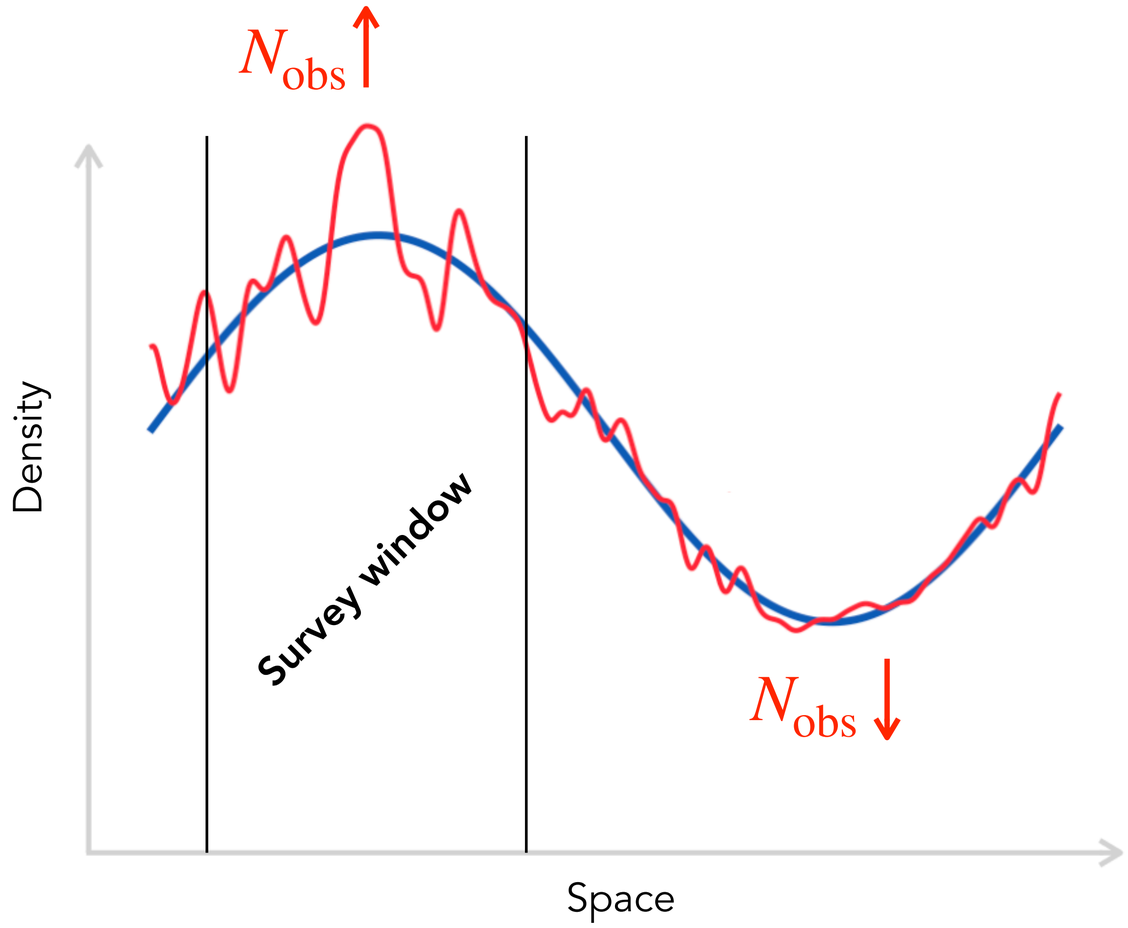
- Count of discrete objects in bins
- Uncorrelated counts
- Intrinsic Poisson variance: Shot Noise $\longrightarrow \sigma^2_{\rm SN} = N$

Super Sample Covariance (Hu & Kravstov, 2003)

$$n_h(\vec{x}) = \bar{n}_h(z)[1 + b\delta(\vec{x})]$$

- We observe a finite region in the sky
- Target an over(-under) dense region
- May increase(-decrease) the number of halos
- All probes react to SSC
- Impact the scattering of estimators (corr. + cov.)

