

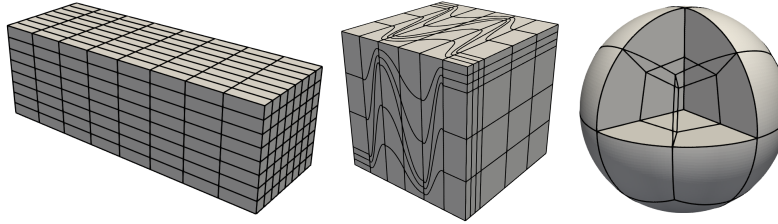
# Cache-optimized and low-overhead implementations of multigrid smoothers for high-order FEM computations

December 11, 2022

This document is the companion material to:

```
@article{asm2022,  
  title = {Data-local and low-overhead implementations of smoothers for multigrid and  
           high-order FEM computations},  
  author = {XXX},  
  year = {2022}  
}
```

It contains raw data of the multigrid section, in which the Poisson equations is solved on three families of meshes (anisotropic Cartesian mesh, Kershaw mesh, 3D ball):



with the statistics:

name	polynomial degree $p = 4$			polynomial degree $p = 7$		
	level $L$	no. cells	no. DoFs	level $L$	no. cells	no. DoFs
anisotropic	7	262,144	16,974,593	6	32,768	11,390,625
Kershaw	5	110,592	7,189,057	4	13824	4,826,809
3D ball	5	131,072	8,438,273	4	16384	5,657,793

We identify the best solver configuration by varying:

1. the Chebyshev degree  $k$ ,
2. the type of decreasing  $p$ ,
3. point Jacobi or ASM (cell-centric patches with  $n = 1$  and  $n = 2$ , vertex-star patches) as preconditioner for the Chebyshev iteration,
4. first or fourth kind of Chebyshev polynomials,
5. one-sided vs. two-sided V-cycles

6. different polynomial degrees  $p$ , and
7. different anisotropy settings.

The meaning of the coloring is as follows:

<b>text</b>	fastest configuration
<i>text</i>	fastest configuration in the current row
	within 20% of the fastest configuration
	within 50% of the fastest configuration
	within 100% of the fastest configuration
	all other configurations

## 1 Anisotropic mesh

Table 1: Anisotropic mesh ( $x_2/x_0 = 1, p = 4$ )

	bisect			go-to-one			decrease-by-one		
	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 3$	$k = 4$	$k = 5$
	#t	#t	#t	#t	#t	#t	#t	#t	#t
cg-diagonal-1st-kind-two-sided	8 3.12e-1	7 3.02e-1	5 3.10e-1	4 2.94e-1	3 2.96e-1	6 3.29e-1	5 3.28e-1	4 3.27e-1	3 3.26e-1
cg-diagonal-4th-kind-two-sided	5 2.37e-1	5 2.52e-1	4 2.40e-1	3 2.56e-1	19 6.05e-1	11 4.90e-1	5 3.79e-1	4 2.50e-1	3 2.37e-1
cg-fm-symm, 1-f, 1st-kind-two-sided	6 2.83e-1	4 2.76e-1	5 4.48e-1	4 4.13e-1	3 3.96e-1	7 2.95e-1	5 3.88e-1	4 4.54e-1	5 5.70e-1
cg-fm-symm, 1-f, 4th-kind-two-sided	5 2.83e-1	4 2.76e-1	5 4.48e-1	3 3.44e-1	3 3.96e-1	5 2.98e-1	4 3.82e-1	3 3.41e-1	4 4.60e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	6 4.33e-1	4 4.70e-1	5 8.18e-1	3 6.31e-1	3 7.69e-1	7 4.41e-1	5 7.00e-1	4 7.06e-1	5 1.09e+0
cg-fm-symm, 2-f, 4th-kind-two-sided	6 4.33e-1	4 4.70e-1	5 8.18e-1	3 6.31e-1	3 7.69e-1	8 5.02e-1	4 5.69e-1	4 7.17e-1	3 8.43e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	5 3.11e-1	7 3.08e-1	5 6.44e-1	4 4.88e-1	3 5.87e-1	7 3.42e-1	5 5.56e-1	4 5.89e-1	5 8.43e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	5 3.11e-1	7 3.08e-1	5 6.44e-1	4 4.88e-1	3 5.87e-1	7 3.42e-1	5 5.56e-1	4 5.89e-1	5 8.43e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	4 2.16e-1	7 5.45e-1	5 5.25e-1	4 5.58e-1	3 5.11e-1	7 3.30e-1	4 4.62e-1	4 5.51e-1	3 6.63e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	4 2.16e-1	7 5.45e-1	5 5.25e-1	4 5.58e-1	3 5.11e-1	7 3.30e-1	4 4.62e-1	4 5.51e-1	3 6.63e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	11 6.10e-1	8 6.21e-1	6 6.18e-1	5 6.51e-1	4 6.42e-1	10 4.90e-1	5 5.47e-1	5 6.02e-1	4 8.37e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	11 6.10e-1	8 6.21e-1	6 6.18e-1	5 6.51e-1	4 6.42e-1	10 4.90e-1	5 5.47e-1	5 6.02e-1	4 8.37e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	8 4.36e-1	6 4.71e-1	5 6.17e-1	5 6.49e-1	5 7.73e-1	9 4.42e-1	7 4.80e-1	6 5.37e-1	5 7.82e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	8 4.36e-1	6 4.71e-1	5 6.17e-1	5 6.49e-1	5 7.73e-1	9 4.42e-1	7 4.80e-1	6 5.37e-1	5 7.82e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	13 1.06e+0	9 1.16e+0	6 1.45e+0	5 1.43e+0	5 1.48e+0	9 6.31e-1	7 1.09e+0	5 1.03e+0	4 1.06e+0
cg-fm-symm, 2-f, 4th-kind-two-sided	13 1.06e+0	9 1.16e+0	6 1.45e+0	5 1.43e+0	5 1.48e+0	9 6.31e-1	7 1.09e+0	5 1.03e+0	4 1.06e+0
cg-fm-symm, 2-f, 1st-kind-two-sided	5 4.12e-1	4 5.01e-1	7 7.87e-1	4 1.01e+0	3 9.83e-1	4 9.83e-1	4 9.83e-1	4 9.83e-1	4 9.83e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	5 4.12e-1	4 5.01e-1	7 7.87e-1	4 1.01e+0	3 9.83e-1	4 9.83e-1	4 9.83e-1	4 9.83e-1	4 9.83e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	6 4.17e-1	8 8.58e-1	6 8.86e-1	5 9.23e-1	4 9.56e-1	7 4.56e-1	6 8.06e-1	5 8.65e-1	4 8.06e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	6 4.17e-1	8 8.58e-1	6 8.86e-1	5 9.23e-1	4 9.56e-1	7 4.56e-1	6 8.06e-1	5 8.65e-1	4 8.06e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	12 8.65e-1	9 9.08e-1	7 1.02e+0	6 1.12e+0	5 1.18e+0	9 9.36e-1	5 9.36e-1	5 9.36e-1	5 9.36e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	12 8.65e-1	9 9.08e-1	7 1.02e+0	6 1.12e+0	5 1.18e+0	9 9.36e-1	5 9.36e-1	5 9.36e-1	5 9.36e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	5 3.61e-1	4 4.61e-1	4 6.25e-1	3 6.23e-1	3 7.61e-1	5 3.37e-1	4 4.20e-1	4 4.20e-1	4 4.20e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	5 3.61e-1	4 4.61e-1	4 6.25e-1	3 6.23e-1	3 7.61e-1	5 3.37e-1	4 4.20e-1	4 4.20e-1	4 4.20e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	7 4.99e-1	7 7.62e-1	6 8.88e-1	6 1.12e+0	6 1.35e+0	6 3.96e-1	7 6.96e-1	6 1.02e+0	5 1.50e+0
cg-fm-symm, 2-f, 4th-kind-two-sided	7 4.99e-1	7 7.62e-1	6 8.88e-1	6 1.12e+0	6 1.35e+0	6 3.96e-1	7 6.96e-1	6 1.02e+0	5 1.50e+0

Table 2: Anisotropic mesh ( $x_2/x_0 = 2, p = 4$ )

