FUNDAMENTALS OF BIOSTATISTICS

SEVENTH EDITION

BERNARD ROSNER

Fundamentals of Biostatistics

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SEVENTH EDITION

Bernard Rosner
Harvard University

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This book is dedicated to my wife, Cynthia,
and my children, Sarah, David, and Laura

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Preface

This introductory-level biostatistics text is designed for upper-level undergraduate or graduate students interested in medicine or other health-related areas. It requires no previous background in statistics, and its mathematical level assumes only a knowledge of algebra.

Fundamentals of Biostatistics evolved from notes that I have used in a biostatistics course taught to Harvard University undergraduates and Harvard Medical School students over the past 30 years. I wrote this book to help motivate students to master the statistical methods that are most often used in the medical literature. From the student's viewpoint, it is important that the example material used to develop these methods is representative of what actually exists in the literature. Therefore, most of the examples and exercises in this book are based either on actual articles from the medical literature or on actual medical research problems I have encountered during my consulting experience at the Harvard Medical School.

The Approach

Most introductory statistics texts either use a completely nonmathematical, cookbook approach or develop the material in a rigorous, sophisticated mathematical framework. In this book, however, I follow an intermediate course, minimizing the amount of mathematical formulation but giving complete explanations of all the important concepts. Every new concept in this book is developed systematically through completely worked-out examples from current medical research problems. In addition, I introduce computer output where appropriate to illustrate these concepts.

I initially wrote this text for the introductory biostatistics course. However, the field has changed rapidly over the past 10 years; because of the increased power of newer statistical packages, we can now perform more sophisticated data analyses than ever before. Therefore, a second goal of this text is to present these new techniques *at an introductory level* so that students can become familiar with them without having to wade through specialized (and, usually, more advanced) statistical texts.

To differentiate these two goals more clearly, I included most of the content for the introductory course in the first 12 chapters. More advanced statistical techniques used in recent epidemiologic studies are covered in Chapter 13, "Design and Analysis Techniques for Epidemiologic Studies" and Chapter 14, "Hypothesis Testing: Person-Time Data."

Changes in the Seventh Edition

For this edition, I have added seven new sections and added new content to one other section. Features new to this edition include the following:

- The data sets are now available on the book's Companion Website at www.cengage.com/statistics/rosner in an expanded set of formats, including Excel, Minitab®, SPSS, JMP, SAS, Stata, R, and ASCII formats.
- Data and medical research findings in Examples have been updated.
- New or expanded coverage of the following topics:
  - Interval estimates for rank correlation coefficients (Section 11.13)
  - Mixed effect models (Section 12.10)
  - Attributable risk (Section 13.4)
  - Extensions to logistic regression (Section 13.9)
  - Regression models for clustered binary data (Section 13.13)
  - Longitudinal data analysis (Section 13.14)
  - Parametric survival analysis (Section 14.13)
  - Parametric regression models for survival data (Section 14.14)

The new sections and the expanded sections for this edition have been indicated by an asterisk in the table of contents.

Exercises

This edition contains 1438 exercises; 244 of these exercises are new. Data and medical research findings in the problems have been updated where appropriate. All problems based on the data sets are included. Problems marked by an asterisk (*) at the end of each chapter have corresponding brief solutions in the answer section at the back of the book. Based on requests from students for more completely solved problems, approximately 600 additional problems and complete solutions are presented in the *Study Guide* available on the Companion Website accompanying this text. In addition, approximately 100 of these problems are included in a Miscellaneous Problems section and are randomly ordered so that they are not tied to a specific chapter in the book. This gives the student additional practice in determining what method to use in what situation. Complete instructor solutions to all exercises are available in secure online format through Cengage's *Solution Builder* service. Adopting instructors can sign up for access at www.cengage.com/solutionbuilder.

Computation Method

The method of handling computations is similar to that used in the sixth edition. All intermediate results are carried to full precision (10+ significant digits), even though they are presented with fewer significant digits (usually 2 or 3) in the text. Thus, intermediate results may seem inconsistent with final results in some instances; this, however, is not the case.

Organization

Fundamentals of Biostatistics, Seventh Edition, is organized as follows.

Chapter 1 is an *introductory* chapter that contains an outline of the development of an actual medical study with which I was involved. It provides a unique sense of the role of biostatistics in medical research.

Chapter 2 concerns *descriptive statistics* and presents all the major numeric and graphic tools used for displaying medical data. This chapter is especially important for both consumers and producers of medical literature because much information is actually communicated via descriptive material.

Chapters 3 through 5 discuss *probability*. The basic principles of probability are developed, and the most common probability distributions—such as the binomial and normal distributions—are introduced. These distributions are used extensively in later chapters of the book. The concepts of prior probability and posterior probability are also introduced.

Chapters 6 through 10 cover some of the basic methods of *statistical inference.*

Chapter 6 introduces the concept of drawing random samples from populations. The difficult notion of a sampling distribution is developed and includes an introduction to the most common sampling distributions, such as the t and chi-square distributions. The basic methods of *estimation,* including an extensive discussion of confidence intervals, are also presented.

Chapters 7 and 8 contain the basic principles of *hypothesis testing*. The most elementary hypothesis tests for normally distributed data, such as the t test, are also fully discussed for one- and two-sample problems. The fundamentals of Bayesian inference are explored.

Chapter 9 covers the basic principles of *nonparametric statistics*. The assumptions of normality are relaxed, and distribution-free analogues are developed for the tests in Chapters 7 and 8.

Chapter 10 contains the basic concepts of *hypothesis testing* as applied to categorical data, including some of the most widely used statistical procedures, such as the chi-square test and Fisher's exact test.

Chapter 11 develops the principles of *regression analysis*. The case of simple linear regression is thoroughly covered, and extensions are provided for the multiple-regression case. Important sections on goodness-of-fit of regression models are also included. Also, rank correlation is introduced. Interval estimates for rank correlation coefficients are covered for the first time. Methods for comparing correlation coefficients from dependent samples are also included.

Chapter 12 introduces the basic principles of the *analysis of variance* (ANOVA). The one-way analysis of variance fixed- and random-effects models are discussed. In addition, two-way ANOVA, the analysis of covariance, and mixed effects models are covered. Finally, we discuss nonparametric approaches to one-way ANOVA. Multiple comparison methods including material on the false discovery rate are also provided. A section of mixed models is also included for the first time.

Chapter 13 discusses methods of design and analysis for *epidemiologic studies*. The most important study designs, including the prospective study, the case–control study, the cross-sectional study, and the cross-over design are introduced. The concept of a confounding variable—that is, a variable related to both the disease and the exposure variable—is introduced, and methods for controlling for confounding, which include the Mantel-Haenszel test and multiple-logistic regression, are discussed in detail. Extensions to logistic regression models, including conditional logistic regression, polytomous logistic regression, and ordinal logistic regression, are discussed for the first time. This discussion is followed by the exploration of topics of current interest in epidemiologic data analysis, including meta-analysis (the combination of results from more than one study); correlated binary data techniques (techniques that can be applied when replicate measures, such as data from multiple teeth from the same person, are available for an individual); measurement error methods (useful when there is substantial measurement error in the exposure data collected); equivalence studies (whose objective it is to establish bioequivalence between two treatment modalities rather than that one treatment is superior to the other); and missing-data methods for how to handle missing data in epidemiologic studies. Longitudinal data analysis and generalized estimating equation (GEE) methods are also briefly discussed.

Chapter 14 introduces methods of analysis for *person-time data*. The methods covered in this chapter include those for incidence-rate data, as well as several methods of survival analysis: the Kaplan-Meier survival curve estimator, the log-rank test, and the proportional-hazards model. Methods for testing the assumptions of the proportional-hazards model have also been included. Parametric survival analysis methods are covered for the first time.

Throughout the text—particularly in Chapter 13—I discuss the elements of study designs, including the concepts of matching; cohort studies; case–control studies; retrospective studies; prospective studies; and the sensitivity, specificity, and predictive value of screening tests. These designs are presented in the context of actual samples. In addition, Chapters 7, 8, 10, 11, 13, and 14 contain specific sections on sample-size estimation for different statistical situations.

A flowchart of appropriate methods of statistical inference (see pages 841–846) is a handy reference guide to the methods developed in this book. Page references for each major method presented in the text are also provided. In Chapters 7–8 and Chapters 10–14, I refer students to this flowchart to give them some perspective on how the methods discussed in a given chapter fit with all the other statistical methods introduced in this book.

In addition, I have provided an index of applications, grouped by *medical specialty,* summarizing all the examples and problems this book covers.

Acknowledgments

I am indebted to Debra Sheldon, the late Marie Sheehan, and Harry Taplin for their invaluable help typing the manuscript, to Dale Rinkel for invaluable help in typing problem solutions, and to Marion McPhee for helping to prepare the data sets on the Companion Website. I am also indebted to Brian Claggett for updating solutions to problems for this edition, and to Daad Abraham for typing the Index of Applications. In addition, I wish to thank the manuscript reviewers, among them: Emilia Bagiella, Columbia University; Ron Brookmeyer, Johns Hopkins University; Mark van der Laan, University of California, Berkeley; and John Wilson, University of Pittsburgh. I would also like to thank my colleagues Nancy Cook, who was instrumental in helping me develop the part of Section 12.4 on the false-discovery rate, and Robert Glynn, who was instrumental in developing Section 13.16 on missing data and Section 14.11 on testing the assumptions of the proportional-hazards model.

In addition, I wish to thank Molly Taylor, Daniel Seibert, Shaylin Walsh, and Laura Wheel, who were instrumental in providing editorial advice and in preparing the manuscript.

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Finally, I would like to acknowledge Leslie Miller, Andrea Wagner, Loren Fishman, and Frank Santopietro, without whose clinical help the current edition of this book would not have been possible.

Bernard Rosner

About the Author

Bernard Rosner is Professor of Medicine (Biostatistics) at Harvard Medical School and Professor of Biostatistics in the Harvard School of Public Health. He received a B.A. in Mathematics from Columbia University in 1967, an M.S. in Statistics from Stanford University in 1968, and a Ph.D. in Statistics from Harvard University in 1971.

He has more than 30 years of biostatistical consulting experience with other investigators at the Harvard Medical School. Special areas of interest include cardiovascular disease, hypertension, breast cancer, and ophthalmology. Many of the examples and exercises used in the text reflect data collected from actual studies in conjunction with his consulting experience. In addition, he has developed new biostatistical methods, mainly in the areas of longitudinal data analysis, analysis of clustered data (such as data collected in families or from paired organ systems in the same person), measurement error methods, and outlier detection methods. You will see some of these methods introduced in this book at an elementary level. He was married in 1972 to his wife, Cynthia, and has three children, Sarah, David, and Laura, each of whom has contributed examples for this book.

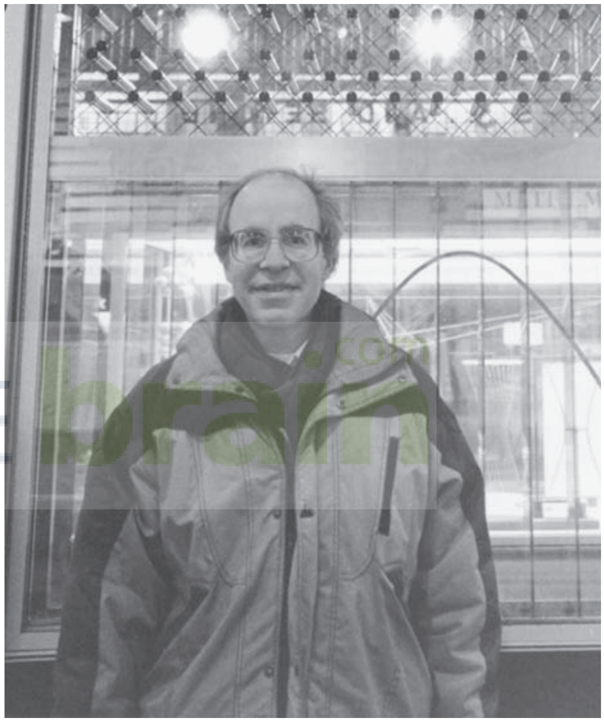


Photo courtesy of the Museum of Science, Boston

APPENDIX
Tables

Table 1 Exact binomial probabilities $Pr(X = k) = \binom{n}{k} p^k q^{n-k}$

n	k	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
2	0	.9025	.8100	.7225	.6400	.5625	.4900	.4225	.3600	.3025	.2500
	1	.0950	.1800	.2550	.3200	.3750	.4200	.4550	.4800	.4950	.5000
	2	.0025	.0100	.0225	.0400	.0625	.0900	.1225	.1600	.2025	.2500
3	0	.8574	.7290	.6141	.5120	.4219	.3430	.2746	.2160	.1664	.1250
	1	.1354	.2430	.3251	.3840	.4219	.4410	.4436	.4320	.4084	.3750
	2	.0071	.0270	.0574	.0960	.1406	.1890	.2389	.2880	.3341	.3750
	3	.0001	.0010	.0034	.0080	.0156	.0270	.0429	.0640	.0911	.1250
4	0	.8145	.6561	.5220	.4096	.3164	.2401	.1785	.1296	.0915	.0625
	1	.1715	.2916	.3685	.4096	.4219	.4116	.3845	.3456	.2995	.2500
	2	.0135	.0486	.0975	.1536	.2109	.2646	.3105	.3456	.3675	.3750
	3	.0005	.0036	.0115	.0256	.0469	.0756	.1115	.1536	.2005	.2500
	4	.0000	.0001	.0005	.0016	.0039	.0081	.0150	.0256	.0410	.0625
5	0	.7738	.5905	.4437	.3277	.2373	.1681	.1160	.0778	.0503	.0313
	1	.2036	.3280	.3915	.4096	.3955	.3602	.3124	.2592	.2059	.1563
	2	.0214	.0729	.1382	.2048	.2637	.3087	.3364	.3456	.3369	.3125
	3	.0011	.0081	.0244	.0512	.0879	.1323	.1811	.2304	.2757	.3125
	4	.0000	.0004	.0022	.0064	.0146	.0283	.0488	.0768	.1128	.1563
	5	.0000	.0000	.0001	.0003	.0010	.0024	.0053	.0102	.0185	.0313
6	0	.7351	.5314	.3771	.2621	.1780	.1176	.0754	.0467	.0277	.0156
	1	.2321	.3543	.3993	.3932	.3560	.3025	.2437	.1866	.1359	.0938
	2	.0305	.0984	.1762	.2458	.2966	.3241	.3280	.3110	.2780	.2344
	3	.0021	.0146	.0415	.0819	.1318	.1852	.2355	.2765	.3032	.3125
	4	.0001	.0012	.0055	.0154	.0330	.0595	.0951	.1382	.1861	.2344
	5	.0000	.0001	.0004	.0015	.0044	.0102	.0205	.0369	.0609	.0938
	6	.0000	.0000	.0000	.0001	.0002	.0007	.0018	.0041	.0083	.0156
7	0	.6983	.4783	.3206	.2097	.1335	.0824	.0490	.0280	.0152	.0078
	1	.2573	.3720	.3960	.3670	.3115	.2471	.1848	.1306	.0872	.0547
	2	.0406	.1240	.2097	.2753	.3115	.3177	.2985	.2613	.2140	.1641
	3	.0036	.0230	.0617	.1147	.1730	.2269	.2679	.2903	.2918	.2734
	4	.0002	.0026	.0109	.0287	.0577	.0972	.1442	.1935	.2388	.2734
	5	.0000	.0002	.0012	.0043	.0115	.0250	.0466	.0774	.1172	.1641
	6	.0000	.0000	.0001	.0004	.0013	.0036	.0084	.0172	.0320	.0547
	7	.0000	.0000	.0000	.0000	.0001	.0002	.0006	.0016	.0037	.0078
8	0	.6634	.4305	.2725	.1678	.1001	.0576	.0319	.0168	.0084	.0039
	1	.2793	.3826	.3847	.3355	.2670	.1977	.1373	.0896	.0548	.0313
	2	.0515	.1488	.2376	.2936	.3115	.2965	.2587	.2090	.1569	.1094
	3	.0054	.0331	.0839	.1468	.2076	.2541	.2786	.2787	.2568	.2188

(continued on next page)

