# UnLimited TRAnsfers for Multi-Modal Route Planning: An Efficient Solution

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#### — Abstract

We study a multi-modal route planning scenario consisting of a public transit network and a transfer graph representing a secondary transportation mode (e.g., walking or taxis). The objective is to compute all journeys that are Pareto-optimal with respect to arrival time and the number of required transfers. While various existing algorithms can efficiently compute optimal journeys in either a pure public transit network or a pure transfer graph, combining the two increases running times significantly. As a result, even walking between stops is typically limited by a maximal duration or distance, or by requiring the transfer graph to be transitively closed. To overcome these shortcomings, we propose a novel preprocessing technique called ULTRA (UnLimited TRAnsfers): Given a complete transfer graph (without any limitations, representing an arbitrary non-schedule-based mode of transportation), we compute a small number of transfer shortcuts that are provably sufficient for computing all Pareto-optimal journeys. We demonstrate the practicality of our approach by showing that these transfer shortcuts can be integrated into a variety of state-of-the-art public transit algorithms, establishing the ULTRA-Query algorithm family. Our extensive experimental evaluation shows that ULTRA is able to improve these algorithms from limited to unlimited transfers without sacrificing query speed, yielding the fastest known algorithms for multi-modal routing. This is true not just for walking, but also for other transfer modes such as cycling or driving.

#### 2012 ACM Subject Classification

Theory of computation  $\rightarrow$  Shortest paths Mathematics of computing  $\rightarrow$  Graph algorithms Applied computing  $\rightarrow$  Transportation

#### Keywords and Phrases

Algorithms, Optimization, Route Planning, Public Transportation

#### Supplement Material

Source code of ULTRA is available at https://github.com/kit-algo/ULTRA.

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# 1 Introduction

Research on efficient route planning algorithms has seen remarkable advances in the past decade. For many types of transportation networks, queries can be solved in a few milliseconds, even on a continental scale [3]. However, combining schedule-based (i.e., public transit) and non-schedule-based (e.g., walking, cycling, driving) transportation modes and solving the resulting multi-modal routing problem is still a challenge [26]. In this work, we consider a multi-modal problem that augments public transit with a transfer graph, which represents an arbitrary non-schedule-based mode of transportation that can be used for transferring between public transit stops. Given a source and target vertex in the transfer graph and a departure time, we want to compute all Pareto-optimal journeys regarding travel time and number of used public transit trips.

Related Work. Most algorithms for public transit routing either impose technical restrictions on the included transfers or have only been evaluated on networks featuring very sparse transfer graphs. Algorithms that were only evaluated for limited transfers include the graph-based techniques in [23] and [18], frequency-based search [5], Transfer Patterns [2] and its accelerated version, Scalable Transfer Patterns [4], Public Transit Labeling [11], and SUBITO [9]. A common restriction, employed by CSA [14, 15], RAPTOR [13], and their corresponding speedup techniques ACSA [25, 15] and HypRAPTOR [12], is to require that the transfer graph is transitively closed. This eliminates the need to search within the transfer graph, as every possible destination can be reached with a single edge. To ensure a reasonably sized transfer graph, transfers are typically limited by a maximal duration (e.g., 15 minutes of walking) or distance before the transitive closure is computed. As shown in [26], choosing a higher limit for the maximal transfer duration increases the size of the resulting transitively closed graph significantly. A limit of only 20 minutes on the maximal transfer duration already leads to a graph that is unsuitable for practical applications. A special case is Trip-Based Routing [27], which precomputes transfers between pairs of trips. This precomputation involves enumerating all possible transfers and then using a limited set of pruning rules to omit some, but not all unnecessary transfers. Trip-Based Routing was only evaluated for transitively closed transfer graphs and likely has prohibitively high preprocessing times on unrestricted transfer graphs.

Using a restricted transfer graph is often justified with the argument that long transfers are rarely useful. However, experiments performed in [26, 22] show that unrestricted walking often significantly reduces the travel time of optimal journeys. This effect is likely even stronger for faster transportation modes, such as bicycle or car. The only algorithms that can handle unrestricted transfer graphs so far are multi-modal techniques such as MCR [10] and UCCH [16]. These techniques work by interleaving a public transit routing algorithm with Dijkstra's algorithm [17] on a contracted transfer network. Accordingly, they are fairly slow compared to pure public transit algorithms. Most recently HLRaptor and HLCSA [22] have been published. Here, RAPTOR/CSA are interleaved with Hub Labeling queries [1] instead of Dijkstra. While requiring more than an hour of preprocessing, this significantly improves query times. However, it is still not as efficient as pure public transit algorithms.

**Our Contribution.** Preliminary experiments [24] have shown that the impact of unrestricted transfers in Pareto-optimal journeys depends heavily on their position in the journey: *Initial transfers*, which connect the source to the first public transit vehicle, and *final transfers*, connecting the final vehicle to the target, are fairly common and often have a large

impact on the travel time. In contrast, intermediate transfers between public transit trips are only occasionally relevant for optimal journeys. This suggests that the number of unique paths in the transfer graph that occur as intermediate transfers of a Pareto-optimal journey is small. Using this insight, we propose a new preprocessing technique called ULTRA (Un-Limited TRAnsfers), which computes a set of shortcut edges representing these paths. The preprocessing step is carefully engineered to ensure that the number of shortcuts remains small. Combined with efficient one-to-many searches for the initial and final transfers, these shortcuts are sufficient to answer all queries in the network correctly. ULTRA shortcuts can be used without adjustment by any algorithm that previously required a transitively closed transfer graph. Our experiments show that this enables unrestricted multi-modal queries with roughly the same performance as restricted queries. In particular, ULTRA-CSA is the first efficient multi-modal variant of CSA. Source code for ULTRA and our experiments is available at https://github.com/kit-algo/ULTRA.

#### 2 Preliminaries

In this section we establish the basic notation and terminology used in this work. Moreover, we introduce the RAPTOR and Bucket-CH algorithms, on which our work is founded.