	bisect			go-to-one			decrease-by-one		
	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 3$	$k = 4$	$k = 5$
	#t	#t	#t	#t	#t	#t	#t	#t	#t
cg-diagonal-1st-kind-two-sided	14 5.45e-1	7 3.53e-1	6 3.71e-1	5 3.67e-1	4 3.41e-1	28 9.82e-1	16 7.16e-1	11 6.01e-1	8 5.94e-1
cg-diagonal-4th-kind-two-sided	17 6.68e-1	9 4.52e-1	6 3.72e-1	5 3.68e-1	4 3.41e-1	44 1.9e+0	19 8.50e-1	13 7.08e-1	8 5.95e-1
cg-fm-symm, 1-f, 1st-kind-two-sided	5 4.23e-1	5 3.42e-1	4 3.50e-1	3 3.45e-1	3 3.38e-1	15 6.25e-1	8 4.76e-1	6 4.46e-1	4 4.54e-1
cg-fm-symm, 1-f, 4th-kind-two-sided	7 5.61e-1	5 8.20e-1	5 8.20e-1	4 8.41e-1	3 7.69e-1	12 7.50e-1	7 7.07e-1	6 8.38e-1	4 8.70e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	9 6.46e-1	4 4.76e-1	4 6.56e-1	4 8.41e-1	3 7.69e-1	9 5.12e-1	6 6.07e-1	5 5.87e-1	4 8.70e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	5 3.12e-1	7 8.29e-1	5 6.46e-1	4 6.52e-1	3 5.90e-1	10 6.25e-1	6 6.95e-1	5 7.38e-1	4 7.15e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	7 4.36e-1	8 5.20e-1	5 5.24e-1	4 5.20e-1	3 4.92e-1	11 5.51e-1	6 5.45e-1	4 4.71e-1	4 5.90e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	7 4.36e-1	8 5.20e-1	5 5.24e-1	4 5.20e-1	3 4.92e-1	11 5.51e-1	6 5.45e-1	4 4.71e-1	4 5.90e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	13 7.66e-1	9 7.01e-1	6 6.13e-1	5 6.38e-1	4 6.13e-1	13 6.68e-1	8 5.78e-1	6 6.22e-1	4 6.68e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	13 7.66e-1	9 7.01e-1	6 6.13e-1	5 6.38e-1	4 6.13e-1	13 6.68e-1	8 5.78e-1	6 6.22e-1	4 6.68e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	10 5.90e-1	7 5.16e-1	6 6.10e-1	6 7.67e-1	6 9.13e-1	14 7.29e-1	8 5.49e-1	7 6.34e-1	6 7.88e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	10 5.90e-1	7 5.16e-1	6 6.10e-1	6 7.67e-1	6 9.13e-1	14 7.29e-1	8 5.49e-1	7 6.34e-1	6 7.88e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	7 5.60e-1	10 1.29e+0	7 1.28e+0	6 1.43e+0	5 1.49e+0	10 1.11e+0	7 1.09e+0	6 1.21e+0	5 1.26e+0
cg-fm-symm, 2-f, 4th-kind-two-sided	7 5.60e-1	10 1.29e+0	7 1.28e+0	6 1.43e+0	5 1.49e+0	10 1.11e+0	7 1.09e+0	6 1.21e+0	5 1.26e+0
cg-fm-symm, 2-f, 1st-kind-two-sided	15 1.24e+0	11 1.43e+0	9 1.62e+0	8 1.65e+0	8 1.78e+0	10 7.07e-1	11 1.24e+0	8 1.24e+0	5 1.26e+0
cg-fm-symm, 2-f, 4th-kind-two-sided	15 1.24e+0	11 1.43e+0	9 1.62e+0	8 1.65e+0	8 1.78e+0	10 7.07e-1	11 1.24e+0	8 1.24e+0	5 1.26e+0
cg-fm-symm, 2-f, 1st-kind-two-sided	8 6.40e-1	5 6.78e-1	4 7.85e-1	4 1.01e+0	4 1.28e+0	9 6.32e-1	4 6.09e-1	4 8.57e-1	4 1.05e+0
cg-fm-symm, 2-f, 4th-kind-two-sided	8 6.40e-1	5 6.78e-1	4 7.85e-1	4 1.01e+0	4 1.28e+0	9 6.32e-1	4 6.09e-1	4 8.57e-1	4 1.05e+0
cg-fm-symm, 2-f, 1st-kind-two-sided	9 7.21e-1	8 1.05e+0	8 1.46e+0	7 1.65e+0	8 2.29e+0	9 6.31e-1	8 9.01e-1	7 1.10e+0	8 1.30e+0
cg-fm-symm, 2-f, 4th-kind-two-sided	9 7.21e-1	8 1.05e+0	8 1.46e+0	7 1.65e+0	8 2.29e+0	9 6.31e-1	8 9.01e-1	7 1.10e+0	8 1.30e+0
cg-fm-symm, 2-f, 1st-kind-two-sided	6 4.39e-1	5 9.65e-1	6 8.89e-1	5 9.54e-1	4 9.56e-1	7 4.50e-1	9 8.82e-1	6 8.08e-1	5 8.68e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	6 4.39e-1	5 9.65e-1	6 8.89e-1	5 9.54e-1	4 9.56e-1	7 4.50e-1	9 8.82e-1	6 8.08e-1	5 8.68e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	13 9.48e-1	10 1.68e+0	8 1.66e+0	7 1.12e+0	5 1.18e+0	9 9.97e-1	11 1.09e+0	8 1.06e+0	5 1.02e+0
cg-fm-symm, 2-f, 4th-kind-two-sided	13 9.48e-1	10 1.68e+0	8 1.66e+0	7 1.12e+0	5 1.18e+0	9 9.97e-1	11 1.09e+0	8 1.06e+0	5 1.02e+0
cg-fm-symm, 2-f, 1st-kind-two-sided	4 5.68e-1	4 5.68e-1	4 5.68e-1	4 5.68e-1	4 5.68e-1	4 5.68e-1	4 5.68e-1	4 5.68e-1	4 5.68e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	4 5.68e-1	4 5.68e-1	4 5.68e-1	4 5.68e-1	4 5.68e-1	4 5.68e-1	4 5.68e-1	4 5.68e-1	4 5.68e-1

Table 3: Anisotropic mesh ( $x_2/x_0 = 5, p = 4$ )

	bisect				go-to-one				decrease-by-one																					
	k = 1	k = 2	k = 3	k = 4	k = 5	k = 1	k = 2	k = 3	k = 4	k = 5																				
	#t	#f	#t	#f	#t	#t	#f	#t	#f	#t																				
cg-diagonal-1st-kind-two-sided	34	1.31e+0	18	8.96e-1	13	7.90e-1	9	7.00e-1	66	2.30e+0	35	1.55e+0	26	1.41e+0	22	1.41e+0	19	1.40e+0	20	1.39e+0	15	9.34e-1	11	8.56e-1	9	8.40e-1	8	8.72e-1		
cg-diagonal-4th-kind-two-sided	39	1.50e+0	22	1.10e+0	16	9.81e-1	12	8.75e-1	10	8.44e-1	78	2.72e+0	46	2.04e+0	32	1.73e+0	25	1.60e+0	20	1.48e+0	33	1.55e+0	19	1.38e+0	14	1.09e+0	10	9.34e-1	4	5.71e-1
cg-fm-symm, 1, f, 1st-kind-two-sided	16	7.45e-1	8	5.43e-1	6	5.36e-1	5	5.53e-1	4	5.27e-1	30	1.24e+0	16	9.45e-1	12	9.24e-1	10	9.48e-1	9	9.02e-1	17	9.82e-1	10	8.59e-1	5	6.87e-1	5	7.13e-1	4	6.85e-1
cg-fm-symm, 1, f, 4th-kind-two-sided	19	8.84e-1	11	7.44e-1	7	6.23e-1	6	6.04e-1	5	6.57e-1	36	1.49e+0	21	1.24e+0	14	1.08e+0	11	1.04e+0	9	1.01e+0	17	1.29e+0	8	1.26e+0	5	1.09e+0	4	1.12e+0	4	1.38e+0
cg-fm-symm, 2, f, 1st-kind-two-sided	18	1.07e+0	8	9.43e-1	6	9.83e-1	4	8.11e-1	4	1.02e+0	21	1.31e+0	11	1.11e+0	10	1.39e+0	8	1.43e+0	6	1.36e+0	17	1.57e+0	9	1.30e+0	6	1.31e+0	5	1.40e+0	4	1.37e+0
cg-fm-symm, 2, f, 4th-kind-two-sided	13	8.03e-1	8	7.02e-1	6	7.19e-1	5	7.08e-1	5	7.08e-1	20	1.34e+0	11	9.34e-1	8	9.47e-1	7	1.03e+0	6	1.06e+0	13	1.26e+0	7	9.08e-1	6	9.96e-1	5	1.05e+0	5	1.27e+0
cg-fm-symm, x, f, 1st-kind-two-sided	15	8.73e-1	8	6.22e-1	6	6.19e-1	5	6.13e-1	4	6.44e-1	28	1.46e+0	15	1.07e+0	11	9.73e-1	9	9.70e-1	8	1.02e+0	14	9.33e-1	8	7.80e-1	5	6.72e-1	4	6.96e-1	4	8.34e-1
cg-fm-symm, x, f, 4th-kind-two-sided	19	1.09e+0	10	7.70e-1	8	8.09e-1	6	7.65e-1	6	9.10e-1	29	1.52e+0	18	1.30e+0	12	1.07e+0	10	1.08e+0	8	1.02e+0	15	1.36e+0	9	8.76e-1	7	9.10e-1	6	9.87e-1	5	1.01e+0
gures-fm, point, 1, f, 1st-kind-one-sided	19	1.10e+0	10	7.70e-1	8	8.13e-1	7	7.93e-1	6	8.11e-1	32	1.70e+0	19	1.36e+0	11	9.84e-1	9	9.71e-1	7	9.05e-1	18	1.26e+0	9	8.75e-1	7	9.08e-1	6	9.83e-1	5	1.09e+0
gures-fm, point, 1, f, 4th-kind-one-sided	13	1.06e+0	10	7.89e-1	8	8.11e-1	7	8.84e-1	6	9.11e-1	32	1.70e+0	19	1.36e+0	11	9.84e-1	9	9.71e-1	7	9.05e-1	18	1.26e+0	9	8.75e-1	7	9.08e-1	6	9.83e-1	5	1.09e+0
gures-fm, point, 2, f, 1st-kind-one-sided	21	1.73e+0	9	1.43e+0	9	1.62e+0	7	1.64e+0	6	1.78e+0	18	1.39e+0	12	1.35e+0	9	1.39e+0	7	1.39e+0	6	1.86e+0	15	1.56e+0	11	1.86e+0	7	1.70e+0	6	1.91e+0	6	2.34e+0
gures-fm, point, 2, f, 4th-kind-one-sided	17	1.44e+0	9	1.16e+0	6	1.11e+0	4	1.23e+0	4	1.28e+0	19	1.38e+0	10	1.15e+0	7	1.09e+0	5	1.07e+0	6	1.26e+0	15	1.35e+0	8	1.36e+0	7	1.48e+0	8	1.66e+0	4	1.66e+0
gures-fm, point, x, f, 1st-kind-one-sided	19	1.09e+0	10	7.70e-1	8	8.13e-1	7	7.93e-1	6	8.11e-1	32	1.70e+0	19	1.36e+0	11	9.84e-1	9	9.71e-1	7	9.05e-1	18	1.26e+0	9	8.75e-1	7	9.08e-1	6	9.83e-1	5	1.09e+0
gures-fm, point, x, f, 4th-kind-one-sided	14	1.02e+0	8	8.61e-1	6	8.91e-1	5	8.61e-1	4	8.61e-1	29	1.29e+0	12	1.07e+0	9	1.07e+0	7	1.07e+0	6	1.29e+0	14	8.72e+0	10	1.35e+0	7	1.32e+0	6	1.45e+0	5	1.50e+0
gures-fm, point, x, f, 1st-kind-one-sided	14	1.02e+0	8	8.61e-1	6	8.91e-1	5	8.61e-1	4	8.61e-1	29	1.29e+0	12	1.07e+0	9	1.07e+0	7	1.07e+0	6	1.29e+0	14	8.72e+0	10	1.35e+0	7	1.32e+0	6	1.45e+0	5	1.50e+0
gures-fm, point, x, f, 4th-kind-one-sided	14	1.02e+0	8	8.61e-1	6	8.91e-1	5	8.61e-1	4	8.61e-1	29	1.29e+0	12	1.07e+0	9	1.07e+0	7	1.07e+0	6	1.29e+0	14	8.72e+0	10	1.35e+0	7	1.32e+0	6	1.45e+0	5	1.50e+0

