Formulation for
Cascading Outage Simulator with Multiprocess Integration Capabilities (COSMIC)

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This document describes the structure of the cascading failure simulator that will be used in this research project.

1 COSMIC Release 0.1 - README

There are two scripts that test the features of cosmic 0.1:

1. test_steady_state.m : This driver integrates a steady state case with 975 buses for 500 seconds. Additionally, it verifies the derivatives of the differential and algebraic equations and their residuals.
2. test_discrete_change.m : This driver simulates the opening and closing of a transmission line on a case with 351 buses. The line is disconnected at $t = 5$ sec. and put back in service at $t = 6$ sec. The total integration time is 40 sec.

2 Generic formulation for the Differential Algebraic Mixed Integer Equations

Let us assume that the state of the power system at time t can be defined by three vectors: $\mathbf{x}(t)$, $\mathbf{y}(t)$, and $\mathbf{z}(t)$, where:

$\mathbf{x}$ is a vector of continuous state variables that change with time according to a set of differential equations

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(t, \mathbf{x}(t), \mathbf{y}(t), \mathbf{z}(t))$$

$\mathbf{y}$ is a vector of continuous state variables that have purely algebraic relationships to other variables in the system:

$$\mathbf{g}(t, \mathbf{x}(t), \mathbf{y}(t), \mathbf{z}(t)) = 0$$

$\mathbf{z}$ is a vector of state variables that can take only integer states ($z_i \in [0, 1, 2, ...]$)

$$\mathbf{h}(t, \mathbf{x}(t), \mathbf{y}(t), \mathbf{z}(t)) = 0$$

where $\mathbf{h}(...)$ is an integer function, such that the correct integer state of the system at time $t + \epsilon$ ($\mathbf{z}(t + \epsilon)$, where ϵ is an arbitrarily small real positive number) can be found given the state of the system at time t:

$$\mathbf{z}(t + \epsilon) = \mathbf{z}(t) + \mathbf{h}(t, \mathbf{x}(t), \mathbf{y}(t), \mathbf{z}(t))$$

3 Problem variables

The following general notation will be used. All per unit values are relative to the system base.

n	The number of buses (nodes) in the network. This does not include internal (non-physical) generator nodes.
N	The set of all buses: $N = \{1, 2, ..., n\}$
m	The number of branches (transmission lines and transformers) in the network.
n_g	The number of generators in the network.
$X_{d,i}$	Direct axis synchronous reactance for the generator at Bus i. $X'_{d,i}$ is the transient reactance. X_q refers to the quadrature axis reactance (per unit)
$r_{g,i}$	Generator equivalent series resistance
$P_{g,i}$	The electrical power produced at the terminal of bus i
M	Generator inertia constant ($M = 2H$) (seconds on system base)
D	Generator damping constant (per unit on system base)
y_{ik}	The i, k'th element of the system admittance ($\mathbf{Y}_{BUS}$) matrix
g_{ik}	$g_{ik} = \Re(y_{ik})$
b_{ik}	$b_{ik} = \Im(y_{ik})$
N_G	The set of all generator buses in the system $N_G \subset N = \{1, 2, ..., n\}$
N_D	The set of all demand/load buses in the system $N_D \subset N = \{1, 2, ..., n\}$

The following tables describe the variables that will be included in the simulator:

Table 1: Dynamic state variables ($\mathbf{x}$)

Symbol	Variable	Support
$E'_{a,i}$	The internal (open circuit) voltage for the generator at Bus i.	$[0, \infty]$
δ_i	The angle corresponding to the position of the rotor open circuit voltage for generator at Bus i, relative to some system angle reference θ_r.	$[-\pi, \pi]$
$\delta_{m,i}$	The angle of the machine relative to its connected bus: $\delta_m = \delta_i - \theta_i$.	
$\overline{\omega_i} = \frac{\omega_i}{2\pi f_0}$	The per unit angular speed of the generator, normalized by the system frequency.	$[-\infty, \infty]$
$P_{m,i}$	The mechanical power forcing the generator rotation (per unit)	$[0, \infty]$
T_i	The temperature of Branch i.	$[0, T_{\max}]$

Table 2: Algebraic variables ($\mathbf{y}$).

