

Assignment 1 - Convex Hull and Plane Sweep Algorithm

Duy Pham - 0980384
Mazen Aly - 0978251
Pattarawat Chormai - 0978675

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Algorithm 1 SmallConvexHull

Require: set of points P

if $|P| < 5$ **then**

 return true

end if

Find P_1, P_2 , the left-most and the right-most points from P

Find $P_3 \in P$, which is the farthest point from the line P_1P_2

Find $P_4 \in P$, which is the farthest point from the triangle $P_1P_2P_3$ and outside the triangle region.

if P_4 does not exist **then**

 return true

end if

if One point in P is outside the polygon $P_1P_2P_3P_4$ **then**

 return false

end if

return true

Proof. We will prove that the algorithm returns the correct result.

The convex hull covers all of the points in the set (P) , by definition. Therefore, it covers the left-most and the right-most points; so P_1 and P_2 belong to the resulting convex hull of the set P .

P_3 is the farthest point from the line P_1P_2 . If P_3 does not belong to the convex hull, then the convex hull does not cover P_3 . It contradicts the definition of the convex hull. Thus, P_3 must belong to the convex hull.

Similarly, P_4 is the farthest point from the triangle $P_1P_2P_3$, which means P_4 must belong to the convex hull.

If in the set P , there is a point outside of the polygon $P_1P_2P_3P_4$, then we need more points to construct the convex hull because these 4 points are proven to be in the resulting convex hull. Thus, the convex hull contains more than 4 vertices. Otherwise, the convex hull obviously contains less than 5 vertices.

Therefore, the algorithm is correct.

Now, we will prove that the algorithm runs in $O(n)$ time.

For finding $P_i, 1 \leq i \leq 4$, it takes $O(n)$ time.

For checking that if any point is outside of the polygon, it takes $O(n)$ time.

Therefore, the overall time complexity is $O(n)$. □

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Our rectangle is a region of $2\delta \times \delta$.

Because δ is the smallest diameter among all three-disks, then the maximum distance among any three points must be at least some constant factor of δ . (Indeed, if the three points create an equilateral triangle, then the distance is $\frac{\sqrt{3} \cdot \delta}{2}$. If the third point is very close to the line created from the first 2 points, then the maximum distance is δ).

Now we try to put as many points as possible into the rectangle region, following the rule: there are no three-disks with diameters less than δ . It is obvious that we have to put the points at a smallest possible distance to each other.

To maximize the number of the points, we will put the first point P_1 in the center of the left border of the rectangle. Then we can insert the next point P_2 at a very close position to P_1 (much smaller than δ). Then the next point P_3 must be at least at a distance of δ from either P_1 or P_2 . There is also the other case when we put P_2, P_3 in which $P_1P_2P_3$ forms an equilateral triangle, and the distance of the edges is $\frac{\sqrt{(3)} \cdot \delta}{2}$. If we put any other point, there will be 3 points that create a smaller disk.

Similarly, we do the same thing for the other half of the rectangle, however we have to take care of the situation when there are 3 points close to the center of the rectangle which can form a smaller three-disk.

Therefore, the number of points in the rectangle is bounded by a constant.