**Public Transit Network.** A public transit network is a 4-tuple  $(S, \mathcal{T}, \mathcal{R}, G)$  consisting of a set of  $stops\ S$ , a set of  $trips\ \mathcal{T}$ , a set of  $routes\ \mathcal{R}$ , and a directed, weighted  $transfer\ graph\ G=(\mathcal{V},\mathcal{E})$ . Every stop in S defines a location in the network where passengers can board or disembark a vehicle (such as buses, trains, ferries, etc.). Furthermore, we associate with each stop  $v\in S$  a non-negative  $departure\ buffer\ time\ \tau_{\rm buf}(v)$ , which defines the minimum amount of time that has to pass after arriving at the stop before a vehicle can be boarded. A trip  $T=\langle v_0,\ldots,v_k\rangle\in\mathcal{T}$  is a sequence of at least two stops which are served consecutively by the same vehicle. For each stop v in the sequence,  $\tau_{\rm arr}(T,v)$  denotes the  $arrival\ time$  of the vehicle at v, and  $\tau_{\rm dep}(T,v)$  denotes its  $departure\ time$ . This, of course, implies that  $\tau_{\rm arr}(T,v)\leq \tau_{\rm dep}(T,v)$  holds for every trip T and stop v. The i-th stop of a trip T is denoted as T[i]. The set of routes  $\mathcal{R}$  defines a partition of the trips such that two trips are part of the same route if they have the same stop sequence and do not overtake each other. A trip  $T_a\in \mathcal{T}$  overtakes the trip  $T_b\in \mathcal{T}$  if two stops  $u,v\in \mathcal{S}$  exist such that  $T_a$  arrives at or departs from u before  $T_b$  and  $T_a$  arrives at or departs from v after  $T_b$ .

The transfer graph  $G = (\mathcal{V}, \mathcal{E})$  consists of a set of vertices  $\mathcal{V}$  with  $\mathcal{S} \subseteq \mathcal{V}$ , and a set of edges  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ . For each edge  $e = (u, v) \in \mathcal{E}$  we define the transfer time  $\tau_{\theta}(e)$  as the time required to transfer from u to v. The notion of transfer time carries over to paths  $P = (v_1, \ldots, v_k)$  in G, using the definition  $\tau_{\theta}(P) := \sum_{i=1}^{k-1} ((v_i, v_{i+1}))$ . Unlike in restricted walking scenarios, we require no special properties for the transfer graph G. It does not need to be transitively closed, it may be strongly connected, and transfer times may represent walking, cycling, or some other non-schedule-based mode of travel.

**Journeys.** A trip leg  $T^{ij}$  is a subsequence of the trip T, representing a passenger boarding the trip T at the *i*-th stop and disembarking at the *j*-th stop. The departure time of  $T^{ij}$  is the departure time at the first stop of the trip leg, i.e.,  $\tau_{\text{dep}}(T^{ij}) := \tau_{\text{dep}}(T, T[i])$ . Similarly, the arrival time is defined as  $\tau_{\text{arr}}(T^{ij}) := \tau_{\text{arr}}(T, T[j])$ . An intermediate transfer  $\vartheta$  between two trip legs  $T_a^{ij}$  and  $T_b^{mn}$  is a path in the transfer graph such that: (1) the path  $\vartheta$  begins with the last stop of  $T_a^{ij}$ , (2) the path ends with the first stop of  $T_b^{mn}$ , and (3) the transfer time of the path is sufficient to reach  $T_b^{mn}$ . The transfer time is sufficient if after

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vacating  $\mathbf{T}_a^{ij}$  and transferring to the departure stop of  $\mathbf{T}_b^{mn}$ , there is still sufficient buffer time to enter  $\mathbf{T}_b^{mn}$ . We can express this formally as  $\tau_{\text{arr}}(\mathbf{T}_a^{ij}) + \tau_{\theta}(\vartheta) + \tau_{\text{buf}}(\mathbf{T}_b^{mn}[m]) \leq \tau_{\text{dep}}(\mathbf{T}_b^{mn})$ . An initial transfer  $\vartheta$  before a trip leg  $\mathbf{T}^{ij}$  is a path in G from the source s to the first stop of  $\mathbf{T}^{ij}$ . Correspondingly, a final transfer  $\vartheta$  after a trip leg  $\mathbf{T}^{ij}$  is a path in G from the last stop of  $\mathbf{T}^{ij}$  to the target t. We use the term transfer on its own to denote the union of all transfer types, or if the actual type of the transfer can be deduced from context. We define a journey  $J = \langle \vartheta_0, \mathbf{T}_0^{ij}, \dots, \mathbf{T}_{k-1}^{mn}, \vartheta_k \rangle$  as an alternating sequence of transfers and trip legs. Note that some or all of the transfers may be empty. The departure time of the journey is defined as  $\tau_{\text{dep}}(J) := \tau_{\text{dep}}(\mathbf{T}_0^{ij}) - \tau_{\text{buf}}(\mathbf{T}_0[i]) - \tau_{\theta}(\vartheta_0)$  and the arrival time as  $\tau_{\text{arr}}(J) := \tau_{\text{arr}}(\mathbf{T}_{k-1}^{mn}) + \tau_{\theta}(\vartheta_k)$ . The number of trips used by the journey is k. A journey J (weakly) dominates a journey J' if  $\tau_{\text{dep}}(J) \geq \tau_{\text{dep}}(J')$ ,  $\tau_{\text{arr}}(J) \leq \tau_{\text{arr}}(J')$ , and J does not use more trips than J'. For strict domination, at least one criterion must be strictly better. A journey J is called Pareto-optimal if no other journey exists that dominates J.

In our journey definition, the departure buffer time at a stop models the time required to reach the right platform and board a trip, regardless of how the stop was reached. Many other works on public transit routing instead use a minimum transfer time, which only needs to be observed if the stop was reached directly via a trip instead of a transfer. This is reasonable for settings with direct transfers between stops, where the buffer time can simply be included in the transfer time. When allowing arbitrary transfers, however, it can lead to inconsistencies. Given a stop with minimum transfer time  $\tau$ , if a path starting and ending at this stop with a transfer time less than  $\tau$  exists, then taking that path would allow passengers to circumvent the minimum transfer time.

