COMP9319 Web Data Compression and Search

An Occ Implementation, RLFM (Compressed FM Index) Revisit

An example Occ implementation

3

FM Inde _L[5])	ex (w	/hen ı	eve	ersing from
	F	L	c	С
0	#	<u>L</u> i	<u>c</u> 0	<u>C</u> # 0
1	i	р	0	i 1
2	i	s	0	m 5
3	i	s	1	p 6
4	i	m	0	LF[5] = 0+0 = 0, i s 8
5	m	#	0	LF[0] = 1+0 = 1, p LF[1] = 6+0 = 6, p
6	p	p	1	LF[6] = 6+1 = 7, i
7	p	i	1	LF[7] = 1+1 = 2, s
8	S	s	2	LF[2] = 8+0 = 8, s LF[8] = 8+2 = 10, i
9	S	s	3	LF[10] = 1+2 = 3, s
10	S	i	2	LF[3] = 8+1 = 9, s LF[9] = 8+3 = 11, i
11	S	1	3	LF[11] = 1+3 = 4, m 5

FM Inde	ex (L)	(x) ≠	c)		
	F		imps	C	
0	#	<u>L</u>	1000	<u>C</u> # 0	
1	i	p	1010	i 1	
2	i	s	1011	m 5	
3	i	s	1012	p 6	
4	i	m	1 1 1 2	s 8	
5	m	#	1 1 1 2		
6	p	р	1 1 2 2		
7	p	i	2 1 2 2		
8	S	s	2 1 2 3		
9	S	s	2 1 2 4		
10	S	i	3 1 2 4		
11	S	i	4 1 2 4		6

5

FMI	ndex (c	on't)			
pss <u>i</u> ⇒ Fst=1 Lst=4	F 0 # 1 i 2 i 3 i 4 i 5 m 6 p 7 p 8 s 9 s 10 s 11 s	L: p s m # p i s s i	imps 1000 1010 1011 1012 1112 1112 1122 2122 2123 2124 3124	C # 0 i 1 m 5 p 6 s 8	7

FM Ir	ndex (co	on't)		
ps <u>si</u> ⇒	F 0 # 1 i 2 i 3 i 4 i 5 m 6 p 7 p 8 s 9 s 10 s 11 s	<u>L</u> i p s s m # p i s s i i	i m p s 1 0 0 0 1 0 1 0 1 0 1 1 1 0 1 2 1 1 1 2 1 1 2 2 2 1 2 2 2 1 2 3 2 1 2 4 3 1 2 4 4 1 2 4	E # 0 i 1 m 5 p 6 s 8 Fst=8+0 Lst=(8+2)-1

7

FM	Index	SS.	ignm	ent Pi	roje	ctM	Index (r	m't	Help	
ps <u>si</u>	0 # 1 i	<u>L</u> i p	<u>i m p s</u> 1 0 0 0 1 0 1 0	C # 0 i 1		p <u>ssi</u>	0 # 1 i	<u>L</u> i p	<u>i m p s</u> 1 0 0 0 1 0 1 0	C # 0 i 1
	3 i 4 i 5 m	s s m #	1 1 1 2 1 1 1 2		WCC	aei	4 i 5 m	∏s m #	1 0 1 1 1 0 1 2 1 1 1 2 1 1 1 2	m 5 p 6 s 8
	6 p 7 p 8 s	p i s	1 1 2 2 A1 2 2 F	.weC	hat	po	6 p P C O	de	1 1 2 2 2 1 2 2 2 1 2 3 F	'st=8+2
	10 s 11 s	i i	3 1 2 4 4 1 2 4	st=(8+2)-1			10 s 11 s	i i	3 1 2 4 4 1 2 4	st=(8+4)-1

10

8

FM Index (con't) F # i i m p s
1 0 0 0
1 0 1 0
1 0 1 1
1 0 1 2
1 1 1 2
1 1 2 2
2 1 2 2
2 1 2 3
2 1 2 4
3 1 2 4
4 1 2 4 <u>C</u> # 0 i 1 L p s s 0 p<u>ssi</u> m 5 p 6 s 8 i m # m p i р р Fst=8+2 Lst=(8+4)-1 9 s 10 s 11 s s i

