Optimised reinforced concrete beams using Peridynamics

Technical milestone report

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1 Summary

This report summarises the work that has been conducted to date, as well as the planned structure for the rest of the project. Project significance and key literature is summarised in section 2. The specific topic of this research project as well as progress and remaining work are discussed in sections 3 and 4.

The aim of the project is to assess the physical validity of Peridynamic (PD) simulations on reinforced concrete beams, and to compare the results to known laboratory experiments. PD is a promising modelling framework for simulation of material behaviour that allows for the natural handling of crack formation and propagation. Part of the aim of the project is to develop a versatile simulation that allows for the analysis of Peridynamic predictions on non-standard beam geometries. The work conducted to date includes creating a bond-based PD simulation in C++ and implementing a data pipeline for geometry imports and data visualisation and analysis. Remaining work includes implementing arbitrary beam geometries and using a parallel optimisation scheme to find efficient beam designs.

2 Motivation & Background

Throughout this report the term *optimised* or *efficient* design is used to broadly mean a beam design that is in some sense cheaper, lighter or with less embodied energy than an equivalent prismatic beam. The approaches and conventions used in the design of reinforced concrete (RC) beams are usually wasteful in terms of material usage and embodied energy [1]. One reason is that in a simple prismatic beam of constant cross section, much of the material is not utilised fully. The reasons these designs are still used are numerous: simpler designs are well covered by existing legislative codes

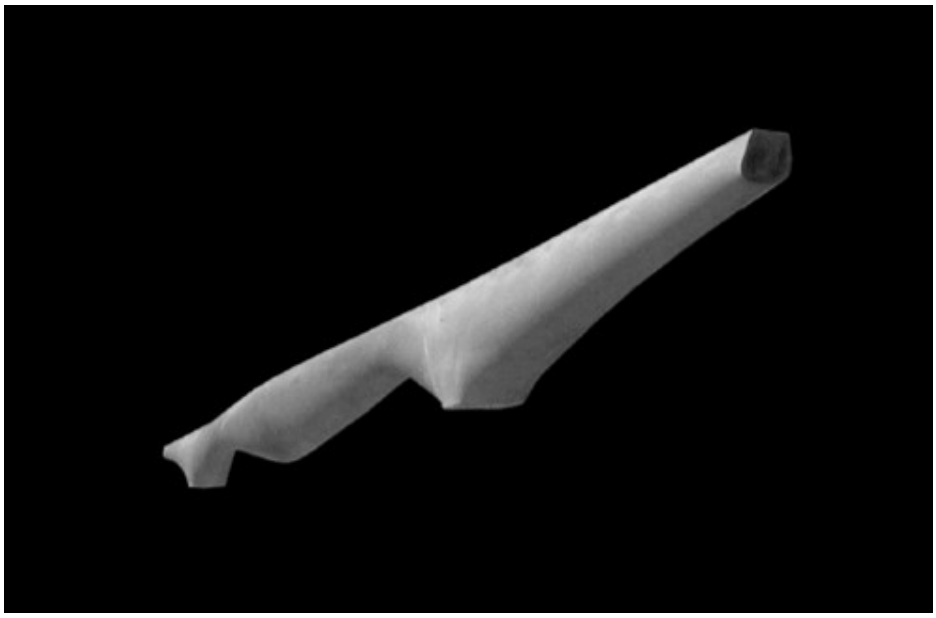


Figure 1: Optimised RC beam design. Credit Mark West C.A.S.T.

governing design. They are simple to verify as safe, and can often be analysed without the use of bespoke software.

A major obstacle to the implementation of more efficient beam designs is the difficulty in predicting the displacement behaviour and failure loads of beams with non-standard geometries. In the vast majority of cases this is not practically possible without the use of complex numerical analysis. However, even using state of the art industrial Finite Element Analysis (FEA) software, predictions are often unreliable, or require a large number of bespoke system parameters to be defined. This is because the failure of concrete, and other brittle materials, is dominated by fracture. However, the classical equations of equilibrium and compatibility in solid mechanics are cast as spatial partial differential equations, meaning that spatial discontinuities like fractures cannot be handled directly. Many FEA's instead treat a crack as a distributed material weakness in the surrounding region. This approach has the obvious drawback of requiring some prior knowledge about crack location [2].

