Assignment 4 - Trapezoidal Map, Arrangements and Duality

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a

Search path to q at D_j becomes longer if q is in a trapezoid that was just created by the latest insertion. We also know that at most 4 line segments define that trapezoids. Thus, the probability that the search part becomes longer is :

$$Pr[\text{Search Path to } q \text{ becomes longer at step } i] = 4/i$$

Hence, the expected increment of the length of the search part at step k, comparing to step j, where $j < k$, is:

$$\begin{aligned}
\text{Expected Length} &\leq \sum_{i=j}^{k}(4/i) \\
&= 4(\sum_{i=1}^{k}(1/i) - \sum_{i=1}^{j}(1/i)) \\
&\leq 4(1 + \ln k - 1 - \ln j) \\
&= O(\log(k/j))
\end{aligned}$$

Therefore, the expected time locating q at D_k is $O(\log(k/j))$.

b

We observe that vertical decomposition lines of e_i intersects properly with P if the edges that they intersect with have not been added into T_i yet. Hence, the number of proper intersections at T_i is :

$$\text{Number of Proper Intersections of } T_i = Pr[T_i \text{ has proper intersections}] = (n{-}j)/(n{-}1)$$

Thus, the expected number of proper intersections, E, between T_j and P is:

$$
\begin{aligned}
\mathrm{E} &= \sum_{i=1}^{n} (n-i)/(n-1) \\
&= \frac{1}{n-1} \frac{(n-1)n}{2} \\
&= \frac{n}{2} \\
&= O(n)
\end{aligned}
$$