9

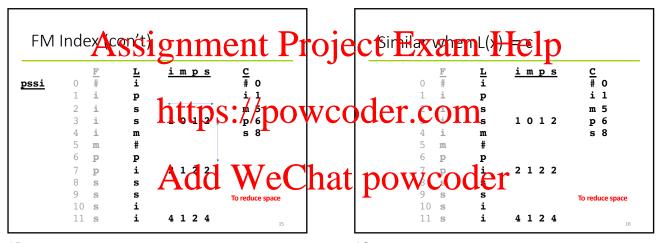
FM Ir	nde	х (с	on't)		
<u>pssi</u> ⇒	0 1 2 3 4 5 6 7 8 9 10	<u>F</u> #: i: i: m p p s s s s s	<u>L</u> i p s s m # p i s s i i	i m p s 1 0 0 0 1 0 1 0 1 0 1 1 1 0 1 2 1 1 1 2 1 1 2 2 2 1 2 2 2 1 2 3 2 1 2 4 3 1 2 4 4 1 2 4	E # 0 i 1 m 5 p 6 s 8 Fst=6+2 Lst=(6+2)-1

11 12

FM Ir	ndex	(con't)		
pssi	0 # i i 2 i i 3 i i 4 i i 5 m 6 p 7 p 8 s 9 s 110 s 111 s	L: p s m # p i s s i	i m p s 1 0 0 0 1 0 1 0 1 0 1 2 1 1 1 2 1 1 2 2 1 2 2 2 1 2 3 2 1 2 4 3 1 2 4 4 1 2 4	E # 0 i 1 m 5 p 6 s 8 Fst=6+2 Lst=(6+2)-1 Fst>Lst => No match

FM	Inde	ex (c	con't)		
pssi	0	<u>F</u>	<u>L</u>	<u>imps</u>	<u>C</u> # 0
	2	i i	p s	1012	i 1 m 5
	5 5	i	s m #	1012	p 6 s 8
	6 7	p p	p i	2 1 2 2	
	8	S	s s	2 1 2 2	
	10	S	i i	4124	To reduce space

13



14

15 16

```
RLFM Index (Revisit)
```

```
RLFM Index (Derive B' from LF)
        <u>L</u> <u>C</u> c a 0 c c 3
                                       \frac{Fs}{a} \frac{Cs}{a} 0
                                  <u>s</u>
                                       а
                                           c 2
                              0 a
        c g 6
a t 8
                              0
                                           g 3
t 4
                                       C
                                  g
                                      g
t
        а
        g
        g
    g a
         t
                          10 0
```

17 18

RLFM Index (con't from the prev lecture) <u>L</u> <u>C</u> c a 0 c c 3 $\frac{Fs}{a} \frac{Cs}{a} 0$ <u>s</u> <u>B</u> <u>B′</u> ī 2 c 2 а 0 а а g 3 t 4 a/c g 6 3 0 g 4 5 4 8 а 1 g 1 а C 0 t 0 1 g g 0 8 а 1 10 0

RLFM Index (con't from the prev lecture) <u>L</u> <u>C</u> a 0 $\frac{Fs}{a} \frac{Cs}{a} 0$ <u>s</u> <u>B</u> <u>B′</u> ī 1 c 2 c 3 а C 0 а 0 g 3 t 4 g 6 c 0 g а g 1 1 а а 0 0 g g 0 а 10 0

19 20

```
RLFM Index (con't from the prey lecture) ASSIGNMENT
                                                                     RLEM Index (con't from the prev
        <u>L</u> <u>C</u> a
                                  Fs Cs a 0
                          <u>В</u>
                             <u>s</u>
             0
                                                                                                               a 0
          c 3
                                                                                                    0
                                                                                                                      0
                                                                                                    0
    C
    g
        t
10
```

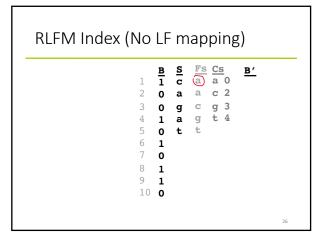
21 22

```
RLFM Index (no L & F, nor LF)
                                 <u>B</u>
                                      <u>s</u>
                                                        <u>B′</u>
                            2
                                 0
                                 0
1
                                      g
                            5
                                 0
                                 0
                            8
                                 1
                            9
                                 1
                            10 0
     If only B and S are stored and given... then how \ref{eq:Barter}
```

```
RLFM Index (no L & F, nor LF)
                                      Fs
                                                  <u>B′</u>
                                      а
                                 c
                                     а
                             0
                                 а
                                      C
                             0
                                 g
                             1
                                      g
                                 а
                             0
                             0
                         10 0
     If only B and S are stored and given... then how \ref{eq:Barter}
```