$$\sigma^2_{\rm SSC}(N) \propto \frac{N^2\sigma^2_W}{\Omega_S} \longrightarrow \boxed{\sigma^2_{\rm tot} = \sigma^2_{\rm SN} + \sigma^2_{\rm SSC}}$$



Adapted from Doux et al. (2017)

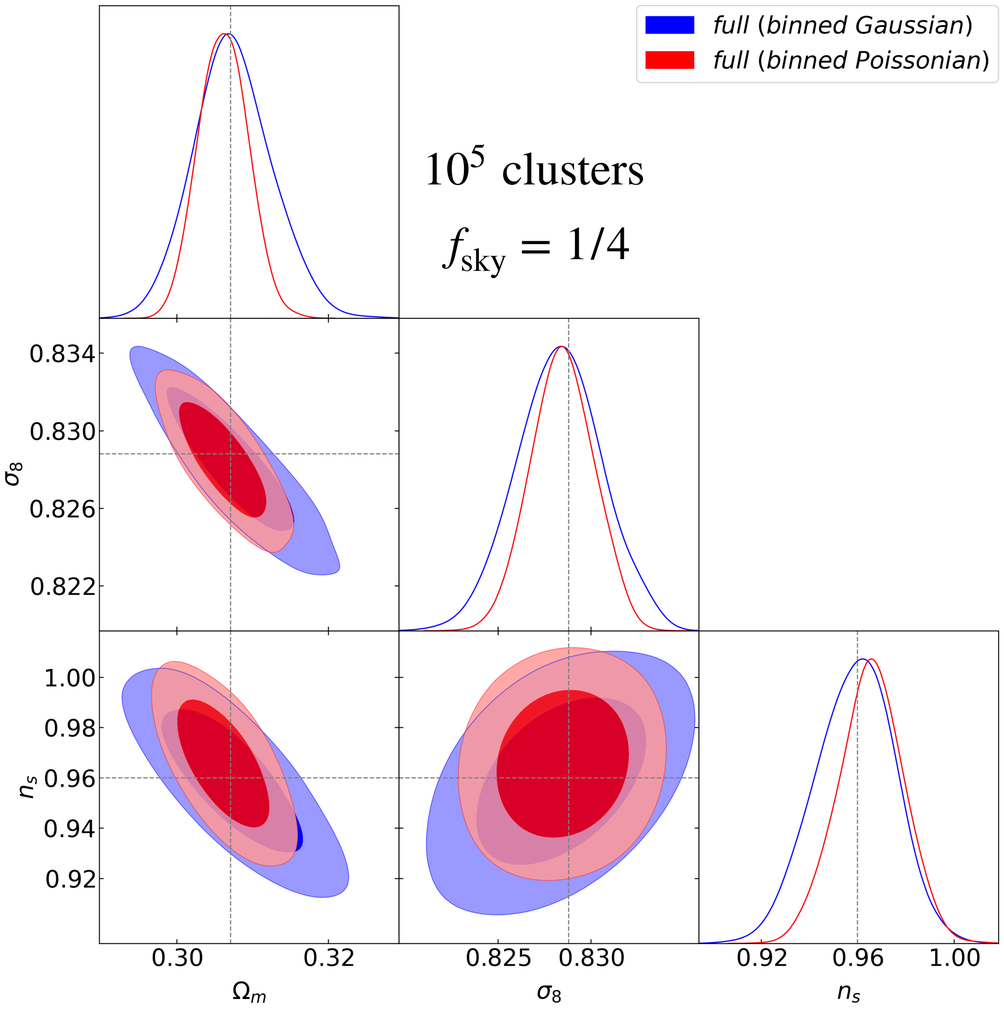
Super Sample Covariance in cosmology

Impact on parameter inference

- Strong impact on cosmological constraints for wide surveys
- -50% on FoM for 3x2pt: Gouyou Beauchamps *et al.* (2021)
- Cluster abundance: Fumagalli *et al.* (2021)
    - Impact binned analysis (count in redshift-proxy bins)
    - $\sim 10^5$ objects: 30-40% of FoM for CL count