2.1 Peridynamics

The basic idea of Peridynamics is to circumvent the problem of spatial derivatives by replacing the governing equations of classical solid mechanics with integral equations. The founding paper *Reformulation of elasticity theory for discontinuities and long-range forces* by Silling et al. [2] introduces the so called *bond-based* Peridynamic model. This is the model type that has been implemented and used so far in this work, although a variety of reformulations and modifications exist, one of the latest of which is the *weak formulation of PD*. These reformulations allow for a more natural modelling of non-linear material properties like plasticity, and circumvent issues arising from the application of displacement boundary conditions and externally applied forces[3].

What is common to all Peridynamic models is that the material being analysed is discretised into a set of points of finite volume and mass. Points within a certain distance of each other interact through f, the *force density function*. This critical distance is called the *horizon* of a point and is usually denoted δ. The dependence of the force density function on node pair displacement and global material parameters is largely what differentiates PD model formulations. In the simple bond-based model, the behaviour is that of a linear weightless spring that connects neighbouring nodes. This allows only for the transmission of axial forces. It can be shown that this form of f restricts the global material behaviour to a Poisson's ratio of 0.25 [5].

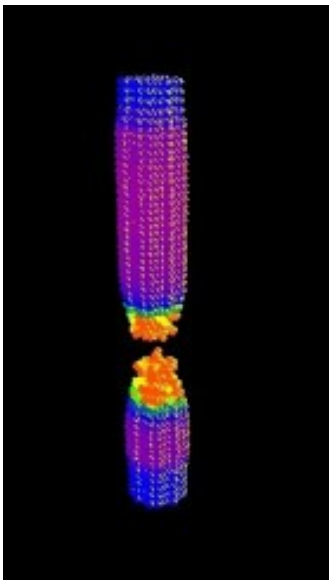


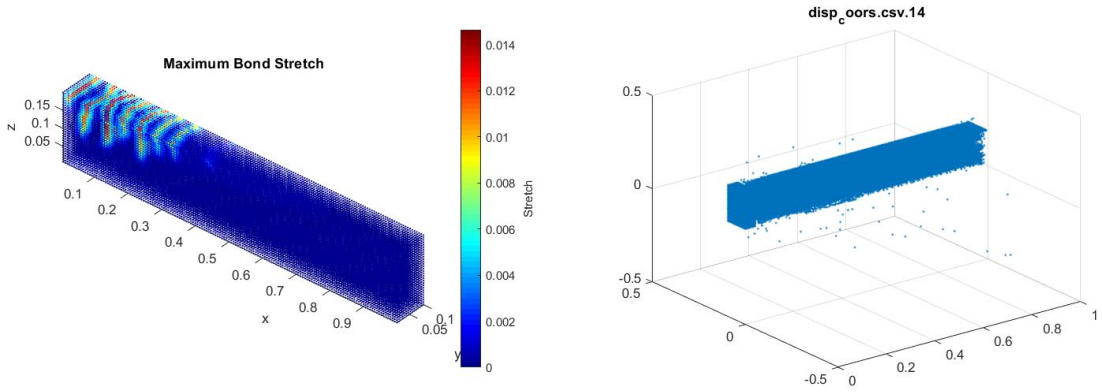
Figure 2: PD simulation of necking failure[4].

Regardless of the exact form of f, the force density function can be used to formulate the equilibrium equation of PD, shown in equation 1. This equation takes the form of Newton's second law and describes the behaviour of a node at position $\boldsymbol{x}$ interacting with neighbouring nodes at $\boldsymbol{x'}$. Here ρ is the material density, $\boldsymbol{u}$ is the node displacement, and $\boldsymbol{b}$ denotes any externally applied body force field. The domain of integration R is in general taken as the whole of the material, however, in most PD formulations this can be replaced by $|\boldsymbol{x} - \boldsymbol{x'}| \leq \delta$ as $f = 0$ outside this region.

$$\rho(\boldsymbol{x})\ddot{\boldsymbol{u}}(\boldsymbol{x},t) = \int_R f(\boldsymbol{u}(\boldsymbol{x'},t) - u(\boldsymbol{x},t), \boldsymbol{x'} - \boldsymbol{x}) dV_{x'} + \boldsymbol{b}(\boldsymbol{x},t) \tag{1}$$

It can be shown that in the limit of an infinitely fine set of points, the integral equations of PD are mathematically equivalent to the classical solid mechanics formulation [6].