23 24

RLFM Index (no L & F, nor LF) $\frac{Fs}{a} \frac{Cs}{a} 0$ <u>s</u> <u>B</u> <u>B′</u> c 2 0 а c g 3 g t 4 0 g 1 а 0 t 1 0 8 1 1 10 0

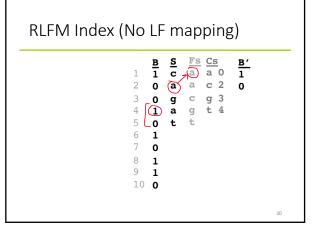


25 26

RLFM Indax (No ignation of the project Physical Phy

27 28

RLFM Index (No LF mapping) Fs Cs a 0 $\begin{array}{ccc} \underline{B} & \underline{S} \\ 1 & c \\ 0 & a \end{array}$ <u>B′</u> a c 2 2 3 4 5 6 7 0 g a t g 3 t 4 C g 1 0 8 1 1 10 0



29 30

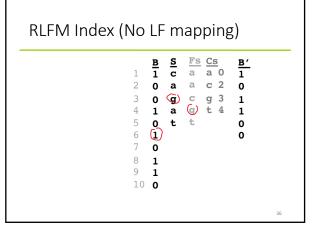
RLFM Index (No LF mapping) $\frac{Fs}{a} \frac{Cs}{a} 0$ <u>B</u> <u>s</u> <u>B'</u> (a) c 2 0 а g 0 1 g 3 t 4 C **a** g 0 1 7 **0** 8 **1** 9 **1** 10 **0**

```
RLFM Index (No LF mapping)
                                       \frac{Fs}{a} \frac{Cs}{a} 0
                              <u>В</u>
                                  <u>s</u>
                                                   <u>B'</u>
                                       (a)
                                           c 2
                              0
                                  а
                                                   0
                                           g 3
t 4
                                       C
                              0
                                  g
                                       g
                              1
                                  a
                              0
                                  t
                         9 1 10 0
```

31 32

33 34

```
RLFM Index (No LF mapping)
                                    Fs
a
a
                                         Cs
a 0
c 2
                                 <u>s</u>
                                                 <u>B'</u>
1
                             <u>В</u>
1
0
                         2
3
4
5
                                a
                                                 0
                                    g 3
t 4
                           0
                                g
a
                                                 1
                                     g
                             0
                             0
                         8
                             1
                         10 0
```

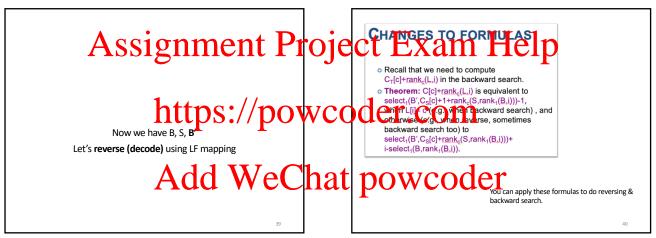


35 36

RLFM Index (No LF mapping) $\frac{Fs}{a} \frac{Cs}{a} 0$ <u>B</u> ī c 1 c 2 0 а а 0 g 3 t 4 **(g)** 0 1 **(g)** 1 а 0 t 0 **(i)** 0 0 1 8 1 n 9 1 10 0

RLFM Index (No LF mapping) Fs Cs a a 0 <u>B'</u> ī c 1 c 2 0 а а 0 g 3 C 0 g 1 g t 1 а 0 (t 0 1 0 9 1 10 o

37 38



39 40

CHANGES TO FORMULAS

- o Recall that we need to compute $C_T[c] + \underbrace{rank_c(L,i)}$ in the backward search.
- o Theorem: C[c]+rank_(L.i) is equivalent to select₁(B',C_S[c]+1+rank_c(S,rank₁(B,i)))-1, when $L[i] \neq c$ (e.g., when backward search) , and otherwise (e.g., when reverse, sometimes backward search too) to select₁(B',C_S[c]+rank_c(S,rank₁(B,i)))+ i-select₁(B,rank₁(B,i)).