Table 1 Exact binomial probabilities $Pr(X = k) = \binom{n}{k} p^k q^{n-k}$ *(continued)*

n	*k*	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
	4	.0004	.0046	.0185	.0459	.0865	.1361	.1875	.2322	.2627	.2734
	5	.0000	.0004	.0026	.0092	.0231	.0467	.0808	.1239	.1719	.2188
	6	.0000	.0000	.0002	.0011	.0038	.0100	.0217	.0413	.0703	.1094
	7	.0000	.0000	.0000	.0001	.0004	.0012	.0033	.0079	.0164	.0313
	8	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0007	.0017	.0039
9	0	.6302	.3874	.2316	.1342	.0751	.0404	.0207	.0101	.0046	.0020
	1	.2985	.3874	.3679	.3020	.2253	.1556	.1004	.0605	.0339	.0176
	2	.0629	.1722	.2597	.3020	.3003	.2668	.2162	.1612	.1110	.0703
	3	.0077	.0446	.1069	.1762	.2336	.2668	.2716	.2508	.2119	.1641
	4	.0006	.0074	.0283	.0661	.1168	.1715	.2194	.2508	.2600	.2461
	5	.0000	.0008	.0050	.0165	.0389	.0735	.1181	.1672	.2128	.2461
	6	.0000	.0001	.0006	.0028	.0087	.0210	.0424	.0743	.1160	.1641
	7	.0000	.0000	.0000	.0003	.0012	.0039	.0098	.0212	.0407	.0703
	8	.0000	.0000	.0000	.0000	.0001	.0004	.0013	.0035	.0083	.0176
	9	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0008	.0020
10	0	.5987	.3487	.1969	.1074	.0563	.0282	.0135	.0060	.0025	.0010
	1	.3151	.3874	.3474	.2684	.1877	.1211	.0725	.0403	.0207	.0098
	2	.0746	.1937	.2759	.3020	.2816	.2335	.1757	.1209	.0763	.0439
	3	.0105	.0574	.1298	.2013	.2503	.2668	.2522	.2150	.1665	.1172
	4	.0010	.0112	.0401	.0881	.1460	.2001	.2377	.2508	.2384	.2051
	5	.0001	.0015	.0085	.0264	.0584	.1029	.1536	.2007	.2340	.2461
	6	.0000	.0001	.0012	.0055	.0162	.0368	.0689	.1115	.1596	.2051
	7	.0000	.0000	.0001	.0008	.0031	.0090	.0212	.0425	.0746	.1172
	8	.0000	.0000	.0000	.0001	.0004	.0014	.0043	.0106	.0229	.0439
	9	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0016	.0042	.0098
	10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0010
11	0	.5688	.3138	.1673	.0859	.0422	.0198	.0088	.0036	.0014	.0005
	1	.3293	.3835	.3248	.2362	.1549	.0932	.0518	.0266	.0125	.0054
	2	.0867	.2131	.2866	.2953	.2581	.1998	.1395	.0887	.0513	.0269
	3	.0137	.0710	.1517	.2215	.2581	.2568	.2254	.1774	.1259	.0806
	4	.0014	.0158	.0536	.1107	.1721	.2201	.2428	.2365	.2060	.1611
	5	.0001	.0025	.0132	.0388	.0803	.1321	.1830	.2207	.2360	.2256
	6	.0000	.0003	.0023	.0097	.0268	.0566	.0985	.1471	.1931	.2256
	7	.0000	.0000	.0003	.0017	.0064	.0173	.0379	.0701	.1128	.1611
	8	.0000	.0000	.0000	.0002	.0011	.0037	.0102	.0234	.0462	.0806
	9	.0000	.0000	.0000	.0000	.0001	.0005	.0018	.0052	.0126	.0269
	10	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0007	.0021	.0054
	11	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0005
12	0	.5404	.2824	.1422	.0687	.0317	.0138	.0057	.0022	.0008	.0002
	1	.3413	.3766	.3012	.2062	.1267	.0712	.0368	.0174	.0075	.0029
	2	.0988	.2301	.2924	.2835	.2323	.1678	.1088	.0639	.0339	.0161
	3	.0173	.0852	.1720	.2362	.2581	.2397	.1954	.1419	.0923	.0537
	4	.0021	.0213	.0683	.1329	.1936	.2311	.2367	.2128	.1700	.1208
	5	.0002	.0038	.0193	.0532	.1032	.1585	.2039	.2270	.2225	.1934
	6	.0000	.0005	.0040	.0155	.0401	.0792	.1281	.1766	.2124	.2256
	7	.0000	.0000	.0006	.0033	.0115	.0291	.0591	.1009	.1489	.1934
	8	.0000	.0000	.0001	.0005	.0024	.0078	.0199	.0420	.0762	.1208
	9	.0000	.0000	.0000	.0001	.0004	.0015	.0048	.0125	.0277	.0537
	10	.0000	.0000	.0000	.0000	.0000	.0002	.0008	.0025	.0068	.0161
	11	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0010	.0029
	12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002
13	0	.5133	.2542	.1209	.0550	.0238	.0097	.0037	.0013	.0004	.0001
	1	.3512	.3672	.2774	.1787	.1029	.0540	.0259	.0113	.0045	.0016
	2	.1109	.2448	.2937	.2680	.2059	.1388	.0836	.0453	.0220	.0095
	3	.0214	.0997	.1900	.2457	.2517	.2181	.1651	.1107	.0660	.0349
	4	.0028	.0277	.0838	.1535	.2097	.2337	.2222	.1845	.1350	.0873
	5	.0003	.0055	.0266	.0691	.1258	.1803	.2154	.2214	.1989	.1571

(continued on next page)

Table 1 Exact binomial probabilities $Pr(X = k) = \binom{n}{k} p^k q^{n-k}$ *(continued)*

n	*k*	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
	6	.0000	.0008	.0063	.0230	.0559	.1030	.1546	.1968	.2169	.2095
	7	.0000	.0001	.0011	.0058	.0186	.0442	.0833	.1312	.1775	.2095
	8	.0000	.0000	.0001	.0011	.0047	.0142	.0336	.0656	.1089	.1571
	9	.0000	.0000	.0000	.0001	.0009	.0034	.0101	.0243	.0495	.0873
	10	.0000	.0000	.0000	.0000	.0001	.0006	.0022	.0065	.0162	.0349
	11	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0012	.0036	.0095
	12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0016
	13	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
14	0	.4877	.2288	.1028	.0440	.0178	.0068	.0024	.0008	.0002	.0001
	1	.3593	.3559	.2539	.1539	.0832	.0407	.0181	.0073	.0027	.0009
	2	.1229	.2570	.2912	.2501	.1802	.1134	.0634	.0317	.0141	.0056
	3	.0259	.1142	.2056	.2501	.2402	.1943	.1366	.0845	.0462	.0222
	4	.0037	.0349	.0998	.1720	.2202	.2290	.2022	.1549	.1040	.0611
	5	.0004	.0078	.0352	.0860	.1468	.1963	.2178	.2066	.1701	.1222
	6	.0000	.0013	.0093	.0322	.0734	.1262	.1759	.2066	.2088	.1833
	7	.0000	.0002	.0019	.0092	.0280	.0618	.1082	.1574	.1952	.2095
	8	.0000	.0000	.0003	.0020	.0082	.0232	.0510	.0918	.1398	.1833
	9	.0000	.0000	.0000	.0003	.0018	.0066	.0183	.0408	.0762	.1222
	10	.0000	.0000	.0000	.0000	.0003	.0014	.0049	.0136	.0312	.0611
	11	.0000	.0000	.0000	.0000	.0000	.0002	.0010	.0033	.0093	.0222
	12	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0019	.0056
	13	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0009
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
15	0	.4633	.2059	.0874	.0352	.0134	.0047	.0016	.0005	.0001	.0000
	1	.3658	.3432	.2312	.1319	.0668	.0305	.0126	.0047	.0016	.0005
	2	.1348	.2669	.2856	.2309	.1559	.0916	.0476	.0219	.0090	.0032
	3	.0307	.1285	.2184	.2501	.2252	.1700	.1110	.0634	.0318	.0139
	4	.0049	.0428	.1156	.1876	.2252	.2186	.1792	.1268	.0780	.0417
	5	.0006	.0105	.0449	.1032	.1651	.2061	.2123	.1859	.1404	.0916
	6	.0000	.0019	.0132	.0430	.0917	.1472	.1906	.2066	.1914	.1527
	7	.0000	.0003	.0030	.0138	.0393	.0811	.1319	.1771	.2013	.1964
	8	.0000	.0000	.0005	.0035	.0131	.0348	.0710	.1181	.1647	.1964
	9	.0000	.0000	.0001	.0007	.0034	.0116	.0298	.0612	.1048	.1527
	10	.0000	.0000	.0000	.0001	.0007	.0030	.0096	.0245	.0515	.0916
	11	.0000	.0000	.0000	.0000	.0001	.0006	.0024	.0074	.0191	.0417
	12	.0000	.0000	.0000	.0000	.0000	.0001	.0004	.0016	.0052	.0139
	13	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0010	.0032
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
16	0	.4401	.1853	.0743	.0281	.0100	.0033	.0010	.0003	.0001	.0000
	1	.3706	.3294	.2097	.1126	.0535	.0228	.0087	.0030	.0009	.0002
	2	.1463	.2745	.2775	.2111	.1336	.0732	.0353	.0150	.0056	.0018
	3	.0359	.1423	.2285	.2463	.2079	.1465	.0888	.0468	.0215	.0085
	4	.0061	.0514	.1311	.2001	.2252	.2040	.1553	.1014	.0572	.0278
	5	.0008	.0137	.0555	.1201	.1802	.2099	.2008	.1623	.1123	.0667
	6	.0001	.0028	.0180	.0550	.1101	.1649	.1982	.1983	.1684	.1222
	7	.0000	.0004	.0045	.0197	.0524	.1010	.1524	.1889	.1969	.1746
	8	.0000	.0001	.0009	.0055	.0197	.0487	.0923	.1417	.1812	.1964
	9	.0000	.0000	.0001	.0012	.0058	.0185	.0442	.0840	.1318	.1746
	10	.0000	.0000	.0000	.0002	.0014	.0056	.0167	.0392	.0755	.1222
	11	.0000	.0000	.0000	.0000	.0002	.0013	.0049	.0142	.0337	.0667
	12	.0000	.0000	.0000	.0000	.0000	.0002	.0011	.0040	.0115	.0278
	13	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0008	.0029	.0085
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0018
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

(continued on next page)

Table 1 Exact binomial probabilities $Pr(X = k) = \binom{n}{k} p^k q^{n-k}$ *(continued)*

n	k	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
17	0	.4181	.1668	.0631	.0225	.0075	.0023	.0007	.0002	.0000	.0000
	1	.3741	.3150	.1893	.0957	.0426	.0169	.0060	.0019	.0005	.0001
	2	.1575	.2800	.2673	.1914	.1136	.0581	.0260	.0102	.0035	.0010
	3	.0415	.1556	.2359	.2393	.1893	.1245	.0701	.0341	.0144	.0052
	4	.0076	.0605	.1457	.2093	.2209	.1868	.1320	.0796	.0411	.0182
	5	.0010	.0175	.0668	.1361	.1914	.2081	.1849	.1379	.0875	.0472
	6	.0001	.0039	.0236	.0680	.1276	.1784	.1991	.1839	.1432	.0944
	7	.0000	.0007	.0065	.0267	.0668	.1201	.1685	.1927	.1841	.1484
	8	.0000	.0001	.0014	.0084	.0279	.0644	.1134	.1606	.1883	.1855
	9	.0000	.0000	.0003	.0021	.0093	.0276	.0611	.1070	.1540	.1855
	10	.0000	.0000	.0000	.0004	.0025	.0095	.0263	.0571	.1008	.1484
	11	.0000	.0000	.0000	.0001	.0005	.0026	.0090	.0242	.0525	.0944
	12	.0000	.0000	.0000	.0000	.0001	.0006	.0024	.0081	.0215	.0472
	13	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0021	.0068	.0182
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0004	.0016	.0052
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0010
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
18	0	.3972	.1501	.0536	.0180	.0056	.0016	.0004	.0001	.0000	.0000
	1	.3763	.3002	.1704	.0811	.0338	.0126	.0042	.0012	.0003	.0001
	2	.1683	.2835	.2556	.1723	.0958	.0458	.0190	.0069	.0022	.0006
	3	.0473	.1680	.2406	.2297	.1704	.1046	.0547	.0246	.0095	.0031
	4	.0093	.0700	.1592	.2153	.2130	.1681	.1104	.0614	.0291	.0117
	5	.0014	.0218	.0787	.1507	.1988	.2017	.1664	.1146	.0666	.0327
	6	.0002	.0052	.0301	.0816	.1436	.1873	.1941	.1655	.1181	.0708
	7	.0000	.0010	.0091	.0350	.0820	.1376	.1792	.1892	.1657	.1214
	8	.0000	.0002	.0022	.0120	.0376	.0811	.1327	.1734	.1864	.1669
	9	.0000	.0000	.0004	.0033	.0139	.0386	.0794	.1284	.1694	.1855
	10	.0000	.0000	.0001	.0008	.0042	.0149	.0385	.0771	.1248	.1669
	11	.0000	.0000	.0000	.0001	.0010	.0046	.0151	.0374	.0742	.1214
	12	.0000	.0000	.0000	.0000	.0002	.0012	.0047	.0145	.0354	.0708
	13	.0000	.0000	.0000	.0000	.0000	.0002	.0012	.0045	.0134	.0327
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0011	.0039	.0117
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0009	.0031
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0006
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
	18	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
19	0	.3774	.1351	.0456	.0144	.0042	.0011	.0003	.0001	.0000	.0000
	1	.3774	.2852	.1529	.0685	.0268	.0093	.0029	.0008	.0002	.0000
	2	.1787	.2852	.2428	.1540	.0803	.0358	.0138	.0046	.0013	.0003
	3	.0533	.1796	.2428	.2182	.1517	.0869	.0422	.0175	.0062	.0018
	4	.0112	.0798	.1714	.2182	.2023	.1491	.0909	.0467	.0203	.0074
	5	.0018	.0266	.0907	.1636	.2023	.1916	.1468	.0933	.0497	.0222
	6	.0002	.0069	.0374	.0955	.1574	.1916	.1844	.1451	.0949	.0518
	7	.0000	.0014	.0122	.0443	.0974	.1525	.1844	.1797	.1443	.0961
	8	.0000	.0002	.0032	.0166	.0487	.0981	.1489	.1797	.1771	.1442
	9	.0000	.0000	.0007	.0051	.0198	.0514	.0980	.1464	.1771	.1762
	10	.0000	.0000	.0001	.0013	.0066	.0220	.0528	.0976	.1449	.1762
	11	.0000	.0000	.0000	.0003	.0018	.0077	.0233	.0532	.0970	.1442
	12	.0000	.0000	.0000	.0000	.0004	.0022	.0083	.0237	.0529	.0961
	13	.0000	.0000	.0000	.0000	.0001	.0005	.0024	.0085	.0233	.0518
	14	.0000	.0000	.0000	.0000	.0000	.0001	.0006	.0024	.0082	.0222
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0022	.0074
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0018
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003

(continued on next page)

Table 1 Exact binomial probabilities $Pr(X = k) = \binom{n}{k} p^k q^{n-k}$ *(continued)*

n	k	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
	18	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	19	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
20	0	.3585	.1216	.0388	.0115	.0032	.0008	.0002	.0000	.0000	.0000
	1	.3774	.2702	.1368	.0576	.0211	.0068	.0020	.0005	.0001	.0000
	2	.1887	.2852	.2293	.1369	.0669	.0278	.0100	.0031	.0008	.0002
	3	.0596	.1901	.2428	.2054	.1339	.0716	.0323	.0123	.0040	.0011
	4	.0133	.0898	.1821	.2182	.1897	.1304	.0738	.0350	.0139	.0046
	5	.0022	.0319	.1028	.1746	.2023	.1789	.1272	.0746	.0365	.0148
	6	.0003	.0089	.0454	.1091	.1686	.1916	.1712	.1244	.0746	.0370
	7	.0000	.0020	.0160	.0546	.1124	.1643	.1844	.1659	.1221	.0739
	8	.0000	.0004	.0046	.0222	.0609	.1144	.1614	.1797	.1623	.1201
	9	.0000	.0001	.0011	.0074	.0271	.0654	.1158	.1597	.1771	.1602
	10	.0000	.0000	.0002	.0020	.0099	.0308	.0686	.1171	.1593	.1762
	11	.0000	.0000	.0000	.0005	.0030	.0120	.0336	.0710	.1185	.1602
	12	.0000	.0000	.0000	.0001	.0008	.0039	.0136	.0355	.0727	.1201
	13	.0000	.0000	.0000	.0000	.0002	.0010	.0045	.0146	.0366	.0739
	14	.0000	.0000	.0000	.0000	.0000	.0002	.0012	.0049	.0150	.0370
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0003	.0013	.0049	.0148
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003	.0013	.0046
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0011
	18	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002
	19	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	20	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Table 2 Exact Poisson probabilities $Pr(X = k) = \frac{e^{-\mu}\mu^k}{k!}$

	μ									
k	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0	.6065	.3679	.2231	.1353	.0821	.0498	.0302	.0183	.0111	.0067
1	.3033	.3679	.3347	.2707	.2052	.1494	.1057	.0733	.0500	.0337
2	.0758	.1839	.2510	.2707	.2565	.2240	.1850	.1465	.1125	.0842
3	.0126	.0613	.1255	.1804	.2138	.2240	.2158	.1954	.1687	.1404
4	.0016	.0153	.0471	.0902	.1336	.1680	.1888	.1954	.1898	.1755
5	.0002	.0031	.0141	.0361	.0668	.1008	.1322	.1563	.1708	.1755
6	.0000	.0005	.0035	.0120	.0278	.0504	.0771	.1042	.1281	.1462
7	.0000	.0001	.0008	.0034	.0099	.0216	.0385	.0595	.0824	.1044
8	.0000	.0000	.0001	.0009	.0031	.0081	.0169	.0298	.0463	.0653
9	.0000	.0000	.0000	.0002	.0009	.0027	.0066	.0132	.0232	.0363
10	.0000	.0000	.0000	.0000	.0002	.0008	.0023	.0053	.0104	.0181
11	.0000	.0000	.0000	.0000	.0000	.0002	.0007	.0019	.0043	.0082
12	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0006	.0016	.0034
13	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0006	.0013
14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0005
15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002
16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

	μ									
k	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
0	.0041	.0025	.0015	.0009	.0006	.0003	.0002	.0001	.0001	.0000
1	.0225	.0149	.0098	.0064	.0041	.0027	.0017	.0011	.0007	.0005
2	.0618	.0446	.0318	.0223	.0156	.0107	.0074	.0050	.0034	.0023
3	.1133	.0892	.0688	.0521	.0389	.0286	.0208	.0150	.0107	.0076

(continued on next page)

Table 2 Exact Poisson probabilities $Pr(X = k) = \dfrac{e^{-\mu}\mu^k}{k!}$ *(continued)*