Table 4: Anisotropic mesh ( $x_2/x_0 = 10, p = 4$ )

	bisect					go-to-one					decrease-by-one																			
	k = 1	k = 2	k = 3	k = 4	k = 5	k = 1	k = 2	k = 3	k = 4	k = 5	k = 1	k = 2	k = 3	k = 4	k = 5															
cg-diagonal-1st-kind-two-sided	64	2.47e+0	35	1.74e+0	26	1.60e+0	22	1.60e+0	19	1.62e+0	126	4.39e+0	60	3.05e+0	52	2.81e+0	43	2.74e+0	38	2.70e+0	53	2.47e+0	29	1.80e+0	21	1.63e+0	15	1.63e+0	15	1.63e+0
cg-diagonal-4th-kind-two-sided	75	2.88e+0	44	2.18e+0	33	1.96e+0	24	1.74e+0	20	1.62e+0	153	5.32e+0	66	3.81e+0	56	3.30e+0	48	3.07e+0	39	2.87e+0	52	2.68e+0	36	2.23e+0	25	1.94e+0	20	1.86e+0	20	1.86e+0
cg-fm-symm, 1, f, 1st-kind-two-sided	38	1.39e+0	17	1.15e+0	12	1.07e+0	9	1.05e+0	58	2.85e+0	40	2.55e+0	29	2.22e+0	24	1.84e+0	19	1.70e+0	17	1.91e+0	30	1.73e+0	17	1.46e+0	12	1.37e+0	10	1.42e+0	7	1.19e+0
cg-fm-symm, 1, f, 4th-kind-two-sided	36	1.87e+0	20	1.63e+0	15	1.63e+0	11	1.21e+0	69	2.85e+0	40	2.55e+0	29	2.22e+0	22	2.07e+0	22	2.07e+0	18	2.02e+0	30	1.73e+0	17	1.46e+0	12	1.37e+0	10	1.42e+0	7	1.19e+0
cg-fm-symm, 2, f, 1st-kind-two-sided	36	1.87e+0	20	1.63e+0	15	1.63e+0	11	1.21e+0	69	2.85e+0	40	2.55e+0	29	2.22e+0	22	2.07e+0	22	2.07e+0	18	2.02e+0	30	1.73e+0	17	1.46e+0	12	1.37e+0	10	1.42e+0	7	1.19e+0
cg-fm-symm, 2, f, 4th-kind-two-sided	30	2.14e+0	18	2.12e+0	13	2.12e+0	8	2.09e+0	47	2.93e+0	27	2.72e+0	19	2.65e+0	14	2.48e+0	12	2.48e+0	12	2.60e+0	27	2.03e+0	15	2.32e+0	10	2.17e+0	8	2.25e+0	6	2.06e+0
cg-fm-symm, x, f, 1st-kind-two-sided	30	1.85e+0	18	1.71e+0	12	1.71e+0	10	1.62e+0	46	2.50e+0	27	2.34e+0	20	2.33e+0	15	2.21e+0	12	2.33e+0	24	1.81e+0	24	1.84e+0	15	1.82e+0	10	1.66e+0	9	1.89e+0	8	2.04e+0
cg-fm-symm, x, f, 4th-kind-two-sided	27	1.54e+0	15	1.50e+0	11	1.51e+0	9	1.52e+0	55	2.86e+0	29	2.65e+0	22	1.97e+0	19	2.09e+0	15	1.91e+0	23	1.57e+0	23	1.57e+0	12	1.38e+0	9	1.16e+0	7	1.14e+0	6	1.18e+0
gures-fm, point, 1, f, 1st-kind-one-sided	29	1.07e+0	15	1.20e+0	12	1.22e+0	10	1.24e+0	9	1.33e+0	57	2.97e+0	29	2.67e+0	23	2.06e+0	19	2.10e+0	17	2.05e+0	25	1.76e+0	13	1.28e+0	10	1.28e+0	8	1.28e+0	7	1.30e+0
gures-fm, point, 1, f, 4th-kind-one-sided	34	1.96e+0	20	1.69e+0	13	1.32e+0	10	1.24e+0	9	1.33e+0	57	2.97e+0	29	2.67e+0	23	2.06e+0	19	2.10e+0	17	2.05e+0	25	1.76e+0	13	1.28e+0	10	1.28e+0	8	1.28e+0	7	1.30e+0
gures-fm, point, 2, f, 1st-kind-one-sided	31	1.82e+0	18	1.47e+0	12	1.29e+0	9	1.12e+0	7	1.05e+0	64	3.36e+0	32	2.32e+0	22	1.98e+0	18	2.01e+0	13	1.66e+0	25	1.71e+0	14	1.38e+0	11	1.28e+0	8	1.28e+0	7	1.35e+0
gures-fm, point, 2, f, 4th-kind-one-sided	22	1.80e+0	12	1.55e+0	9	1.62e+0	7	1.64e+0	6	1.74e+0	29	2.10e+0	21	2.28e+0	12	1.83e+0	9	1.76e+0	8	1.91e+0	22	2.27e+0	12	2.02e+0	8	1.91e+0	7	2.19e+0	5	2.00e+0
gures-fm, point, x, f, 1st-kind-one-sided	26	2.13e+0	13	1.69e+0	10	1.70e+0	9	2.06e+0	7	2.00e+0	29	2.11e+0	20	2.29e+0	12	1.84e+0	10	1.95e+0	8	1.92e+0	24	2.47e+0	13	2.10e+0	10	2.37e+0	8	2.47e+0	7	2.68e+0
gures-fm, point, x, f, 4th-kind-one-sided	25	2.04e+0	13	1.68e+0	9	1.69e+0	8	1.85e+0	6	1.74e+0	33	2.42e+0	19	2.17e+0	13	1.98e+0	10	1.94e+0	8	1.92e+0	25	2.57e+0	13	2.18e+0	9	2.14e+0	8	2.18e+0	6	2.33e+0
gures-fm, point, 2, f, 1st-kind-one-sided	20	2.13e+0	13	1.69e+0	10	1.70e+0	9	2.06e+0	8	2.00e+0	31	2.31e+0	18	2.09e+0	12	1.84e+0	9	1.76e+0	8	1.92e+0	25	2.58e+0	13	2.19e+0	9	2.14e+0	8	2.18e+0	7	2.68e+0
gures-fm, point, 2, f, 4th-kind-one-sided	23	1.68e+0	13	1.45e+0	10	1.45e+0	9	1.64e+0	8	1.77e+0	31	2.15e+0	20	2.00e+0	12	1.57e+0	10	1.63e+0	9	1.77e+0	19	1.71e+0	11	1.46e+0	9	1.66e+0	8	1.88e+0	7	2.01e+0
gures-fm, point, x, f, 1st-kind-one-sided	24	1.75e+0	12	1.52e+0	9	1.50e+0	9	1.80e+0	8	1.98e+0	32	2.22e+0	20	2.02e+0	14	1.84e+0	11	1.80e+0	9	1.97e+0	22	1.96e+0	13	1.70e+0	11	2.01e+0	10	2.32e+0	8	2.28e+0
gures-fm, point, x, f, 4th-kind-one-sided	25	1.82e+0	14	1.53e+0	11	1.50e+0	10	1.80e+0	9	1.96e+0	35	2.39e+0	20	2.02e+0	13	1.71e+0	11	1.80e+0	10	1.97e+0	22	1.95e+0	13	1.70e+0	11	2.01e+0	10	2.32e+0	9	2.54e+0



Table 6: Anisotropic mesh ( $x_2/x_0 = 50, p = 4$ )

