Assignment 2 - Quadtree and WSPD

Duy Pham - 0980384
Mazen Aly - 0978251
Pattarawat Chormai - 0978675

November 28, 2015

1

Assume A and B are two adjacent squares in a quadtree whose size are different by factor of 4 and B is bigger than A. If we split B into 4 smaller squares, one of them will become adjacent to A with size differing by 2. Because of the number of triangles after triangulating a subdivision is $O(1)$ when using factor of 2 as a balanced condition. Thus, the number of triangles can increase at most $4O(1)$ when splitting B.

Therefore, the number of triangles is still $O(1)$ when using the factor of 4 as a balanced condition.

2

a

Because $p \in A$ and $q \in B$ where A, B is a pair in the s-WSPD, the distance $|pq|$ should be large. In particular, $|pq| \geq sr$ where r is the radius of A and B. As we know, $s > 2$, so $sr > d$ where $d = 2s$ is the diameter of the circles.

If there is another point p' in A, then $|pp'| \leq d < sr < |pq|$. Thus q is not the nearest neighbor of p anymore. This contradicts our proposal, which completes the proof.

If p and q are the closest pair in P, then q is the nearest neighbor of p and vice versa. Thus if $p \in A$ and $q \in B$ where A, B is a pair in the s-WSPD, A and B contain only 1 element as being proven above. From the lecture, we know that the number of pairs in the s-WSPD is $O(s^d \dot{n})$. s and d do not depend on n, so we can consider this as $O(n)$. Therefore, in $O(n)$ time, we can extract all pairs A_i, B_i in which both groups contain only 1 element. Then we can compare the distances between the items in each pair, and pick the one with the smallest distance, which also takes $O(n)$ because the number of such pairs is at most the number of the s-WSPD. In general, we can do that in $O(n)$ time.

b

According to (a) if $s > 2$, for each point p there will be a pair $A, B \in WSPD$ in which A contains only p. Thus, we have n such circles A. Therefore, the number of pairs between the circles is at least $n/2$.