	μ									
k	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
4	.1558	.1339	.1118	.0912	.0729	.0573	.0443	.0337	.0254	.0189
5	.1714	.1606	.1454	.1277	.1094	.0916	.0752	.0607	.0483	.0378
6	.1571	.1606	.1575	.1490	.1367	.1221	.1066	.0911	.0764	.0631
7	.1234	.1377	.1462	.1490	.1465	.1396	.1294	.1171	.1037	.0901
8	.0849	.1033	.1188	.1304	.1373	.1396	.1375	.1318	.1232	.1126
9	.0519	.0688	.0858	.1014	.1144	.1241	.1299	.1318	.1300	.1251
10	.0285	.0413	.0558	.0710	.0858	.0993	.1104	.1186	.1235	.1251
11	.0143	.0225	.0330	.0452	.0585	.0722	.0853	.0970	.1067	.1137
12	.0065	.0113	.0179	.0263	.0366	.0481	.0604	.0728	.0844	.0948
13	.0028	.0052	.0089	.0142	.0211	.0296	.0395	.0504	.0617	.0729
14	.0011	.0022	.0041	.0071	.0113	.0169	.0240	.0324	.0419	.0521
15	.0004	.0009	.0018	.0033	.0057	.0090	.0136	.0194	.0265	.0347
16	.0001	.0003	.0007	.0014	.0026	.0045	.0072	.0109	.0157	.0217
17	.0000	.0001	.0003	.0006	.0012	.0021	.0036	.0058	.0088	.0128
18	.0000	.0000	.0001	.0002	.0005	.0009	.0017	.0029	.0046	.0071
19	.0000	.0000	.0000	.0001	.0002	.0004	.0008	.0014	.0023	.0037
20	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0006	.0011	.0019
21	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0003	.0005	.0009
22	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0004
23	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002
24	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
25	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

	μ									
k	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0
0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1	.0003	.0002	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000
2	.0015	.0010	.0007	.0004	.0003	.0002	.0001	.0001	.0001	.0000
3	.0053	.0037	.0026	.0018	.0012	.0008	.0006	.0004	.0003	.0002
4	.0139	.0102	.0074	.0053	.0038	.0027	.0019	.0013	.0009	.0006
5	.0293	.0224	.0170	.0127	.0095	.0070	.0051	.0037	.0027	.0019
6	.0513	.0411	.0325	.0255	.0197	.0152	.0115	.0087	.0065	.0048
7	.0769	.0646	.0535	.0437	.0353	.0281	.0222	.0174	.0135	.0104
8	.1009	.0888	.0769	.0655	.0551	.0457	.0375	.0304	.0244	.0194
9	.1177	.1085	.0982	.0874	.0765	.0661	.0563	.0473	.0394	.0324
10	.1236	.1194	.1129	.1048	.0956	.0859	.0760	.0663	.0571	.0486
11	.1180	.1194	.1181	.1144	.1087	.1015	.0932	.0844	.0753	.0663
12	.1032	.1094	.1131	.1144	.1132	.1099	.1049	.0984	.0910	.0829
13	.0834	.0926	.1001	.1056	.1089	.1099	.1089	.1060	.1014	.0956
14	.0625	.0728	.0822	.0905	.0972	.1021	.1050	.1060	.1051	.1024
15	.0438	.0534	.0630	.0724	.0810	.0885	.0945	.0989	.1016	.1024
16	.0287	.0367	.0453	.0543	.0633	.0719	.0798	.0866	.0920	.0960
17	.0177	.0237	.0306	.0383	.0465	.0550	.0633	.0713	.0785	.0847
18	.0104	.0145	.0196	.0255	.0323	.0397	.0475	.0554	.0632	.0706
19	.0057	.0084	.0119	.0161	.0213	.0272	.0337	.0409	.0483	.0557
20	.0030	.0046	.0068	.0097	.0133	.0177	.0228	.0286	.0350	.0418
21	.0015	.0024	.0037	.0055	.0079	.0109	.0146	.0191	.0242	.0299
22	.0007	.0012	.0020	.0030	.0045	.0065	.0090	.0121	.0159	.0204
23	.0003	.0006	.0010	.0016	.0024	.0037	.0053	.0074	.0100	.0133
24	.0001	.0003	.0005	.0008	.0013	.0020	.0030	.0043	.0061	.0083
25	.0001	.0001	.0002	.0004	.0006	.0010	.0016	.0024	.0035	.0050
26	.0000	.0000	.0001	.0002	.0003	.0005	.0008	.0013	.0020	.0029
27	.0000	.0000	.0000	.0001	.0001	.0002	.0004	.0007	.0011	.0016
28	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0003	.0005	.0009

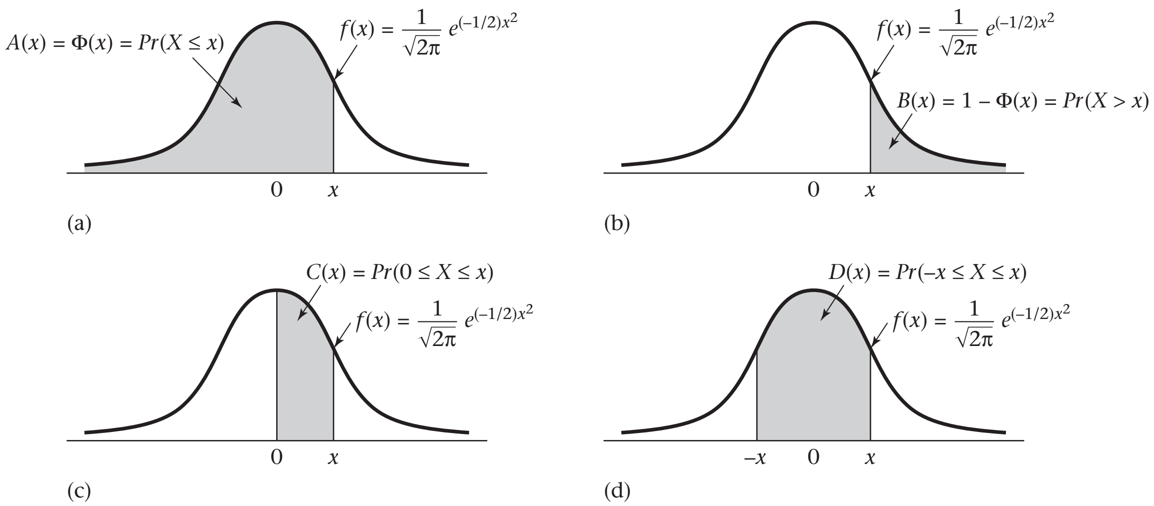
(continued on next page)

Table 2 Exact Poisson probabilities $Pr(X = k) = \frac{e^{-\mu}\mu^k}{k!}$ *(continued)*

	μ									
k	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0
29	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0003	.0004
30	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002
31	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001
32	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
33	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

	μ									
k	15.5	16.0	16.5	17.0	17.5	18.0	18.5	19.0	19.5	20.0
0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
4	.0004	.0003	.0002	.0001	.0001	.0001	.0000	.0000	.0000	.0000
5	.0014	.0010	.0007	.0005	.0003	.0002	.0002	.0001	.0001	.0001
6	.0036	.0026	.0019	.0014	.0010	.0007	.0005	.0004	.0003	.0002
7	.0079	.0060	.0045	.0034	.0025	.0019	.0014	.0010	.0007	.0005
8	.0153	.0120	.0093	.0072	.0055	.0042	.0031	.0024	.0018	.0013
9	.0264	.0213	.0171	.0135	.0107	.0083	.0065	.0050	.0038	.0029
10	.0409	.0341	.0281	.0230	.0186	.0150	.0120	.0095	.0074	.0058
11	.0577	.0496	.0422	.0355	.0297	.0245	.0201	.0164	.0132	.0106
12	.0745	.0661	.0580	.0504	.0432	.0368	.0310	.0259	.0214	.0176
13	.0888	.0814	.0736	.0658	.0582	.0509	.0441	.0378	.0322	.0271
14	.0983	.0930	.0868	.0800	.0728	.0655	.0583	.0514	.0448	.0387
15	.1016	.0992	.0955	.0906	.0849	.0786	.0719	.0650	.0582	.0516
16	.0984	.0992	.0985	.0963	.0929	.0884	.0831	.0772	.0710	.0646
17	.0897	.0934	.0956	.0963	.0956	.0936	.0904	.0863	.0814	.0760
18	.0773	.0830	.0876	.0909	.0929	.0936	.0930	.0911	.0882	.0844
19	.0630	.0699	.0761	.0814	.0856	.0887	.0905	.0911	.0905	.0888
20	.0489	.0559	.0628	.0692	.0749	.0798	.0837	.0866	.0883	.0888
21	.0361	.0426	.0493	.0560	.0624	.0684	.0738	.0783	.0820	.0846
22	.0254	.0310	.0370	.0433	.0496	.0560	.0620	.0676	.0727	.0769
23	.0171	.0216	.0265	.0320	.0378	.0438	.0499	.0559	.0616	.0669
24	.0111	.0144	.0182	.0226	.0275	.0328	.0385	.0442	.0500	.0557
25	.0069	.0092	.0120	.0154	.0193	.0237	.0285	.0336	.0390	.0446
26	.0041	.0057	.0076	.0101	.0130	.0164	.0202	.0246	.0293	.0343
27	.0023	.0034	.0047	.0063	.0084	.0109	.0139	.0173	.0211	.0254
28	.0013	.0019	.0028	.0038	.0053	.0070	.0092	.0117	.0147	.0181
29	.0007	.0011	.0016	.0023	.0032	.0044	.0058	.0077	.0099	.0125
30	.0004	.0006	.0009	.0013	.0019	.0026	.0036	.0049	.0064	.0083
31	.0002	.0003	.0005	.0007	.0010	.0015	.0022	.0030	.0040	.0054
32	.0001	.0001	.0002	.0004	.0006	.0009	.0012	.0018	.0025	.0034
33	.0000	.0001	.0001	.0002	.0003	.0005	.0007	.0010	.0015	.0020
34	.0000	.0000	.0001	.0001	.0002	.0002	.0004	.0006	.0008	.0012
35	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0003	.0005	.0007
36	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0003	.0004
37	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002
38	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001
39	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
40	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Table 3 The normal distribution



x	A[a]	B[b]	C[c]	D[d]	x	A	B	C	D
0.0	.5000	.5000	.0	.0	0.32	.6255	.3745	.1255	.2510
0.01	.5040	.4960	.0040	.0080	0.33	.6293	.3707	.1293	.2586
0.02	.5080	.4920	.0080	.0160	0.34	.6331	.3669	.1331	.2661
0.03	.5120	.4880	.0120	.0239	0.35	.6368	.3632	.1368	.2737
0.04	.5160	.4840	.0160	.0319	0.36	.6406	.3594	.1406	.2812
0.05	.5199	.4801	.0199	.0399	0.37	.6443	.3557	.1443	.2886
0.06	.5239	.4761	.0239	.0478	0.38	.6480	.3520	.1480	.2961
0.07	.5279	.4721	.0279	.0558	0.39	.6517	.3483	.1517	.3035
0.08	.5319	.4681	.0319	.0638	0.40	.6554	.3446	.1554	.3108
0.09	.5359	.4641	.0359	.0717	0.41	.6591	.3409	.1591	.3182
0.10	.5398	.4602	.0398	.0797	0.42	.6628	.3372	.1628	.3255
0.11	.5438	.4562	.0438	.0876	0.43	.6664	.3336	.1664	.3328
0.12	.5478	.4522	.0478	.0955	0.44	.6700	.3300	.1700	.3401
0.13	.5517	.4483	.0517	.1034	0.45	.6736	.3264	.1736	.3473
0.14	.5557	.4443	.0557	.1113	0.46	.6772	.3228	.1772	.3545
0.15	.5596	.4404	.0596	.1192	0.47	.6808	.3192	.1808	.3616
0.16	.5636	.4364	.0636	.1271	0.48	.6844	.3156	.1844	.3688
0.17	.5675	.4325	.0675	.1350	0.49	.6879	.3121	.1879	.3759
0.18	.5714	.4286	.0714	.1428	0.50	.6915	.3085	.1915	.3829
0.19	.5753	.4247	.0753	.1507	0.51	.6950	.3050	.1950	.3899
0.20	.5793	.4207	.0793	.1585	0.52	.6985	.3015	.1985	.3969
0.21	.5832	.4168	.0832	.1663	0.53	.7019	.2981	.2019	.4039
0.22	.5871	.4129	.0871	.1741	0.54	.7054	.2946	.2054	.4108
0.23	.5910	.4090	.0910	.1819	0.55	.7088	.2912	.2088	.4177
0.24	.5948	.4052	.0948	.1897	0.56	.7123	.2877	.2123	.4245
0.25	.5987	.4013	.0987	.1974	0.57	.7157	.2843	.2157	.4313
0.26	.6026	.3974	.1026	.2051	0.58	.7190	.2810	.2190	.4381
0.27	.6064	.3936	.1064	.2128	0.59	.7224	.2776	.2224	.4448
0.28	.6103	.3897	.1103	.2205	0.60	.7257	.2743	.2257	.4515
0.29	.6141	.3859	.1141	.2282	0.61	.7291	.2709	.2291	.4581
0.30	.6179	.3821	.1179	.2358	0.62	.7324	.2676	.2324	.4647
0.31	.6217	.3783	.1217	.2434	0.63	.7357	.2643	.2357	.4713

(continued on next page)

Table 3 The normal distribution *(continued)*

x	A[a]	B[b]	C[c]	D[d]	x	A	B	C	D
0.64	.7389	.2611	.2389	.4778	1.23	.8907	.1093	.3907	.7813
0.65	.7422	.2578	.2422	.4843	1.24	.8925	.1075	.3925	.7850
0.66	.7454	.2546	.2454	.4907	1.25	.8944	.1056	.3944	.7887
0.67	.7486	.2514	.2486	.4971	1.26	.8962	.1038	.3962	.7923
0.68	.7517	.2483	.2517	.5035	1.27	.8980	.1020	.3980	.7959
0.69	.7549	.2451	.2549	.5098	1.28	.8997	.1003	.3997	.7995
0.70	.7580	.2420	.2580	.5161	1.29	.9015	.0985	.4015	.8029
0.71	.7611	.2389	.2611	.5223	1.30	.9032	.0968	.4032	.8064
0.72	.7642	.2358	.2642	.5285	1.31	.9049	.0951	.4049	.8098
0.73	.7673	.2327	.2673	.5346	1.32	.9066	.0934	.4066	.8132
0.74	.7703	.2297	.2703	.5407	1.33	.9082	.0918	.4082	.8165
0.75	.7734	.2266	.2734	.5467	1.34	.9099	.0901	.4099	.8198
0.76	.7764	.2236	.2764	.5527	1.35	.9115	.0885	.4115	.8230
0.77	.7793	.2207	.2793	.5587	1.36	.9131	.0869	.4131	.8262
0.78	.7823	.2177	.2823	.5646	1.37	.9147	.0853	.4147	.8293
0.79	.7852	.2148	.2852	.5705	1.38	.9162	.0838	.4162	.8324
0.80	.7881	.2119	.2881	.5763	1.39	.9177	.0823	.4177	.8355
0.81	.7910	.2090	.2910	.5821	1.40	.9192	.0808	.4192	.8385
0.82	.7939	.2061	.2939	.5878	1.41	.9207	.0793	.4207	.8415
0.83	.7967	.2033	.2967	.5935	1.42	.9222	.0778	.4222	.8444
0.84	.7995	.2005	.2995	.5991	1.43	.9236	.0764	.4236	.8473
0.85	.8023	.1977	.3023	.6047	1.44	.9251	.0749	.4251	.8501
0.86	.8051	.1949	.3051	.6102	1.45	.9265	.0735	.4265	.8529
0.87	.8078	.1922	.3078	.6157	1.46	.9279	.0721	.4279	.8557
0.88	.8106	.1894	.3106	.6211	1.47	.9292	.0708	.4292	.8584
0.89	.8133	.1867	.3133	.6265	1.48	.9306	.0694	.4306	.8611
0.90	.8159	.1841	.3159	.6319	1.49	.9319	.0681	.4319	.8638
0.91	.8186	.1814	.3186	.6372	1.50	.9332	.0668	.4332	.8664
0.92	.8212	.1788	.3212	.6424	1.51	.9345	.0655	.4345	.8690
0.93	.8238	.1762	.3238	.6476	1.52	.9357	.0643	.4357	.8715
0.94	.8264	.1736	.3264	.6528	1.53	.9370	.0630	.4370	.8740
0.95	.8289	.1711	.3289	.6579	1.54	.9382	.0618	.4382	.8764
0.96	.8315	.1685	.3315	.6629	1.55	.9394	.0606	.4394	.8789
0.97	.8340	.1660	.3340	.6680	1.56	.9406	.0594	.4406	.8812
0.98	.8365	.1635	.3365	.6729	1.57	.9418	.0582	.4418	.8836
0.99	.8389	.1611	.3389	.6778	1.58	.9429	.0571	.4429	.8859
1.00	.8413	.1587	.3413	.6827	1.59	.9441	.0559	.4441	.8882
1.01	.8438	.1562	.3438	.6875	1.60	.9452	.0548	.4452	.8904
1.02	.8461	.1539	.3461	.6923	1.61	.9463	.0537	.4463	.8926
1.03	.8485	.1515	.3485	.6970	1.62	.9474	.0526	.4474	.8948
1.04	.8508	.1492	.3508	.7017	1.63	.9484	.0516	.4484	.8969
1.05	.8531	.1469	.3531	.7063	1.64	.9495	.0505	.4495	.8990
1.06	.8554	.1446	.3554	.7109	1.65	.9505	.0495	.4505	.9011
1.07	.8577	.1423	.3577	.7154	1.66	.9515	.0485	.4515	.9031
1.08	.8599	.1401	.3599	.7199	1.67	.9525	.0475	.4525	.9051
1.09	.8621	.1379	.3621	.7243	1.68	.9535	.0465	.4535	.9070
1.10	.8643	.1357	.3643	.7287	1.69	.9545	.0455	.4545	.9090
1.11	.8665	.1335	.3665	.7330	1.70	.9554	.0446	.4554	.9109
1.12	.8686	.1314	.3686	.7373	1.71	.9564	.0436	.4564	.9127
1.13	.8708	.1292	.3708	.7415	1.72	.9573	.0427	.4573	.9146
1.14	.8729	.1271	.3729	.7457	1.73	.9582	.0418	.4582	.9164
1.15	.8749	.1251	.3749	.7499	1.74	.9591	.0409	.4591	.9181
1.16	.8770	.1230	.3770	.7540	1.75	.9599	.0401	.4599	.9199
1.17	.8790	.1210	.3790	.7580	1.76	.9608	.0392	.4608	.9216
1.18	.8810	.1190	.3810	.7620	1.77	.9616	.0384	.4616	.9233
1.19	.8830	.1170	.3830	.7660	1.78	.9625	.0375	.4625	.9249
1.20	.8849	.1151	.3849	.7699	1.79	.9633	.0367	.4633	.9265
1.21	.8869	.1131	.3869	.7737	1.80	.9641	.0359	.4641	.9281
1.22	.8888	.1112	.3888	.7775	1.81	.9649	.0351	.4649	.9297

