Statistical Inference Course Project Part 1

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Contents

Overview

In this report we're going to study the exponential distribution and the Central Limit Theorem. By using simulation we will show a comparison of the sample mean and variance with respect to the theoretical values as well as their distributions.

Simulations

Values used for the simulations:

```
myLambda <- 0.2
myNumExp <- 40
myNumSim <- 1000
set.seed(43)
```

```
myMeans <- NULL
for (i in 1:myNumSim) {
    myExp <- rexp(n= myNumExp, rate = myLambda)
    myMeans <- c(myMeans, mean(myExp))
}
```

Sample mean and theoretical mean

The theoretical mean of our exponential distribution is $1/\lambda$= `5`. The observed mean of 1000 repetitions of the mean of 40 exponentials is `5.001`.

Sample variance and theoretical variance

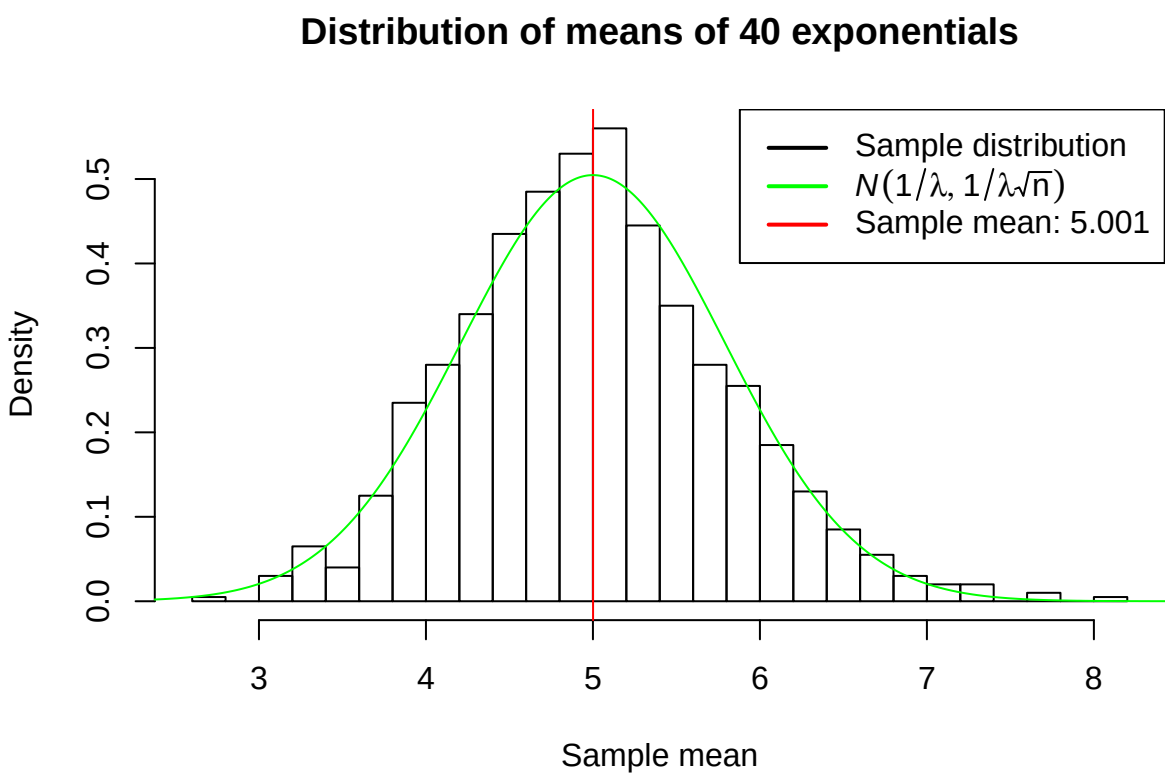
```
mySigma <- 1/myLambda/sqrt(myNumExp)
mySd <- sd(myMeans)
```

Theoretical standard deviation of the distribution of the mean of 40 exponentials is $1/\lambda\sqrt{n}$= `0.791` and the observed standard deviation of the samples is `0.792`.

Distributions

To show the distribution of 1000 means computed on 40 samples from an exponential distribution we plotted the histogram of the means along with the theoretical normal distribution with mean $1/\lambda$ and standard deviation $1/\lambda\sqrt{1000}$.

```
hist(myMeans,
     breaks= "Scott",
     freq= FALSE,
     main= "Distribution of means of 40 exponentials",
     xlab= "Sample mean",
     ylab= "Density")
abline(v= mean(myMeans), col= "red")
mySigma <- 1/myLambda/sqrt(myNumExp)
x <- seq(0, 9, by= 0.01)
lines(x, dnorm(x,
               mean = 1/myLambda,
               sd= mySigma),
      type= "l",
      col= "green"
      )
legend("topright",
       legend= c("Sample distribution",
                 expression(italic(N)(1/lambda, 1/lambda*sqrt(n))),
                 paste("Sample mean:", round(mean(myMeans), 3))),
       lty= c("solid", "solid", "solid"),
       col= c("black", "green", "red"),
       lwd= 2
       )
```



Conclusions

By means of simulating the repetition of 1000 means of 40 random samples from an exponential distribution of mean $1/\lambda$ and and standard deviation $1/\lambda$ we showed that:

1. the mean of the samples (`5.001`) is very close to the theoretical mean of the distribution $1/\lambda = 5$.
2. the distribution of the means of the samples is aproximately normal with mean corresponding to the theoretical mean and standard distribution
$$\frac{1}{\lambda\sqrt{n}}$$
3. the standard deviation of the samples (`0.792`) is very close to the theoretical standard deviation `0.791`