**Algorithms.** Since our algorithm is strongly influenced by the RAPTOR algorithm family, we now introduce the basic concepts of these algorithms. The RAPTOR [13] algorithm can be used to solve one-to-one and one-to-many queries on a public transit network with limited transfers. The algorithm operates in rounds, where the i-th round finds all journeys using exactly i trips. For this, each round extends journeys found in the previous round by one trip, which can be done via a single scan of all routes in the network. An extension of this algorithm for multi-modal scenarios with unlimited transfers is MCR [10]. In this algorithm the RAPTOR rounds are alternated with Dijkstra's algorithm on a contracted transfer graph, in order to propagate arrival times through the transfer graph. Another extension, rRAPTOR [13], can be used to answer range queries, which ask for all Pareto-optimal journeys that depart within a given time interval. The rRAPTOR algorithm operates in iterations, where every iteration handles a possible departure time using the basic RAPTOR algorithm. The possible departure times are handled in descending order, and the data structures used by RAPTOR are not cleared in between iterations. As a result, journeys found by the current iteration are implicitly pruned by journeys that depart later and neither arrive later nor have more trips. This property of the rRAPTOR algorithm is called self-pruning.

Besides public transit routing algorithms, we also require efficient one-to-many algorithms for the transfer graph. Especially the Bucket-CH [21, 19, 20] algorithm is useful for our purposes. This algorithm is based on Contraction Hierarchies (CH) [19, 20] and operates in three phases. First the CH is computed, requiring only the graph. Second, given the set of targets, a bucket containing distances to the targets is computed for every vertex. This is done by adding every target to the buckets of all vertices in its reverse CH search space. Finally, the distance from a source to all targets is computed by performing the forward part of a CH search. For each vertex v in the forward search space, the bucket is evaluated by combining the distance to v with the distance from v to the targets in the bucket.

# 3 Shortcut Computation

Our preprocessing technique aims at finding a small number of transfer shortcuts that are sufficient to answer every point-to-point query correctly. This is achieved if for every Pareto-optimal journey there exists a journey with the same departure time, arrival time, and number of trips that uses only the precomputed shortcuts to transfer between trips. Next, we present a high-level overview of the ULTRA preprocessing, followed by an in-depth description of important algorithmic details.

## 3.1 Overview

The basic idea of ULTRA is as follows. We enumerate all possible journeys that use exactly two trips and require neither an initial nor a final transfer. The transfers between the two trips of these journeys are then considered as *candidates* for shortcuts. For each of these *candidate journeys*, we check if there is another journey that dominates it. If this is the case, we can replace the candidate journey with the dominating journey without losing Pareto-optimality. Note that if the candidate journey is contained in a longer journey, then it still can be replaced without affecting the Pareto-optimality of the longer journey. We call such a dominating journey a *witness* since its existence proves that the candidate shortcut is not needed. Unlike the candidate journey, the witness journey can make use of the transfer graph before the first trip or after the second trip. If no witness is found, then the candidate shortcut is added to the resulting shortcut graph.

A naive implementation of this idea would be to first enumerate all candidate journeys and subsequently search for witnesses. However, this would be impractical due to the sheer number of possible journeys. We therefore propose to interweave the candidate enumeration and the witness search, with the goal of eliminating as many candidates as early as possible. Pseudocode for the result of these considerations is given by Algorithm 1. The algorithm resembles invoking rRAPTOR [13] once per stop, restricted to the first two rounds per iteration. Remember that the original rRAPTOR algorithm already answers one-to-all range queries. Restricting this algorithm to the first two rounds enables an efficient enumeration of candidate journeys. Moreover, many dominated candidates are eliminated early on, due to self-pruning. We will now continue with a detailed discussion of Algorithm 1, showing step by step what has changed in comparison to the original rRAPTOR and how this helps with computing the transfer shortcuts.

#### 3.2 Implementation Details

A first important difference is due to the fact that rRAPTOR requires a transitively closed transfer graph. As we want to allow arbitrary transfer graphs, we replace the RAPTOR that is invoked in every iteration of rRAPTOR with  $MR-\infty$ , the variant of MCR that optimizes arrival time and number of used trips. Because of this change, the relaxation of transfers in lines 8 and 11 is not done by relaxing outgoing edges of updated stops. Instead, Dijkstra's algorithm is performed in order to propagate arrival times found by the preceding route scanning step. Furthermore, MCR would also use Dijkstra's algorithm in order to collect all routes reachable from the source stop in line 6. In the context of rRAPTOR this leads to many redundant computations, as the source stop does not change between iterations. We therefore compute distances from the source stop to all other stops once in line 3, again using Dijkstra's algorithm. These distances can then be used in line 6.

**Algorithm 1:** ULTRA transfer shortcut computation.

```
Input: Public transit network (S, \mathcal{T}, \mathcal{R}, G), with unrestricted transfer
              graph G = (\mathcal{V}, \mathcal{E})
    Output: Shortcut graph G' = (S, \mathcal{E}')
 1 for each s \in \mathcal{S} do
 2
        Clear all arrival labels and Dijkstra queues
        d(s,*) \leftarrow \text{Compute distances from } s \text{ to all stops in } G
 3
        D \leftarrow \text{Collect departure times of trips at } s
 4
        for each \tau_{dep} \in D in descending order do
                                                                                  // rRAPTOR iteration
             Collect routes reachable from s at \tau_{\rm dep}
                                                                                // first RAPTOR round
 6
             Scan routes
             Relax transfers
 8
             Collect routes serving updated stops
                                                                               // second RAPTOR round
             Scan routes
10
             C \leftarrow \text{Relax transfers}, thereby collecting unwitnessed candidates
11
            \mathcal{E}' \leftarrow \mathcal{E}' \cup C
12
```

**Departure Time Collection.** In line 4, standard rRAPTOR would collect all departure events that are reachable from the source stop s. However, given a transfer graph without any restrictions, this could possibly be every departure event in the network. Since we are primarily interested in finding candidate journeys, which do not have initial transfers, we collect only those departure events which depart directly at the source stop s. However, in order to find witness journeys, we still need to explore initial transfers in line 6. A naive implementation would check for each stop v reachable from s and for each route containing the stop v whether a trip that was not scanned in a previous iteration can be reached given the departure time  $\tau_{\rm dep}$  at s.

A more efficient approach combines lines 4 and 6 into a single operation. For this, we first sort all departure triplets  $(v, \tau_{\text{dep}}, r)$  of departure stop v, departure time  $\tau_{\text{dep}}$ , and route r by their corresponding departure time at the source,  $\tau_{\text{dep}} - \tau_{\text{buf}}(v) - d(s, v)$ . Afterwards, we iterate through this sorted list in descending order of departure time. If the next triplet to be processed has a departure stop  $v \neq s$ , then its route is added to a set  $\mathcal{R}'$ . In the case that the next triplet actually has the source stop s as departure stop v, we proceed with lines 6 through 12. Now the routes that have to be collected in line 6 are exactly the routes in  $\mathcal{R}'$ . Thus we simply scan all routes in  $\mathcal{R}'$  and then reset  $\mathcal{R}' = \emptyset$  for the next iteration.