(continued on next page)

Table 3 The normal distribution *(continued)*

x	A[a]	B[b]	C[c]	D[d]	x	A	B	C	D
1.82	.9656	.0344	.4656	.9312	2.39	.9916	.0084	.4916	.9832
1.83	.9664	.0336	.4664	.9327	2.40	.9918	.0082	.4918	.9836
1.84	.9671	.0329	.4671	.9342	2.41	.9920	.0080	.4920	.9840
1.85	.9678	.0322	.4678	.9357	2.42	.9922	.0078	.4922	.9845
1.86	.9686	.0314	.4686	.9371	2.43	.9925	.0075	.4925	.9849
1.87	.9693	.0307	.4693	.9385	2.44	.9927	.0073	.4927	.9853
1.88	.9699	.0301	.4699	.9399	2.45	.9929	.0071	.4929	.9857
1.89	.9706	.0294	.4706	.9412	2.46	.9931	.0069	.4931	.9861
1.90	.9713	.0287	.4713	.9426	2.47	.9932	.0068	.4932	.9865
1.91	.9719	.0281	.4719	.9439	2.48	.9934	.0066	.4934	.9869
1.92	.9726	.0274	.4726	.9451	2.49	.9936	.0064	.4936	.9872
1.93	.9732	.0268	.4732	.9464	2.50	.9938	.0062	.4938	.9876
1.94	.9738	.0262	.4738	.9476	2.51	.9940	.0060	.4940	.9879
1.95	.9744	.0256	.4744	.9488	2.52	.9941	.0059	.4941	.9883
1.96	.9750	.0250	.4750	.9500	2.53	.9943	.0057	.4943	.9886
1.97	.9756	.0244	.4756	.9512	2.54	.9945	.0055	.4945	.9889
1.98	.9761	.0239	.4761	.9523	2.55	.9946	.0054	.4946	.9892
1.99	.9767	.0233	.4767	.9534	2.56	.9948	.0052	.4948	.9895
2.00	.9772	.0228	.4772	.9545	2.57	.9949	.0051	.4949	.9898
2.01	.9778	.0222	.4778	.9556	2.58	.9951	.0049	.4951	.9901
2.02	.9783	.0217	.4783	.9566	2.59	.9952	.0048	.4952	.9904
2.03	.9788	.0212	.4788	.9576	2.60	.9953	.0047	.4953	.9907
2.04	.9793	.0207	.4793	.9586	2.61	.9955	.0045	.4955	.9909
2.05	.9798	.0202	.4798	.9596	2.62	.9956	.0044	.4956	.9912
2.06	.9803	.0197	.4803	.9606	2.63	.9957	.0043	.4957	.9915
2.07	.9808	.0192	.4808	.9615	2.64	.9959	.0041	.4959	.9917
2.08	.9812	.0188	.4812	.9625	2.65	.9960	.0040	.4960	.9920
2.09	.9817	.0183	.4817	.9634	2.66	.9961	.0039	.4961	.9922
2.10	.9821	.0179	.4821	.9643	2.67	.9962	.0038	.4962	.9924
2.11	.9826	.0174	.4826	.9651	2.68	.9963	.0037	.4963	.9926
2.12	.9830	.0170	.4830	.9660	2.69	.9964	.0036	.4964	.9929
2.13	.9834	.0166	.4834	.9668	2.70	.9965	.0035	.4965	.9931
2.14	.9838	.0162	.4838	.9676	2.71	.9966	.0034	.4966	.9933
2.15	.9842	.0158	.4842	.9684	2.72	.9967	.0033	.4967	.9935
2.16	.9846	.0154	.4846	.9692	2.73	.9968	.0032	.4968	.9937
2.17	.9850	.0150	.4850	.9700	2.74	.9969	.0031	.4969	.9939
2.18	.9854	.0146	.4854	.9707	2.75	.9970	.0030	.4970	.9940
2.19	.9857	.0143	.4857	.9715	2.76	.9971	.0029	.4971	.9942
2.20	.9861	.0139	.4861	.9722	2.77	.9972	.0028	.4972	.9944
2.21	.9864	.0136	.4864	.9729	2.78	.9973	.0027	.4973	.9946
2.22	.9868	.0132	.4868	.9736	2.79	.9974	.0026	.4974	.9947
2.23	.9871	.0129	.4871	.9743	2.80	.9974	.0026	.4974	.9949
2.24	.9875	.0125	.4875	.9749	2.81	.9975	.0025	.4975	.9950
2.25	.9878	.0122	.4878	.9756	2.82	.9976	.0024	.4976	.9952
2.26	.9881	.0119	.4881	.9762	2.83	.9977	.0023	.4977	.9953
2.27	.9884	.0116	.4884	.9768	2.84	.9977	.0023	.4977	.9955
2.28	.9887	.0113	.4887	.9774	2.85	.9978	.0022	.4978	.9956
2.29	.9890	.0110	.4890	.9780	2.86	.9979	.0021	.4979	.9958
2.30	.9893	.0107	.4893	.9786	2.87	.9979	.0021	.4979	.9959
2.31	.9896	.0104	.4896	.9791	2.88	.9980	.0020	.4980	.9960
2.32	.9898	.0102	.4898	.9797	2.89	.9981	.0019	.4981	.9961
2.33	.9901	.0099	.4901	.9802	2.90	.9981	.0019	.4981	.9963
2.34	.9904	.0096	.4904	.9807	2.91	.9982	.0018	.4982	.9964
2.35	.9906	.0094	.4906	.9812	2.92	.9982	.0018	.4982	.9965
2.36	.9909	.0091	.4909	.9817	2.93	.9983	.0017	.4983	.9966
2.37	.9911	.0089	.4911	.9822	2.94	.9984	.0016	.4984	.9967
2.38	.9913	.0087	.4913	.9827	2.95	.9984	.0016	.4984	.9968

(continued on next page)

Table 3 The normal distribution *(continued)*

x	A[a]	B[b]	C[c]	D[d]	x	A	B	C	D
2.96	.9985	.0015	.4985	.9969	3.49	.9998	.0002	.4998	.9995
2.97	.9985	.0015	.4985	.9970	3.50	.9998	.0002	.4998	.9995
2.98	.9986	.0014	.4986	.9971	3.51	.9998	.0002	.4998	.9996
2.99	.9986	.0014	.4986	.9972	3.52	.9998	.0002	.4998	.9996
3.00	.9987	.0013	.4987	.9973	3.53	.9998	.0002	.4998	.9996
3.01	.9987	.0013	.4987	.9974	3.54	.9998	.0002	.4998	.9996
3.02	.9987	.0013	.4987	.9975	3.55	.9998	.0002	.4998	.9996
3.03	.9988	.0012	.4988	.9976	3.56	.9998	.0002	.4998	.9996
3.04	.9988	.0012	.4988	.9976	3.57	.9998	.0002	.4998	.9996
3.05	.9989	.0011	.4989	.9977	3.58	.9998	.0002	.4998	.9997
3.06	.9989	.0011	.4989	.9978	3.59	.9998	.0002	.4998	.9997
3.07	.9989	.0011	.4989	.9979	3.60	.9998	.0002	.4998	.9997
3.08	.9990	.0010	.4990	.9979	3.61	.9998	.0002	.4998	.9997
3.09	.9990	.0010	.4990	.9980	3.62	.9999	.0001	.4999	.9997
3.10	.9990	.0010	.4990	.9981	3.63	.9999	.0001	.4999	.9997
3.11	.9991	.0009	.4991	.9981	3.64	.9999	.0001	.4999	.9997
3.12	.9991	.0009	.4991	.9982	3.65	.9999	.0001	.4999	.9997
3.13	.9991	.0009	.4991	.9983	3.66	.9999	.0001	.4999	.9997
3.14	.9992	.0008	.4992	.9983	3.67	.9999	.0001	.4999	.9998
3.15	.9992	.0008	.4992	.9984	3.68	.9999	.0001	.4999	.9998
3.16	.9992	.0008	.4992	.9984	3.69	.9999	.0001	.4999	.9998
3.17	.9992	.0008	.4992	.9985	3.70	.9999	.0001	.4999	.9998
3.18	.9993	.0007	.4993	.9985	3.71	.9999	.0001	.4999	.9998
3.19	.9993	.0007	.4993	.9986	3.72	.9999	.0001	.4999	.9998
3.20	.9993	.0007	.4993	.9986	3.73	.9999	.0001	.4999	.9998
3.21	.9993	.0007	.4993	.9987	3.74	.9999	.0001	.4999	.9998
3.22	.9994	.0006	.4994	.9987	3.75	.9999	.0001	.4999	.9998
3.23	.9994	.0006	.4994	.9988	3.76	.9999	.0001	.4999	.9998
3.24	.9994	.0006	.4994	.9988	3.77	.9999	.0001	.4999	.9998
3.25	.9994	.0006	.4994	.9988	3.78	.9999	.0001	.4999	.9998
3.26	.9994	.0006	.4994	.9989	3.79	.9999	.0001	.4999	.9998
3.27	.9995	.0005	.4995	.9989	3.80	.9999	.0001	.4999	.9999
3.28	.9995	.0005	.4995	.9990	3.81	.9999	.0001	.4999	.9999
3.29	.9995	.0005	.4995	.9990	3.82	.9999	.0001	.4999	.9999
3.30	.9995	.0005	.4995	.9990	3.83	.9999	.0001	.4999	.9999
3.31	.9995	.0005	.4995	.9991	3.84	.9999	.0001	.4999	.9999
3.32	.9995	.0005	.4995	.9991	3.85	.9999	.0001	.4999	.9999
3.33	.9996	.0004	.4996	.9991	3.86	.9999	.0001	.4999	.9999
3.34	.9996	.0004	.4996	.9992	3.87	.9999	.0001	.4999	.9999
3.35	.9996	.0004	.4996	.9992	3.88	.9999	.0001	.4999	.9999
3.36	.9996	.0004	.4996	.9992	3.89	.9999	.0001	.4999	.9999
3.37	.9996	.0004	.4996	.9992	3.90	1.0000	.0000	.5000	.9999
3.38	.9996	.0004	.4996	.9993	3.91	1.0000	.0000	.5000	.9999
3.39	.9997	.0003	.4997	.9993	3.92	1.0000	.0000	.5000	.9999
3.40	.9997	.0003	.4997	.9993	3.93	1.0000	.0000	.5000	.9999
3.42	.9997	.0003	.4997	.9994	3.94	1.0000	.0000	.5000	.9999
3.43	.9997	.0003	.4997	.9994	3.95	1.0000	.0000	.5000	.9999
3.45	.9997	.0003	.4997	.9994	3.96	1.0000	.0000	.5000	.9999
3.46	.9997	.0003	.4997	.9995	3.97	1.0000	.0000	.5000	.9999
3.47	.9997	.0003	.4997	.9995	3.98	1.0000	.0000	.5000	.9999
3.48	.9997	.0003	.4997	.9995	3.99	1.0000	.0000	.5000	.9999

[a] $A(x) = \Phi(x) = Pr(X \leq x)$, where X is a standard normal distribution.
[b] $B(x) = 1 - \Phi(x) = Pr(X > x)$, where X is a standard normal distribution.
[c] $C(x) = Pr(0 \leq X \leq x)$, where X is a standard normal distribution.
[d] $D(x) = Pr(-x \leq X \leq x)$, where X is a standard normal distribution.

Table 4 Table of 1000 random digits

01	32924	22324	18125	09077	26	96772	16443	39877	04653
02	54632	90374	94143	49295	27	52167	21038	14338	01395
03	88720	43035	97081	83373	28	69644	37198	00028	98195
04	21727	11904	41513	31653	29	71011	62004	81712	87536
05	80985	70799	57975	69282	30	31217	75877	85366	55500
06	40412	58826	94868	52632	31	64990	98735	02999	35521
07	43918	56807	75218	46077	32	48417	23569	59307	46550
08	26513	47480	77410	47741	33	07900	65059	48592	44087
09	18164	35784	44255	30124	34	74526	32601	24482	16981
10	39446	01375	75264	51173	35	51056	04402	58353	37332
11	16638	04680	98617	90298	36	39005	93458	63143	21817
12	16872	94749	44012	48884	37	67883	76343	78155	67733
13	65419	87092	78596	91512	38	06014	60999	87226	36071
14	05207	36702	56804	10498	39	93147	88766	04148	42471
15	78807	79243	13729	81222	40	01099	95731	47622	13294
16	69341	79028	64253	80447	41	89252	01201	58138	13809
17	41871	17566	61200	15994	42	41766	57239	50251	64675
18	25758	04625	43226	32986	43	92736	77800	81996	45646
19	06604	94486	40174	10742	44	45118	36600	68977	68831
20	82259	56512	48945	18183	45	73457	01579	00378	70197
21	07895	37090	50627	71320	46	49465	85251	42914	17277
22	59836	71148	42320	67816	47	15745	37285	23768	39302
23	57133	76610	89104	30481	48	28760	81331	78265	60690
24	76964	57126	87174	61025	49	82193	32787	70451	91141
25	27694	17145	32439	68245	50	89664	50242	12382	39379

Table 5 Percentage points of the *t* distribution ($t_{d,u}$)[a]

Degrees of freedom, d	u .75	.80	.85	.90	.95	.975	.99	.995	.9995
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.598
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.767
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
60	0.679	0.848	1.046	1.296	1.671	2.000	2.390	2.660	3.460
120	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	3.373
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

[a]The uth percentile of a t distribution with d degrees of freedom.

Source: Table 5 is taken from Table III of Fisher and Yates: "Statistical Tables for Biological, Agricultural and Medical Research," published by Longman Group Ltd., London (previously published by Oliver and Boyd Ltd., Edinburgh). Reprinted by permission of Pearson Education Ltd.

