Another Economic Application of Semialgebraic Parametric Analysis by Metaprogramming

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Abstract

One example of rediscovering the efficient frontier in portfolio optimization using a self-developed point-wise prover program and SPAM.

1 Introduction

Computer-assisted proof programs have become more and more useful in many areas of research. As computing power increases, the capabilities of such programs continue to expand. One such method of generating computer-assisted proofs is termed Semialgebraic Parametric Analysis by Metaprogramming, or SPAM [2]. Potential applications for this method are many, and indeed exist in the field of economics.

This paper introduces an additional application for SPAM, generating the efficient frontier for the problem of portfolio optimization. While this application involves only one specific instance of generating such a frontier, it should become clear that this can be applied to additional sorts of portfolio optimization problems.

2 Parametric Simplex Method

In developing this SPAM application, I first investigated the simplex method and its parametric variant [1] in Vanderbei's book. The parametric simplex method solves the portfolio optimization problem for one concrete value of μ (the risk parameter) and then generalizes the solution to every value of mu (from 0 to 10). This process is replicated in my work using SPAM.

In rediscovering the efficient frontier for this specific problem, SPAM provides an alternative method to the traditional parametric simplex method in which the same solution is obtained.

3 Methods

In order to utilize SPAM, one must create (or obtain) a "point-wise prover" program. This program must only function using semialgebraic operations in order to remain suitable for SPAM. For this application, I created a point-wise prover of my own, dubbed "portfolio2" that takes as input a concrete value of μ and returns the optimal weights for each investment. The sum of all such weights is one, so they represent proportions of wealth invested into each security.

Once verifying this program's validity, I used it as an input for SPAM. This process involves giving portfolio2 a rational-valued argument. In my case, I chose to use $\mu = 2$ as the starting point for SPAM. All of my analysis takes place in this one-dimensional parameter space. I chose this parameter value somewhat arbitrarily because the existing work from Vanderbei chooses μ values between one and ten, so I could check my result against the existing frontier.

Once SPAM has this concrete-valued tuple it begins its work. **I used the breadth-first search (BFS) method from SPAM. With this method, the BFS first executes ineq with the given tuple and records both the Boolean result from ineq as well as the inequalities that are checked. This data is stored in a proof cell by SPAM and eventually constitutes one cell of the complex. Once BFS completion is achieved, analysis of the proof cell complex can begin. In this paper, the description of a proof cell is identical to the set of inequalities which define it, and vice versa.**

4 Results and Discussion

While I have not yet perfected the BFS process for portfolio2, I have checked many values for μ between 0 and 10 and obtained answers consistent with Vanderbei's each time. Once I tune the BFS for this program I suspect I will obtain a proof cell complex identical to the existing efficient frontier for this specific case.

Since my results are obtained on the real line (a result of this program only taking one argument) the interpretation is simple. SPAM would return an optimal weight for each investment for every value of μ between 0 and 10, replicating the existing results.

5 Conclusion

As I said, I have yet to perfect the BFS and obtain a functional proof cell complex for portfolio2, but once I do I suspect having interesting conclusions to report.

References

[1] Vanderbei, R.: Linear Programming, Foundations and Extensions. *Fourth Edition.*. 185–195 (2014)