

n: number of control points
 n: number of basis functions
 dim: dimension of state
 p: degree of BSpline curve
 k: order of BSpline curve $k = p - 1$
 nKnot: number of knots $nKnot = n + k$
 nx: number of variables $nx = n * dim + 1$
 nc: number of constraints $nc = nEquaC + inequality * 2$
 nEquaC: number of equality constraints $nEquaC = 4$
 nInequaC: number of inequality constraints

$$\begin{aligned}
 J &= X_{1*nx}^T * Q_{nx*nx} * X_{nx*1} + q_{1*nx}^T * X_{nx*1} \\
 s.t. \quad L_{(nc*dim)*1} &\leq A_{(nc*dim)*nx} X_{nx*1} \leq U_{(nc*dim)*1}
 \end{aligned}$$

$$J = T * Q_T * T + \sum_{i=0}^{n-1} (p_i - p_{ir})^T * r * (p_i - p_{ir})$$

$$s.t. \quad q(0) = q_{start};$$

$$q(1) = q_{goal};$$

$$\dot{q}(0) = 0;$$

$$\dot{q}(1) = 0;$$

$$Q_{min} \leq \dot{q}(i) \quad i \in [0, nInequaC];$$

$$\dot{q}(i) \leq Q_{max} \quad i \in [0, nInequaC]$$

$$X = [T, p_0, \dots, p_{n-1}]^T = [T, p]_{nx*1}^T$$

$$R = \begin{bmatrix} r & & \\ & \ddots & \\ & & r \end{bmatrix}_{(n*dim)*(n*dim)}$$

$$Q = \begin{bmatrix} Q_T & \\ & R \end{bmatrix}_{nx*nx}$$

$$P_r = [p_{0r}, \dots, p_{(n-1)r}]_{(n*dim)*1}^T$$

$$q = [0, -R * P_r]_{nx*1}^T$$

$$B_i(t)_{dim*dim} = BasisFunctionI(t).asDiagonal()$$

$$B(t) = [B_0(t), B_1(t), \dots, B_{n-1}(t)]_{dim*(n*dim)}$$

Aq: equality constraints matrix

$$Aq * X = Lq = Uq$$

$$Aq_0 = [0, B(0)]_{dim*nx}$$

$$(Aq_0 * X)_{dim*1} = q_0$$

$$Aq_1 = [0, B(1)]_{dim*nx}$$

$$(Aq_1 * X)_{dim*1} = q_1$$

$$M = \begin{bmatrix} 0 & -E_{dim*dim} & E_{dim*dim} & 0 & \cdots & 0 \\ 0 & 0 & -E_{dim*dim} & E_{dim*dim} & \cdots & 0 \\ \vdots & \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & \cdots & -E_{dim*dim} & E_{dim*dim} \end{bmatrix}_{((n-1)*dim)*nx}$$

$$\dot{B}_i(t)_{dim*dim}$$

$$\dot{B}(t) = [\dot{B}_0(t), \dot{B}_1(t), \dots, \dot{B}_{n-1}(t)]_{dim*(n*dim)}$$

$$r(t) = \sum_{i=0}^{n-1} B_i^p(t) P_i$$

$$DotBspline : dB = Spline(k-1, knot_1, \dots, kont_{nKnot-1})$$

$$r(\dot{t}) = \sum_{i=0}^{n-2} B_{i+1}^{p-1}(t) Q_i = \sum_{i=0}^{n-2} dB_i^{p-1}(t) Q_i$$

$$Q_i = \frac{p}{t_{i+p+1} - t_{i+1}} (P_{i+1} - P_i)$$

$$dBWeighted_i(t) = \frac{p}{t_{i+p+1} - t_{i+1}} * dB_i^{p-1}(t)$$

$$dBWeighted(t) = [dBWeighted_0(t), dBWeighted_1(t), \dots, dBWeighted_{n-2}(t)]_{dim*((n-1)*dim)}$$

$$dBWSequence = [dBWeighted(t_0), \dots, dBWeighted(t_m)]$$

$$Aqdq(t)_{dim*nx} = dBWeighted(t)_{dim*((n-1)*dim)} * M_{((n-1)*dim)*nx}$$

$$\dot{r}(t)_{dim*1} = Aqdq(t) * X$$

$$r(\dot{0}) = 0, \quad Aqdq_0 * X = 0$$

$$r(\dot{1}) = 0, \quad Aqdq_1 * X = 0$$

$$Aq = [Aq_0; Aq_1; Aqdq_0; Aqdq_1]_{(4*dim)*nx}$$

$$\begin{aligned}
Lq &= Uq = [q_0; q_1; 0; 0]_{(4*dim)*1} \\
dqmin &= [q_{min}; \dots; q_{min}]_{(nInequaC*dim)*1} \\
dqmax &= [q_{max}; \dots; q_{max}]_{(nInequaC*dim)*1} \\
dM &= [1, 0, \dots, 0]_{1*nx} \\
Aq dq &= [Aq dq(t_0); Aq dq(t_1); \dots; Aq dq(t_{nInequaC-1})]_{(nInequaC*dim)*nx} \\
Aq dq * X - dqmin * dM * X &\geq 0_{(nInequaC*dim)*1} \\
(Aq dq - dqmin * dM) * X &\geq 0 \\
AiqL &= (Aq dq - dqmin * dM)_{(nInequaC*dim)*nx} \\
AiqU &= (Aq dq - dqmax * dM)_{(nInequaC*dim)*nx} \\
A &= [Aq; AiqL; AiqU]_{((4+2*nInequaC)*dim)*nx} \\
L &= [Lq; 0; +inf]_{((4+2*nInequaC)*dim)*1} \\
U &= [Uq; -inf; 0]_{((4+2*nInequaC)*dim)*1}
\end{aligned}$$