Symbol	Variable	Support
θ	A vector of voltage phase angles in radians. For power flow, one bus will be chosen as the reference bus: $\theta_r = 0$. $\tilde{V}_i = \|V_i\|e^{j\theta}$	$\mathbb{R}^n \in [-\pi, \pi]$
$\|\mathbf{V}\|$	A vector of voltage magnitudes. The complex bus voltage for Bus i will be noted as $\tilde{V}_i$. Normalized such that 1.0 is nominal.	$\mathbb{R}^n \in [0, \infty]$
	The variables below are algebraic, but not in $\mathbf{y}$.	
$\tilde{I}_{f,i}$	Complex current on branch i at the "from" end.	$\mathbb{C}^m$
$\tilde{I}_{t,i}$	Complex current on branch i at the "to" end.	$\mathbb{C}^m$

4 Differential equations

- Equations for rotor speed:

$$M \frac{d\overline{\omega_i}}{dt} = P_{m,i} - P_{g,i} - D(\overline{\omega_i} - 1) \tag{1}$$

where $P_{g,i}(t)$ is

$$P_{g,i}(t) = \frac{|E'_{a,i}||V_i|}{X'_{d,i}} \sin \delta_{m,i} + \frac{|V_i|^2}{2} \left(\frac{1}{X_{q,i}} - \frac{1}{X'_{d,i}} \right) \sin 2\delta_{m,i} \tag{2}$$

- Equation for rotor angle:

$$\frac{d\delta_i(t)}{dt} = 2\pi f_0 (\overline{\omega_i} - 1) \tag{3}$$

Table 3: Integer variables ($\mathbf{z}$). TO BE DEVELOPED

Symbol	Variable	Support
$\mathbf{z}_B$	The vector of binary status variables for branches in the system.	$\mathbb{Z}^m \in [0, 1]$
$\mathbf{z}_F$	A vector indicating whether a fault is applied at each bus	$\mathbb{Z}^n \in [0, 1]$

- Exciter/open circuit voltage model.
  - For the moment we are assuming that $|E'_a|$ is held fixed.

$$\frac{dE'_{a,i}(t)}{dt} = 0$$

- Governor model. For the moment we are assuming the generator mechanical power is fixed.

$$\frac{dP_m}{dt} = 0$$

- Temperature relays. For the moment we assume that the temperature of the branches do not change therefore not tripping relays.

$$\frac{dT_i}{dt} = 0$$

5 Algebraic equations

Real/active power balance equation for all buses:

$$P_{g,i}(\delta_{m,i}, |E'_{a,i}|, |V_i|) - P_{d,i}(|V_i(t)|, t) = \sum_{k=1}^{n} |V_i||V_k|(g_{ik}\cos\theta_{ik} + b_{ik}\sin\theta_{ik}) \quad (4)$$

where $P_{g,i}(\delta_{m,i}, |E'_{a,i}|, |V_i|)$ is defined in Eq. 4 ($P_{g,i} = 0$ for non generator buses), and $P_{d,,i}(|V_i(t)|, t)$ represents the load at bus i. The load can be constant power ($P_{d,i}$ is independent of $|V|$), constant current, constant impedance, exponential, or any combination of these. We will need to do some work to figure out the best load model for our application. Loads will have complex power $S_{d,i}(|V_i|, t)$.

The reactive power equation for all buses:

$$Q_{g,i}(\delta_{m,i}, |E'_{a,i}|, |V_i|) - Q_{d,i}(|V_i|, t) = \sum_{k=1}^{n} |V_i||V_k|(g_{ik}\sin\theta_{ik} - b_{ik}\cos\theta_{ik})$$

where $Q_{g,i}$ is the reactive power generation at bus i, defined as:

$$Q_{g,i}(t) = \begin{cases} \frac{|E'_{a,i}||V_i|}{X'_{d,i}}\cos(\delta_{m,i}) + |V_i|^2\left(\frac{\cos^2\delta_{m,i}}{X'_{d,i}} + \frac{\sin^2\delta_{m,i}}{X_{q,i}}\right) & \forall i \in N_G \\ 0 & \forall i \notin N_G \end{cases} \quad (5)$$