Standard binned likelihood choice

- Depends on the relative importance between SSC and SN
- Poisson or SSC dominated regime



Landscape of (binned) likelihoods

Survey	Analysis	$N_{\rm tot}$
ROSAT	WtG, Mantz et al. (2015)	224
XMM	XXV, Pacaud et al. (2018)	178
	XLVI, Garrel et al. (2022)	178
eROSITA	eFEDS, Chiu et al. (2023)	455
ACT	S11, Sehgal et al. (2011)	9
	H13, Hasselfield et al. (2013)	15
Planck	XX, Ade et al. (2014)	189
	XXIV, Ade et al. (2016)	439
	S18, Salvati et al. (2018)	439
	×SPT, Salvati et al. (2022)	782
SPT	H16, de Haan et al. (2016)	377
	B19, Bocquet et al. (2019)	343
	C22, Chaubal et al. (2022)	343
SDSS	R10, Rozo et al. (2010)	10,810
	M13, Mana et al. (2013)	13,823
	C19, Costanzi et al. (2019)	7,000
	A20, Abdullah et al. (2020)	756
	P23, Park et al. (2023)	8,379
	S23, Sunayama et al. (2023)	8,379
	F23, Fumagalli et al. (2023)	6,964
DES	Y1, Abbott et al. (2020)	7,000
	T21, To et al. (2021)	4,794
	×SPT, Costanzi et al. (2021)	7,000
KiDS	DR3, Lesci et al. (2022)	3,652

Poisson likelihood

- Poisson sampling, no SSC
- Valid for low number of clusters - SN >> SSC

Gaussian-SSC likelihood

- Includes SN and SSC $(\Sigma_{\rm tot})_{kl} = N_k \delta^K_{kl} + (\Sigma_{\rm SSC})_{kl}$
- Limited to continuous approximation
- Valid for high number of clusters - SN ~ SSC

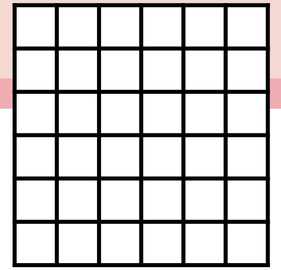
Gauss-Poisson Compound/GPC

- Poisson sampling and SSC
- Discrete description of counts

$$\mathscr{L} = \int d\vec{x}\ \mathscr{N}_{\rm SSC}[\vec{x}\,|\,\vec{N}] \times \prod_{k=1}^{n} \mathscr{P}[\,\widehat{N}_k\,|\,x_k]$$

Unbinned regime

Mass-redshift binning plane



$\widehat{N}_k = \{0,\ 1,\ 24,\ 245,\ \ldots\}$

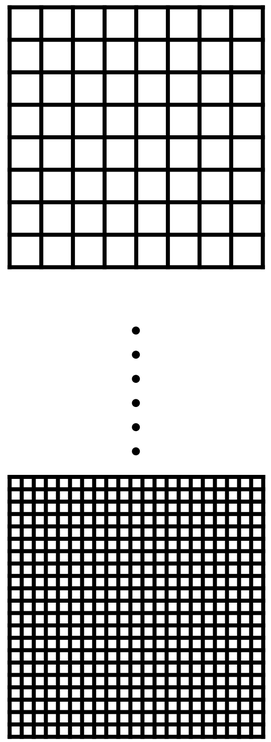
Principle

- Infinitesimal mass-redshift bins, until the count is at most 1 cluster

$$N_k \sim n_h(m_k, z_k) dm_k dV_k \ll 1$$

- Generally assumed that SN is the dominant source of uncertainties
- Use only Poisson statistics

$$\mathscr{L} = \prod_{k=1}^{\text{all bins}} \mathscr{P}(\widehat{N}_k | N_k) = \prod_{k'=1}^{\text{empty bins}} \mathscr{P}(0 | N_{k'}) \times \prod_{k=1}^{\text{full bins}} \mathscr{P}(1 | N_k) = e^{-N_{\text{th}}} \prod_{k=1}^{\widehat{N}_{\text{tot}}} N_k$$