Limited Transfer Relaxation. Another part of ULTRA that differs from rRAPTOR is the final relaxation of transfers in line 11. This is the part of the algorithm where we actually determine the candidate journeys for which have not found a witness. As usual, relaxing the transfers is done by Dijkstra's algorithm, initialized with the arrival times from the preceding route scanning step. Whenever a stop is settled during this execution of Dijkstra's algorithm, we look at the corresponding journey and check whether it is a candidate journey, i.e., does not require initial or final transfers. If so, we know that there is no witness journey dominating this candidate, because otherwise the search would have reached the stop via this witness journey instead. Thus, we extract the intermediate transfer of the found candidate journey and add it as an edge to the shortcut graph.

We further increase the practical performance of our algorithm by adding a stopping criterion to the final transfer relaxation in line 11. For this purpose, we count the number of stops which were newly reached via a candidate journey in the preceding route scanning step. Whenever such a stop is settled in line 11, we decrease our counter. Once the counter reaches zero, we can stop settling further vertices as we know that no more candidates can be found in this iteration. We can apply a similar stopping criterion to the intermediate transfer relaxation in line 8. In this case, we count the stops which were reached via a route directly from s, without an initial transfer, since only these stops can later become part of a candidate journey. As in line 11, we can stop settling vertices as soon as no such stops are left in the Dijkstra queue. This does not affect the correctness of the algorithm, as we still process all candidates. However, it might cause some witnesses to be pruned and thus lead to superfluous shortcuts in the result. To counteract this, we take the arrival time  $\tau_{\rm arr}$  of the last stop representing a candidate that is settled. Instead of stopping the transfer relaxation immediately, we continue until the queue head has an arrival time greater than  $\tau_{\rm arr} + \bar{\tau}$  for some parameter  $\bar{\tau}$  (which we call witness limit). With these changes, the only remaining part of the algorithm that performs an unlimited search on the transfer graph is the initial transfer relaxation in line 3, which is only done once per source stop.

The success of our pruning rule for the transfer relaxation in lines 8 and 11 depends on the presence of candidate journeys in the Dijkstra queues. Fewer candidate journeys could therefore lead to an earlier application of the pruning rule. We exploit this by further restricting the notion of candidate journeys. As before, a candidate journey must not contain any initial or final transfers. In addition, we now require that the intermediate transfer of a candidate journey is not contained in the set of already computed transfer shortcuts.

**Cyclic Witnessing.** Since witnesses are only required to dominate candidate journeys weakly, there may be journeys J, J' that dominate each other. If J has an initial transfer of length > 0, then J without the initial transfer is not dominated by J' extended by the reverse initial transfer. Therefore, the shortcut required by J will be added. Thus, cyclic domination is only problematic between journeys with initial transfers of length 0. We prevent this by temporarily contracting groups of stops with transfer distance 0 during the preprocessing.

Transfer Graph Contraction. As shown for MCR [10], the transfer relaxation is often the bottleneck of multi-modal routing algorithms. Since ULTRA only needs to compute journeys between stops, rather than arbitrary vertices of the transfer graph, only transfers that start and end at stops are relevant. Therefore, any overlay graph that preserves the distances between all stops can be used instead of the transfer graph in our preprocessing algorithm. An easy way of obtaining such an overlay graph is to construct a partial CH that only contracts vertices that do not correspond to stops of the public transit network. This, of course, leads to a suboptimal contraction order and thus makes it infeasible to contract all vertices that are not stops. As done in many other algorithms [6, 16, 10, 8, 7], we therefore stop the contraction once the uncontracted *core* graph surpasses a certain average vertex degree.

Parallelization. Finally, we observe that ULTRA allows for trivial parallelization. Our algorithm searches for candidate journeys once for every possible source stop (line 1 of Algorithm 1). As these searches are mostly independent of each other, we can distribute them to parallel threads and combine the results in a final sequential step. Only the usage of the restricted candidate notion introduces a dependence between the searches for different source stops. As this is only a heuristic performance optimization, we simply relax the notion of candidate journeys again, only requiring that no shortcut representing the intermediate transfer has been found by the same thread yet.

Algorithm 2: Query algorithm, using transfer shortcuts computed by ULTRA

Input: Public transit network  $(S, \mathcal{T}, \mathcal{R}, G)$ , shortcut graph  $G' = (S, \mathcal{E})$ ,

Bucket-CH of G, source vertex s, departure time  $\tau_{\text{dep}}$ , and target vertex tOutput: All Pareto-optimal journeys from s to t for departure time  $\tau_{\text{dep}}$ 1  $d(s, *) \leftarrow \text{Run}$  Bucket-CH query from s2  $d(*, t) \leftarrow \text{Run}$  reverse Bucket-CH query from t3  $G'' \leftarrow G'$ 4 for each  $v \in S$  do

5 Add edge (s, v) to G'' with travel time d(s, v)6 Add edge (v, t) to G'' with travel time d(v, t)7 Run black box public transit algorithm on  $(S \cup \{s, t\}, \mathcal{T}, \mathcal{R}, G'')$ 