	bisect			go-to-one			decrease-by-one																							
	k = 1	k = 2	k = 3	k = 4	k = 5	k = 6	k = 7	k = 8	k = 9																					
cg-diagonal,1st-kind-two-sided	269	1,02e+1	148	7,30e+0	111	6,76e+0	91	6,77e+0	78	6,51e+0	423	1,46e+1	236	1,40e+1	201	1,08e+1	156	9,91e+0	125	9,77e+0	212	9,82e+0	117	7,20e+0	86	6,66e+0	71	6,50e+0	63	6,88e+0
cg-diagonal,4th-kind-two-sided	319	1,21e+1	183	9,02e+0	125	7,60e+0	97	7,02e+0	78	6,57e+0	423	1,46e+1	236	1,40e+1	201	1,08e+1	156	9,91e+0	125	9,77e+0	212	9,82e+0	117	7,20e+0	86	6,66e+0	71	6,50e+0	63	6,88e+0
cg-fm,1st-kind-two-sided	122	5,64e+0	68	4,58e+0	51	4,32e+0	43	4,71e+0	38	4,97e+0	103	7,93e+0	79	6,05e+0	67	6,33e+0	59	6,02e+0	80	4,77e+0	47	4,02e+0	34	3,89e+0	24	4,11e+0	20	4,29e+0	24	4,11e+0
cg-fm,1st-kind-two-sided	148	6,86e+0	85	5,74e+0	59	5,25e+0	46	5,04e+0	37	4,89e+0	129	9,42e+0	133	7,82e+0	74	6,98e+0	59	6,02e+0	80	4,77e+0	47	4,02e+0	34	3,89e+0	24	4,11e+0	20	4,29e+0	24	4,11e+0
cg-fm,1st-kind-two-sided	81	5,77e+0	45	5,28e+0	34	5,06e+0	27	5,05e+0	23	5,06e+0	128	9,94e+0	71	7,14e+0	42	7,53e+0	38	8,27e+0	66	6,13e+0	38	5,80e+0	28	6,13e+0	22	6,21e+0	20	6,94e+0	19	6,94e+0
cg-fm,1st-kind-two-sided	96	6,85e+0	56	6,58e+0	40	6,59e+0	30	6,92e+0	26	6,92e+0	151	9,37e+0	86	8,05e+0	49	8,70e+0	39	8,40e+0	78	7,20e+0	45	6,98e+0	33	7,18e+0	25	7,05e+0	19	6,94e+0	19	6,94e+0
cg-fm,1st-kind-two-sided	86	6,28e+0	48	6,55e+0	37	6,55e+0	27	6,77e+0	23	6,77e+0	134	7,33e+0	70	6,04e+0	46	6,76e+0	40	7,00e+0	40	7,00e+0	40	7,00e+0	40	7,00e+0	40	7,00e+0	40	7,00e+0	40	7,00e+0
cg-fm,1st-kind-two-sided	151	8,80e+0	66	5,29e+0	46	4,82e+0	37	4,65e+0	31	4,70e+0	262	1,38e+1	120	8,57e+0	82	7,38e+0	64	7,01e+0	54	6,91e+0	41	4,05e+0	30	3,90e+0	27	3,89e+0	22	4,19e+0	15	4,19e+0
cg-fm,1st-kind-two-sided	139	8,59e+0	65	5,22e+0	45	4,62e+0	38	4,71e+0	32	4,82e+0	253	1,42e+1	121	8,71e+0	82	7,38e+0	64	7,01e+0	54	6,91e+0	41	4,05e+0	30	3,90e+0	27	3,89e+0	22	4,19e+0	15	4,19e+0
cg-fm,1st-kind-two-sided	199	1,16e+1	81	6,51e+0	55	5,64e+0	41	5,11e+0	32	4,82e+0	259	1,37e+1	154	1,10e+1	84	7,93e+0	63	6,90e+0	55	7,04e+0	109	7,17e+0	41	4,02e+0	36	4,68e+0	27	4,31e+0	23	4,31e+0
cg-fm,1st-kind-two-sided	166	9,67e+0	74	4,91e+0	44	4,51e+0	34	4,30e+0	27	4,01e+0	252	1,33e+1	124	8,89e+0	79	6,00e+0	58	6,30e+0	45	6,06e+0	96	6,06e+0	47	4,54e+0	21	4,99e+0	15	4,50e+0	13	4,50e+0
cg-fm,1st-kind-two-sided	71	5,84e+0	34	4,57e+0	25	4,51e+0	22	5,07e+0	19	5,38e+0	115	8,28e+0	58	6,37e+0	39	6,02e+0	31	6,20e+0	52	5,39e+0	59	4,72e+0	20	5,07e+0	17	5,07e+0	14	5,14e+0	14	5,14e+0
cg-fm,1st-kind-two-sided	74	6,13e+0	37	4,85e+0	27	4,86e+0	22	5,06e+0	18	5,13e+0	138	1,01e+1	64	7,31e+0	43	6,62e+0	31	6,20e+0	52	5,39e+0	59	4,72e+0	20	5,07e+0	17	5,07e+0	14	5,14e+0	14	5,14e+0
cg-fm,1st-kind-two-sided	79	5,85e+0	39	4,27e+0	30	4,35e+0	25	4,51e+0	21	4,57e+0	123	8,41e+0	57	5,67e+0	41	5,40e+0	34	5,63e+0	30	5,89e+0	63	5,46e+0	33	4,54e+0	25	4,57e+0	20	4,48e+0	18	5,14e+0
cg-fm,1st-kind-two-sided	76	5,69e+0	40	4,30e+0	30	4,35e+0	26	4,06e+0	22	4,70e+0	123	8,48e+0	58	5,55e+0	42	5,56e+0	35	5,82e+0	30	5,89e+0	63	5,46e+0	33	4,54e+0	25	4,57e+0	20	4,48e+0	18	5,14e+0
cg-fm,1st-kind-two-sided	94	6,94e+0	50	5,50e+0	35	5,10e+0	27	4,86e+0	23	4,90e+0	147	1,01e+1	75	7,49e+0	51	6,76e+0	38	6,24e+0	31	6,21e+0	72	6,40e+0	40	5,43e+0	28	5,13e+0	22	5,10e+0	18	5,14e+0
cg-fm,1st-kind-two-sided	91	6,78e+0	43	4,69e+0	30	4,34e+0	24	4,38e+0	20	4,30e+0	140	9,65e+0	64	6,47e+0	41	5,44e+0	32	5,37e+0	26	5,12e+0	67	5,97e+0	40	5,43e+0	28	5,13e+0	22	5,10e+0	18	5,14e+0

Table 7: Anisotropic mesh ( $x_2/x_0 = 50, p = 7$ )

	bisect			go-to-one			decrease-by-one																							
	k = 1	k = 2	k = 3	k = 4	k = 5	k = 6	k = 7	k = 8	k = 9																					
cg-diagonal,1st-kind-two-sided	393	7,69e+0	184	5,31e+0	135	4,79e+0	110	4,51e+0	105	4,50e+0	550	1,10e+1	300	8,19e+0	222	7,20e+0	185	7,13e+0	161	7,19e+0	190	6,95e+0	107	5,35e+0	75	4,92e+0	59	4,51e+0	52	4,51e+0
cg-diagonal,4th-kind-two-sided	397	8,11e+0	225	6,05e+0	150	5,75e+0	120	5,08e+0	101	4,79e+0	658	1,43e+1	379	1,13e+1	285	8,80e+0	208	8,03e+0	168	7,46e+0	225	8,57e+0	130	6,38e+0	86	5,20e+0	69	4,91e+0	53	4,51e+0
cg-fm,1st-kind-two-sided	123	3,30e+0	66	2,49e+0	52	2,53e+0	42	2,90e+0	36	2,54e+0	202	5,07e+0	112	3,93e+0	83	3,76e+0	67	3,70e+0	60	3,92e+0	61	2,72e+0	36	2,40e+0	26	2,31e+0	18	2,39e+0	18	2,39e+0
cg-fm,1st-kind-two-sided	147	3,94e+0	85	3,21e+0	50	2,87e+0	44	2,62e+0	35	2,46e+0	238	5,99e+0	137	4,31e+0	96	4,33e+0	75	4,13e+0	65	4,24e+0	75	3,35e+0	42	2,81e+0	31	2,76e+0	22	2,44e+0	18	2,82e+0
cg-fm,1st-kind-two-sided	79	3,19e+0	45	2,92e+0	33	2,95e+0	28	3,20e+0	25	3,47e+0	126	4,70e+0	73	4,35e+0	62	4,28e+0	44	4,62e+0	38	4,85e+0	44	3,25e+0	19	2,40e+0	16	2,86e+0	10	2,82e+0	11	3,11e+0
cg-fm,1st-kind-two-sided	94	3,80e+0	52	3,36e+0	39	3,40e+0	31	3,54e+0	25	3,54e+0	152	5,69e+0	87	5,20e+0	62	5,12e+0	49	5,16e+0	39	5,00e+0	44	3,29e+0	23	2,91e+0	17	3,28e+0	14	3,50e+0	12	3,66e+0
cg-fm,1st-kind-two-sided	76	3,47e+0	44	3,30e+0	34	3,54e+0	27	3,60e+0	24	3,90e+0	126	5,66e+0	86	6,11e+0	62	6,12e+0	48	6,08e+0	38	5,88e+0	46	3,68e+0	29	3,55e+0	19	3,66e+0	15	3,74e+0	12	3,66e+0
cg-fm,1st-kind-two-sided	92	4,20e+0	53	3,97e+0	39	4,02e+0	29	3,87e+0	25	4,08e+0	150	6,49e+0	86	6,11e+0	62	6,12e+0	48	6,08e+0	38	5,88e+0	46	3,68e+0	29	3,55e+0	19	3,66e+0	15	3,74e+0	12	3,66e+0
cg-fm,1st-kind-two-sided	135	4,69e+0	65	2,96e+0	45	2,57e+0	37	2,58e+0	31	2,54e+0	356	1,18e+1	138	5,94e+0	89	4,77e+0	69	4,42e+0	56	4,18e+0	57	3,98e+0	31	3,20e+0	20	2,78e+0	17	2,57e+0	17	2,57e+0
cg-fm,1st-kind-two-sided	149	5,02e+0	85	2,96e+0	54	2,57e+0	42	2,87e+0	33	2,67e+0	405	1,61e+1	171	7,34e+0	112	5,90e+0	80	5,14e+0	58	4,33e+0	62	3,93e+0	38	2,80e+0	23	2,27e+0	20	2,48e+0	17	2,56e+0
cg-fm,1st-kind-two-sided	189	6,45e+0	88	4,02e+0	54	3,07e+0	42	2,87e+0	33	2,67e+0	405	1,61e+1	171	7,34e+0	112	5,90e+0	80	5,14e+0	58	4,33e+0	62	3,93e+0	38	2,80e+0	23	2,27e+0	21	2,61e+0	18	2,71e+0
cg-fm,1st-kind-two-sided	173	5,89e+0	71	3,22e+0	45	2,57e+0	33	2,29e+0	27	2,18e+0	406	1,32e+1	142	6,10e+0	87	6,05e+0	61	5,96e+0	41	3,96e+0	42	3,63e+0	69	3,52e+0	23	2,28e+0	18	2,28e+0	13	1,88e+0
cg-fm,1st-kind-two-sided	66	3,11e+0	33	2,43e+0	25	2,45e+0	21	2,64e+0	19	2,93e+0	125	5,53e+0	55	3,69e+0	41	3,72e+0	33	3,84e+0	28	3,86e+0	29	2,38e+0	19	2,65e+0	10	2,48e+0	9	2,75e+0	9	2,75e+0
cg-fm,1st-kind-two-sided	81	3,35e+0	34	2,50e+0	26	2,66e+0	21	2,87e+0	19	2,93e+0	130	5,77e+0	55	3,79e+0	40	3,65e+0	33	3,84e+0	28	3,86e+0	29	2,38e+0	19	2,65e+0	10	2,48e+0	9	2,75e+0	9	2,75e+0
cg-fm,1st-kind-two-sided	80	3,80e+0	40	2,90e+0	29	2,84e+0	23	2,87e+0	19	2,93e+0	155	6,09e+0	75	5,06e+0	49	4,40e+0	37	4,20e+0	30	4,13e+0	35	2,98e+0	20	2,77e+0	11	2,72e+0	9	2,72e+0	7	2,72e+0
cg-fm,1st-kind-two-sided	75	3,57e+0	36	2,64e+0	26	2,56e+0	19	2,41e+0	15	2,28e+0	139	6,20e+0	62	4,25e+0	40	3,60e+0	30	3,44e+0	25	3,38e+0	33	2,98e+0	19	2,64e+0	12	2,29e+0	9	2,29e+0	7	2,29e+0
cg-fm,1st-kind-two-sided	53	2,58e+0	25	2,38e+0	21	2,40e+0	15	2,28e+0	10	2,10e+0	77	4,01e+0	40	3,32e+0	29	3,17e+0	24	3,37e+0	21	3,37e+0	28	2,53e+0	15	2,53e+0	11	2,32e+0	9	2,32e+0	9	2,32e+0
cg-fm,1st-kind-two-sided	64	3,45e+0	34	2,90e+0	24	2,77e+0	19	2,84e+0	15	2,65e+0	97	4,01e+0	40	3,32e+0	29	3,17e+0	24	3,37e+0	21	3,37e+0	28	2,53e+0	15	2,53e+0	11	2,32e+0	9	2,32e+0	7	2,32e+0
cg-fm,1st-kind-two-sided	58	3,13e+0	30	2,57e+0	21	2,46e+0	18	2,72e+0	13	2,50e+0	113	4,20e+0	42	3,40e+0	28	3,09e+0	22	3,10e+0	19	3,29e+0	30	2,70e+0	18	2,77e+0	11	2,32e+0	9	2,48e+0	7	2,47e+0