But I promised that I would explain why/how

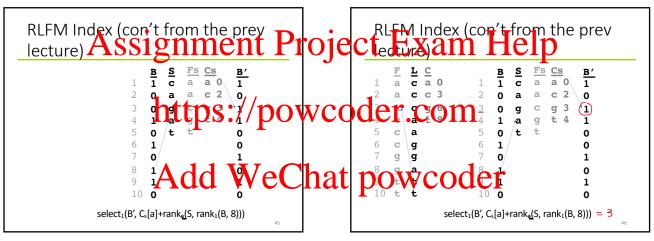
these formulas actually work

RLFM Index (con't from the prev lecture) <u>L</u> C Fs Cs <u>c</u> a 0 a a 0 1 а 1 C **c** c 3 а c 2 а 2 0 а 0 g 3 С a **c** g 6 0 g 1 ť а t. 8 1 g а а 0 g g 0 g 1 8 g a 8 0 9 t 9 10 t t 10 0 0 Suppose reverse from L[8]

41 42

RLFM Index (con't from the prev lecture) $\frac{Fs}{a} \frac{Cs}{a} 0$ <u>B</u> 1 c 2 а 0 0 g 1 $rank_{\underline{a}}(S, rank_1(B, 8)) = 2$ 0 8 1 1 1 10 0 0

43 4



45 46

Good, but not good enough

RLFM Index (con't from the prev lecture) Cs <u>B'</u> a 0 а C а c 2 0 0 g g g 10 0 $select_1(B', C_s[c]+rank_c(S, rank_1(B, 3)))$ =select₁(B', 2 + 1)=4

47 48

RLFM Index (con't from the prev lecture) <u>L</u> <u>C</u> a $\frac{Fs}{a} \frac{Cs}{a}_0$ <u>B</u> 0 а 1 **₄**C 1 c 2 2 а C C 3 0 а а 0 3 g t C g 6 0 g 1 8 C g а 1 а 1 C а 0 t C g g g 0 1 8 8 g а 0 9 t 1 10 0 $select_1(B', C_s[c]+rank_c(S, rank_1(B, 3)))$ =select₁(B', 2 + 1)=4

RLFM Index (con't from the prev lecture) $\frac{\mathbf{L}}{\mathbf{c}} \stackrel{\mathbf{C}}{=} 0$ Fs Cs <u>B</u> <u>B′</u> a 0 a а 1 **₄**C 1 c 2 а C c 3 0 а а 0 g 6 3 C 0 g g 1 а g t 1 а 1 а 0 t g g g 0 1 g а 1 0 1 10 0 $select_1(B', C_s[c]+rank_c(S, rank_1(B, 3)))$ =select₁(B', 2 + 1)=4 + 2

49 50

```
RLFM Index (con't from the prey lecture) ASSIGNMENT
                                                    roject Exam Help
         <u>C</u>
       <u>L</u>
C
                          <u>S</u>
                       <u>B</u>
            0
                                  a 0
    a
       С
            3
                    2
                                             powcoder.com
       C
            6
                                                                                 Another example, LF[5] = ?
                    6
    c,
    g
                                                     hat powcoder
   g
       t
10
       t
              select_1(B', C_s[c]+rank_c(S, rank_1(B, 3)))
              =select<sub>1</sub>(B', 2 + 1)=4 + (i -rank<sub>1</sub>(B, i))
```

51 52

```
RLFM Index (con't from the prev
lecture)
                                       Cs
                                       a 0
                                    a
                           ī
                               c
                                              1
                                       c 2
                       2
                           0
                               ₄a
                                    а
                                               0
                                    С
                                          3
                           0
                                       g
                                               1
                               g
                                        t
                           1
                                    g
                               а
                           0
                           1
                           0
                                               1
                       8
                       9
                       10 0
            select_1(B', C_s[a]+rank_a(S, rank_1(B, 5)))
            =select<sub>1</sub>(B', 0 + 1)=1 + (i -rank<sub>1</sub>(B, i))
```