#### 3.3 Proof of Correctness

Before continuing with the query algorithms, we want to justify that ULTRA computes a shortcut graph that is sufficient to answer all queries correctly. For this purpose we assume that a journey  $J = \langle \vartheta_0, \mathbf{T}_0^{ij}, \dots, \mathbf{T}_{k-1}^{mn}, \vartheta_k \rangle$  exists that requires an intermediate transfer not contained in the shortcut graph and cannot be replaced with a journey of equal travel time and number of trips that solely uses transfers from the shortcut graph. In this case, the journey J must contain at least two trips, since otherwise it would not contain any intermediate transfers. Since the journey contains two or more trips, it can be disassembled into candidate journeys  $\langle \mathbf{T}_0^{ij}, \vartheta_1, \mathbf{T}_1^{gh} \rangle, \langle \mathbf{T}_1^{gh}, \vartheta_2, \mathbf{T}_2^{pq} \rangle, \dots, \langle \mathbf{T}_{k-2}^{uv}, \vartheta_{k-1}, \mathbf{T}_{k-1}^{mn} \rangle$ . As J requires a transfer not contained in the shortcut graph, at least one of these candidates must also contain a transfer not contained in the shortcut graph. Let  $J^c = \langle T_x^{gh}, \vartheta_{x+1}, T_{x+1}^{pq} \rangle$  be such a candidate journey. Since the main loop of ULTRA is executed for every stop in the network, it was also executed for the source stop  $T_x[g]$  of this candidate journey. Derived from the correctness of rRAPTOR, we know that for a given source stop our algorithm computes Pareto-optimal arrival labels for all stops reachable with two trips or less. Thus we also reached the target stop  $T_{x+1}[q]$  of the candidate journey. The journey J' corresponding to the target's arrival label is in this case either the candidate journey or a journey that dominates the candidate journey. In the first case, we have added the transfer  $\vartheta_{x+1}$  of the candidate journey to the shortcut graph. In the second case, the candidate journey  $J^c$ can be replaced by the journey J' corresponding to the target's arrival label, leading to a journey that is not worse than the original journey and does not require the missing transfer. Therefore both cases contradict our assumption.

# 4 Query Algorithms

The shortcuts obtained by ULTRA can in principle be combined with any public transit query algorithm that normally requires a transitively closed transfer graph, such as RAPTOR [13], CSA [14, 15], or Trip-Based Routing [27]. The basic idea of the query algorithm is to simply use one of the above algorithms together with our precomputed shortcut graph instead of the original transfer graph. However, our shortcut graph only represents transfers between two trips, and does not provide any information for transferring from the source to the first trip or from the last trip to the target. In this section we describe how the public transit algorithms can be modified in order to handle initial and final transfers efficiently.

Basic Query Algorithm. Our approach is based on the observation that for initial and final transfers one endpoint of the transfer is fixed. All initial transfers start at the source vertex, and all final transfers end at the target vertex. Therefore, we can use two additional one-to-many queries (one of them performed in reverse) to cover initial and final transfers. These queries have to be performed on the original transfer graph, where they compute the distances from the source to all stops and from all stops to the target. While any one-to-many algorithm might be used to perform this task, we decided to use Bucket-CH, as it is one of the fastest known one-to-many algorithms and allows for optimization of local queries. Pseudocode for the resulting query algorithm using Bucket-CH and our transfer shortcuts is shown in Algorithm 2.

Our algorithm begins with performing the two Bucket-CH queries from the source and target stop in lines 1 and 2. Afterwards a temporary copy of the shortcut graph G'' is initialized. In lines 5 and 6, this temporary graph is complemented with edges from the source to all other stops and edges from all stops to the target, using the distances obtained from the Bucket-CH queries. Finally, a public transit algorithm is invoked as a black box on the public transit network with the temporary graph instead of the shortcut graph in line 7. The temporary graph is sufficient for the query to yield correct results, as it contains edges from the source to any possible first stop, all edges required to transfer between trips, and edges from any possible last stop to the target. Since there are no additional requirements on the black box public transit algorithm, it is easy to see that any existing public transit algorithm can be used with our shortcuts.

Running Time Optimizations. We can further improve the performance of this query algorithm in practice by introducing some adjustments. First, we observe that we actually do not need edges from the source to every other stop. If the distance d(s,v) from s to a stop v is greater than the distance d(s,t) from s to t, every journey that requires a transfer from s to v is dominated by simply transferring directly from s to t. Thus, we do not need to add the edge (s, v) to the temporary graph. The same argument can be made for edges from some stop u to the target t if the distance d(u,t) is greater than d(s,t). Moreover, if we know that a stop v is further away from the source than the target, then we do not even need to compute the actual distance d(s, v). We can use this fact to prune the search space of the Bucket-CH queries in lines 1 and 2. For this purpose, we first perform a standard bidirectional CH query from source to target that stops settling vertices from the forward (respectively backward) queue if the corresponding key is greater than the tentative distance from the source to the target. As a result we obtain the distance d(s,t), as well as the partial forward (backward) CH search space from s(t), containing no vertices that have a greater distance from s (to t) than d(s,t). We then perform the second phase of the Bucket-CH query (i.e., scanning the buckets) only for the vertices in the partial search spaces of the CH query. Furthermore, we store the entries in each bucket sorted by the distance to their target. Thus we can stop scanning through the bucket of a vertex u once we reach a stop v within the bucket with  $d(s,u) + d(u,v) \ge d(s,t)$ . Doing so can drastically improve local queries, as we do not need to look at all stops, but only at stops that are close to the source or target.

If we do not treat the underlying public transit algorithm as a black box, we can further improve practical performance by omitting the construction of the temporary graph G''. Instead of adding edges from s to stops v, we can directly initialize the tentative arrival times used by most public transit algorithms with  $\tau_{\rm dep} + d(s,v)$ . Instead of adding edges to t, we try to update the tentative arrival time at the target with the arrival time at v plus d(v,t) whenever the arrival time at v is updated.

**Table 1** Sizes of the used public transit networks and their transfer graphs (full and transitive).

Network	Stops	Routes	Trips	Stop events	Vertices	Full edges	Tran. edges
Switzerland	25426	13934	369534	4740929	604167	1847140	4687016
Germany	244055	231089	2387297	48495169	6872105	21372360	22645480

# **Experiments**

All algorithms were implemented in C++17 compiled with GCC version 7.3.1 and optimization flag -O3. All experiments were conducted on a machine with two 8-core Intel Xeon Skylake SP Gold 6144 CPUs clocked at 3.5 GHz, 192 GiB of DDR4-2666 RAM, and 24.75 MiB of L3 cache. The shortcut preprocessing was performed in parallel on all 16 cores. The transfer graph contraction and the queries were performed on a single core.