$\widehat{N}_k = \{0,\ 1\}$

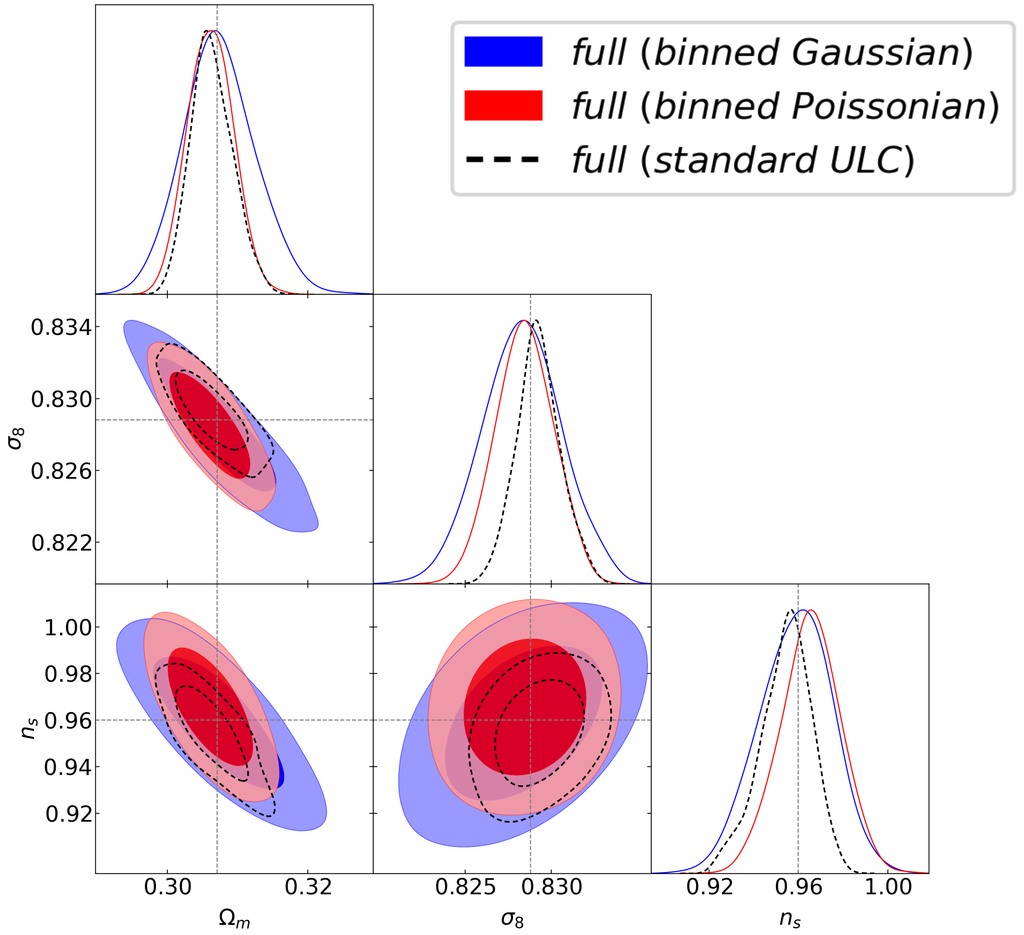
Standard unbinned likelihood

Advantages

- « *Halo-to-halo* » description
- Sensitive to *local* variation of halo mass function
- Takes account more *naturally* of individual source of scatters (Penna-Lima *et al.*, 2014)
- Account easily for multiple mass proxies (Bocquet et al. 2023)
- More intuitive to combine with lensing profiles, etc.
- Used with e.g. XMM, ACT, SPT clusters

Disadvantages

- Numerical complexity:
  - Models computed N_{cluster} times
  - Difficulty with more systematics, more clusters
- Based on Poisson statistics, no SSC



Impact of SSC on unbinned likelihood

Impact of SSC on parameter error bars ?