Table 6 Percentage points of the chi-square distribution ($\chi^2_{d,u}$)[a]

	u													
d	.005	.01	.025	.05	.10	.25	.50	.75	.90	.95	.975	.99	.995	.999
1	$0.0^{4}393$[b]	$0.0^{3}157$[c]	$0.0^{3}982$[d]	0.00393	0.02	0.10	0.45	1.32	2.71	3.84	5.02	6.63	7.88	10.83
2	0.0100	0.0201	0.0506	0.103	0.21	0.58	1.39	2.77	4.61	5.99	7.38	9.21	10.60	13.81
3	0.0717	0.115	0.216	0.352	0.58	1.21	2.37	4.11	6.25	7.81	9.35	11.34	12.84	16.27
4	0.207	0.297	0.484	0.711	1.06	1.92	3.36	5.39	7.78	9.49	11.14	13.28	14.86	18.47
5	0.412	0.554	0.831	1.15	1.61	2.67	4.35	6.63	9.24	11.07	12.83	15.09	16.75	20.52
6	0.676	0.872	1.24	1.64	2.20	3.45	5.35	7.84	10.64	12.59	14.45	16.81	18.55	22.46
7	0.989	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.02	14.07	16.01	18.48	20.28	24.32
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.22	13.36	15.51	17.53	20.09	21.95	26.12
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	2.60	3.05	3.82	4.57	5.58	7.58	10.34	13.70	17.28	19.68	21.92	24.72	26.76	31.26
12	3.07	3.57	4.40	5.23	6.30	8.44	11.34	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	3.57	4.11	5.01	5.89	7.04	9.30	12.34	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	4.07	4.66	5.63	6.57	7.79	10.17	13.34	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	4.60	5.23	6.27	7.26	8.55	11.04	14.34	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	5.14	5.81	6.91	7.96	9.31	11.91	15.34	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	5.70	6.41	7.56	8.67	10.09	12.79	16.34	20.49	24.77	27.59	30.19	33.41	35.72	40.79
18	6.26	7.01	8.23	9.39	10.86	13.68	17.34	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	6.84	7.63	8.91	10.12	11.65	14.56	18.34	22.72	27.20	30.14	32.85	36.19	38.58	43.82
20	7.43	8.26	9.59	10.85	12.44	15.45	19.34	23.83	28.41	31.41	34.17	37.57	40.00	45.32
21	8.03	8.90	10.28	11.59	13.24	16.34	20.34	24.93	29.62	32.67	35.48	38.93	41.40	46.80
22	8.64	9.54	10.98	12.34	14.04	17.24	21.34	26.04	30.81	33.92	36.78	40.29	42.80	48.27
23	9.26	10.20	11.69	13.09	14.85	18.14	22.34	27.14	32.01	35.17	38.08	41.64	44.18	49.73
24	9.89	10.86	12.40	13.85	15.66	19.04	23.34	28.24	33.20	36.42	39.36	42.98	45.56	51.18
25	10.52	11.52	13.12	14.61	16.47	19.94	24.34	29.34	34.38	37.65	40.65	44.31	46.93	52.62
26	11.16	12.20	13.84	15.38	17.29	20.84	25.34	30.43	35.56	38.89	41.92	45.64	48.29	54.05
27	11.81	12.88	14.57	16.15	18.11	21.75	26.34	31.53	36.74	40.11	43.19	46.96	49.64	55.48
28	12.46	13.56	15.31	16.93	18.94	22.66	27.34	32.62	37.92	41.34	44.46	48.28	50.99	56.89
29	13.12	14.26	16.05	17.71	19.77	23.57	28.34	33.71	39.09	42.56	45.72	49.59	52.34	58.30
30	13.79	14.95	16.79	18.49	20.60	24.48	29.34	34.80	40.26	43.77	46.98	50.89	53.67	59.70
40	20.71	22.16	24.43	26.51	29.05	33.66	39.34	45.62	51.81	55.76	59.34	63.69	66.77	73.40
50	27.99	29.71	32.36	34.76	37.69	42.94	49.33	56.33	63.17	67.50	71.42	76.15	79.49	86.66
60	35.53	37.48	40.48	43.19	46.46	52.29	59.33	66.98	74.40	79.08	83.30	88.38	91.95	99.61
70	43.28	45.44	48.76	51.74	55.33	61.70	69.33	77.58	85.53	90.53	95.02	100.42	104.22	112.32
80	51.17	53.54	57.15	60.39	64.28	71.14	79.33	88.13	96.58	101.88	106.63	112.33	116.32	124.84
90	59.20	61.75	65.65	69.13	73.29	80.62	89.33	98.64	107.56	113.14	118.14	124.12	128.30	137.21
100	67.33	70.06	74.22	77.93	82.36	90.13	99.33	109.14	118.50	124.34	129.56	135.81	140.17	149.45

[a] $\chi^2_{d,u}$ = *u*th percentile of a χ^2 distribution with *d* degrees of freedom.
[b] = 0.0000393
[c] = 0.000157
[d] = 0.000982

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Table 7a Exact two-sided 100% × (1 − α) confidence limits for binomial proportions (α = .05)

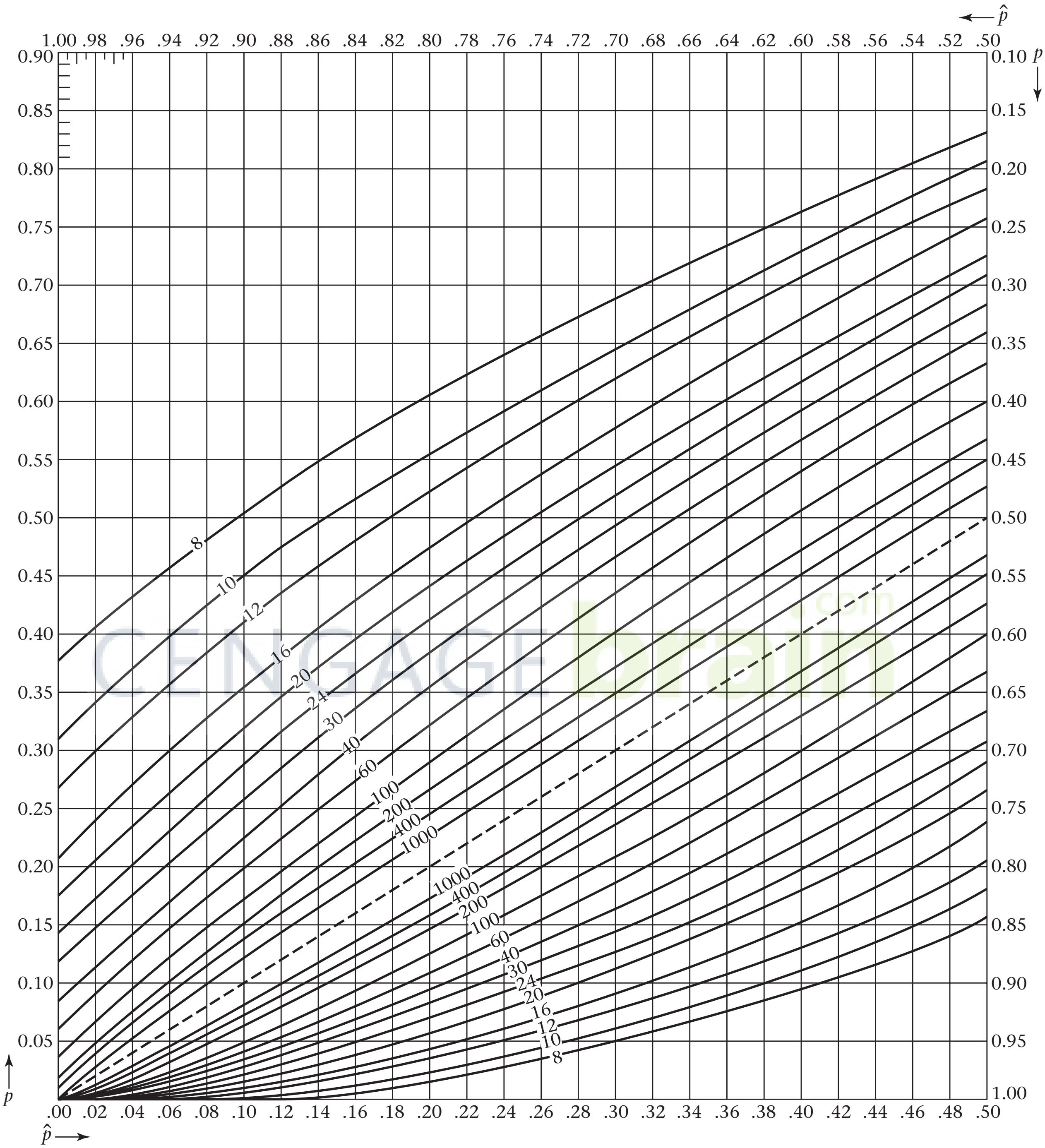

Table 7b Exact two-sided 100% × (1 − α) confidence limits for binomial proportions (α = .01)

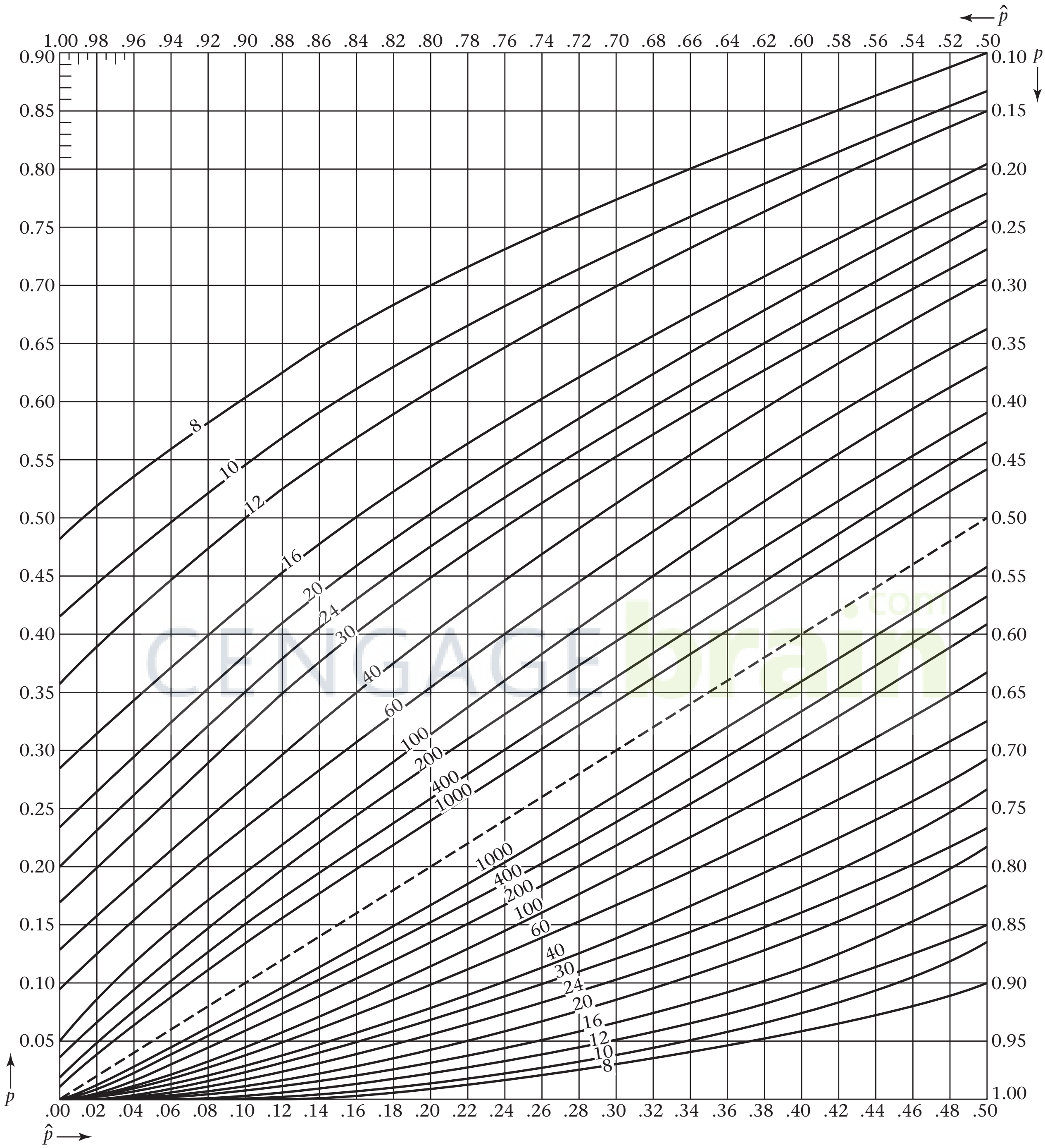

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Table 8 Confidence limits for the expectation of a Poisson variable (μ)

	Confidence level $(1-\alpha)$										
$(1-\alpha)$	0.998		0.99		0.98		0.95		0.90		$(1-2\alpha)$
x	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	x
0	0.00000	6.91	0.00000	5.30	0.0000	4.61	0.0000	3.69	0.0000	3.00	**0**
1	.00100	9.23	.00501	7.43	.0101	6.64	.0253	5.57	.0513	4.74	**1**
2	.0454	11.23	.103	9.27	.149	8.41	.242	7.22	.355	6.30	**2**
3	.191	13.06	.338	10.98	.436	10.05	.619	8.77	.818	7.75	**3**
4	.429	14.79	.672	12.59	.823	11.60	1.09	10.24	1.37	9.15	**4**
5	0.739	16.45	1.08	14.15	1.28	13.11	1.62	11.67	1.97	10.51	**5**
6	1.11	18.06	1.54	15.66	1.79	14.57	2.20	13.06	2.61	11.84	**6**
7	1.52	19.63	2.04	17.13	2.33	16.00	2.81	14.42	3.29	13.15	**7**
8	1.97	21.16	2.57	18.58	2.91	17.40	3.45	15.76	3.98	14.43	**8**
9	2.45	22.66	3.13	20.00	3.51	18.78	4.12	17.08	4.70	15.71	**9**
10	2.96	24.13	3.72	21.40	4.13	20.14	4.80	18.39	5.43	16.96	**10**
11	3.49	25.59	4.32	22.78	4.77	21.49	5.49	19.68	6.17	18.21	**11**
12	4.04	27.03	4.94	24.14	5.43	22.82	6.20	20.96	6.92	19.44	**12**
13	4.61	28.45	5.58	25.50	6.10	24.14	6.92	22.23	7.69	20.67	**13**
14	5.20	29.85	6.23	26.84	6.78	25.45	7.65	23.49	8.46	21.89	**14**
15	5.79	31.24	6.89	28.16	7.48	26.74	8.40	24.74	9.25	23.10	**15**
16	6.41	32.62	7.57	29.48	8.18	28.03	9.15	25.98	10.04	24.30	**16**
17	7.03	33.99	8.25	30.79	8.89	29.31	9.90	27.22	10.83	25.50	**17**
18	7.66	35.35	8.94	32.09	9.62	30.58	10.67	28.45	11.63	26.69	**18**
19	8.31	36.70	9.64	33.38	10.35	31.85	11.44	29.67	12.44	27.88	**19**
20	8.96	38.04	10.35	34.67	11.08	33.10	12.22	30.89	13.25	29.06	**20**
21	9.62	39.38	11.07	35.95	11.82	34.36	13.00	32.10	14.07	30.24	**21**
22	10.29	40.70	11.79	37.22	12.57	35.60	13.79	33.31	14.89	31.42	**22**
23	10.96	42.02	12.52	38.48	13.33	36.84	14.58	34.51	15.72	32.59	**23**
24	11.65	43.33	13.25	39.74	14.09	38.08	15.38	35.71	16.55	33.75	**24**
25	12.34	44.64	14.00	41.00	14.85	39.31	16.18	36.90	17.38	34.92	**25**
26	13.03	45.94	14.74	42.25	15.62	40.53	16.98	38.10	18.22	36.08	**26**
27	13.73	47.23	15.49	43.50	16.40	41.76	17.79	39.28	19.06	37.23	**27**
28	14.44	48.52	16.24	44.74	17.17	42.98	18.61	40.47	19.90	38.39	**28**
29	15.15	49.80	17.00	45.98	17.96	44.19	19.42	41.65	20.75	39.54	**29**
30	15.87	51.08	17.77	47.21	18.74	45.40	20.24	42.83	21.59	40.69	**30**
35	19.52	57.42	21.64	53.32	22.72	51.41	24.38	48.68	25.87	46.40	**35**
40	23.26	63.66	25.59	59.36	26.77	57.35	28.58	54.47	30.20	52.07	**40**
45	27.08	69.83	29.60	65.34	30.88	63.23	32.82	60.21	34.56	57.69	**45**
50	30.96	75.94	33.66	71.27	35.03	69.07	37.11	65.92	38.96	63.29	**50**

Note: If X is the random variable denoting the observed number of events and μ_1, μ_2 are the lower and upper confidence limits for its expectation, μ, then $Pr(\mu_1 \leq \mu \leq \mu_2) = 1 - \alpha$.

Source: Biometrika Tables for Statisticians, 3rd edition, Volume 1, edited by E. S. Pearson and H. O. Hartley. Published for the Biometrika Trustees, Cambridge University Press, Cambridge, England, 1966.

Table 9 Percentage points of the *F* distribution ($F_{d_1,d_2,p}$)

df for denominator, d_2	*p*	*df* for numerator, d_1: 1	2	3	4	5	6	7	8	12	24	∞
1	.90	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	60.71	62.00	63.33
	.95	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	243.9	249.1	254.3
	.975	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	976.7	997.2	1018.
	.99	4052.	5000.	5403.	5625.	5764.	5859.	5928.	5981.	6106.	6235.	6366.
	.995	16211.	20000.	21615.	22500.	23056.	23437.	23715.	23925.	24426.	24940.	25464.
	.999	405280.	500000.	540380.	562500.	576400.	585940.	592870.	598140.	610670.	623500.	636620.
2	.90	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.41	9.45	9.49
	.95	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.41	19.45	19.50
	.975	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.42	39.46	39.50
	.99	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.42	99.46	99.50
	.995	198.5	199.0	199.2	199.2	199.3	199.3	199.4	199.4	199.4	199.5	199.5
	.999	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	999.5	999.5
3	.90	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.22	5.18	5.13
	.95	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.74	8.64	8.53
	.975	17.44	16.04	15.44	15.10	14.88	14.74	14.62	14.54	14.34	14.12	13.90
	.99	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.05	26.60	26.13
	.995	55.55	49.80	47.47	46.20	45.39	44.84	44.43	44.13	43.39	42.62	41.83
	.999	167.00	148.5	141.1	137.1	134.6	132.8	131.6	130.6	128.3	125.9	123.5
4	.90	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.90	3.83	3.76
	.95	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	5.91	5.77	5.63
	.975	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.75	8.51	8.26
	.99	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.37	13.93	13.46
	.995	31.33	26.28	24.26	23.16	22.46	21.98	21.62	21.35	20.70	20.03	19.32
	.999	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	47.41	45.77	44.05
5	.90	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.27	3.19	3.10
	.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.68	4.53	4.36
	.975	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.52	6.28	6.02
	.99	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	9.89	9.47	9.02
	.995	22.78	18.31	16.53	15.56	14.94	14.51	14.20	13.96	13.38	12.78	12.14
	.999	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	26.42	25.13	23.79
6	.90	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.90	2.82	2.72
	.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.00	3.84	3.67
	.975	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.37	5.12	4.85
	.99	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.72	7.31	6.88
	.995	18.64	14.54	12.92	12.03	11.46	11.07	10.79	10.57	10.03	9.47	8.88
	.999	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	17.99	16.90	15.75
7	.90	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.67	2.58	2.47
	.95	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.57	3.41	3.23
	.975	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.67	4.42	4.14
	.99	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.47	6.07	5.65
	.995	16.24	12.40	10.88	10.05	9.52	9.16	8.89	8.68	8.18	7.65	7.08
	.999	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	13.71	12.73	11.70
8	.90	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.50	2.40	2.29
	.95	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.28	3.12	2.93
	.975	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.20	3.95	3.67
	.99	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.67	5.28	4.86
	.995	14.69	11.04	9.60	8.81	8.30	7.95	7.69	7.50	7.01	6.50	5.95
	.999	25.42	18.49	15.83	14.39	13.49	12.86	12.40	12.04	11.19	10.30	9.33
9	.90	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.38	2.28	2.16
	.95	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.07	2.90	2.71
	.975	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	3.87	3.61	3.33
	.99	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.11	4.73	4.31
	.995	13.61	10.11	8.72	7.96	7.47	7.13	6.88	6.69	6.23	5.73	5.19
	.999	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	9.57	8.72	7.81
10	.90	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.28	2.18	2.06
	.95	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.91	2.74	2.54
	.975	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.62	3.37	3.08
	.99	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.71	4.33	3.91
	.995	12.83	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.66	5.17	4.64
	.999	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.45	7.64	6.76
12	.90	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.15	2.04	1.90
	.95	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.69	2.51	2.30
	.975	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.28	3.02	2.72

