Complexity classes

The following list contains common time complexities of algorithms:

$O(1)$ The running time of a **constant-time** algorithm does not depend on the input size. A typical constant-time algorithm is a direct formula that calculates the answer.

$O(\log n)$ A **logarithmic** algorithm often halves the input size at each step. The running time of such an algorithm is logarithmic, because $\log_2 n$ equals the number of times n must be divided by 2 to get 1.

$O(\sqrt{n})$ A **square root algorithm** is slower than $O(\log n)$ but faster than $O(n)$. A special property of square roots is that $\sqrt{n} = n/\sqrt{n}$, so the square root $\sqrt{n}$ lies, in some sense, in the middle of the input.

$O(n)$ A **linear** algorithm goes through the input a constant number of times. This is often the best possible time complexity, because it is usually necessary to access each input element at least once before reporting the answer.

$O(n \log n)$ This time complexity often indicates that the algorithm sorts the input, because the time complexity of efficient sorting algorithms is $O(n \log n)$. Another possibility is that the algorithm uses a data structure where each operation takes $O(\log n)$ time.

$O(n^2)$ A **quadratic** algorithm often contains two nested loops. It is possible to go through all pairs of the input elements in $O(n^2)$ time.

$O(n^3)$ A **cubic** algorithm often contains three nested loops. It is possible to go through all triplets of the input elements in $O(n^3)$ time.

$O(2^n)$ This time complexity often indicates that the algorithm iterates through all subsets of the input elements. For example, the subsets of $\{1,2,3\}$ are $\emptyset$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$ and $\{1,2,3\}$.

$O(n!)$ This time complexity often indicates that the algorithm iterates through all permutations of the input elements. For example, the permutations of $\{1,2,3\}$ are $(1,2,3)$, $(1,3,2)$, $(2,1,3)$, $(2,3,1)$, $(3,1,2)$ and $(3,2,1)$.

An algorithm is **polynomial** if its time complexity is at most $O(n^k)$ where k is a constant. All the above time complexities except $O(2^n)$ and $O(n!)$ are polynomial. In practice, the constant k is usually small, and therefore a polynomial time complexity roughly means that the algorithm is *efficient*.

Most algorithms in this book are polynomial. Still, there are many important problems for which no polynomial algorithm is known, i.e., nobody knows how to solve them efficiently. **NP-hard** problems are an important set of problems, for which no polynomial algorithm is known[1].

[1] A classic book on the topic is M. R. Garey's and D. S. Johnson's *Computers and Intractability: A Guide to the Theory of NP-Completeness* [28].