**Networks.** We evaluated our technique on the public transit networks of Switzerland and Germany, which were previously used in [26]. The Switzerland network was extracted from a publicly available GTFS feed<sup>1</sup> and consists of two successive business days (30th and 31st of May 2017). The Germany network is based on data from bahn.de for Winter 2011/2012, comprising two successive identical days. For both networks, stops and connections outside of the country borders were removed. As unrestricted transfer graphs, we used the road networks of Switzerland and Germany, including pedestrian zones and stairs, which were obtained from OpenStreetMap<sup>2</sup> data. Vertices with degree one and two were contracted unless they coincided with stops. Unless stated otherwise, we used walking as the transfer mode, assuming a walking speed of 4.5 km/h on each edge. To obtain transitively closed transfer graphs (for comparison with standard RAPTOR and CSA), we inserted an edge between all stops whose distance in the transfer graph lies below a certain threshold (15 minutes for Switzerland, 8 minutes for Germany) and then computed the transitive closure. An overview of the networks is given in Table 1.

#### 5.1 **Preprocessing**

In this section we evaluate the performance of the ULTRA preprocessing phase, including the transfer graph contraction and the shortcut computation.

Core Degree and Witness Limit. The two main parameters influencing the performance of the ULTRA preprocessing are the average vertex degree of the contracted transfer graph and the witness limit  $\bar{\tau}$ . Figure 1 shows the impact of these two parameters on the Switzerland network. The lowest preprocessing times are achieved with a core degree of 14. While the actual shortcut computation still becomes slightly faster for higher core degrees, this is offset by the increased time required to contract the transfer graph. Contracting up to a core degree of 14 took 1:29 minutes and yielded a graph with 32 683 vertices and 466 331 edges. By contrast, the witness limit  $\bar{\tau}$  only has a minor impact on the number of computed shortcuts, with a difference of fewer than 600 shortcuts between  $\bar{\tau} = 0$  and  $\bar{\tau} = \infty$ . For all following experiments, we chose a witness limit of 15 minutes, yielding 139 669 shortcuts for Switzerland.

 $<sup>^1</sup>$  http://gtfs.geops.ch/

<sup>&</sup>lt;sup>2</sup> http://download.geofabrik.de/

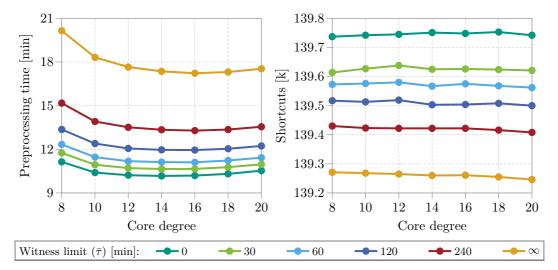
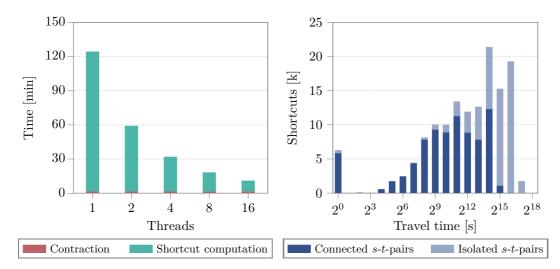


Figure 1 Impact of core degree and witness limit on the running time of the preprocessing algorithm and the number of computed shortcuts, measured on the Switzerland network. Preprocessing time includes both contracting the transfer graph and computing the shortcuts.

For the Germany network, we chose to contract up to a core degree of 20, since the share of the core computation in the overall preprocessing time decreases as the network size increases. Contraction took 24:56 minutes and produced a core graph with  $314\,021$  vertices and  $6\,280\,440$  edges. As before, we used a witness limit of 15 minutes for the shortcut computation, which yielded  $2\,077\,374$  shortcuts.

**Parallelization.** We used all 16 cores of our machine in parallel to accelerate the shortcut computation. On the Switzerland network this reduced the shortcut computation time from 2:02:55 hours sequentially to 9:35 minutes, which corresponds to a speedup of 12.8. Thus, we obtain a total preprocessing time of 11:05 minutes, including the time for the contraction, which was not parallelized. This yields an overall speedup for the preprocessing phase of 11.2. Figure 2 (left side) shows the impact of the parallelization on the Switzerland network for different numbers of used cores. For the Germany network the sequential shortcut computation would take several days, while computing the shortcuts in parallel using all 16 cores took 10:53:35 hours.

Transfer Speed. In order to test the impact of the used transfer mode on the shortcut computation, we changed the transfer speed in the Switzerland network from 4.5 km/h to different values between 1 km/h and 140 km/h. We considered two ways of applying the transfer speed: In the first version, we did not allow the transfer speed on an edge to exceed the speed limit given in the road network. This allowed us to model fast transfer modes such as cars fairly realistically. In the second version, we ignored speed limits and assumed a constant speed on every edge. Thus, we can analyze to which extend the effects observed in the first version are caused by the speed limit data. Figure 3 reports the preprocessing time and number of computed shortcuts measured for each configuration. In all measurements, the preprocessing time remained below 15 minutes. A peak in the number of shortcuts is reached between 10 and 20 km/h, which roughly corresponds to the speed of a bicycle. If speed limits are ignored, the number of shortcuts then starts decreasing again for higher transfer speeds and reaches a plateau at around 188 000 shortcuts. If speed limits are obeyed,



**Figure 2** Preprocessing results for the Switzerland network using a core degree of 14, a witness limit of 15 minutes, and a transfer speed of  $4.5 \,\mathrm{km/h}$ . Left: Preprocessing times depending on the number of threads used in parallel. The contraction of the transfer graph only takes a fraction of the overall time and was not parallelized. Right: Histogram of shortcut travel times. The bars with the labels  $2^i$  for i>0 include all shortcuts with travel time  $\tau$  in the half-closed interval  $[2^{i-1},2^i)$ . The  $2^0$  bar represents the interval  $[0,2^0)$ , i.e., all shortcuts with travel time exactly zero. The dark blue portion of the bar represents shortcuts where the source and the target of the corresponding candidate journey are connected by a path in the transfer graph.

the number of shortcuts eventually rises again and reaches the overall peak at 140 km/h, which was the highest speed limit observed in the network.

