

Assignment 2 - Quadtree and WSPD

Duy Pham - 0980384

Mazen Aly - 0978251

Pattarawat Chormai - 0978675

November 30, 2015

1

Assume A and B are two adjacent squares in a quadtree whose size are different by factor of 4 and B is bigger than A . If we split B into 4 smaller squares, one of them will become adjacent to A with size differing by 2. Because of the number of triangles after triangulating a subdivision is $O(1)$ when using factor of 2 as a balanced condition. Thus, the number of triangles can increase at most $4O(1)$ when splitting B .

Therefore, the number of triangles is still $O(1)$ when using the factor of 4 as a balanced condition.

2

a

Because $p \in A$ and $q \in B$ where A, B is a pair in the s -WSPD, the distance $|pq|$ should be large. In particular, $|pq| \geq sr$ where r is the radius of A and B . As we know, $s > 2$, so $sr > d$ where $d = 2s$ is the diameter of the circles.

If there is another point p' in A , then $|pp'| \leq d < sr < |pq|$. Thus q is not the nearest neighbor of p anymore. This contradicts our proposal, which completes the proof.

If p and q are the closest pair in P , then q is the nearest neighbor of p and vice versa. Thus if $p \in A$ and $q \in B$ where A, B is a pair in the s -WSPD, A and B contain only 1 element as being proven above. From the lecture, we know that the number of pairs in the s -WSPD is $O(s^d n)$. s and d do not depend on n , so we can consider this as $O(n)$. Therefore, in $O(n)$ time, we can extract all pairs A_i, B_i in which both groups contain only 1 element. Then we can compare the distances between the items in each pair, and pick the one with the smallest distance, which also takes $O(n)$ because the number of such pairs is at most the

number of the s -WSPD. In general, we can do that in $O(n)$ time.

b

According to (a) if $s > 2$, for each point p there will be a pair $A, B \in WSPD$ in which A contains only p . Thus, we have n such circles A . Therefore, the number of pairs between the circles is at least $n/2$.

In the construction of a t -spanner, we use $s = 4(t + 1)/(t - 1)$, when $t > 1$, $s > 4$. In the spanner, every vertex should be accessible, which means the number of edges should be at least $n - 1$, otherwise there is at least an unreachable vertex. The construction algorithm builds an edge in the spanner if there is a pair in the s -WSPD. Therefore, the number of pairs should be at least $n - 1$, when $s > 4$.

c

As we observe from (a), each point in P will have its own circle. If $s > 0$, it means all circles, having the same size, are disjoint. With this property, the number of circles of a WSPD, $\sum |A_i| + |B_i|$, is maximum when the algorithm can always find a pair of circles with exactly 2 smaller circles inside. Note that, if there are more than 2 smaller circles in a circle, the total number of circles reduces.

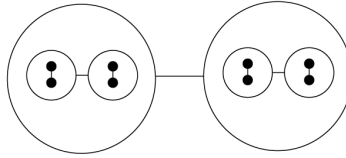


Figure 1: A WSPD whose circles contains 2 smaller circles.

It is obvious to see that if every circle of the WSPD contains exactly 2 smaller circles. The relation of circles is similar to binary tree whose leaves are the points of P . Hence,

$$\begin{aligned} \text{No. Circles in the WSPD} &= \text{No. Nodes in the tree} - 1 \\ &= O(n \log n) \end{aligned}$$

Therefore, this proves $\sum |A_i| + |B_i|$ is bounded by $O(n \log n)$.