Binned

- Great impact of Euclid like statistics:
- (Fumagalli *et al.* (2021) for $\sim 5 - 100$ bins)
- Payerne *et al.* (2023): the Gaussian/SSC captures well the SSC for Euclid-like surveys

$$\mathrm{Var}(N_k) = N_k + b_k^2 N_k^2 S_{kk}$$

$$\propto dV_k \ll 1 \qquad \propto dV_k^2 \ll 1$$

Unbinned

- SSC decreases but to which value ?
- Objectives:
  1. Add SSC to unbinned analysis
  2. Small: validate that the effect is negligeable ?
  3. Large: Is it a big concern for large cluster surveys ?

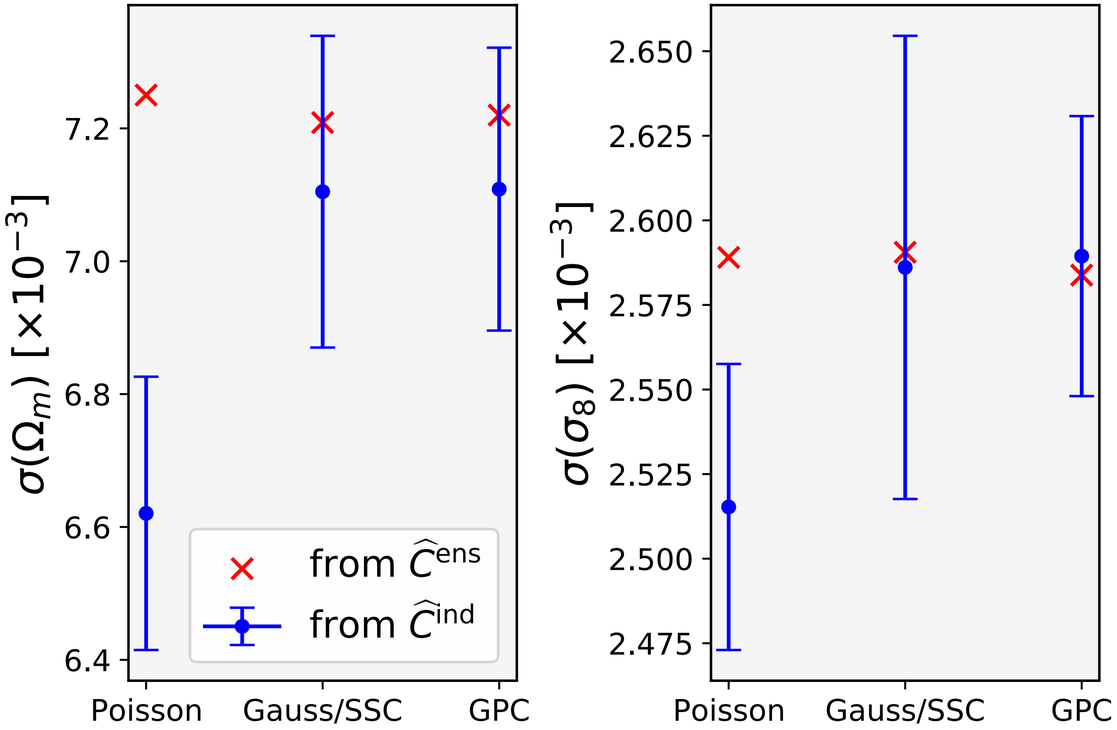
1. Testing the impact of SSC with simulations

Using dark matter catalogs at fixed cosmology

1. Use « standard » unbinned likelihood
2. For each simulation, compute $\langle \theta | \widehat{N} \rangle \pm \sqrt{\mathrm{Var}(\theta | \widehat{N})}$
3. Compare the spread of $\langle \theta | \widehat{N} \rangle$ to $\sqrt{\mathrm{Var}(\theta | \widehat{N})}$

- Payerne et al. (2023):
  - Used 1000 PINOCCHIO mocks (Monaco et al., 2013)
  - Test the « correctness » of binned approaches
  - Direct test of the lack of SSC
  - Could be extended to the unbinned framework