## 2 Kershaw mesh



Table 8: Kershaw mesh ( $\epsilon = 1.0$ ,  $p = 4$ )

	bisect						go-to-one						decrease-by-one					
	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
	#t	#i	#f	#t	#i	#f	#t	#i	#f	#t	#i	#f	#t	#i	#f	#t	#i	#f
cg-diagonal-1st-kind-two-sided	8	1,48e-1	7	1,40e-1	5	1,32e-1	4	1,25e-1	3	1,09e-1	16	2,35e-1	9	1,79e-1	6	1,38e-1	5	1,37e-1
cg-diagonal-4th-kind-two-sided	5	1,02e-1	5	1,07e-1	4	1,05e-1	4	1,05e-1	3	1,09e-1	19	2,79e-1	11	2,07e-1	8	1,84e-1	5	1,58e-1
cg-fm-symm, 1-f, 1st-kind-two-sided	6	1,22e-1	4	1,19e-1	4	1,15e-1	4	1,19e-1	3	1,72e-1	9	1,61e-1	7	1,79e-1	5	1,67e-1	4	1,94e-1
cg-fm-symm, 1-f, 4th-kind-two-sided	5	1,55e-1	6	3,03e-1	5	3,51e-1	3	2,71e-1	3	3,30e-1	5	1,29e-1	4	1,41e-1	4	1,65e-1	3	1,47e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	6	1,86e-1	3	1,53e-1	4	2,84e-1	3	2,72e-1	3	3,30e-1	8	2,13e-1	6	2,59e-1	4	3,06e-1	3	3,72e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	5	1,55e-1	7	2,88e-1	5	2,78e-1	4	2,78e-1	3	2,53e-1	7	1,46e-1	4	2,00e-1	4	2,63e-1	3	2,80e-1
cg-fm-symm, x, 1st-kind-two-sided	4	9,62e-2	7	2,35e-1	4	2,25e-1	4	2,30e-1	3	2,19e-1	7	1,42e-1	4	1,96e-1	3	2,31e-1	4	2,35e-1
cg-fm-symm, x, 4th-kind-one-sided	11	2,59e-1	7	2,32e-1	6	2,62e-1	5	2,76e-1	4	2,72e-1	8	2,32e-1	5	2,38e-1	4	1,90e-1	3	2,32e-1
cg-fm-symm, x, 4th-kind-two-sided	7	1,67e-1	4	1,41e-1	4	1,85e-1	3	1,83e-1	3	1,28e-1	8	1,63e-1	4	1,60e-1	4	1,97e-1	3	1,84e-1
cg-fm-symm, 2-f, 1st-kind-one-sided	6	2,07e-1	9	4,97e-1	5	5,46e-1	5	5,21e-1	4	5,27e-1	9	1,87e-1	7	4,63e-1	5	5,28e-1	5	5,28e-1
cg-fm-symm, 2-f, 4th-kind-one-sided	13	4,51e-1	9	4,98e-1	8	6,17e-1	6	6,10e-1	5	6,35e-1	10	4,74e-1	7	4,66e-1	6	5,10e-1	5	5,37e-1
cg-fm-symm, x, 1st-kind-one-sided	5	1,75e-1	4	2,40e-1	4	2,48e-1	4	2,42e-1	3	2,42e-1	7	2,57e-1	4	2,60e-1	4	2,84e-1	4	2,48e-1
cg-fm-symm, x, 4th-kind-one-sided	6	1,53e-1	8	3,08e-1	6	3,89e-1	5	4,07e-1	4	4,07e-1	8	3,34e-1	6	3,92e-1	5	3,68e-1	5	3,68e-1
cg-fm-symm, x, 4th-kind-two-sided	12	3,69e-1	9	4,13e-1	7	4,37e-1	6	4,77e-1	5	4,96e-1	9	3,77e-1	8	3,96e-1	5	3,72e-1	5	4,52e-1
cg-fm-symm, x, 4th-kind-one-sided	5	1,54e-1	4	1,96e-1	4	2,68e-1	3	2,69e-1	3	3,25e-1	4	1,78e-1	4	1,78e-1	4	2,54e-1	4	2,54e-1
cg-fm-symm, x, 4th-kind-one-sided	7	2,12e-1	7	3,26e-1	6	3,78e-1	6	4,77e-1	6	5,73e-1	7	1,67e-1	6	2,95e-1	6	5,20e-1	6	5,20e-1

Table 9: Kershaw mesh ( $\epsilon = 0.99$ ,  $p = 4$ )

	bisect						go-to-one						decrease-by-one					
	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
	#t	#i	#f	#t	#i	#f	#t	#i	#f	#t	#i	#f	#t	#i	#f	#t	#i	#f
cg-diagonal-1st-kind-two-sided	8	1,03e-1	7	2,44e-1	5	2,39e-1	4	2,28e-1	3	2,03e-1	16	3,33e-1	9	2,66e-1	5	2,37e-1	5	2,85e-1
cg-diagonal-4th-kind-two-sided	5	2,17e-1	5	1,70e-1	4	1,58e-1	3	1,70e-1	3	2,03e-1	19	3,96e-1	11	3,26e-1	6	2,87e-1	5	2,85e-1
cg-fm-symm, 1-f, 1st-kind-two-sided	6	1,71e-1	4	1,77e-1	4	2,39e-1	3	2,23e-1	3	2,09e-1	9	2,21e-1	7	1,87e-1	4	2,46e-1	3	2,22e-1
cg-fm-symm, 1-f, 4th-kind-two-sided	5	1,96e-1	6	3,91e-1	5	4,55e-1	3	3,50e-1	3	4,29e-1	8	2,43e-1	6	3,78e-1	4	3,80e-1	4	4,76e-1
cg-fm-symm, 2-f, 1st-kind-two-sided	6	2,35e-1	3	1,96e-1	4	3,67e-1	3	3,51e-1	3	4,29e-1	4	2,19e-1	4	3,94e-1	3	3,54e-1	3	3,54e-1
cg-fm-symm, 2-f, 4th-kind-two-sided	5	1,81e-1	7	3,95e-1	5	3,86e-1	4	3,90e-1	3	3,51e-1	6	1,94e-1	8	3,99e-1	4	3,30e-1	4	4,12e-1
cg-fm-symm, x, 1st-kind-two-sided	5	1,52e-1	4	2,28e-1	4	3,11e-1	3	2,93e-1	3	3,54e-1	7	2,27e-1	4	2,71e-1	3	2,56e-1	3	3,07e-1
cg-fm-symm, x, 4th-kind-one-sided	4	1,27e-1	7	3,21e-1	5	3,29e-1	4	3,38e-1	3	3,21e-1	8	3,10e-1	4	2,00e-1	4	2,77e-1	4	3,33e-1
cg-fm-symm, x, 4th-kind-one-sided	11	3,53e-1	8	3,02e-1	6	3,73e-1	5	4,01e-1	4	4,02e-1	9	3,42e-1	6	3,15e-1	5	3,30e-1	4	3,34e-1
cg-fm-symm, x, 4th-kind-one-sided	7	2,17e-1	6	2,70e-1	5	3,29e-1	4	4,02e-1	5	4,83e-1	8	2,67e-1	7	3,10e-1	5	3,30e-1	5	4,02e-1
cg-fm-symm, x, 4th-kind-one-sided	6	2,53e-1	9	6,12e-1	7	6,70e-1	5	6,48e-1	4	6,59e-1	7	2,73e-1	6	3,15e-1	6	3,90e-1	5	4,01e-1
cg-fm-symm, 2-f, 1st-kind-one-sided	13	5,42e-1	9	6,13e-1	8	7,64e-1	6	7,93e-1	5	7,93e-1	10	5,73e-1	7	5,66e-1	6	5,37e-1	5	5,45e-1
cg-fm-symm, 2-f, 4th-kind-one-sided	5	2,15e-1	4	2,98e-1	4	4,13e-1	4	5,00e-1	3	5,27e-1	4	2,49e-1	4	3,88e-1	4	4,46e-1	4	5,45e-1
cg-fm-symm, x, 1st-kind-one-sided	8	3,31e-1	7	4,84e-1	7	6,70e-1	6	7,57e-1	7	1,00e-1	7	4,08e-1	7	5,66e-1	7	7,23e-1	7	8,80e-1
cg-fm-symm, x, 4th-kind-one-sided	12	4,63e-1	9	5,33e-1	8	4,70e-1	6	5,01e-1	5	5,37e-1	4	5,30e-1	5	4,44e-1	5	4,40e-1	4	4,72e-1
cg-fm-symm, x, 4th-kind-one-sided	4	2,55e-1	4	3,44e-1	4	4,44e-1	4	5,29e-1	5	6,40e-1	7	5,75e-1	8	5,03e-1	7	5,03e-1	7	5,03e-1
cg-fm-symm, x, 4th-kind-one-sided	7	2,71e-1	7	4,20e-1	6	4,07e-1	6	6,28e-1	6	7,57e-1	7	3,72e-1	6	4,38e-1	6	5,51e-1	6	6,66e-1

