Assignment 4 - Trapezoidal Map, Arrangements and Duality

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Search path to q at D_j becomes longer if q is in a trapezoid that was just created by the latest insertion. We also know that at most 4 line segments define that trapezoids. Thus, the probability the search part becomes longer is :

$$Pr[\text{Search Path to } q \text{ become longer at step } i] = 4/i$$

Hence, the expected length of the search part increases when comparing at step j and k where $j < k$ is:

$$\begin{aligned}
\text{Expected Length} &\leq \sum_{i=j}^{k}(4/i) \\
&= 4(\sum_{i=1}^{k}(1/i) - \sum_{i=1}^{j}(1/i)) \\
&= 4(1 + \log_e k - 1 - log_e j) \\
&= O(\log(k/j))
\end{aligned}$$

Therefore, the expected time locating q at D_k is $O(\log(k/j))$.