

Assignment 4 - Trapezoidal Map, Arrangements and Duality

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a

Search path to q at D_j becomes longer if q is in a trapezoid that was just created by the latest insertion. We also know that at most 4 line segments define that trapezoids. Thus, the probability the search part becomes longer is :

$$Pr[\text{Search Path to } q \text{ become longer at step } i] = 4/i$$

Hence, the expected length of the search part increases when comparing at step j and k where $j < k$ is:

$$\begin{aligned} \text{Expected Length} &\leq \sum_{i=j}^k (4/i) \\ &= 4 \left(\sum_{i=1}^k (1/i) - \sum_{i=1}^j (1/i) \right) \\ &= 4(1 + \log_e k - 1 - \log_e j) \\ &= O(\log(k/j)) \end{aligned}$$

Therefore, the expected time locating q at D_k is $O(\log(k/j))$.

b

We observe that vertical decomposition lines will intersect properly with P if the edges that they intersect with have not added into T yet. Hence, the probability that T_j have proper intersection with P is :

$$Pr[\text{Proper Intersection between } T_j \text{ and } P] = (n - j)/(n - 1)$$

Thus, the expected number of proper intersection, E , between T_j and P is :

$$\begin{aligned}
 E &= \sum_{i=1}^n (n-i)/(n-1) \\
 &= \frac{1}{n-1} \frac{(n-1)(n-2)}{2} \\
 &= \frac{n-2}{2} \\
 &= O(n)
 \end{aligned}$$