Table 10: Kershaw mesh ( $\epsilon = 0.9, p = 4$ )

	bisect					go-to-one					decrease-by-one									
	k = 1	k = 2	#t	#f	#i	k = 4	k = 5	k = 1	k = 2	#t	#f	#i	k = 4	k = 5						
cg-diagonal-1st-kind-two-sided	12	2.01e+1	7	2.44e+1	6	2.79e+1	5	2.86e+1	4	2.71e+1	22	4.53e+1	12	3.36e+1	6	2.83e+1	4	2.81e+1	4	3.39e+1
cg-diagonal-4th-kind-two-sided	14	3.37e+1	8	2.82e+1	6	2.79e+1	5	2.86e+1	4	2.71e+1	26	4.53e+1	15	3.36e+1	6	2.83e+1	4	2.81e+1	4	3.39e+1
cg-fm-symm-1-f-1st-kind-two-sided	8	2.28e+1	7	3.12e+1	5	2.95e+1	4	2.98e+1	4	3.03e+1	11	2.69e+1	8	2.99e+1	6	3.00e+1	5	3.12e+1	4	2.83e+1
cg-fm-symm-1-f-4th-kind-two-sided	9	2.85e+1	6	2.68e+1	4	2.37e+1	4	3.00e+1	4	3.03e+1	14	3.43e+1	8	2.99e+1	5	2.70e+1	4	2.98e+1	4	3.38e+1
cg-fm-symm-2-f-1st-kind-two-sided	9	3.58e+1	5	3.39e+1	4	4.02e+1	4	4.72e+1	4	5.82e+1	14	3.71e+1	8	4.03e+1	4	4.78e+1	4	5.49e+1	4	6.02e+1
cg-fm-symm-2-f-4th-kind-two-sided	11	4.37e+1	6	3.98e+1	4	4.72e+1	4	4.75e+1	4	5.81e+1	14	4.71e+1	8	4.44e+1	5	4.96e+1	4	5.49e+1	4	6.23e+1
cg-fm-symm-2-f-1st-kind-one-sided	8	2.03e+1	7	3.08e+1	5	2.88e+1	4	3.39e+1	4	4.78e+1	9	3.58e+1	6	3.16e+1	5	4.06e+1	4	4.88e+1	4	5.96e+1
cg-fm-symm-2-f-4th-kind-one-sided	10	2.85e+1	6	2.68e+1	4	2.37e+1	4	3.00e+1	4	3.03e+1	14	3.43e+1	8	2.99e+1	5	2.70e+1	4	2.98e+1	4	3.38e+1
cg-fm-symm-2-f-1st-kind-one-sided	8	2.43e+1	7	3.23e+1	5	3.23e+1	4	3.36e+1	4	4.05e+1	11	2.97e+1	6	3.19e+1	5	3.30e+1	4	3.38e+1	4	4.29e+1
cg-fm-symm-2-f-4th-kind-one-sided	11	3.37e+1	9	4.12e+1	7	4.36e+1	4	4.75e+1	5	4.90e+1	12	3.25e+1	9	3.53e+1	6	3.98e+1	4	4.77e+1	5	5.16e+1
cg-fm-symm-2-f-1st-kind-one-sided	10	3.06e+1	5	2.40e+1	4	2.68e+1	4	3.37e+1	4	4.08e+1	13	3.57e+1	7	2.77e+1	5	2.73e+1	5	3.30e+1	4	3.39e+1
cg-fm-symm-2-f-4th-kind-one-sided	11	3.37e+1	7	3.24e+1	4	4.35e+1	6	4.75e+1	4	5.72e+1	13	3.56e+1	8	3.75e+1	7	3.68e+1	6	4.78e+1	5	6.05e+1
cg-fm-symm-2-f-1st-kind-one-sided	8	3.37e+1	8	5.57e+1	6	5.99e+1	5	6.58e+1	4	6.70e+1	9	3.24e+1	8	4.99e+1	5	5.45e+1	4	5.55e+1	6	7.81e+1
cg-fm-symm-2-f-4th-kind-one-sided	12	5.06e+1	9	6.20e+1	7	6.88e+1	5	6.59e+1	5	8.05e+1	10	3.62e+1	9	5.26e+1	7	5.75e+1	6	6.98e+1	11	7.89e+1
cg-fm-symm-2-f-1st-kind-one-sided	10	4.19e+1	5	3.68e+1	4	4.25e+1	4	4.28e+1	4	6.68e+1	11	3.98e+1	7	4.27e+1	4	4.58e+1	5	5.34e+1	8	6.67e+1
cg-fm-symm-2-f-4th-kind-one-sided	12	5.06e+1	9	6.20e+1	7	6.88e+1	5	6.59e+1	5	8.05e+1	10	3.62e+1	9	5.26e+1	7	5.75e+1	6	6.98e+1	11	7.89e+1
cg-fm-symm-2-f-1st-kind-one-sided	8	3.42e+1	8	4.82e+1	6	5.05e+1	5	5.46e+1	4	5.69e+1	9	3.17e+1	6	4.81e+1	5	4.80e+1	5	5.79e+1	7	7.31e+1
cg-fm-symm-2-f-4th-kind-one-sided	12	4.71e+1	9	5.40e+1	7	5.79e+1	6	6.38e+1	4	6.93e+1	10	3.53e+1	9	5.34e+1	7	5.11e+1	6	6.77e+1	12	5.57e+1
cg-fm-symm-2-f-1st-kind-one-sided	10	3.89e+1	5	3.16e+1	4	3.58e+1	4	4.52e+1	4	5.40e+1	11	3.87e+1	6	3.29e+1	5	3.79e+1	4	4.81e+1	9	4.25e+1
cg-fm-symm-2-f-4th-kind-one-sided	10	3.90e+1	7	4.27e+1	4	5.81e+1	7	7.31e+1	4	8.83e+1	11	3.90e+1	8	4.31e+1	7	5.33e+1	5	5.96e+1	4	7.02e+1
cg-fm-symm-2-f-1st-kind-one-sided	10	3.90e+1	7	4.27e+1	4	5.81e+1	7	7.31e+1	4	8.83e+1	11	3.90e+1	8	4.31e+1	7	5.33e+1	5	5.96e+1	4	7.02e+1
cg-fm-symm-2-f-4th-kind-one-sided	10	3.90e+1	7	4.27e+1	4	5.81e+1	7	7.31e+1	4	8.83e+1	11	3.90e+1	8	4.31e+1	7	5.33e+1	5	5.96e+1	4	7.02e+1

Table 11: Kershaw mesh ( $\epsilon = 0.7, p = 4$ )

