David A. Huffman

David Huffman is best known for the invention of Huffman code, a highly important compression scheme for lossless variable length encoding. It was the result of a term paper he wrote while a graduate student at the Massachusetts Institute of Technology (MIT)…

From: Wikipedia

Huffman coding algorithm

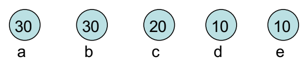
1. Take the two least probable symbols in the alphabet
   (longest code words, equal length, differing in last digit)
2. Combine these two symbols into a single symbol
3. Repeat

Example: Huffman coding

S	Freq
a	30
b	30
c	20
d	10
e	10

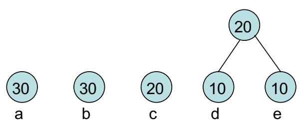
Example

S	Freq	Huffman
a	30	
b	30	
c	20	
d	10	
e	10	



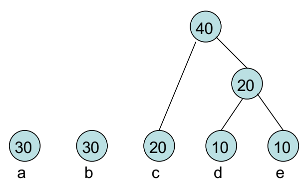
Example

S	Freq	Huffman
a	30	
b	30	
c	20	
d	10	
e	10	



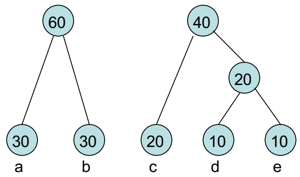
Example

S	Freq	Huffman
a	30	
b	30	
c	20	
d	10	
e	10	



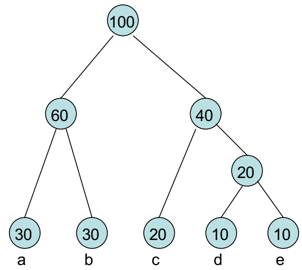
Example

S	Freq	Huffman
a	30	
b	30	
c	20	
d	10	
e	10	



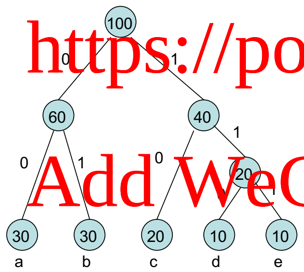
Example

S	Freq	Huffman
a	30	
b	30	
c	20	
d	10	
e	10	



Example

S	Freq	Huffman
a	30	00
b	30	01
c	20	10
d	10	110
e	10	111



Average length L

= (30*2 + 30*2 + 20*2 + 10*3 + 10*3) / 100
= 220 / 100
= 2.2

Average length L

= (30*2 + 30*2 + 20*2 + 10*3 + 10*3) / 100
= 220 / 100
= 2.2

Better than using fixed length 3 bits for 5 symbols.

Entropy

H = -0.3 * log 0.3 + -0.3 * log 0.3 + -0.2 * log 0.2 + -0.1 * log 0.1 + -0.1 * log 0.1
= -0.3*(-1.737) + -0.3*(-1.737) + -0.2 * (-2.322) + -0.1 * (-3.322) + -0.1 * (-3.322)

= 0.3 log 10/3 + 0.3 log 10/3 + 0.2 log 5 + 0.1 log 10 + 0.1 log 10
= 0.3*1.737 + 0.3*1.737 + 0.2* 2.322 + 0.1*3.322 + 0.1*3.322
= 2.17

Another example

- S={a, b, c, d} with freq {4, 2, 1, 1}
- $H = 4/8*\log_2 2 + 2/8*\log_2 4 + 1/8*\log_2 8 + 1/8*\log_2 8$
- H = 1/2 + 1/2 + 3/8 + 3/8 = 1.75
- a => 0   b => 10   c => 110   d => 111
- Message: {abcdabaa} => {0 10 110 111 0 10 0 0}
- Average length L = 14 bits / 8 chars = 1.75
- If equal probability, i.e. fixed length, need $\log_2 4$ = 2 bits

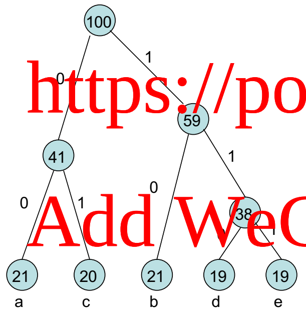
Huffman coding

S	Freq	Huffman
a	~~30~~ 21	
b	~~30~~ 21	
c	~~20~~ 20	
d	~~10~~ 19	
e	~~10~~ 19	

Total: 100

Huffman coding

S	Freq	Huffman
a	21	00
b	21	10
c	20	01
d	19	110
e	19	111



Huffman optimal?

H = 0.21 log 100/21 + 0.21 log 100/21 + 0.2 log 5 + 0.19 log 100/19 + 0.19 log 100/19

= 0.21*2.252 + 0.21*2.252 + 0.2* 2.322 + 0.19*2.396 + 0.19*2.396

= 2.32

L = (21*2 + 21*2 + 20*2 + 19*3 + 19*3)/100

= 2.38

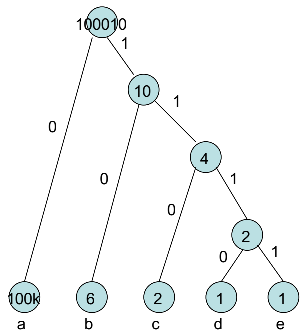
Huffman coding

S	Freq	Huffman
a	~~30~~ 100000	
b	~~30~~ 6	
c	~~20~~ 2	
d	~~10~~ 1	
e	~~10~~ 1	

Total: 100010

Huffman coding

S	Freq	Huffman
a	100000	0
b	6	10
c	2	110
d	1	1110
e	1	1111



Huffman optimal?