Binned likelihood, $M > 5 \times 10^{14} M_\odot$



2. Incorporating SSC analytically

SSC in the Poisson dominated regime

- ✓ We used the GPC formalism (GPC, Hu & Kravtsov, 2003, Lacasa & Grain, 2019)
- ✓ Low abundance regime (work based on Takada & Spergel, 2014)
- ✓ Push toward the un-binned regime

✓ $$\mathscr{L}_{\rm tot} = \left\langle \prod_{k=1}^{n} \mathscr{P}[\,\widehat{N}_k \,|\, N_k(1 + b_k\delta_k)] \right\rangle$$

$$\delta_k \sim \mathcal{N}(0\,|\,\sigma_{\rm SSC,k})$$

✓ $$N_k b_k \ll 1 \qquad \mathscr{L}_{\rm tot} \sim \mathscr{L}_{\rm Poisson} F_{\rm SSC}^{\rm binned} \equiv 1 + \frac{1}{2}\sum_{k,l=1}^{c} b_k b_l S_{kl}[(N_k - \widehat{N}_k)(N_l - \widehat{N}_l) - \widehat{N}_k \delta_{kl}^K]$$

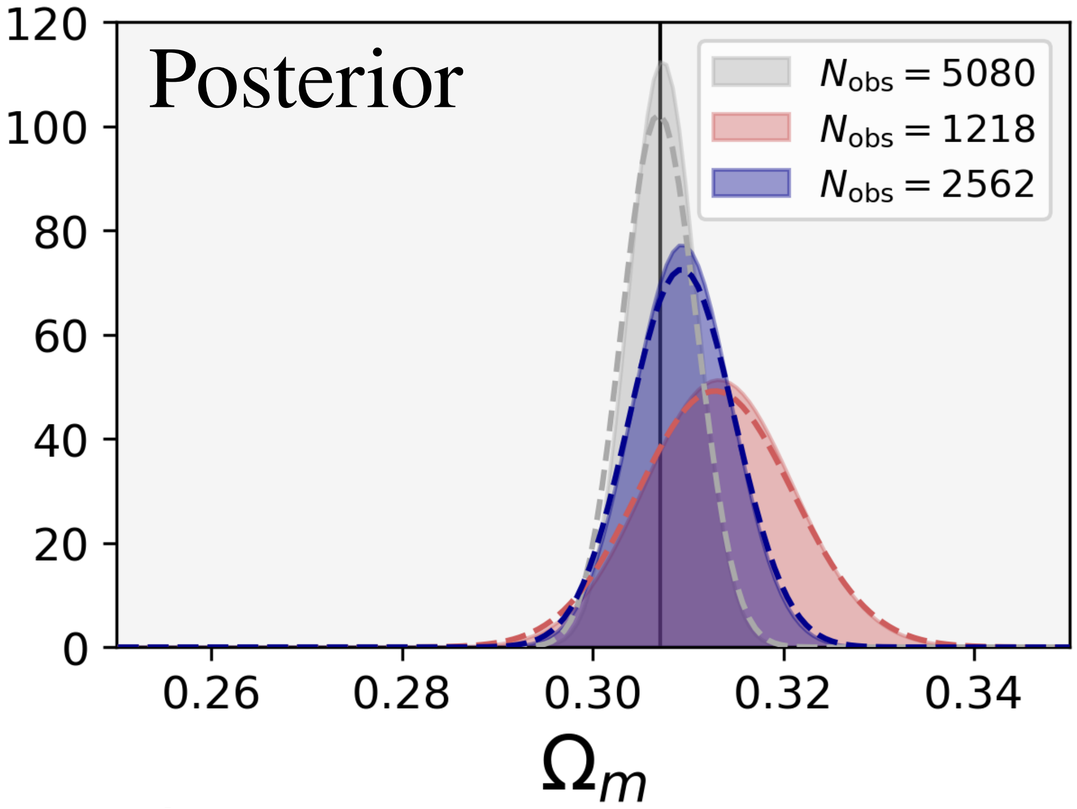
Separated SSC contribution

✓ $$\lim_{V_k \to dVk} F_{\rm SSC}^{\rm binned} \to F_{\rm SSC}^{\rm unbinned}$$

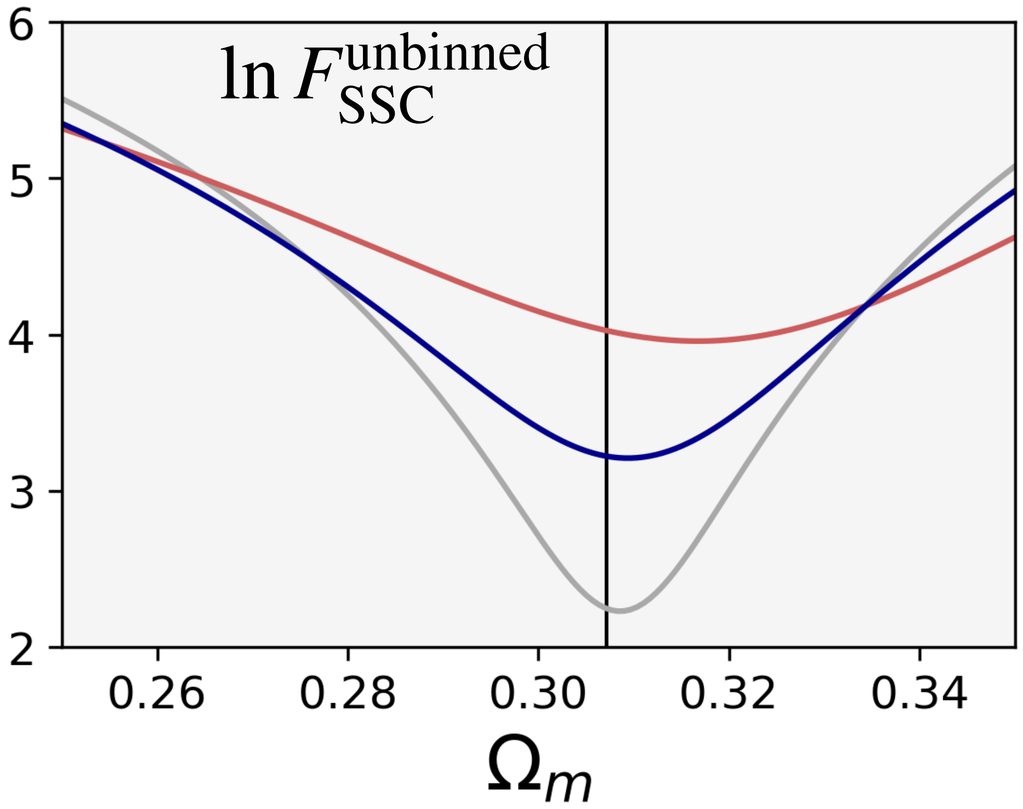
$$\widehat{N}_K = \{0,1\}$$

2. Incorporating SSC analytically - results

Full line: Standard Poisson likelihood
Dashed line: Poisson + SSC likelihood



Amplitude of matter fluctuations: PySSC (Lacasa et al, 2019)
https://github.com/fabienlacasa/PySSC



More clusters

- more tighter constraints without SSC
- SSC contribution: convex
- $\sigma_{\rm tot}(\Omega_m) = [104\,\%, 106\,\%, 110\%]\sigma_{\rm Poisson}$

Summary and conclusions

Summary

- Abundance of clusters - gain insight in cosmological models
- Standard unbinned likelihood: no SSC included
- Maybe be important for future large surveys $\sim 10^5$ clusters

In this talk

- We aim to test how SSC impact unbinned statistics
    1. We develop a formalism to include SSC in unbinned likelihood
    2. Test how the standard unbinned approach is lacking of SSC (using simulations)
- *Preliminary results*
    - As for binned approach, SSC impacts errors
    - SSC contribution increases with $N_{\rm tot}$
    - Push further ~ 5000 clusters, computationally demanding (idealistic dataset)
    - How does it compare to SSC in binned regime ?
- Paper Payerne et al. in prep.