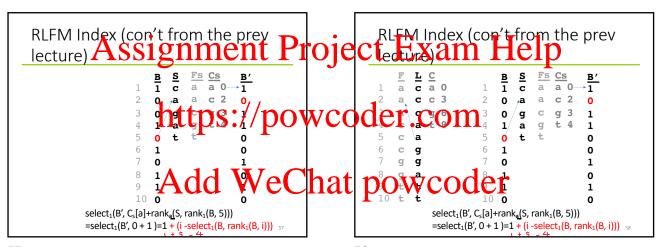
```
RLFM Index (con't from the prev
lecture)
                                    Fs Cs
                                        a 0
                            ī
                                    а
                                               1
                                C
                                    а
                                        c 2
                            0
                               ₄a
                                                0
                                    C
                                           3
                                        g
                            0
                                g
                                                1
                                        ŧ
                            1
                                    g
                                а
                            0
                                                0
            select_1(B', C_s[a]+rank_1(S, rank_1(B, 5)))
            =select<sub>1</sub>(B', 0 + 1)=1 + (i -rank<sub>1</sub>(B, i))
```

53 54

RLFM Index (con't from the prev lecture) Fs Cs a 0 <u>B</u> <u>B'</u> c 1 1 c 2 0 ₄a а 0 g 3 t 4 0 g 1 g 1 а o t 1 0 1 8 1 0 1 10 0 $select_1(B', C_s[a]+rank_a(S, rank_1(B, 5)))$ =select₁(B', 0 + 1)=1 + (i -rank₁(B, i))

RLFM Index (con't from the prev lecture) Fs Cs <u>B</u> <u>B′</u> a 0 a c 1 1 c 2 0 ₁a а 0 3 0 g g 1 g t 1 а 0 ŧ. 1 0 1 1 0 1 10 0
$$\begin{split} & select_1(B', C_s[a] + rank_{\underline{a}}(S, rank_1(B, 5))) \\ & = select_1(B', O + 1) = 1 + (i - select_1(B, rank_1(B, i))) \end{split} \label{eq:select_1}$$

55 56



57 58

RLFM Index (con't from the prev lecture)

CHANGES TO FORMULAS

• Recall that we need to compute $C_T[c]+rank_c(L_i)$ in the backward search.
• Theorem: $C[c]+rank_c(L_i)$ is equivalent to select, $(B', C_s[c]+1+rank_c(S, rank_1(B_i)))-1$, when $L[i] \neq c$ (e.g., when backward search), and otherwise (e.g., when reverse, sometimes backward search too) to select, $(B', C_s[c]+rank_c(S, rank_1(B_i)))+i-select_1(B', C_s[a]+rank_a(S, rank_1(B, j)))$ $select_1(B', C_s[a]+rank_a(S, rank_1(B, 5)))$ $=select_1(B', O+1)=1+(i-select_1(B, rank_1(B, i)))$

Backward Search

59 60

Backward sear	ch f	or	"sj'	,		
1 2 3 4 5 6 7 8 9	B 1 1 0 1 1 1 1 1 0 0 1	<u>s</u> i p s m # p i s i	F _s # i i i m p p s s	C # 0 i 1 m 4 p 5 s 7	B' 1 1 1 1 0 1 1 1 1 0 1	

61

63

Backward search for "si" <u>В</u> 1 <u>s</u> i <u>B′</u> 1 0 i 1 -1 s i 0 m p i р 1 11 **1** 12 **0** 0

```
Backward search grisment Project Backward search grisment Project
```

64

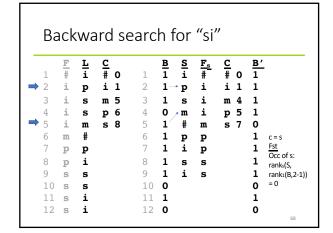
62

```
Backward search for "si"
             <u>C</u>
# 0
i 1
                                                 <u>B′</u>
1
•1
        <u>L</u>
                            <u>В</u>
1
1
                                <u>s</u>
                                           <u>C</u>
#
                                              0
                       2
         р
                                     i
                                           i 1
                                р
                                           m 4
p 5
                                                 1
                                s
m
         s
s
                       3
                            1
                                     i
                            0
              p 6
                                     i
                       4
5
6
                                 #
                                                  0
         m
                                     m
         #
                            1
                                р
                                    р
                                                  1
                                i p
                                                      Fst =
                            1
                                                  1
    р
        р
                                                      C[c] + Occ(c,
                       8
                            1
         i
                                                  1
    р
                                s
                                     s
                                                     Fst – 1) + 1
= ?
                            1
        s
                            0
                                                  0
11 s
                       11 1
                                                  1
                       12 0
                                                  0
```