	bisect					go-to-one					decrease-by-one					
	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	
cg-diagonal-1st-kind-two-sided	45	1.12e+0	25	9.10e-1	10	9.20e-1	16	1.42e+0	34	1.39e+0	25	1.51e+0	38	1.18e+0	13	9.68e-1
cg-diagonal-4th-kind-two-sided	55	1.38e+0	31	1.14e+0	22	1.07e+0	22	1.07e+0	15	9.49e-1	101	2.11e+0	58	1.60e+0	32	1.67e+0
cg-fm-symm-1-f-1st-kind-two-sided	30	8.31e-1	18	8.28e-1	13	8.11e-1	10	7.84e-1	10	9.53e-1	48	1.22e+0	28	1.10e+0	20	1.06e+0
cg-fm-symm-1-f-4th-kind-two-sided	25	1.01e+0	14	9.47e-1	11	1.04e+0	10	1.22e+0	9	1.34e+0	39	1.14e+0	23	1.02e+0	14	1.11e+0
cg-fm-symm-2-f-1st-kind-two-sided	21	7.91e-1	13	7.72e-1	11	8.88e-1	11	1.35e+0	9	1.34e+0	39	1.35e+0	23	1.31e+0	16	1.27e+0
cg-fm-symm-2-f-4th-kind-two-sided	25	9.43e-1	15	8.92e-1	12	9.73e-1	10	1.02e+0	9	1.10e+0	29	9.73e-1	17	8.90e-1	14	9.93e-1
cg-fm-symm-2-f-1st-kind-one-sided	25	7.93e-1	14	6.59e-1	12	6.89e-1	10	7.71e-1	9	8.35e-1	40	1.13e+0	23	9.39e-1	13	9.84e-1
cg-fm-symm-2-f-4th-kind-one-sided	26	8.24e-1	19	9.15e-1	14	8.79e-1	12	9.27e-1	11	1.02e+0	40	1.33e+0	23	9.37e-1	14	9.85e-1
cg-fm-symm-2-f-1st-kind-one-sided	28	8.93e-1	19	9.17e-1	14	8.72e-1	12	9.27e-1	10	9.36e-1	43	1.22e+0	24	9.78e-1	18	9.89e-1
cg-fm-symm-2-f-4th-kind-one-sided	23	9.86e-1	12	8.59e-1	10	9.02e-1	9	1.12e+0	8	1.22e+0	28	1.04e+0	10	1.14e+0	11	1.13e+0
cg-fm-symm-2-f-1st-kind-one-sided	25	1.07e+0	15	1.04e+0	12	1.15e+0	10	1.23e+0	9	1.34e+0	39	1.35e+0	23	1.31e+0	16	1.27e+0
cg-fm-symm-2-f-4th-kind-one-sided	26	1.11e+0	15	1.04e+0	12	1.15e+0	10	1.23e+0	9	1.34e+0	39	1.35e+0	23	1.31e+0	16	1.27e+0
cg-fm-symm-2-f-1st-kind-one-sided	24	9.58e-1	14	8.45e-1	12	9.75e-1	10	1.02e+0	9	1.10e+0	30	1.09e+0	20	1.06e+0	14	1.13e+0
cg-fm-symm-2-f-4th-kind-one-sided	27	1.03e+0	20	1.23e+0	14	1.34e+0	12	1.23e+0	11	1.38e+0	32	1.16e+0	14	1.15e+0	13	1.08e+0
cg-fm-symm-2-f-1st-kind-one-sided	27	1.03e+0	20	1.23e+0	14	1.34e+0	12	1.23e+0	11	1.38e+0	32	1.16e+0	14	1.15e+0	13	1.08e+0
cg-fm-symm-2-f-4th-kind-one-sided	26	1.01e+0	19	1.18e+0	14	1.14e+0	13	1.32e+0	12	1.40e+0	34	1.25e+0	21	1.11e+0	15	1.13e+0

Table 12: Kershaw mesh ( $\epsilon = 0.5, p = 4$ )

	bisect				go-to-one				decrease-by-one			
	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 1$	$k = 2$
	#t	#t	#t	#t	#t	#t	#t	#t	#t	#t	#t	#t
cg-diagonal-1st-kind-two-sided	107	2.65e+0	59	2.16e+0	36	2.19e+0	32	2.35e+0	199	4.27e+0	108	3.34e+0
cg-diagonal-4th-kind-two-sided	131	3.25e+0	73	2.68e+0	52	2.54e+0	43	2.44e+0	136	4.22e+0	136	4.22e+0
cg-film-symm_1.f-1st-kind-two-sided	55	1.62e+0	30	1.39e+0	23	1.44e+0	18	1.78e+0	94	2.40e+0	51	2.06e+0
cg-film-symm_1.f-4th-kind-two-sided	66	1.95e+0	38	1.76e+0	27	1.70e+0	18	1.78e+0	114	2.92e+0	65	2.56e+0
cg-film-symm_2.f-1st-kind-two-sided	52	2.11e+0	29	1.97e+0	22	2.10e+0	19	2.33e+0	16	2.40e+0	72	2.48e+0
cg-film-symm_2.f-4th-kind-two-sided	63	2.96e+0	36	2.47e+0	26	2.40e+0	21	2.58e+0	17	2.59e+0	39	2.22e+0
cg-film-symm_2.f-1st-kind-two-sided	47	1.77e+0	28	1.67e+0	22	1.78e+0	19	1.95e+0	18	2.28e+0	38	1.96e+0
cg-film-symm_2.f-4th-kind-two-sided	54	2.54e+0	33	2.14e+0	25	2.16e+0	21	2.36e+0	15	2.40e+0	31	2.10e+0
cg-film-post_1.f-1st-kind-two-sided	53	1.71e+0	29	1.59e+0	23	1.66e+0	18	1.78e+0	104	2.98e+0	40	2.04e+0
cg-film-post_1.f-4th-kind-one-sided	68	2.20e+0	36	1.72e+0	26	1.64e+0	21	1.67e+0	110	3.15e+0	63	2.63e+0
cg-film-post_1.f-4th-kind-one-sided	65	2.11e+0	32	1.56e+0	24	1.52e+0	20	1.92e+0	132	3.70e+0	54	2.23e+0
cg-film-post_2.f-1st-kind-two-sided	45	1.95e+0	27	1.76e+0	20	1.98e+0	14	2.20e+0	65	2.44e+0	34	2.04e+0
cg-film-post_2.f-4th-kind-two-sided	53	2.30e+0	27	1.90e+0	22	2.16e+0	19	2.42e+0	68	2.55e+0	34	2.05e+0
cg-film-post_2.f-1st-kind-two-sided	57	2.47e+0	30	2.10e+0	22	2.15e+0	19	2.41e+0	83	3.11e+0	42	2.48e+0
cg-film-post_2.f-4th-kind-two-sided	49	2.00e+0	30	1.82e+0	22	2.01e+0	19	2.31e+0	71	2.60e+0	41	2.25e+0
cg-film-post_2.f-1st-kind-two-sided	55	2.22e+0	32	2.01e+0	26	2.15e+0	23	2.42e+0	20	2.50e+0	78	2.86e+0
cg-film-post_2.f-4th-kind-one-sided	57	2.31e+0	34	2.12e+0	25	2.07e+0	21	2.20e+0	82	3.01e+0	45	2.47e+0
cg-film-post_2.f-4th-kind-one-sided	57	2.31e+0	33	2.06e+0	26	2.15e+0	24	2.22e+0	81	3.09e+0	42	2.30e+0

Table 13: Kershaw mesh ( $\epsilon = 0.3, p = 4$ )

	bisect						go-by-one						decrease-by-one					
	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
cg-diagonal-f-4th-kind-two-sided	200	4.51e+0	103	3.56e+0	75	3.11e+0	62	3.90e+0	55	3.72e+0	353	7.33e+0	190	5.90e+0	140	5.37e+0	102	5.70e+0
cg-diagonal-f-4th-kind-two-sided	231	5.50e+0	129	4.48e+0	90	4.11e+0	70	3.97e+0	57	3.86e+0	429	8.92e+0	238	7.05e+0	168	6.48e+0	130	6.18e+0
cg-fm-symm,1-f-1st-kind-two-sided	96	2.75e+0	51	2.47e+0	38	2.29e+0	28	2.56e+0	168	4.17e+0	88	3.32e+0	85	3.20e+0	64	3.42e+0	54	3.87e+0
cg-fm-symm,1-f-4th-kind-two-sided	116	3.31e+0	65	2.89e+0	46	2.77e+0	36	2.72e+0	30	2.73e+0	204	5.06e+0	113	4.27e+0	79	4.00e+0	50	3.82e+0
cg-fm-symm,2-f-1st-kind-two-sided	89	3.53e+0	47	3.11e+0	36	3.36e+0	27	3.59e+0	124	4.18e+0	66	4.66e+0	49	3.76e+0	41	4.06e+0	36	4.36e+0
cg-fm-symm,2-f-4th-kind-two-sided	107	4.26e+0	60	3.98e+0	43	4.00e+0	34	4.06e+0	28	4.08e+0	194	5.06e+0	84	4.06e+0	46	4.50e+0	38	4.59e+0
cg-fm-symm,3-f-1st-kind-two-sided	85	3.10e+0	49	3.52e+0	37	3.48e+0	29	3.48e+0	20	3.48e+0	126	4.10e+0	57	3.53e+0	41	3.83e+0	40	4.21e+0
cg-fm-symm,3-f-4th-kind-two-sided	121	4.87e+0	62	4.60e+0	46	4.52e+0	32	4.31e+0	27	4.30e+0	158	6.16e+0	92	4.70e+0	52	4.38e+0	44	4.22e+0
gures-fm,post,1-f-1st-kind-one-sided	102	3.22e+0	48	2.96e+0	36	2.82e+0	30	2.98e+0	27	2.95e+0	178	4.13e+0	68	3.58e+0	44	3.84e+0	32	4.11e+0
gures-fm,post,1-f-1st-kind-one-sided	112	3.54e+0	51	3.38e+0	37	3.25e+0	34	2.93e+0	28	2.91e+0	224	6.32e+0	102	4.43e+0	68	3.90e+0	47	3.60e+0
gures-fm,post,1-f-4th-kind-one-sided	137	4.48e+0	64	4.30e+0	43	4.16e+0	39	4.06e+0	26	3.98e+0	241	6.84e+0	111	4.50e+0	62	4.15e+0	49	3.82e+0
gures-fm,post,2-f-1st-kind-one-sided	101	3.36e+0	47	2.77e+0	30	2.86e+0	27	3.26e+0	24	3.57e+0	133	4.95e+0	60	3.53e+0	43	3.45e+0	36	3.71e+0
gures-fm,post,2-f-1st-kind-one-sided	114	4.87e+0	62	4.60e+0	46	4.52e+0	32	4.31e+0	27	4.30e+0	158	6.16e+0	92	4.70e+0	52	4.38e+0	44	4.22e+0
gures-fm,post,2-f-4th-kind-one-sided	137	4.48e+0	64	4.30e+0	43	4.16e+0	39	4.06e+0	26	3.98e+0	241	6.84e+0	111	4.50e+0	62	4.15e+0	49	3.82e+0
gures-fm,post,3-f-1st-kind-one-sided	110	4.30e+0	59	3.75e+0	42	3.61e+0	34	3.90e+0	29	3.55e+0	157	5.69e+0	73	3.99e+0	43	3.88e+0	38	4.17e+0
gures-fm,post,3-f-1st-kind-one-sided	115	4.60e+0	59	3.75e+0	42	3.61e+0	34	3.90e+0	29	3.55e+0	157	5.69e+0	73	3.99e+0	43	3.88e+0	38	4.17e+0
gures-fm,post,3-f-4th-kind-one-sided	133	5.31e+0	64	3.90e+0	44	3.57e+0	35	3.50e+0	39	3.50e+0	209	7.57e+0	97	5.16e+0	61	4.45e+0	40	4.22e+0
gures-fm,post,3-f-4th-kind-one-sided	123	4.93e+0	60	3.63e+0	44	3.56e+0	36	3.68e+0	33	3.68e+0	234	7.11e+0	95	5.28e+0	59	4.25e+0	45	4.05e+0