H = 0.9999 log 1.0001 + 0.00006 log 16668.333
+ … + 1/100010 log 100010
≈ 0.00

L = (100000*1 + …)/100010
≈ 1

Problems of Huffman coding

- Huffman codes have an integral # of bits.
  - E.g., log (3) = 1.585 while Huffman may need 2 bits
- Noticeable non-optimality when prob of a symbol is high.

=> Arithmetic coding

Arithmetic coding

Message to encode:
BILL GATES

Character	Probability
SPACE	1/10
A	1/10
B	1/10
E	1/10
G	1/10
I	1/10
L	2/10
S	1/10
T	1/10

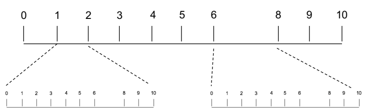
Example extracted from February, 1991 issue of Dr. Dobb's Journal

Arithmetic coding

Character	Probability	Range
SPACE	1/10	0.00 - 0.10
A	1/10	0.10 - 0.20
B	1/10	0.20 - 0.30
E	1/10	0.30 - 0.40
G	1/10	0.40 - 0.50
I	1/10	0.50 - 0.60
L	2/10	0.60 - 0.80
S	1/10	0.80 - 0.90
T	1/10	0.90 - 1.00

Arithmetic coding

Character	Probability	Range
SPACE	1/10	0.00 - 0.10
A	1/10	0.10 - 0.20
B	1/10	0.20 - 0.30
E	1/10	0.30 - 0.40
G	1/10	0.40 - 0.50
I	1/10	0.50 - 0.60
L	2/10	0.60 - 0.80
S	1/10	0.80 - 0.90
T	1/10	0.90 - 1.00



Arithmetic coding algorithm

Set low to 0.0
Set high to 1.0
While there are still input symbols do
  get an input symbol
  code_range = high - low.
  high = low + range*high_range(symbol)
  low = low + range*low_range(symbol)
End of While
output low or a number within the range

Arithmetic coding

New Character	Low value	High Value
	0.0	1.0
B	0.2	0.3
I	0.25	0.26
L	0.256	0.258
L	0.2572	0.2576
SPACE	0.25720	0.25724
G	0.257216	0.257220
A	0.2572164	0.2572168
T	0.25721676	0.2572168
E	0.257216772	0.257216776
S	0.2572167752	0.2572167756

Example

Consider the second L as new char:

code_range = 0.258 – 0.256 = 0.002
high = 0.256 + 0.002*0.8 = 0.2576
low = 0.256 + 0.002*0.6 = 0.2572

Decoding algorithm

get encoded number
Do
  find symbol whose range straddles the encoded number
  output the symbol
  range = symbol high value - symbol low value
  subtract symbol low value from encoded number
  divide encoded number by range
until no more symbols

Arithmetic coding

Encoded Number	Output Symbol	Low	High	Range
0.2572167752	B	0.2	0.3	0.1
0.572167752	I	0.5	0.6	0.1
0.72167752	L	0.6	0.8	0.2
0.6083876	L	0.6	0.8	0.2
0.041938	SPACE	0.0	0.1	0.1
0.41938	G	0.4	0.5	0.1
0.1938	A	0.2	0.3	0.1
0.938	T	0.9	1.0	0.1
0.38	E	0.3	0.4	0.1
0.8	S	0.8	0.9	0.1
0.0				

Example

At the first L, encoded number is 0.72167752.
output the first L

range = 0.8 – 0.6 = 0.2

encoded number = (0.72167752 – 0.6) / 0.2
= 0.6083876

Advantage of arithmetic coding

Assume: A 90% END 10%
To encode: AAAAAAA

New Character	Low value	High Value
	0.0	1.0
A	0.0	0.9
A	0.0	0.81
A	0.0	0.729
A	0.0	0.6561
A	0.0	0.59049
A	0.0	0.531441
A	0.0	0.4782969
END	0.43046721	0.4782969

Advantage of arithmetic coding

Assume: A 90% END 10%

To encode: AAAAAAA

New Character	Low value	High Value
	0.0	1.0
A	0.0	0.9
A	0.0	0.81
A	0.0	0.729
A	0.0	0.6561
A	0.0	0.59049
A	0.0	0.531441
A	0.0	0.4782969
END	0.43046721	0.4782969

e.g., 0.45

98

98

Patents on AC

- Bzip2 and JPG use Huffman as AC protected by patents
- PackJPG using AC shows 25% of size saving

99

99

Some AC patents (expiring)

U.S. Patent 4,122,440 — (IBM) Filed 4 March 77, Granted 24 October 78 (Now expired)
U.S. Patent 4,286,256 — (IBM) Granted 25 August 81 (Now expired)
U.S. Patent 4,467,317 — (IBM) Granted 21 August 84 (Now expired)
U.S. Patent 4,652,856 — (IBM) Granted 4 February 86 (Now expired)
U.S. Patent 4,891,643 — (IBM) Filed 15 September 86, granted 2 January 90 (Now expired)
U.S. Patent 4,905,297 — (IBM) Filed 18 November 88, granted 27 February 90 (Now expired)
U.S. Patent 4,933,883 — (IBM) Filed 3 May 88, granted 12 June 90 (Now expired)
U.S. Patent 4,935,882 — (IBM) Filed 20 July 88, granted 19 June 90 (Now expired)
U.S. Patent 4,989,000 — Filed 19 June 89, granted 29 January 91 (Now expired)
U.S. Patent 5,099,440 — (IBM) Filed 5 January 90, granted 24 March 92 (Now expired)
U.S. Patent 5,272,478 — (Ricoh) Filed 17 August 92, granted 21 December 93 (Now expired)

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