A practical challenge that is limiting the implementation of PD simulations is the often high computational cost compared to FEA solutions. This is a known issue with most particle simulations, as explicit numerical integration over all particles is needed. In PD this means that a summation over all neighbour nodes is required to compute the internal force on each individual node at each time step. Because of this nested summation, the computational cost is heavily dependent on the number of nodes used for discretisation [7].



(a) Bond stretch in cantilevered RC beam. MATLAB implementation.

(b) Simulation instability from wave propagation and tear-off. C++ implementation.

Figure 3: Example PD simulations conducted in Michaelmas term of prismatic beam.

3 Research Focus

A large portion of the existing body of scientific literature on PD is concerned with the mathematical properties and validity of various PD formulations. For example showing that a formulation has desirable features like wave damping and energy dissipation. Comparative experiments are often in the form of comparing the PD formulation in question to a classical FEA analysis of a well explored problem, like the uniaxial tension of a rod or a rectangular plate with a central circular hole. Few papers are concerned chiefly with the practical implementation of PD or the physical validity of predictions compared to laboratory experiments. That is why the the focus of this work is to assess the *physical* validity and predictive accuracy of PD for RC beams of non-standard geometries. This research focus is broken down into key questions:

- Does a coarse bond-based model accurately predict the deflection and failure of a standard geometry RC beam?
- Is the computational cost of a reasonably accurate PD solution for a non-standard geometry such that an iterative optimisation scheme is practical?
- What is the accuracy of a bond-based PD failure analysis of a non-standard RC beam when compared to laboratory tests?

3.1 Expected Results and Challenges

Previous work on PD simulations have shown that the qualitative aspect of fracture initialisation and growth are well-modelled. Failure modes displayed for simple beams and known geometries agree well with known failure modes of such members. However, like many particle based models, PD simulation needs careful handling of edge effects, specifically displacement boundary conditions and load application. With

highly coarse discretisations, PD models often suffer from *tear-off* close to clamped supports and unphysical strain wave-propagation through the bulk of the material. These problems have been encountered in the simulation work conducted (see figure 3b) and are expected to be more pronounced with beams of complicated geometries. Or equivalently, the discretisation resolution will most likely need to be higher for such geometries. These issues are however well known and explored in the existing literature, both for PD and other particle simulation models, and a variety of techniques exist to address the issues.

4 Project Roadmap

One of the aims of this project is to develop and implement a versatile PD simulation script that allows for the analysis of beams of complicated geometries and varying material composition. Using an existing bond-based implementation in MATLAB as a basis, a C++ implementation was made using the Standard C++ library during Michaelmas term 2018. In order to modulerise and generalise the simulation process a geometry import script has been implemented to store pre- and post-simulation geometries as standardised *.csv* files. This separates one of the more computationally costly steps of the PD analysis from the simulation process, and allows easy graphical data analysis using a host of possible packages, including Paraview and MATLAB.

Due to the high computational cost of performing PD simulations, the planned optimisation scheme will be constrained by limiting beam cross sections to *parameterised function* sections. This means that the shape and axial variation of the beam is described by a set of polynomial functions with a combined set of $3 - 6$ shape controlling parameters. An example is a parabolic cross section with a depth that varies as a symmetric cubic function along the longitudinal axis of the beam. This greatly cheapens the optimisation process, and allows a parallel implementation on High Performance Computers (HPCs) using standard *black-box* optimisation schemes like *Evolution Strategies*.

4.1 Remaining Work

- Assess and remedy stability issues in C++ implementation of bond-based PD model.
- Implement the most costly aspects of the PD simulation code as vectorised C++ code using the library *Eigen*.
- Validate simulation predictions on prismatic cantilevered RC beams.
- Implement the generation of arbitrary beam-geometry *.csv* files using parameterised beam cross sections.
- Validate simulation on optimised beam geometries, and compare against known laboratory results.

- Implement a parallel optimisation scheme for parameterised cross sections on HPCs.

References

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