Table 14: Kershaw mesh ( $\epsilon = 0.3, p = 7$ )

	bisect						go-by-one						decrease-by-one					
	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$
cg-diagonal-f-4th-kind-two-sided	286	3.21e+0	158	2.37e+0	115	2.17e+0	91	2.15e+0	57	2.15e+0	573	5.94e+0	315	4.31e+0	201	3.95e+0	100	3.88e+0
cg-diagonal-f-4th-kind-two-sided	333	3.85e+0	199	2.99e+0	139	2.62e+0	106	2.41e+0	85	2.45e+0	686	7.11e+0	396	5.11e+0	281	4.79e+0	174	4.70e+0
cg-fm-symm,1-f-1st-kind-two-sided	135	1.81e+0	72	1.41e+0	53	1.30e+0	44	1.30e+0	39	1.40e+0	266	3.27e+0	144	2.57e+0	107	2.40e+0	88	2.33e+0
cg-fm-symm,1-f-4th-kind-two-sided	163	2.18e+0	91	1.78e+0	64	1.66e+0	49	1.55e+0	41	1.51e+0	323	3.97e+0	180	3.21e+0	129	3.00e+0	99	2.84e+0
cg-fm-symm,2-f-1st-kind-two-sided	96	1.85e+0	52	1.62e+0	39	1.67e+0	30	1.93e+0	20	1.98e+0	160	2.80e+0	87	2.45e+0	64	2.40e+0	53	2.62e+0
cg-fm-symm,2-f-4th-kind-two-sided	117	2.60e+0	66	2.05e+0	47	2.02e+0	36	1.97e+0	30	1.90e+0	195	3.42e+0	109	3.07e+0	78	3.03e+0	59	2.93e+0
cg-fm-symm,3-f-1st-kind-two-sided	140	3.10e+0	79	2.96e+0	46	2.80e+0	38	2.52e+0	34	2.70e+0	202	4.27e+0	107	3.75e+0	79	3.86e+0	65	4.08e+0
cg-fm-symm,3-f-4th-kind-two-sided	179	3.98e+0	79	2.96e+0	46	2.80e+0	38	2.52e+0	34	2.70e+0	202	4.27e+0	107	3.75e+0	79	3.86e+0	65	4.08e+0
gures-fm,post,1-f-1st-kind-one-sided	159	2.40e+0	79	1.75e+0	54	1.82e+0	44	1.57e+0	30	1.47e+0	185	3.76e+0	136	3.16e+0	96	2.71e+0	73	4.50e+0
gures-fm,post,1-f-1st-kind-one-sided	179	2.40e+0	79	1.75e+0	54	1.82e+0	44	1.57e+0	30	1.47e+0	185	3.76e+0	136	3.16e+0	96	2.71e+0	73	4.50e+0
gures-fm,post,1-f-4th-kind-one-sided	207	3.37e+0	97	2.18e+0	61	1.76e+0	46	1.61e+0	37	1.57e+0	—	—	—	—	—	—	—	—
gures-fm,post,2-f-1st-kind-one-sided	178	2.88e+0	85	1.80e+0	54	1.92e+0	46	1.44e+0	35	1.42e+0	—	—	—	—	—	—	—	—
gures-fm,post,2-f-1st-kind-one-sided	101	2.22e+0	45	1.53e+0	33	1.54e+0	27	1.57e+0	24	1.60e+0	169	3.43e+0	82	2.55e+0	54	2.27e+0	44	2.32e+0
gures-fm,post,2-f-4th-kind-one-sided	120	2.42e+0	49	1.68e+0	39	1.67e+0	30	1.74e+0	26	1.83e+0	221	4.40e+0	96	3.01e+0	56	2.36e+0	40	2.48e+0
gures-fm,post,2-f-4th-kind-one-sided	124	2.76e+0	55	1.87e+0	39	1.80e+0	29	1.62e+0	24	1.60e+0	276	5.60e+0	112	3.48e+0	56	2.60e+0	40	2.60e+0
gures-fm,post,3-f-1st-kind-one-sided	208	3.80e+0	71	2.40e+0	43	2.07e+0	31	2.06e+0	31	2.06e+0	—	—	—	—	—	—	—	—
gures-fm,post,3-f-1st-kind-one-sided	284	7.35e+0	93	3.86e+0	56	3.15e+0	41	2.93e+0	39	3.04e+0	—	—	—	—	—	—	—	—
gures-fm,post,3-f-4th-kind-one-sided	231	6.00e+0	89	3.66e+0	56	3.15e+0	39	2.79e+0	30	2.59e+0	—	—	—	—	—	—	—	—

### 3 3D ball

Table 15: 3D ball ( $p = 4$ )

	biset						go-to-one						decrease-by-one					
	$k = 1$		$k = 2$		$k = 3$		$k = 4$		$k = 5$		$k = 1$		$k = 2$		$k = 3$		$k = 4$	
	#i	$t[s]$	#i	$t[s]$	#i	$t[s]$	#i	$t[s]$	#i	$t[s]$	#i	$t[s]$	#i	$t[s]$	#i	$t[s]$	#i	$t[s]$
cg-diagonal-1a-kind-two-sided	9	2.80e-1	7	3.15e-1	5	2.80e-1	4	3.30e-1	17	4.06e-1	10	3.90e-1	7	3.55e-1	6	3.70e-1	5	3.71e-1
cg-diagonal-4b-kind-two-sided	12	3.86e-1	6	2.69e-1	5	2.87e-1	4	2.80e-1	4	3.20e-1	21	6.12e-1	12	4.78e-1	8	3.68e-1	5	3.61e-1
cg-fm-symm-1-f-1st-kind-two-sided	6	2.69e-1	7	3.92e-1	5	3.71e-1	4	3.08e-1	4	4.40e-1	9	3.02e-1	8	3.92e-1	5	3.23e-1	4	3.20e-1
cg-fm-symm-1-f-4th-kind-two-sided	7	2.64e-1	4	2.25e-1	4	2.97e-1	3	2.77e-1	3	3.31e-1	10	3.30e-1	6	2.95e-1	4	3.20e-1	4	3.82e-1
gures-fm-pow-1-f-1st-kind-one-sided	6	2.69e-1	7	4.45e-1	5	4.37e-1	4	4.51e-1	4	5.41e-1	8	3.08e-1	8	4.46e-1	4	4.48e-1	4	4.69e-1
gures-fm-pow-1-f-1st-kind-one-sided	11	4.69e-1	9	5.61e-1	7	5.84e-1	5	5.30e-1	5	6.46e-1	7	5.12e-1	6	5.92e-1	5	5.63e-1	12	6.25e-1
gures-fm-pow-1-f-4th-kind-one-sided	7	3.06e-1	4	2.73e-1	4	3.62e-1	3	3.68e-1	3	4.31e-1	10	3.86e-1	6	3.42e-1	4	3.93e-1	4	4.69e-1
gures-fm-pow-1-f-4th-kind-one-sided	9	5.61e-1	7	4.41e-1	6	5.67e-1	6	6.28e-1	6	7.06e-1	11	4.22e-1	8	4.42e-1	7	5.11e-1	6	6.38e-1

Table 16: 3D ball ( $p = 7$ )

	biset						go-to-one						decrease-by-one					
	$k = 1$		$k = 2$		$k = 3$		$k = 4$		$k = 5$		$k = 1$		$k = 2$		$k = 3$		$k = 4$	
	#i	$t[s]$	#i	$t[s]$	#i	$t[s]$	#i	$t[s]$	#i	$t[s]$	#i	$t[s]$	#i	$t[s]$	#i	$t[s]$	#i	$t[s]$
cg-diagonal-1a-kind-two-sided	16	2.79e-1	9	2.10e-1	7	2.06e-1	6	2.12e-1	5	2.06e-1	37	6.06e-1	21	4.61e-1	15	4.13e-1	12	3.97e-1
cg-diagonal-4b-kind-two-sided	19	3.32e-1	10	2.34e-1	7	2.06e-1	6	2.12e-1	5	2.06e-1	44	7.20e-1	25	5.47e-1	18	4.94e-1	14	4.62e-1
cg-fm-symm-1-f-1st-kind-two-sided	8	1.67e-1	8	2.30e-1	5	1.87e-1	4	1.84e-1	4	2.18e-1	14	2.60e-1	8	2.13e-1	6	2.07e-1	5	2.13e-1
cg-fm-symm-1-f-4th-kind-two-sided	10	2.00e-1	5	1.44e-1	4	1.49e-1	4	1.84e-1	3	1.63e-1	17	3.10e-1	10	2.66e-1	7	2.42e-1	5	2.13e-1
gures-fm-pow-1-f-1st-kind-one-sided	7	1.62e-1	8	2.62e-1	5	2.21e-1	4	2.26e-1	4	2.67e-1	13	2.88e-1	8	2.43e-1	6	2.41e-1	5	2.32e-1
gures-fm-pow-1-f-1st-kind-one-sided	12	2.68e-1	9	2.90e-1	6	2.65e-1	4	2.66e-1	3	2.66e-1	15	3.38e-1	9	2.72e-1	6	2.70e-1	6	2.69e-1
gures-fm-pow-1-f-4th-kind-one-sided	11	2.68e-1	6	1.87e-1	4	1.82e-1	3	2.06e-1	3	2.06e-1	11	3.30e-1	9	2.73e-1	7	2.76e-1	6	2.76e-1
gures-fm-pow-1-f-4th-kind-one-sided	10	2.30e-1	7	2.25e-1	6	2.57e-1	6	3.15e-1	6	3.75e-1	15	3.30e-1	9	2.73e-1	7	2.76e-1	6	2.94e-1