(continued on next page)

Table 9 Percentage points of the *F* distribution ($F_{d_1,d_2,p}$) *(continued)*

df for denominator, d_2	p	*df* for numerator, d_1: 1	2	3	4	5	6	7	8	12	24	∞
	.99	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.16	3.78	3.36
	.995	11.75	8.51	7.23	6.52	6.07	5.76	5.52	5.35	4.91	4.43	3.90
	.999	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.00	6.25	5.42
14	.90	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.05	1.94	1.80
	.95	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.53	2.35	2.13
	.975	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.05	2.79	2.49
	.99	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	3.80	3.43	3.00
	.995	11.06	7.92	6.68	6.00	5.56	5.26	5.03	4.86	4.43	3.96	3.44
	.999	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.13	5.41	4.60
16	.90	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	1.99	1.87	1.72
	.95	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.42	2.24	2.01
	.975	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	2.89	2.63	2.32
	.99	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.55	3.18	2.75
	.995	10.58	7.51	6.30	5.64	5.21	4.91	4.69	4.52	4.10	3.64	3.11
	.999	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.55	4.85	4.06
18	.90	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	1.93	1.81	1.66
	.95	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.34	2.15	1.92
	.975	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.77	2.50	2.19
	.99	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.37	3.00	2.57
	.995	10.22	7.21	6.03	5.37	4.96	4.66	4.44	4.28	3.86	3.40	2.87
	.999	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.13	4.45	3.67
20	.90	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.89	1.77	1.61
	.95	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.28	2.08	1.84
	.975	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.68	2.41	2.09
	.99	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.23	2.86	2.42
	.995	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.68	3.22	2.69
	.999	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	4.82	4.15	3.38
30	.90	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.77	1.64	1.46
	.95	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.09	1.89	1.62
	.975	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.41	2.14	1.79
	.99	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	2.84	2.47	2.01
	.995	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.18	2.73	2.18
	.999	13.29	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.00	3.36	2.59
40	.90	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.71	1.57	1.38
	.95	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.00	1.79	1.51
	.975	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.29	2.01	1.64
	.99	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.66	2.29	1.80
	.995	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	2.95	2.50	1.93
	.999	12.61	8.25	6.59	5.70	5.13	4.73	4.44	4.21	3.64	3.01	2.23
60	.90	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.66	1.51	1.29
	.95	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	1.92	1.70	1.39
	.975	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.17	1.88	1.48
	.99	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.50	2.12	1.60
	.995	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	2.74	2.29	1.69
	.999	11.97	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.32	2.69	1.89
120	.90	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.60	1.45	1.19
	.95	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.83	1.61	1.25
	.975	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.05	1.76	1.31
	.99	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.34	1.95	1.38
	.995	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.54	2.09	1.43
	.999	11.38	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.02	2.40	1.54
∞	.90	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.55	1.38	1.00
	.95	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.75	1.52	1.00
	.975	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	1.94	1.64	1.00
	.99	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.18	1.79	1.00
	.995	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.36	1.90	1.00
	.999	10.83	6.91	5.42	4.62	4.10	3.74	3.47	3.27	2.74	2.13	1.00

Note: $F_{d_1,d_2,p}$ = pth percentile of an *F* distribution with d_1 and d_2 degrees of freedom.

Source: This table has been reproduced in part with the permission of the Biometrika Trustees, from *Biometrika Tables for Statisticians,* Volume 2, edited by E. S. Pearson and H. O. Hartley, published for the Biometrika Trustees, Cambridge University Press, Cambridge, England, 1972.

Table 10 Critical values for the ESD (Extreme Studentized Deviate) outlier statistic ($ESD_{n,1-\alpha}$, α = .05, .01)

n	$1-\alpha$ = .95	$1-\alpha$ = .99	n	$1-\alpha$ = .95	$1-\alpha$ = .99
5	1.72	1.76	**25**	2.82	3.14
6	1.89	1.97	**26**	2.84	3.16
7	2.02	2.14	**27**	2.86	3.18
8	2.13	2.28	**28**	2.88	3.20
9	2.21	2.39	**29**	2.89	3.22
10	2.29	2.48	**30**	2.91	3.24
11	2.36	2.56	**35**	2.98	3.32
12	2.41	2.64	**40**	3.04	3.38
13	2.46	2.70	**45**	3.09	3.44
14	2.51	2.75	**50**	3.13	3.48
15	2.55	2.81	**60**	3.20	3.56
16	2.59	2.85	**70**	3.26	3.62
17	2.62	2.90	**80**	3.31	3.67
18	2.65	2.93	**90**	3.35	3.72
19	2.68	2.97	**100**	3.38	3.75
20	2.71	3.00	**150**	3.52	3.89
21	2.73	3.03	**200**	3.61	3.98
22	2.76	3.06	**300**	3.72	4.09
23	2.78	3.08	**400**	3.80	4.17
24	2.80	3.11	**500**	3.86	4.23

Note: For values of n not found in the table, the percentiles can be evaluated using the formula $ESD_{n,1-\alpha} = \dfrac{t_{n-2,p}(n-1)}{\sqrt{n(n-2+t_{n-2,p}^2)}}$, where $p = 1-[\alpha/(2n)]$.

Table 11 Two-tailed critical values for the Wilcoxon signed-rank test

n[a]	.10 Lower	.10 Upper	.05 Lower	.05 Upper	.02 Lower	.02 Upper	.01 Lower	.01 Upper
1	—		—		—		—	
2	—		—		—		—	
3	—		—		—		—	
4	—		—		—		—	
5	0	15	—		—		—	
6	2	19	0	21	—		—	
7	3	25	2	26	0	28	—	
8	5	31	3	33	1	35	0	36
9	8	37	5	40	3	42	1	44
10	10	45	8	47	5	50	3	52
11	13	53	10	56	7	59	5	61
12	17	61	13	65	9	69	7	71
13	21	70	17	74	12	79	9	82
14	25	80	21	84	15	90	12	93
15	30	90	25	95	19	101	15	105

[a] n = number of untied pairs.
Source: Figures from "Documenta Geigy Scientific Tables," 6th edition. Reprinted with the kind permission of CIBA-GEIGY Limited, Basel, Switzerland.

Table 12 Two-tailed critical values for the Wilcoxon rank-sum test

	α = .10 n_1[a]						α = .05 n_1					
n_2[b]	4	5	6	7	8	9	4	5	6	7	8	9
	T_l[c] T_r[d]	T_l T_r	T_l T_r	T_l T_r	T_l T_r	T_l T_r	T_l T_r	T_l T_r	T_l T_r	T_l T_r	T_l T_r	T_l T_r
4	11–25	17–33	24–42	32–52	41–63	51–75	10–26	16–34	23–43	31–53	40–64	49–77
5	12–28	19–36	26–46	34–57	44–68	54–81	11–29	17–38	24–48	33–58	42–70	52–83
6	13–31	20–40	28–50	36–62	46–74	57–87	12–32	18–42	26–52	34–64	44–76	55–89
7	14–34	21–44	29–55	39–66	49–79	60–93	13–35	20–45	27–57	36–69	46–82	57–96
8	15–37	23–47	31–59	41–71	51–85	63–99	14–38	21–49	29–61	38–74	49–87	60–102
9	16–40	24–51	33–63	43–76	54–90	66–105	14–42	22–53	31–65	40–79	51–93	62–109
10	17–43	26–54	35–67	45–81	56–96	69–111	15–45	23–57	32–70	42–84	53–99	65–115
11	18–46	27–58	37–71	47–86	59–101	72–117	16–48	24–61	34–74	44–89	55–105	68–121
12	19–49	28–62	38–76	49–91	62–106	75–123	17–51	26–64	35–79	46–94	58–110	71–127
13	20–52	30–65	40–80	52–95	64–112	78–129	18–54	27–68	37–83	48–99	60–116	73–134
14	21–55	31–69	42–84	54–100	67–117	81–135	19–57	28–72	38–88	50–104	62–122	76–140
15	22–58	33–72	44–88	56–105	69–123	84–141	20–60	29–76	40–92	52–109	65–127	79–146
16	24–60	34–76	46–92	58–110	72–128	87–147	21–63	30–80	42–96	54–114	67–133	82–152
17	25–63	35–80	47–97	61–114	75–133	90–153	21–67	32–83	43–101	56–119	70–138	84–159
18	26–66	37–83	49–101	63–119	77–139	93–159	22–70	33–87	45–105	58–124	72–144	87–165
19	27–69	38–87	51–105	65–124	80–144	96–165	23–73	34–91	46–110	60–129	74–150	90–171
20	28–72	40–90	53–109	67–129	83–149	99–171	24–76	35–95	48–114	62–134	77–155	93–177
21	29–75	41–94	55–113	69–134	85–155	102–177	25–79	37–98	50–118	64–139	79–161	95–184
22	30–78	43–97	57–117	72–138	88–160	105–183	26–82	38–102	51–123	66–144	81–167	98–190
23	31–81	44–101	58–122	74–143	90–166	108–189	27–85	39–106	53–127	68–149	84–172	101–196
24	32–84	45–105	60–126	76–148	93–171	111–195	27–89	40–110	54–132	70–154	86–178	104–202
25	33–87	47–108	62–130	78–153	96–176	114–201	28–92	42–113	56–136	72–159	89–183	107–208
26	34–90	48–112	64–134	81–157	98–182	117–207	29–95	43–117	58–140	74–164	91–189	109–215
27	35–93	50–115	66–138	83–162	101–187	120–213	30–98	44–121	59–145	76–169	93–195	112–221
28	36–96	51–119	67–143	85–167	103–193	123–219	31–101	45–125	61–149	78–174	96–200	115–227
29	37–99	53–122	69–147	87–172	106–198	126–225	32–104	47–128	63–153	80–179	98–206	118–233
30	38–102	54–126	71–151	89–177	109–203	129–231	33–107	48–132	64–158	82–184	101–211	121–239
31	39–105	55–130	73–155	92–181	111–209	132–237	34–110	49–136	66–162	84–189	103–217	123–246
32	40–108	57–133	75–159	94–186	114–214	135–243	34–114	50–140	67–167	86–194	106–222	126–252
33	41–111	58–137	77–163	96–191	117–219	138–249	35–117	52–143	69–171	88–199	108–228	129–258
34	42–114	60–140	78–168	98–196	119–225	141–255	36–120	53–147	71–175	90–204	110–234	132–264
35	43–117	61–144	80–172	100–201	122–230	144–261	37–123	54–151	72–180	92–209	113–239	135–270
36	44–120	62–148	82–176	102–206	124–236	148–266	38–126	55–155	74–184	94–214	115–245	137–277
37	45–123	64–151	84–180	105–210	127–241	151–272	39–129	57–158	76–188	96–219	117–251	140–283
38	46–126	65–155	85–185	107–215	130–246	154–278	40–132	58–162	77–193	98–224	120–256	143–289
39	47–129	67–158	87–189	109–220	132–252	157–284	41–135	59–166	79–197	100–229	122–262	146–295
40	48–132	68–162	89–193	111–225	135–257	160–290	41–139	60–170	80–202	102–234	125–267	149–301
41	49–135	69–166	91–197	114–229	138–262	163–296	42–142	61–174	82–206	104–239	127–273	151–308
42	50–138	71–169	93–201	116–234	140–268	166–302	43–145	63–177	84–210	106–244	129–279	154–314
43	51–141	72–173	95–205	118–239	143–273	169–308	44–148	64–181	85–215	108–249	132–284	157–320
44	52–144	74–176	96–210	120–244	146–278	172–314	45–151	65–185	87–219	110–254	134–290	160–326
45	53–147	75–180	98–214	123–248	148–284	175–320	46–154	66–189	88–224	112–259	137–295	163–332
46	55–149	77–183	100–218	125–253	151–289	178–326	47–157	68–192	90–228	114–264	139–301	165–339
47	56–152	78–187	102–222	127–258	154–294	181–332	48–160	69–196	92–232	116–269	141–307	168–345
48	57–155	79–191	104–226	129–263	156–300	184–338	48–164	70–200	93–237	118–274	144–312	171–351
49	58–158	81–194	106–230	132–267	159–305	187–344	49–167	71–204	95–241	120–279	146–318	174–357
50	59–161	82–198	107–235	134–272	162–310	190–350	50–170	73–207	97–245	122–284	149–323	177–363

[a] n_1 = minimum of the two sample sizes.
[b] n_2 = maximum of the two sample sizes.
[c] T_l = lower critical value for the rank sum in the first sample.
[d] T_r = upper critical value for the rank sum in the first sample.

Table 12 Two-tailed critical values for the Wilcoxon rank-sum test *(continued)*

	$\alpha = .02$ n_1[a]						$\alpha = .01$ n_1					
n_2[b]	4	5	6	7	8	9	4	5	6	7	8	9
	T_l[c] T_r[d]	T_l T_r	T_l T_r	T_l T_r	T_l T_r	T_l T_r	T_l T_r	T_l T_r	T_l T_r	T_l T_r	T_l T_r	T_l T_r
4	— —	15–35	22–44	29–55	38–66	48–78	— —	— —	21–45	28–56	37–67	46–80
5	10–30	16–39	23–49	31–60	40–72	50–85	— —	15–40	22–50	29–62	38–74	48–87
6	11–33	17–43	24–54	32–66	42–78	52–92	10–34	16–44	23–55	31–67	40–80	50–94
7	11–37	18–47	25–59	34–71	43–85	54–99	10–38	16–49	24–60	32–73	42–86	52–101
8	12–40	19–51	27–63	35–77	45–91	56–106	11–41	17–53	25–65	34–78	43–93	54–108
9	13–43	20–55	28–68	37–82	47–97	59–112	11–45	18–57	26–70	35–84	45–99	56–115
10	13–47	21–59	29–73	39–87	49–103	61–119	12–48	19–61	27–75	37–89	47–105	58–122
11	14–50	22–63	30–78	40–93	51–109	63–126	12–52	20–65	28–80	38–95	49–111	61–128
12	15–53	23–67	32–82	42–98	53–115	66–132	13–55	21–69	30–84	40–100	51–117	63–135
13	15–57	24–71	33–87	44–103	56–120	68–139	13–59	22–73	31–89	41–106	53–123	65–142
14	16–60	25–75	34–92	45–109	58–126	71–145	14–62	22–78	32–94	43–111	54–130	67–149
15	17–63	26–79	36–96	47–114	60–132	73–152	15–65	23–82	33–99	44–117	56–136	69–156
16	17–67	27–83	37–101	49–119	62–138	76–158	15–69	24–86	34–104	46–122	58–142	72–162
17	18–70	28–87	39–105	51–124	64–144	78–165	16–72	25–90	36–108	47–128	60–148	74–169
18	19–73	29–91	40–110	52–130	66–150	81–171	16–76	26–94	37–113	49–133	62–154	76–176
19	19–77	30–95	41–115	54–135	68–156	83–178	17–79	27–98	38–118	50–139	64–160	78–183
20	20–80	31–99	43–119	56–140	70–162	85–185	18–82	28–102	39–123	52–144	66–166	81–189
21	21–83	32–103	44–124	58–145	72–168	88–191	18–86	29–106	40–128	53–150	68–172	83 196
22	21–87	33–107	45–129	59–151	74–174	90–198	19–89	29–111	42–132	55–155	70–178	85–203
23	22–90	34 111	47–133	61–156	76–180	93–204	19–93	30–115	43–137	57–160	71–185	88–209
24	23–93	35 115	48 138	63–161	78–186	95–211	20–96	31–119	44–142	58–166	73–191	90–216
25	23–97	36 119	50 142	64–167	81–191	98–217	20–100	32–123	45–147	60–171	75–197	92–223
26	24–100	37–123	51–147	66–172	83–197	100–224	21–103	33–127	46–152	61–177	77–203	94–230
27	25–103	38–127	52–152	68–177	85–203	103–230	22–106	34–131	48–156	63–182	79–209	97–236
28	26–106	39–131	54–156	70–182	87–209	105–237	22–110	35–135	49–161	64–188	81–215	99–243
29	26–110	40–135	55–161	71–188	89–215	108–243	23–113	36–139	50–166	66–193	83–221	101–250
30	27–113	41–139	56–166	73–193	91–221	110–250	23–117	37–143	51–171	68–198	85–227	103–257
31	28–116	42–143	58–170	75–198	93–227	112–257	24–120	37–148	53–175	68–204	87–233	106–263
32	28–120	43–147	59–175	77–203	95–233	115–263	24–124	38–152	54–180	71–209	89–239	108–270
33	29–123	44–151	61–179	78–209	97–239	117–270	25–127	39–156	55–185	72–215	90–246	110–277
34	30–126	45–155	62–184	79–215	99–245	120–276	26–130	40–160	56–190	73–221	92–252	112–284
35	30–130	46–159	63–189	81–220	101–251	122–283	26–134	41–164	57–195	75–226	94–258	114–291
36	31–133	47–163	65–193	83–225	103–257	125–289	27–137	42–168	58–200	76–232	96–264	117–297
37	32–136	48–167	66–198	84–231	105–263	127–296	28–140	43–172	60–204	78–237	98–270	119–304
38	32–140	49–171	67–203	86–236	107–269	129–303	28–144	44–176	61–209	79–243	100–276	121–311
39	33–143	50–175	69–207	88–241	109–275	132–309	29–147	45–180	62–214	81–248	102–282	123–318
40	34–146	51–179	70–212	90–246	111–281	134–316	29–151	46–184	63–219	82–254	103–289	126–324
41	34–150	52–183	72–216	91–252	113–287	137–322	30–154	46–189	65–223	84–259	105–295	128–331
42	35–153	53–187	73–221	93–257	116–292	139–329	31–157	47–193	66–228	85–265	107–301	130–338
43	35–157	54–191	74–226	95–262	118–298	142–335	31–161	48–197	67–233	87–270	109–307	133–344
44	36–160	55–195	76–230	97–267	120–304	144–342	32–164	49–201	68–238	88–276	111–313	135–351
45	37–163	56–199	77–235	98–273	122–310	147–348	32–168	50–205	69–243	90–281	113–319	137–358
46	37–167	57–203	78–240	100–278	124–316	149–355	33–171	51–209	71–247	91–287	115–325	139–365
47	38–170	58–207	80–244	102–283	126–322	152–361	34–174	52–213	72–252	93–292	117–331	142–371
48	39–173	59–211	81–249	103–289	128–328	154–368	34–178	53–217	73–257	95–297	118–338	144–378
49	39–177	60–215	82–254	105–294	130–334	157–374	35–181	54–221	74–262	96–303	120–344	146–385
50	40–180	61–219	84–258	107–299	132–340	159–381	36–184	55–225	76–266	98–308	122–350	148–392

Source: The data of this table are from *Documenta Geigy Scientific Tables,* 6th edition. Reprinted with the kind permission of CIBA–GEIGY Limited, Basel, Switzerland.

