Assignment 2 - Quadtree and WSPD

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Assume A and B are two adjacent squares in a quadtree whose size are different by factor of 4 and B is bigger than A. If we split B into 4 equal squares, one of them will become adjacent to A with size differing by a factor of 2. Because of the number of triangles after triangulating a subdivision is $O(1)$ when using factor of 2 as a balanced condition. Thus, the number of triangles can increase at most $4O(1)$ when splitting B.

Therefore, the number of triangles is still $O(1)$ when using the factor of 4 as a balanced condition.

The idea is to choose a Steiner point in the center of B, and choose other Steiner points in the center of the edge connecting the first Steiner point to the corners of B, when neccessary.

2

a

Because $p \in A$ and $q \in B$ where A, B is a pair in the s-WSPD, the distance $|pq|$ should be large. In particular, $|pq| \geq sr$ where r is the radius of A and B. As we know, $s > 2$, so $sr > d$ where $d = 2s$ is the diameter of the circles.

If there is another point p' in A, then $|pp'| \leq d < sr < |pq|$. Thus q is not the nearest neighbor of p anymore. This contradicts our proposal, which completes the proof.

If p and q are the closest pair in P, then q is the nearest neighbor of p and vice versa. Thus if $p \in A$ and $q \in B$ where A, B is a pair in the s-WSPD, A and B contain only 1 element as being proven above. From the lecture, we know that the number of pairs in the s-WSPD is $O(s^d n)$. s and d do not depend on n, so we can consider this as $O(n)$. Therefore, in $O(n)$ time, we can extract all pairs A_i, B_i in which both groups contain only 1 element. Then we can compare the distances between the items in each pair, and pick the one with the smallest distance, which also takes $O(n)$ because the number of such pairs is at most the number of the s-WSPD. In general, we can do that in $O(n)$ time.

b

According to (a) if $s > 2$, for each point p there will be a pair $A, B \in WSPD$ in which A contains only p. Thus, we have n such circles A. Therefore, the number of pairs between the circles is at least $n/2$.

In the construction of a t-spanner, we use $s = 4(t+1)/(t-1)$. From this, we see that when $t > 1$, $s > 4$. In the spanner, every vertex should be accessible, which means the number of edges should be at least $n-1$, otherwise there is at least an unreachable vertex. The construction algorithm builds an edge in the spanner if there is a pair in the s-WSPD. Therefore, the number of pairs should be at least $n-1$, when $s > 4$.

c

As we observe from (a), each point in P will have its own circle. If $s > 0$, it means all circles with the same size are disjoint. With this property, the number of circles of a WSPD, $\sum |A_i| + |B_i|$, is maximum when the algorithm can always find a pair of circles with exactly 2 smaller circles inside. Note that, if there are more than 2 smaller circles in a circle, the total number of circles reduces.

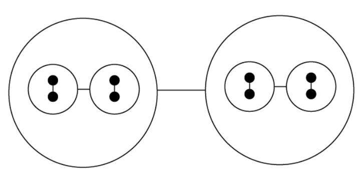

Figure 1: A WSPD whose circles contains 2 smaller circles.

It is obvious to see that if every circle of the WSPD contains exactly 2 smaller circles, the relation of circles is similar to binary tree whose leaves are the points of P. Hence,

$$
\begin{aligned}
\text{No. Circles in the WSPD} &\leq \text{No. Nodes in the tree} - 1 \\
&= O(n \log n)
\end{aligned}
$$

Therefore, this proves $\sum |A_i| + |B_i| = O(n \log n)$.