Backward search for "si" <u>C</u> # 0 i 1 <u>s</u> <u>F</u>s # i <u>B'</u> 1 <u>L</u> <u>В</u> 1 1 <u>C</u> р р s m i i 1 1 s 1 Lr. 5 6 **p** 6 0 # 1 m 0 c = i Fst = 2 Lst = 5 1 р 1 7 1 i р р р 8 i 1 р s s 9 1 s 0 11 s i 11 12 s **i** 12 **0** 0

65 66

	ıcı	VVC	ard se	Juic		O1	51			
	F	L	С		В	s	Fs	С	<u>B′</u>	
1	#	<u>L</u> i	<u>C</u> # 0	1	<u>В</u> 1	<u>s</u> i	<u>F</u> s	<u>C</u> # 0	1	
2	i	р	i 1	2	1-	• p	i	i 1	1	
3	i	s	m 5	3	1	s	i	m 4	1	
4	i	s	p 6	4	0	, m	i	p 5	1	
5	i	m	s 8	5	1	#	m	s 7	0	
6	m	#		6	1	р	р		1	c = s
7	p	р		7	1	i	р		1	Fst = ??
8	р	i		8	1	s	s		1	
_	S	s		9	1	i	s		1	
9		~		10	0				0	
10	S	s								



67 68

```
\begin{array}{ccc} \underline{C} & \underline{B'} \\ \# & 0 & 1 \\ i & 1 & 1 \\ m & 4 & 1 \end{array}
                       <u>C</u>
# 0
                                                                                                                                                                                                                   <u>C</u>
# 0
i 1
               <u>L</u>
i
                                       2
               р
                      i 1
                                               1 → p
                                                                                                                                                                                                                             1
                                                                                                                                                                                               • p
                                               1
                                                                                                                                                                                                         i
               s
                                                                                                                                                                                                 s
                                       4
5
                       p 6
               m
                                               1
                                                       #
                                                              m-
                                                                                                                                                                                                         m
                                       6
               #
                                              1 p
1 i
                                               1
                                                             р
        m
                                                                                                                                                                                                                                   Fst
Occ of s:
        р
               р
                                                                                                                                                                                                                              1
                                       8
                                               1
        р
               i
                                                                                                                                                                                                                                    rank<sub>s</sub>(S,
rank<sub>1</sub>(B,2-1))
                                       9
                                               1
        S
               s
                                                                                   0 = 0

1 selecti(B',7+

1+0)

0 So Fst = 9
10 s
                                       10
                                               0
               s
                                                                                                                                                                                                                                    selecti(B',7+
               i
                                               1
11
       S
                                        11
                                                                                                                                                                                                                                   1+0)
So Fst = 9
                                        12 0
                                                                                                                                           12 s
                                                                                                                                                                                  12 0
```

70

Backward search for "si" <u>B'</u> 1 1 <u>C</u> # 0 i 1 <u>C</u> # 0 i 1 <u>F</u>s # i <u>L</u> <u>В</u> 1 i 1 → p 2 i р m 4 p 5 1 1 s s m 5 3 1 s i ranks(S, ranks(B,5)) p 6 4 i i 0 " m 5 0 i 1 # 7 m s 8 m s m # 6 1 р р 1 1 i 1 р р р 1 8 8 1 р i s s 9 9 1 1 S s 0 0 11 S i 11 1 1 12 s i 12 **0** 0

