Assignment 1 - Convex Hull and Plane Sweep Algorithm

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Algorithm 1 SmallConvexHull

Require: set of points P
  if $|P| < 5$ **then**
    return true
  end if
  Find P_1, P_2, the left-most and the right-most points from P
  Find $P_3 \in P$, which is the farthest point from the line P_1P_2
  Find $P_4 \in P$, which is the farthest point from the triangle $P_1P_2P_3$ and outside the triangle region.
  if $P4$ does not exist **then**
    return true
  end if
  if One point in P is outside the polygon $P_1P_2P_3P_4$ **then**
    return false
  end if
  return true

Proof. We will prove that the algorithm returns the correct result.

The convex hull covers all of the points in the set (P), by definition. Therefore, it covers the left-most and the right-most points; so P_1 and P_2 belong to the resulting convex hull of the set P.

P_3 is the farthest point from the line P_1P_2. If P_3 does not belong to the convex hull, then the convex hull does not cover P_3. It contradicts the definition of the convex hull. Thus, P_3 must belong to the convex hull.

Similarly, $P4$ is the farthest point from the triangle $P_1P_2P_3$, which means $P4$ must belong to the convex hull.

If in the set P, there is a point outside of the polygon $P_1P_2P_3P_4$, then we need more points to construct the convex hull because these 4 points are proven to be in the resulting convex hull. Thus, the convex hull contains more than 4 vertices. Otherwise, the convex hull obviously contains less than 5 vertices.

Therefore, the algorithm is correct.

Now, we will prove that the algorithm runs in $O(n)$ time.

For finding $P_i, 1 \leq i \leq 4$, it takes $O(n)$ time.

For checking that if any point is outside of the polygon, it takes $O(n)$ time.

Therefore, the overall time complexity is $O(n)$. $\square$