Table 13 Fisher's *z* transformation

r	*z*	*r*	*z*	*r*	*z*	*r*	*z*	*r*	*z*
.00	.000								
.01	.010	.21	.213	.41	.436	.61	.709	.81	1.127
.02	.020	.22	.224	.42	.448	.62	.725	.82	1.157
.03	.030	.23	.234	.43	.460	.63	.741	.83	1.188
.04	.040	.24	.245	.44	.472	.64	.758	.84	1.221
.05	.050	.25	.255	.45	.485	.65	.775	.85	1.256
.06	.060	.26	.266	.46	.497	.66	.793	.86	1.293
.07	.070	.27	.277	.47	.510	.67	.811	.87	1.333
.08	.080	.28	.288	.48	.523	.68	.829	.88	1.376
.09	.090	.29	.299	.49	.536	.69	.848	.89	1.422
.10	.100	.30	.310	.50	.549	.70	.867	.90	1.472
.11	.110	.31	.321	.51	.563	.71	.887	.91	1.528
.12	.121	.32	.332	.52	.576	.72	.908	.92	1.589
.13	.131	.33	.343	.53	.590	.73	.929	.93	1.658
.14	.141	.34	.354	.54	.604	.74	.950	.94	1.738
.15	.151	.35	.365	.55	.618	.75	.973	.95	1.832
.16	.161	.36	.377	.56	.633	.76	.996	.96	1.946
.17	.172	.37	.388	.57	.648	.77	1.020	.97	2.092
.18	.182	.38	.400	.58	.662	.78	1.045	.98	2.298
.19	.192	.39	.412	.59	.678	.79	1.071	.99	2.647
.20	.203	.40	.424	.60	.693	.80	1.099		

Table 14 Two-tailed upper critical values for the Spearman rank-correlation coefficient (r_s)

	α			
n	.10	.05	.02	.01
1	—	—	—	—
2	—	—	—	—
3	—	—	—	—
4	1.0	—	—	—
5	.900	1.0	1.0	—
6	.829	.886	.943	1.0
7	.714	.786	.893	.929
8	.643	.738	.833	.881
9	.600	.683	.783	.833

Source: The data for this table have been adapted with permission from E. G. Olds (1938), "Distributions of Sums of Squares of Rank Differences for Small Numbers of Individuals," *Annals of Mathematical Statistics, 9,* 133–148.

Table 15 Critical values for the Kruskal-Wallis test statistic (H) for selected sample sizes for $k = 3$

			α			
n_1	n_2	n_3	.10	.05	.02	.01
1	1	2	—	—	—	—
1	1	3	—	—	—	—
1	1	4	—	—	—	—
1	1	5	—	—	—	—
1	2	2	—	—	—	—
1	2	3	4.286	—	—	—
1	2	4	4.500	—	—	—
1	2	5	4.200	5.000	—	—
1	3	3	4.571	5.143	—	—
1	3	4	4.056	5.389	—	—
1	3	5	4.018	4.960	6.400	—
1	4	4	4.167	4.967	6.667	—
1	4	5	3.987	4.986	6.431	6.954
1	5	5	4.109	5.127	6.146	7.309
2	2	2	4.571	—	—	—
2	2	3	4.500	4.714	—	—
2	2	4	4.500	5.333	6.000	—
2	2	5	4.373	5.160	6.000	6.533
2	3	3	4.694	5.361	6.250	—
2	3	4	4.511	5.444	6.144	6.444
2	3	5	4.651	5.251	6.294	6.909
2	4	4	4.554	5.454	6.600	7.036
2	4	5	4.541	5.273	6.541	7.204
2	5	5	4.623	5.338	6.469	7.392
3	3	3	5.067	5.689	6.489	7.200
3	3	4	4.709	5.791	6.564	7.000
3	3	5	4.533	5.648	6.533	7.079
3	4	4	4.546	5.598	6.712	7.212
3	4	5	4.549	5.656	6.703	7.477
3	5	5	4.571	5.706	6.866	7.622
4	4	4	4.654	5.692	6.962	7.654
4	4	5	4.668	5.657	6.976	7.760
4	5	5	4.523	5.666	7.000	7.903
5	5	5	4.580	5.780	7.220	8.000

Source: The data for this table have been adapted from Table F of *A Nonparametric Introduction to Statistics* by C.H. Kraft and C. Van Eeden, Macmillan, New York, 1968.

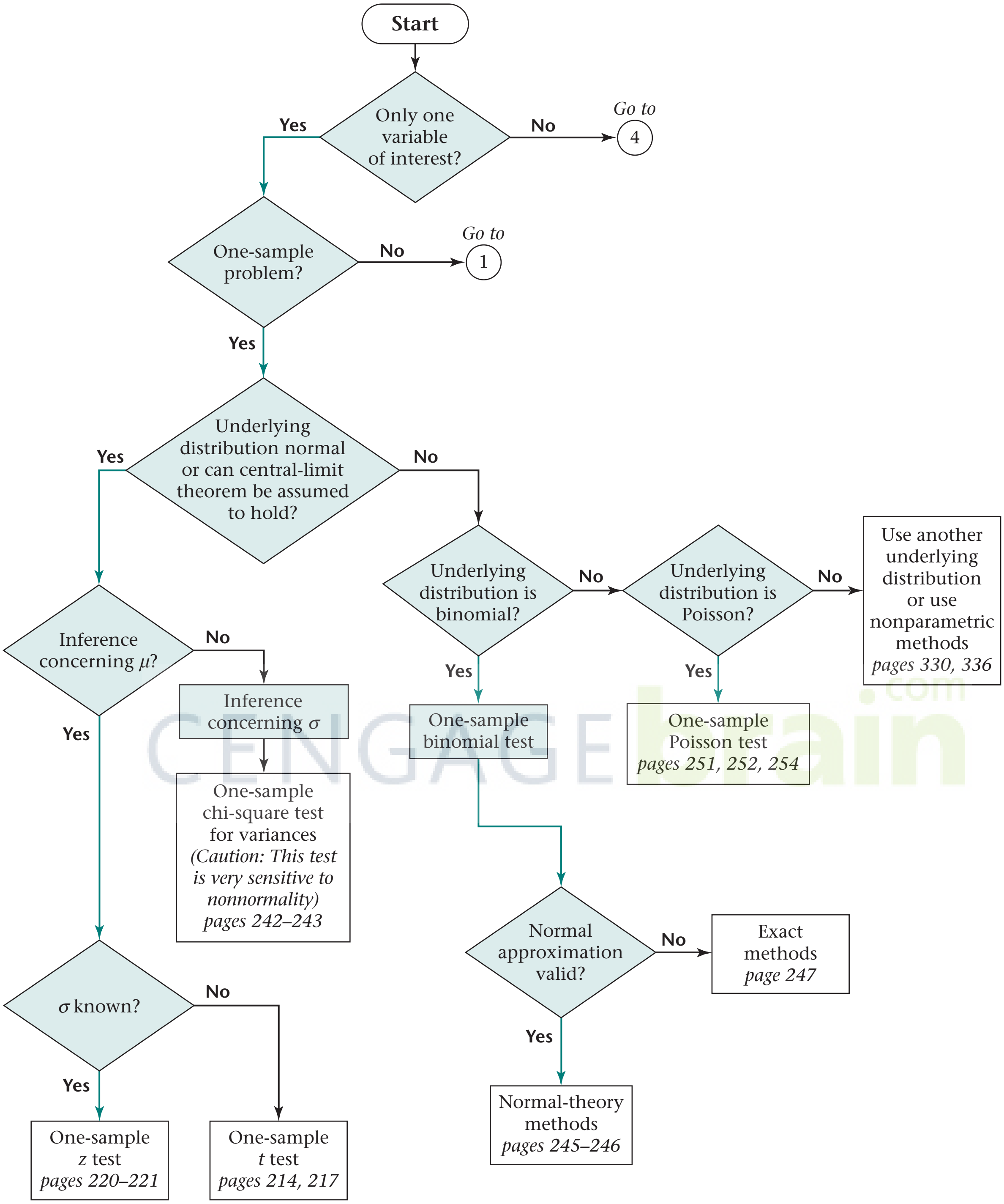
Table 16 Critical values for the studentized range statistic q^*, α = .05

ν	k: 2	3	4	5	6	7	8	9	10
1	17.97	26.98	32.82	37.08	40.41	43.12	45.40	47.36	49.07
2	6.085	8.331	9.798	10 88	11.74	12.44	13.03	13.54	13.99
3	4.501	5.910	6.825	7.502	8.037	8.478	8.853	9.177	9.462
4	3.927	5.040	5.757	6.287	6.707	7.053	7.347	7.602	7.826
5	3.635	4.602	5.218	5.673	6.033	5.330	6.582	6.802	6.995
6	3.461	4.339	4.896	5.305	5.628	5.895	6.122	6.319	6.493
7	3.344	4.165	4.681	5.060	5.359	5 606	5.815	5.998	6.158
8	3.261	4.041	4.529	4.886	5.167	5.399	5.597	5.767	5.918
9	3.199	3.949	4.415	4.756	5.024	5.244	5.432	5.595	5.739
10	3.151	3.877	4.327	4.654	4.912	5.124	5.305	5.461	5.599
11	3.113	3.820	4.256	4.574	4.823	5.028	5.202	5.353	5.487
12	3.082	3.773	4.199	4.508	4.751	4.950	5.119	5.265	5.395
13	3.055	3.735	4.151	4.453	4.690	4.885	5.049	5.192	5.318
14	3.033	3.702	4.111	4.407	4.639	4.829	4.990	5.131	5.254
15	3.014	3.674	4.076	4.367	4.595	4.782	4.940	5.077	5.198
16	2.998	3.649	4.046	4.333	4.557	4.741	4.897	5.031	5.150
17	2.984	3.628	4.020	4.303	4.524	4.705	4.858	4.991	5.108
13	2.971	3.609	3.997	4.277	4.495	4.673	4.824	4.956	5.071
19	2.960	3.593	3.977	4.253	4.469	4.645	4.794	4.924	5.038
20	2.950	3.578	3.958	4.232	4.445	4.620	4.768	4.896	5.008
24	2.919	3.532	3.901	4.166	4.373	4.541	4.684	4.807	4.915
30	2.888	3.486	3.845	4.102	4.302	4.464	4.602	4.720	4.824
40	2.858	3.442	3.791	4.039	4.232	4.389	4.521	4.635	4.735
6O	2.829	3.399	3.737	3.977	4.163	4.314	4.441	4.550	4.646
120	2.800	3.356	3.685	3.917	4.096	4.241	4.363	4.468	4.560
∞	2.772	3.314	3.633	3.858	4.030	4.170	4.286	4.387	4.474

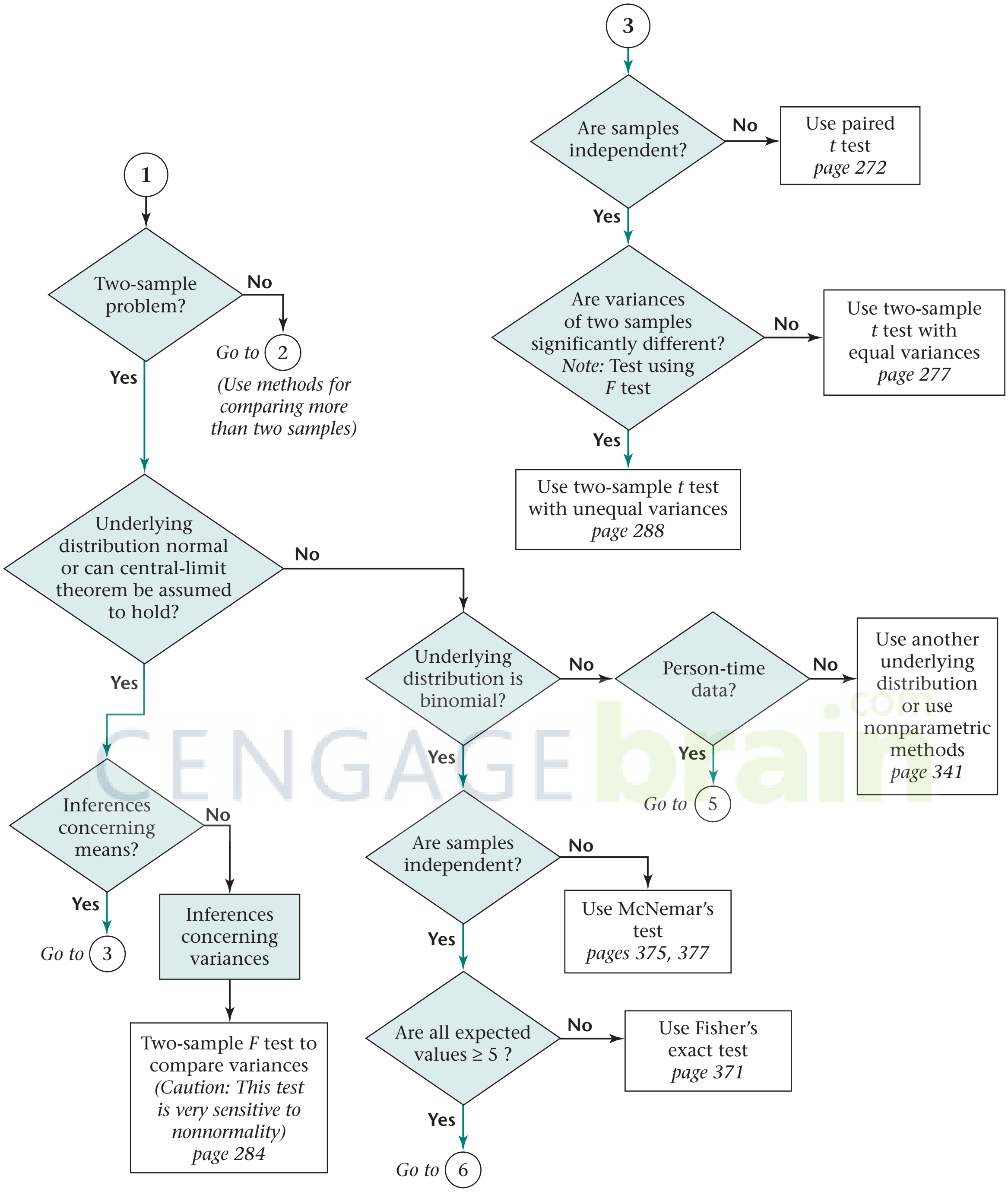
ν	k: 11	12	13	14	15	16	17	18	19
1	5.059	51.96	53.20	54.33	55.36	56.32	57.22	58.04	58.83
2	14.39	14.75	15.08	15.38	15.65	1.591	16.14	16.37	16.57
3	9.717	9.946	10.15	10.35	10.53	10.69	10.84	10 98	11.11
4	8.027	8.208	8.373	8.525	8664	8.794	8.914	9.028	9.134
5	7.168	7.324	7.466	7.596	7.717	7.828	7.932	8.030	8.122
6	6.649	6.789	6.917	7.034	7.143	7.244	7.338	7.426	7.508
7	6.302	6.431	6.550	6.658	6.759	6.852	6.939	7.020	7.097
8	6.054	6.175	6.287	6.389	6.483	6.571	6.653	6.729	6.802
9	5.867	5.983	6.089	6.186	6.276	6.359	6.437	6.510	6.579
10	5.722	5.833	5.935	6.028	6.114	6.194	6.269	6.339	6.405
11	5.605	5.713	5.811	5.901	5.984	6.062	6.134	6.202	6.265
12	5.511	5.615	5.710	5.798	5.878	5.953	6.023	6.089	6.151
13	5.431	5.533	5.625	5.711	5.789	5.862	5.931	5.995	6.055
14	5.364	5.463	5.554	5.637	5.714	5.786	5.852	5.915	5.974
15	5.306	5.404	5.493	5.574	5.649	5.720	5.785	5.846	5.904
16	5.256	5.352	5.439	5.520	5.593	5.662	5.727	5.786	5.843
17	5.212	5.307	5.392	5.471	5.544	5.612	5.675	5.734	5.790
18	5.174	5.267	5.352	5.429	5.501	5.568	5.630	5.688	5.242
19	5.140	5.231	5.315	5.391	5.462	5.528	5.589	5.647	5.701
20	5.108	5.199	5.282	5.357	5.427	5.493	5.553	5.610	5.663
24	5.012	5.099	5.179	5.251	5.319	5.381	5.439	5.494	5.545
30	4.917	5.001	5.077	5.147	5.211	5.271	5.327	5.379	5.42S
40	4.824	4.904	4.977	5.044	5.106	5.163	5.216	5.266	5.313
50	4.732	4.808	4.878	4.942	5.001	5.056	5.107	5.154	5.199
120	4.641	4.714	4.781	4.842	4.893	4.950	4.998	5.044	5.086
∞	4.522	4.622	4.685	4.743	4.796	4.845	4.891	4.934	4.974

