n: number of control points

n: number of basis functions

dim: dimension of state

p: degree of BSpline curve

k: order of BSpline curve k = p - 1

nKnot: number of knots nKnot = n + k

nx: number of variables nx = n * dim + 1

nc: number of constraints nc = nEquaC + inequality * 2

nEquaC: number of equality constraints nEquaC = 4

nInequaC: number of inequality constraints

$$
\begin{gathered}
J = X_{1*nx}^{T} * Q_{nx*nx} * X_{nx*1} + q_{1*nx}^{T} * X_{nx*1} \\
s.t. \quad L_{(nc*dim)*1} \leq A_{(nc*dim)*nx} X_{nx*1} \leq U_{(nc*dim)*1}
\end{gathered}
$$

$$
\begin{gathered}
J = T * Q_T * T + \sum_{i=0}^{n-1} (p_i - p_{ir})^T * r * (p_i - p_{ir}) \\
s.t. \quad q(0) = q_{start}; \\
q(1) = q_{goal}; \\
q\dot{(0)} = 0; \\
q\dot{(1)} = 0; \\
qMin \leq \dot{(q)}(i) \quad i \in [0, nInequaC]; \\
\dot{(q)}(i) \leq qMax \quad i \in [0, nInequaC]
\end{gathered}
$$

****

$$
\begin{gathered}
X = [T, p_0, \dots, p_{n-1}]^T = [T, p]_{nx*1}^T \\
R = \begin{bmatrix} r & & \\ & \ddots & \\ & & r \end{bmatrix}_{(n*dim)*(n*dim)} \\
Q = \begin{bmatrix} Q_T & \\ & R \end{bmatrix}_{nx*nx} \\
P_r = [p_{0r}, \dots, p_{(n-1)r}]_{(n*dim)*1}^T
\end{gathered}
$$

$$q = [0, -R * P_r]^T_{nx*1}$$

$$B_i(t)_{dim*dim} = BasisFunctionI(t).asDiagonal()$$
$$B(t) = [B_0(t), B_1(t), \ldots, B_{n-1}(t)]_{dim*(n*dim)}$$

Aq: equality constraints matrix

$$Aq * X = Lq = Uq$$
$$Aq_0 = [0, B(0)]_{dim*nx}$$
$$(Aq_0 * X)_{dim*1} = q_0$$
$$Aq_1 = [0, B(1)]_{dim*nx}$$
$$(Aq_1 * X)_{dim*1} = q_1$$

$$M = \begin{bmatrix} 0 & -E_{dim*dim} & E_{dim*dim} & 0 & \cdots & 0 \\ 0 & 0 & -E_{dim*dim} & E_{dim*dim} & \cdots & 0 \\ \vdots & \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & \cdots & -E_{dim*dim} & E_{dim*dim} \end{bmatrix}_{((n-1)*dim)*nx}$$

$$\dot{B_i}(t)_{dim*dim}$$
$$\dot{B}(t) = [\dot{B_0}(t), \dot{B_1}(t), \ldots, \dot{B_{n-1}}(t)]_{dim*(n*dim)}$$

$$r(t) = \Sigma_{i=0}^{n-1} B_i^p(t) P_i$$
$$DotBspline : dB = Spline(k-1, knot_1, \ldots, kont_{nKnot-1})$$
$$\dot{r}(t) = \Sigma_{i=0}^{n-2} B_{i+1}^{p-1}(t) Q_i = \Sigma_{i=0}^{n-2} dB_i^{p-1}(t) Q_i$$
$$Q_i = \frac{p}{t_{i+p+1} - t_{i+1}}(P_{i+1} - P_i)$$
$$dBWeighted_i(t) = \frac{p}{t_{i+p+1} - t_{i+1}} * dB_i^{p-1}(t)$$
$$dBWeighted(t) = [dBWeighted_0(t), dBWeighted_1(t), \ldots, dBWeighted_{n-2}(t)]_{dim*((n-1)*dim)}$$
$$dBWSequence = [dBWeighted(t_0), \ldots, dBWeighted(t_m)]$$

$$Aqdq(t)_{dim*nx} = dBWeighted(t)_{dim*((n-1)*dim)} * M_{((n-1)*dim)*nx}$$
$$\dot{r}(t)_{dim*1} = Aqdq(t) * X$$
$$\dot{r}(0) = 0, \quad Aqdq_0 * X = 0$$
$$\dot{r}(1) = 0, \quad Aqdq_1 * X = 0$$
$$Aq = [Aq_0; Aq_1; Aqdq_0; Aqdq_1]_{(4*dim)*nx}$$

$$Lq = Uq = [q_0; q_1; 0; 0]_{(4*dim)*1}$$

$$dqmin = [\dot{q_{min}}; \ldots; \dot{q_{min}}]_{(nInequaC*dim)*1}$$

$$dqmax = [\dot{q_{max}}; \ldots; \dot{q_{max}}]_{(nInequaC*dim)*1}$$

$$dM = [1, 0, \ldots, 0]^T_{1*nx}$$

$$Aqdq = [Aqdq(t_0); Aqdq(t_1); \ldots; Aqdq(t_{nInequaC-1})]_{(nInequaC*dim)*nx}$$

$$Aqdq * X - dqmin * dM * X \geq 0_{nInequaC*dim}$$

$$(Aqdq - dqmin * dM) * X \geq 0$$

$$AiqL = (Aqdq - dqmin * dM)_{(nInequaC*dim)*nx}$$

$$AiqU = (Aqdq - dqmax * dM)_{(nInequaC*dim)*nx}$$

$$A = [Aq; AiqL; AiqU]_{((4+2*nInequaC)*dim)*nx}$$

$$L = [Lq; 0; +inf]_{((4+2*nInequaC)*dim)*1}$$

$$U = [Uq; -inf; 0]_{((4+2*nInequaC)*dim)*1}$$