For low to medium transfer speeds, the results conformed with our expectations. As the transfer speed increases, it becomes increasingly feasible to cover large distances in the transfer graph quickly, making it possible to transfer between trips that are further away from each other. Accordingly, new shortcuts appear between these trips. However, once the transfer speed becomes competitive with the public transit vehicles, it eventually becomes preferable to avoid the public transit network altogether and transfer directly from source to target. In this case, all journeys using trips from the public transit network are dominated by the journey corresponding to the direct transfer. Since no shortcuts are required for such pairs of source and target stop, we would expect a sharp decrease in the number of shortcuts. The reason why this decrease is not observed in our measurements is that not all stops in our network instances are connected to the transfer graph. Consider what happens in the shortcut computation for journeys between stops s and t that are isolated from each other and the rest of the transfer graph. In this case, a direct transfer is not possible, regardless of transfer speed. In fact, unless there is a route that serves both s and t, any optimal journey from s to t will include at least two trips. If a transfer is necessary between these two trips, then the journey is an nondominated candidate journey and a shortcut is added for the corresponding transfer. In our Switzerland network, 625 stops are isolated from the transfer graph, usually as a result of incomplete or imperfect data. To assess the impact of these stops on the number of computed shortcuts, we repeated our experiments, this time not adding shortcuts to the result if the source and target stop of the corresponding candidate journey were not connected in the transfer graph. This resulted in much fewer shortcuts, especially for high transfer speeds. If speed limits are ignored, the amount of necessary shortcuts becomes negligible at around 60 km/h and eventually reaches 0. If speed limits are obeyed, the number of shortcuts stagnates at 17000.

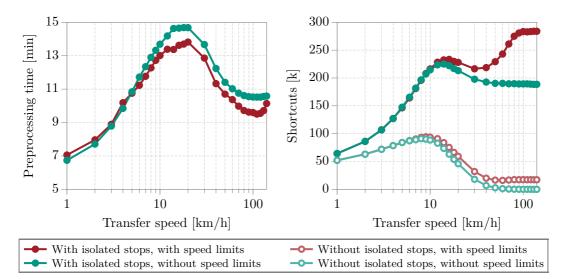


Figure 3 Impact of transfer speed on preprocessing time and number of computed shortcuts, measured on the Switzerland network with a core degree of 14, a witness limit of 15 minutes, and a transfer speed of 4.5 km/h. Speed limits in the network were obeyed for the red lines and ignored for the green lines. For the two lines at the bottom on the right side, shortcuts were only added to the result if the source and target stop for which they were found were connected by a path in the transfer graph. This shows that reducing the number of isolated stops can drastically improve the number of required shortcuts. However, this does not impact the preprocessing time.

Overall, these experiments show that our shortcut computation remains feasible regardless of the speed of the used transfer mode. Moreover, if the network does not include many stops that are isolated from the transfer graph, transferring between stops is most useful for transfer speeds between 10 and 20 km/h.

**Shortcut Graph Structure.** The shortcut graph we obtained for Switzerland is structurally very different from the transitively closed transfer graph we used for standard RAPTOR and CSA. This is already evidenced by the fact that it is much less dense, containing only 3% as many edges as the transitively closed graph. Furthermore, the transitive graph consists of many small fully connected components, with the largest one containing 1233 vertices. By contrast, the largest strongly connected component in the shortcut graph contains  $10\,186$  vertices, which is 40% of all stops. Accordingly, a transitive closure of the shortcut graph would contain more than 100 million edges. Figure 2 (right side) shows that most of the shortcuts are longer than the 15 minutes ( $\approx 2^{10}$  seconds) time limit that was used during the construction of the transitively closed graph. Only 33 868 edges are shared between the two graphs, which represent 0.7% of all transitive edges and 24.2% of all shortcuts. Altogether, this shows that the transitively closed graph fails to represent most of the relevant intermediate transfers, at the expense of many superfluous ones. Note that the high number of shortcuts with travel time 0 is caused by cases where several stops model the same physical location. The bars in Figure 2 are subdivided into shortcuts that arise from candidate journeys where the source and target stop are not connected by a path in the transfer graph (dark blue) and shortcuts where this is not the case (light blue). As before, we make this distinction in order to identify effects caused by imperfect or incomplete data. We observe that stops which are isolated from the rest of the transfer graph not only cause many additional shortcuts, but also that these shortcuts are disproportionately long compared to the other shortcuts.

**Table 2** Query performance for RAPTOR, MR- $\infty$ , and ULTRA-RAPTOR. Query times are divided into phases: scanning initial transfers, collecting routes, scanning routes, and relaxing transfers. All results are averaged over 10 000 random queries. Note that RAPTOR (marked with  $^*$ ) only supports stop-to-stop queries with transitive transfers, whereas the other two algorithms support vertex-to-vertex queries on the full graph.

Network	Algorithm	Full graph	Scans [k]		Time [ms]				
	1118011111111		Routes	Edges	Init.	Collect	Scan	Relax	Total
Switzerland	RAPTOR*	0	27.2	3527	0.0	3.7	6.4	7.8	18.4
	$MR-\infty$	•	34.9	769	11.6	5.9	8.2	12.3	39.3
	ULTRA-RAPTOR	•	37.7	148	1.6	4.9	7.9	1.9	16.7
Germany	$RAPTOR^*$	0	480.4	25798	0.0	166.9	178.0	85.1	436.5
	$MR-\infty$	•	555.8	12571	191.1	250.7	202.2	272.2	944.1
	ULTRA-RAPTOR	•	610.6	2224	26.8	204.5	202.9	37.0	477.8

#### 5.2 ULTRA Queries

To evaluate the impact of our shortcuts on the query performance, we tested them with two public transit algorithms, RAPTOR and CSA. For each algorithm, we compared three query variants: one using our ULTRA approach, one using a transitively closed transfer graph, and one using a multi-modal variant of the algorithm on an unrestricted transfer graph.

**RAPTOR Queries.** In the case of RAPTOR, we used the MR- $\infty$  variant of MCR as the multi-modal algorithm, employing the same core graph that was used by the ULTRA preprocessing. The results of our comparison are shown in Table 2. Using ULTRA-RAPTOR drastically reduces the time consumption for exploring the transfer graph compared to MR- $\infty$ , from 50–60% of the overall running time to 10–20%. The reason for this is that both scanning the initial/final transfers and relaxing the intermediate transfers are an order of magnitude faster in ULTRA-RAPTOR compared to MR- $\infty$ . For the initial and final transfers, the Core-CH search of MR- $\infty$  is replaced by a Bucket-CH query in ULTRA-RAPTOR. Similarly, ULTRA-RAPTOR uses shortcuts for relaxing the intermediate transfers whereas MR- $\infty$  performs a Dijkstra search in the core graph. Overall, ULTRA-RAPTOR is twice as fast as MR- $\infty$  and has a similar running time to RAPTOR with transitive transfers. Note that comparing the running times of RAPTOR and ULTRA-RAPTOR has to be done with caution, as they were measured for a different set of queries. Hence, our shortcut technique enables RAPTOR to use unrestricted transfers without incurring the performance loss that is associated with MCR.