6 Implementation in MATLAB

Each matrix within the "ps" structure defines a set of network elements with similar properties. For instance, "ps.gen" contains all the information about the generators. A row in that matrix represents a specific generator, whereas a column defines a given variable. In order to translate from column number to human readable variable (or vice-versa) we utilize "psconstants.m" to define a shortcut variable "C". E.g., the column that indexes the maximum real power that generators can produce would be column number "C.gen.Pmax" (or 9). The location of the problem variables that we have defined so far are detailed in the following tables:

Table 4: ps.bus (n,C.bu.cols)

Col.	C	Symbol	Variable	Support
1	C.bu.id	i	Bus identifier.	$[i_{min}, i_{max}]$
2	C.bu.type	-	Defines whether the bus is type PQ, PV, slack, isolated or the participation factor.	$[1, 5]$
8	C.bu.Vmag	$\|V_i\|$	The p.u. voltage magnitude for this bus.	$[-\infty, \infty]$
9	C.bu.Vang	θ_d	The bus angle in degrees.	$[-180, 180]$

*C.bus = C.bu

Table 5: ps.branch (m,C.br.cols)

Col.	C	Symbol	Variable	Support
1	C.br.from	F	"from" end bus number.	$[i_{min}, i_{max}]$
2	C.br.to	T	"to" end bus number.	$[i_{min}, i_{max}]$
3	C.br.R	$R_{F,T}$	The p.u. resistance of the line.	$[-\infty, \infty]$
4	C.br.X	$X_{F,T}$	The p.u. reactance of the line.	$[-\infty, \infty]$
5	C.br.B	$B_{F,T}$	The p.u. charging of the line.	$[-\infty, \infty]$

*C.branch = C.br

Table 6: ps.gen (n_g,C.ge.cols)

Col.	C	Symbol	Variable	Support
1	C.ge.bus	i_g	The location of the generator.	$[i_{min}, i_{max}]$
2	C.ge.Pg	$P_{G,i}$	The actual output of the generator.	$[0, P_{max,i}]$
3	C.ge.Qg	$Q_{G,i}$	The actual reactive output of the generator.	$[0, Q_{max,i}]$
4	C.ge.Qmax	$Q_{max,i}$	The max reactive output of the generator.	$[0, \infty]$
5	C.ge.Qmin	$Q_{min,i}$	The min reactive output of the generator.	$[0, \infty]$
9	C.ge.Pmax	$P_{max,i}$	The max output of the generator.	$[0, \infty]$
10	C.ge.Pmin	$P_{min,i}$	The min output of the generator.	$[0, \infty]$
15	C.ge.type	-	Defines whether the bus is type PQ, PV, slack, isolated or the participation factor.	$[1, 5]$

*C.gen = C.ge

Table 7: ps.mac (n_g,C.ma.cols)

Col.	C	Symbol	Variable	Support
1	C.ma.gen	i_g	The location of the generator.	$[i_{min}, i_{max}]$
2	C.ma.r	r_i	The coil-winding AC resistance.	$[0, \infty]$
3	C.ma.Xd	$X_{d,i}$	The synchronous reactance (p.u).	$[0, \infty]$
9	C.ma.D	D	Damping constant.	$[0, \infty]$
10	C.ma.M	M	Machine starting time.	$[0, \infty]$
11	C.ma.Ea	$E_{a,i}$	The internal (open circuit) voltage for the machine.	$[0, \infty]$
12	C.ma.Eap	$E'_{a,i}$	The transient internal voltage for the machine.	$[0, \infty]$
13	C.ma.Pm	$P_{m,i}$	The mechanical power forcing the generator rotation	$[0, \infty]$
14	C.ma.delta	δ_i	The angle corresponding to the position of the rotor, relative to the system reference angle θ_r.	$[-\pi, \pi]$
15	C.ma.omega	ω_i	The angular speed of the generator, relative to a 60 Hz reference.	$[-\infty, \infty]$

*C.mac = C.ma