69

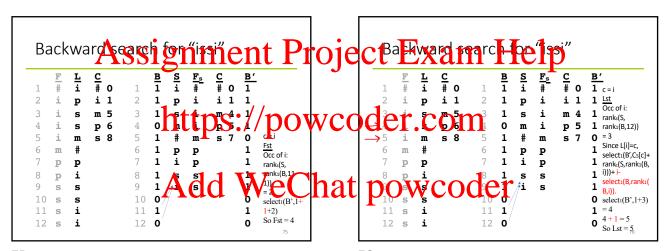
```
Backward search for "si"
                                                                  <u>C</u>
# 0
i 1
                                                                            <u>B'</u>
1
                     <u>C</u>
# 0
i 1
                                                 <u>s</u>
i
                                                         <u>F</u>s
#
i
              <u>L</u>
                                           <u>В</u>
1
       i
                                    2
                                           \mathbf{1}^{\longrightarrow}\,\mathbf{p}
                                                                            1
              р
                                                         i
i
             s
                     m 5
                                    3
                                           1
                                                 s
                                                                                   rank<sub>s</sub>(S,
rank<sub>1</sub>(B,5))
                     p 6
                                                                            1
       i
                                    4
                                           0
                                                                   p 5
                                                🛪 m
                                    5
       i
                     s 8
                                                  #
                                                                             0
                                           1
                                                         m
                                                                   s
                                                                                   = 1
select<sub>1</sub>(B',7+
1+1) = 11
11 - 1 = 10
So Lst = 10
       m
                                           1
                                                  р
                                                         р
                                    7
                                           1
                                                  i
       р
                                                         р
              р
                                    8
8
                                           1
       р
              i
                                                  s
                                                          s
                                    9
                                           1
                                                                                   -1: since
inclusively,
e.g., Lst-Fst+1
= #matches
       S
              s
                                    10
                                           0
      S
              i
                                    11
                                           1
12 s
                                    12 0
                                                                             0
                                                                                           72
```

71 72

Bá	ack	Wa	ard se	earc	h f	or	"SS	i"		
1 2 3 4 5 6 7 8 9 10 11 12	F# i i i i m p p s s s s	Lipssm#Pissii	C # 0 1 m 5 p 6 s 8	1 2 3 4 5 6 7 8 9 10 11	B 1 1 1 0 1 1 1 1 0 0 1 0 0	<u>s</u> i p s m # p i	F _s # i i m p p s	C # 0 i 1 m 4 p 5 s 7	B' 1 1 1 1 0 1 1 1 0 1 0	c = s <u>Fst</u> Occ of s: rank ₄ (S, rank ₁ (B,9-1)) = 1 select ₁ (B',7+ 1+1) So Fst = 11

Ba	ack	Wā	ard s	earc	h f	or	"ss	i"	
1 2 3 4 5 6 7 8 9 10 11 12	F#:i:i:m ppssss	Li Pssm# Pissii	C # 0 i 1 m 5 p 6 s 8	1 2 3 4 5 6 7 8 9 10 11	B/1 1 1 0 1 1 1 1 0 1 0 1	<u>s</u> ; ps m # p; i	F _s # i i m p s s	C # 0 i 1 m 4 p 5 s 7	$\begin{array}{c} {\bf B'} \\ {\bf 1} \\ {\bf c} = s \\ {\bf 1} \\ {\bf Lst} \\ {\bf Corof s:} \\ {\bf 1} \\ {\bf corof s:} \\ {\bf 1} \\ {\bf rank_s(S, 0)} \\ {\bf 0} = {\bf 1} \\ {\bf 1} \\ {\bf since L(i) = c, select_l(B',Cs[c] + 1 \\ {\bf 1} \\ {\bf mink_s(S,rank_s(B, 1))} \\ {\bf 1} \\ {\bf 1} \\ {\bf 0} \\ {\bf 1} \\ {\bf 2} \\ {\bf 5o Lst} = \frac{1}{12} \\ {\bf 2} \\ {\bf 5o Lst} = \frac{1}{12} \\ {\bf 1} \\$

73



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75 76

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Therefore ...

CHANGES TO FORMULAS

• Recall that we need to compute C_T[c]+rank_c(L_i) in the backward search.
• Theorem: C[c]+rank_c(L_i) is equivalent to (select_1(B^i,C_s[c]+1+rank_c(S,rank_1(B_i)))-1) when L[i]\neq c (e.g., when backward search), and otherwise (e.g., when reverse, sometimes backward search too) to select_1(B^i,C_s[c]+rank_c(S,rank_1(B_i)))+i-select_1(B,rank_1(B_i)).
```