**CSA Queries.** For CSA, incorporating unrestricted transfers efficiently is more challenging than for RAPTOR. Since no multi-modal variant of CSA has been published thus far, we implemented a naive multi-modal version of CSA, which we call MCSA, as a baseline for our comparison. This algorithm alternates connection scans with Dijkstra searches on the contracted core graph, in a similar manner to MCR. Query times for all three CSA variants are reported in Table 3. Note that CSA solves an easier problem than RAPTOR, since it only minimizes the arrival time and not the number of transfers. When using a transitively closed transfer graph, it is thus approximately three times as fast as RAPTOR.

■ Table 3 Query performance for CSA, MCSA, and ULTRA-CSA. Query times are divided into two phases: initialization including initial transfers (Init.), and connection scans including intermediate transfers (Scan). All results are averaged over 10 000 random queries. Note that CSA (marked with \*) only supports stop-to-stop queries with transitive transfers, instead of vertex-to-vertex queries on the full graph.

Network	Algorithm	Full graph	Scans	Time [ms]			
TYCOWOTK	7115011011111		Connections	Edges	Init.	Scan	Total
Switzerland	CSA*	0	126.7	1 307	0.2	5.0	5.1
	MCSA	•	88.0	5337	12.9	48.4	61.3
	ULTRA-CSA	•	87.3	52	1.8	3.1	4.9
Germany	$CSA^*$	0	2620.3	6216	2.9	162.1	165.1
	MCSA	•	1568.2	118026	233.6	1462.5	1696.1
	ULTRA-CSA	•	1562.5	665	25.7	116.8	142.5

With unrestricted transfers, we observe that MCSA is slower than  $MR-\infty$ . This is because the performance advantage of CSA over RAPTOR mainly stems from the high memory locality of its sequential connection scan. By contrast, MCSA loses this memory locality, as it has to perform a Dijkstra search every time an arrival time is updated after scanning a connection. When using ULTRA-CSA, however, this advantage is restored because only a few shortcut edges have to be relaxed after scanning each connection. Altogether, ULTRA-CSA is only slightly slower than transitive CSA and about three times as fast as RAPTOR with shortcuts, making it the only efficient multi-modal variant of CSA known so far.

Overall, our query times for ULTRA-RAPTOR and ULTRA-CSA are significantly faster than those reported for the state-of-the-art techniques HLRaptor and HLCSA [22], by a factor of 3.6 and 11.1, respectively. With respect to preprocessing time and space consumption, HL-based techniques are also outperformed by ULTRA.

Transfer Speed Impact on Travel Time. In addition to overall performance, we also measured how query times for RAPTOR are impacted by the transfer speed. Results are shown in Figure 4 (left side). The performance gains for ULTRA-RAPTOR compared to MR- $\infty$  are similar for all transfer speeds, and in fact slightly better for higher speeds. To explain this, observe that the time required for the route scanning phase decreases as the transfer speed increases. This is because the total number of rounds and thus the number of scanned routes decreases for higher transfer speeds. ULTRA-RAPTOR benefits more from this since the share of the route scanning phase in the overall running time is greater for ULTRA-RAPTOR than for MR- $\infty$ . In all cases, the entire query time for ULTRA-RAPTOR is similar to or lower than the time that MR- $\infty$  takes for the route scanning phases only.

The impact of the transfer speed on the travel time of the fastest journey is shown in Figure 4 (right side). As the transfer speed increases, the overall travel time decreases and the share of the travel time that is spent on an initial or final transfer becomes larger. From around 50 km/h onward, transferring directly from source to target is the best option in most cases. In contrast to initial and final transfers, intermediate transfers have a very small impact on the overall travel time, further demonstrating that long intermediate transfers are rarely needed.

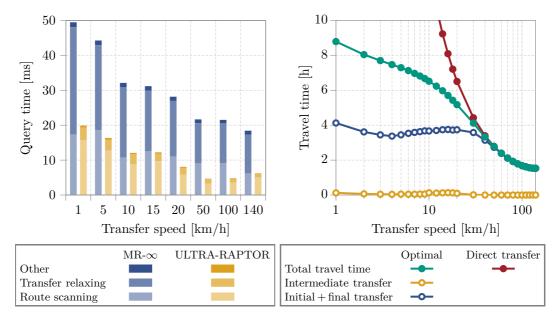


Figure 4 Impact of transfer speed on query times and travel times, measured on the Switzerland network with a core degree of 14 and a witness limit of 15 minutes. All results were averaged over 10 000 random queries. Left: Query performance of MR- $\infty$  and ULTRA-RAPTOR. Speed limits were obeyed. Query times are divided into route collecting/scanning, transfer relaxation, and remaining time. Right: Total travel time and time spent on initial/final and intermediate transfers for the journey with minimal arrival time. Also shown for reference is the time required for a direct transfer from source to target.

#### 6 Conclusion

We developed a technique which significantly speeds up the computation of Pareto-optimal journeys in a public transit network with an unrestricted transfer graph. We achieved this by efficiently computing shortcuts that provably represent all necessary transfers. Parallelization enables fast precomputation, taking a few minutes on the network of Switzerland. Our evaluation showed that the number of computed shortcuts is low, regardless of the underlying transfer mode. The shortcuts can be used without adjustments by any public transit algorithm that previously required a transitively closed transfer graph. For RAPTOR and CSA, we showed that using shortcuts leads to similar query times as using a transitively closed transfer graph. Consequently, shortcuts enable the computation of unrestricted multi-modal journeys without incurring the performance losses of existing multi-modal algorithms. In particular, combining shortcuts with CSA yields the first efficient multi-modal variant of CSA.

For future work, we would like to develop a shortcut-based query algorithm that can answer many-to-many queries. It would also be interesting to adapt our shortcut precomputation to scenarios with additional Pareto criteria, such as walking distance or cost. Furthermore, it should be possible to extend the ULTRA approach to more complicated transfer modes, including bike or car sharing.

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