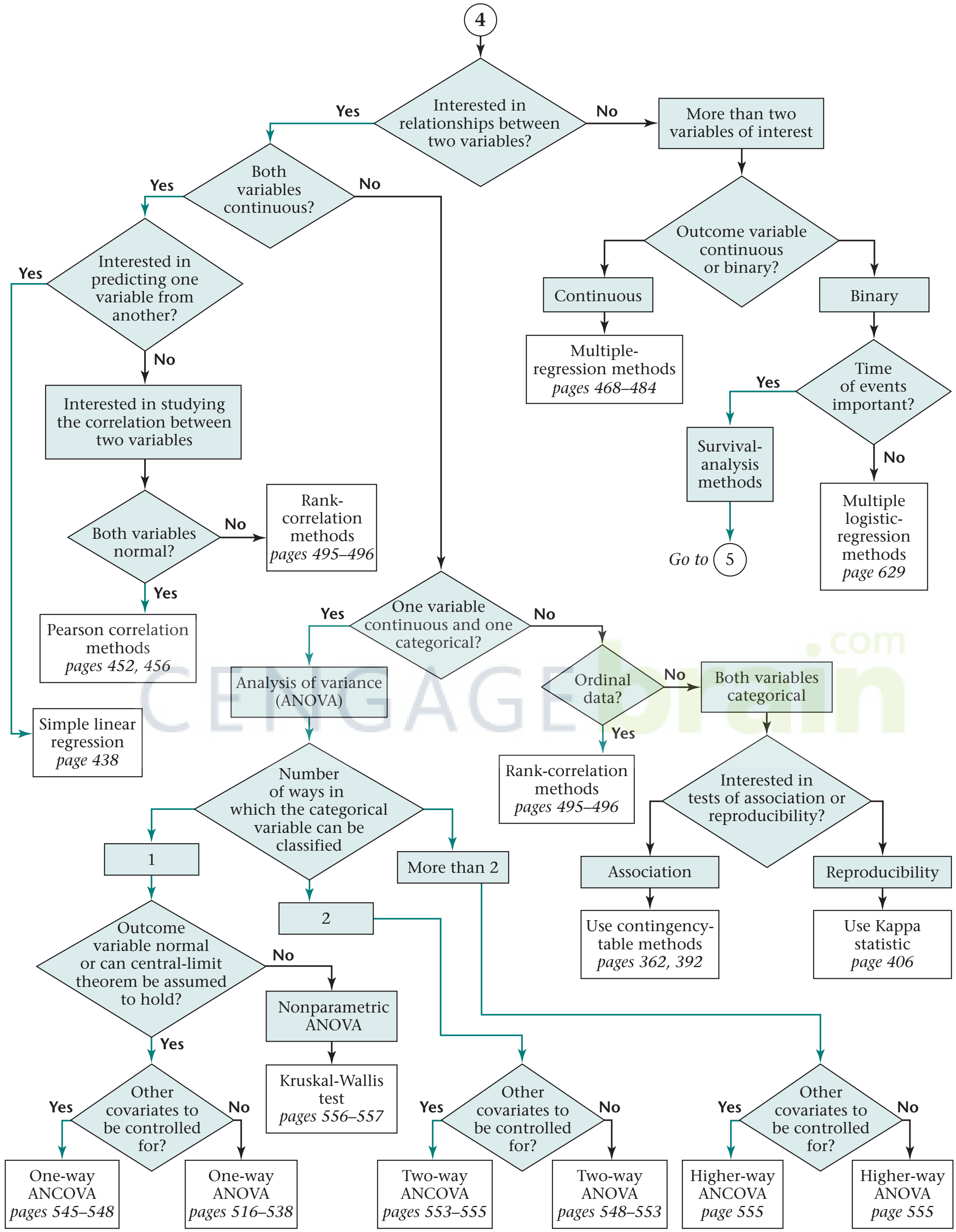
*$q_{k,v,.05}$ = upper 5th percentile of a $q_{k,v}$ distribution

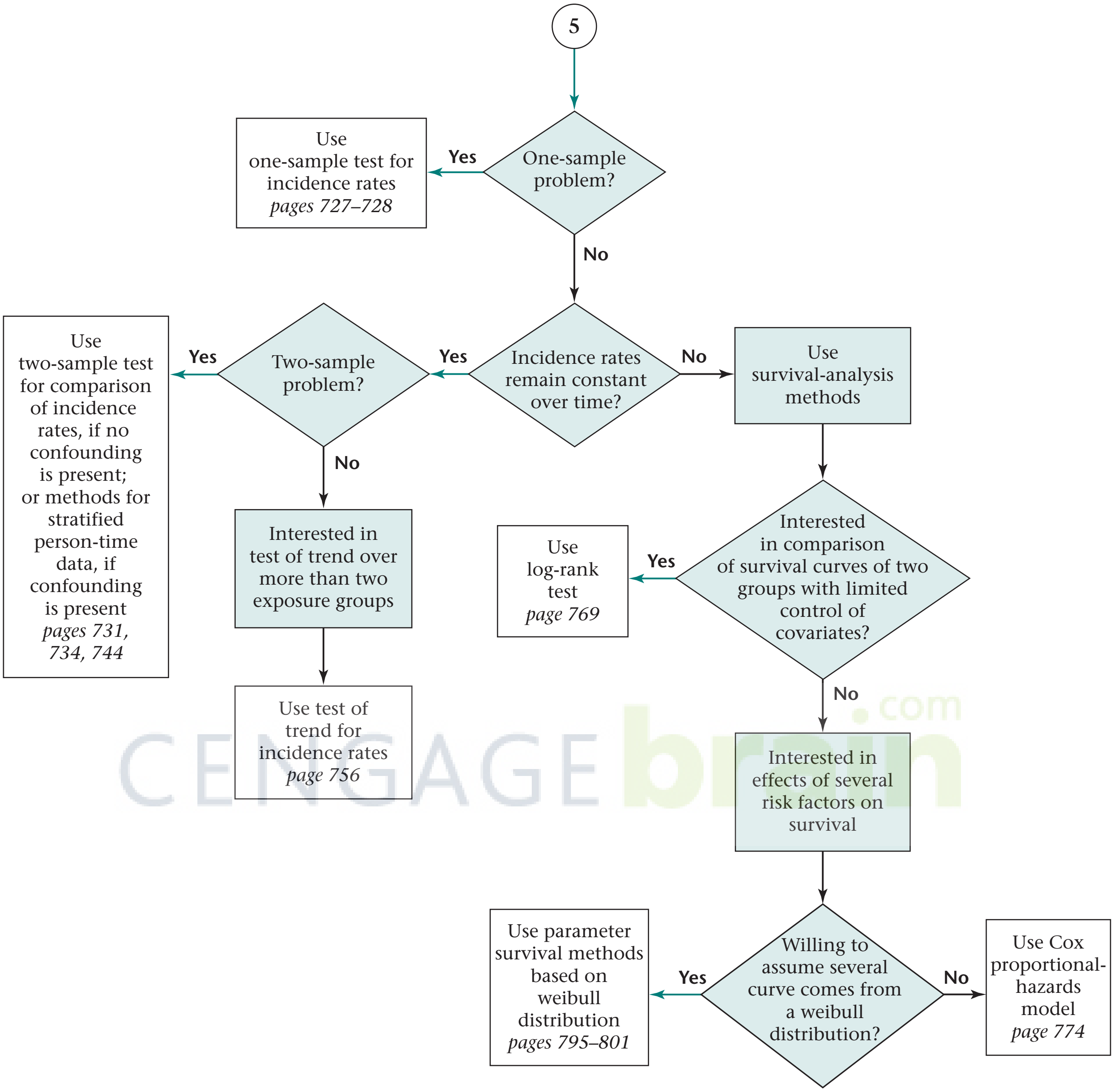
FLOWCHART
Methods of Statistical Inference

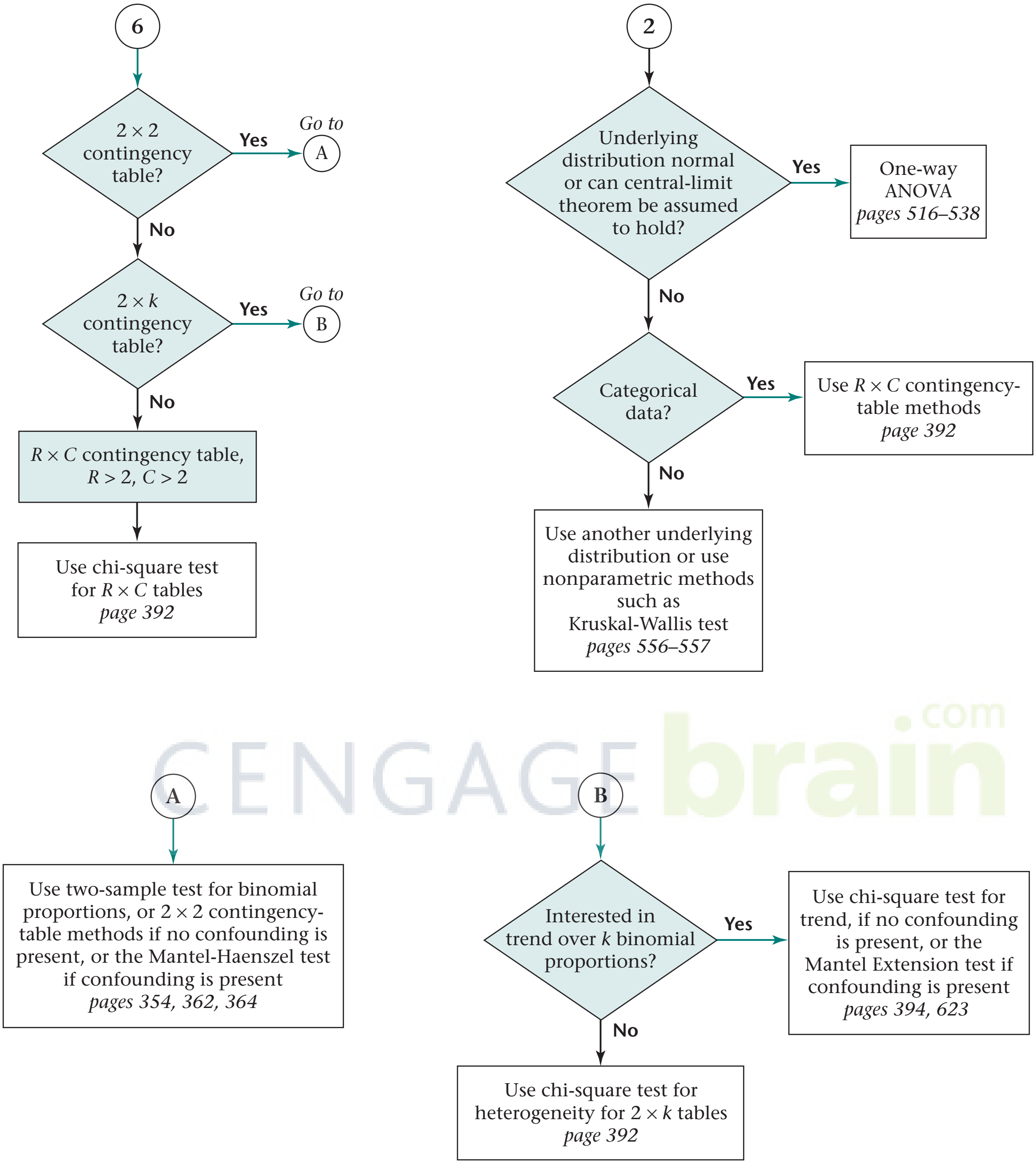


Start
Only one variable of interest?
Yes
No
Go to 4
One-sample problem?
No
Go to 1
Yes
Underlying distribution normal or can central-limit theorem be assumed to hold?
Yes
No
Inference concerning μ?
No
Inference concerning σ
One-sample chi-square test for variances (Caution: This test is very sensitive to nonnormality) pages 242–243
Yes
σ known?
No
Yes
One-sample z test pages 220–221
One-sample t test pages 214, 217
Underlying distribution is binomial?
No
Yes
One-sample binomial test
Underlying distribution is Poisson?
No
Yes
One-sample Poisson test pages 251, 252, 254
Use another underlying distribution or use nonparametric methods pages 330, 336
Normal approximation valid?
No
Exact methods page 247
Yes
Normal-theory methods pages 245–246


3
Are samples independent?
No
Use paired t test page 272
Yes
Are variances of two samples significantly different? Note: Test using F test
No
Use two-sample t test with equal variances page 277
Yes
Use two-sample t test with unequal variances page 288
1
Two-sample problem?
No
Go to 2
(Use methods for comparing more than two samples)
Yes
Underlying distribution normal or can central-limit theorem be assumed to hold?
No
Underlying distribution is binomial?
No
Person-time data?
No
Use another underlying distribution or use nonparametric methods page 341
Yes
Go to 5
Yes
Are samples independent?
No
Use McNemar's test pages 375, 377
Yes
Are all expected values ≥ 5 ?
No
Use Fisher's exact test page 371
Yes
Go to 6
Yes
Inferences concerning means?
No
Inferences concerning variances
Two-sample F test to compare variances (Caution: This test is very sensitive to nonnormality) page 284
Yes
Go to 3


4
Interested in relationships between two variables?
Yes
No
More than two variables of interest
Outcome variable continuous or binary?
Continuous
Multiple-regression methods pages 468–484
Binary
Time of events important?
Yes
Survival-analysis methods
Go to 5
No
Multiple logistic-regression methods page 629
Both variables continuous?
Yes
Interested in predicting one variable from another?
Yes
Simple linear regression page 438
No
Interested in studying the correlation between two variables
Both variables normal?
No
Rank-correlation methods pages 495–496
Yes
Pearson correlation methods pages 452, 456
No
One variable continuous and one categorical?
Yes
Analysis of variance (ANOVA)
Number of ways in which the categorical variable can be classified
1
Outcome variable normal or can central-limit theorem be assumed to hold?
No
Nonparametric ANOVA
Kruskal-Wallis test pages 556–557
Yes
Other covariates to be controlled for?
Yes
One-way ANCOVA pages 545–548
No
One-way ANOVA pages 516–538
2
Other covariates to be controlled for?
Yes
Two-way ANCOVA pages 553–555
No
Two-way ANOVA pages 548–553
More than 2
Other covariates to be controlled for?
Yes
Higher-way ANCOVA page 555
No
Higher-way ANOVA page 555
No
Ordinal data?
Yes
Rank-correlation methods pages 495–496
No
Both variables categorical
Interested in tests of association or reproducibility?
Association
Use contingency-table methods pages 362, 392
Reproducibility
Use Kappa statistic page 406


5
One-sample problem?
Yes
Use one-sample test for incidence rates *pages 727–728*
No
Incidence rates remain constant over time?
Yes
Two-sample problem?
Yes
Use two-sample test for comparison of incidence rates, if no confounding is present; or methods for stratified person-time data, if confounding is present *pages 731, 734, 744*
No
Interested in test of trend over more than two exposure groups
Use test of trend for incidence rates *page 756*
No
Use survival-analysis methods
Interested in comparison of survival curves of two groups with limited control of covariates?
Yes
Use log-rank test *page 769*
No
Interested in effects of several risk factors on survival
Willing to assume several curve comes from a weibull distribution?
Yes
Use parameter survival methods based on weibull distribution *pages 795–801*
No
Use Cox proportional-hazards model *page 774*


6
2 × 2 contingency table?
Yes
Go to
A
No
2 × k contingency table?
Yes
Go to
B
No
R × C contingency table, R > 2, C > 2
Use chi-square test for R × C tables
page 392
2
Underlying distribution normal or can central-limit theorem be assumed to hold?
Yes
One-way ANOVA
pages 516–538
No
Categorical data?
Yes
Use R × C contingency-table methods
page 392
No
Use another underlying distribution or use nonparametric methods such as Kruskal-Wallis test
pages 556–557
A
Use two-sample test for binomial proportions, or 2 × 2 contingency-table methods if no confounding is present, or the Mantel-Haenszel test if confounding is present
pages 354, 362, 364
B
Interested in trend over k binomial proportions?
Yes
Use chi-square test for trend, if no confounding is present, or the Mantel Extension test if confounding is present
pages 394, 623
No
Use chi-square test for heterogeneity for 2 × k tables
page 392

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