BEAM DYNAMICS WITH A CRAB CAVITY

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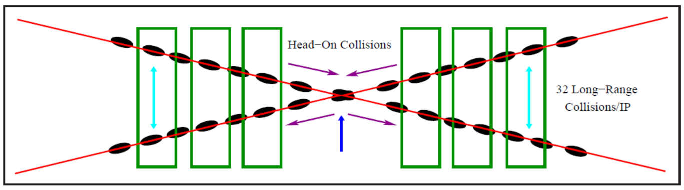
ABSTRACT. CERN is considering the use of crab cavities for its future luminosity upgrade. Compact crab cavities are being designed and a thorough study of the impact of the cavity is needed. In this paper, we study the effects of the nonlinear electromagnetic fields in the simulation of the ODU JLab model at 26 GeV. An interpolation method is introduced and simulation results of several beam parameters with or without a crab cavity are compared.

1. INTRODUCTION

1.1. **Motivation.** In the LHC, two beams have to travel at an angle near the interaction point to avoid parasitic collisions. However, it reduces the cross-sectional area of the interaction, thus reducing the luminosity.



Schematic of the LHC interaction region triplets to depict the crossing scheme required to minimize parasitic collisions with reducing β^*.

Inefficient overlap

The idea of crab crossing was first proposed by R. Palmer in an attempt to enable effective head-on collisions in linear accelerators ([7]). Shortly afterwards, K. Oide and K. Yokoya proposed a scheme for storage-ring colliders ([6]). Crab crossings have been implemented at KEKB in 2007 and succeeded in improving the luminosity. Various laboratories worldwide are devoted to the design and research of crab cavities.

According to K. Ohmi at KEK, crab cavities have the potential to boost the beam-beam parameter higher than 0.15. At the same time, it alleviates the requirement to substantially increase the beam bunch intensity or reduce emittance, which can usually be very challenging and problematic.

CERN is considering the use of crab cavity during the LHC's next luminosity upgrade, to enable a larger crossing angle without luminosity loss. However, existing models can only fit in one site of LHC (IR4). Accelerator physicists worldwide have decided to focus on designing compact crab cavities in the recent years.

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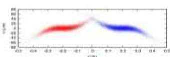
1.2. **Research objectives.** Despite its alluring prospects, crab cavities are not without their problems. Even at KEKB where crab cavities has first succeeded, the actual luminosity improvement is still not clear and under tuning. One potential problem related to our research is that the EM fields inside a crab cavity is not linear. The nonlinear fields may give rise to subtle changes in the beam parameters, resulting in particle loss and degradation of luminosity.

The goal of this project is to study the possible negative effects of the ODU-JLab cavity design on the tune footprint, dynamic aperture and emittance of the beam using simulations at the energy level of 26 GeV. We have obtained the numerical values of discrete electrical and magnetic fields of the ODU-JLab model from CSD Microwave Studio simulations. EM fields anywhere inside the cavity are obtained through interpolating the known data. Equations of motion of particles inside the cavity can be obtained by combining the equation of Lorentz force and the EM fields, thus providing a numerical way of calculating the crab cavity kicks.

2. JLab cavity design

We particularly research on the ODU-JLab model. In its simplest form, it is a parallel bar cavity with TEM mode. The cavity intended for LHC has to be much different from the KEKB model. Apart from the apparent size constraint, a lot of important specifications are also different, such as frequency, horizontal and vertical crossing and electric field. A lot of redesigning will be needed to fit the cavity into the LHC.

	Baseline	Unit	LHC	KEK-B
RF	Frequency	MHz	400 (800)	509
RF	Deflecting Voltage	MV/Cav	5	2.0 (0.9-1.5)
RF	Peak E-field	MV/m	< 45	28
RF	Peak B-field	mT	< 80 mT	82 mT
Geometrical	Aperture (diameter)	mm	84	130
Geometrical	Cav Outer Envelope	mm	< 150	866/483
Geometrical	Module length	m	~ 1m	1.5 m
	HV crossing	-	Desirable	N/A
Optics	β* (IR1/IR5)	cm	15-25	63/0.7
Optics	β crab	km	~ 5	0.2/0.04
	Non-linear harmonics	Units [10^{-4}]	2-3	N/A
	Impedance Budget	Longitudinal, Transverse	60kΩ, 2.5MΩ/m	-



$\sigma_z = 7.55$ cm

Beam-beam separation

FIGURE 1. Comparison of cavity specifications between KEKB and LHC

The original cavity design has a rectangular shape, but a more recent design took the cylindrical shape instead.

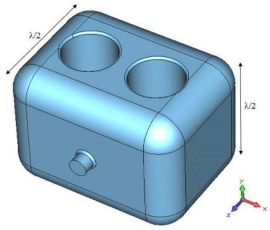


FIGURE 2. Initial design of ODU-JLab cavity with round parallel bars (left)

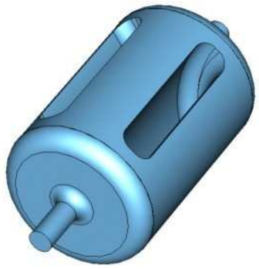


FIGURE 3. Present (August 2011) form of the cavity evolved to minimize higher order modes (right)

3. The algorithm

3.1. Crab cavity kicks. EM fields in a TEM resonance structure are

$$\mathbf{E}(x,y,\sigma,t)=\mathbf{E}(x,\sigma)\cos\left(\frac{2\pi y}{\lambda}\right)\sin(\omega t),$$

$$\mathbf{B}(x,y,\sigma,t)=\frac{\mathbf{E}(x,\sigma)}{Z_0}\times\hat{y}\sin\left(\frac{2\pi y}{\lambda}\right)\cos(\omega t)$$

where $Z_0=\sqrt{\epsilon/\mu}$.

In an analytical model as in [7], consider two infinite rods parallel to the y-axis with uniform charge density q, and crossing the (x,σ) plane at $x=\pm a,\ \sigma=0$. The potential is given by

$$V(x,\sigma)=\frac{q}{4\pi\epsilon_0}\ln\left(\frac{r_-^2}{r_+^2}\right),$$

where

$$r_-^2=(x-a)^2+\sigma^2,\ r_+^2=(x+a)^2+\sigma^2.$$

The electric fields are

$$E_x(x,\sigma)=-\frac{\partial V}{\partial x}=-\frac{aq}{\pi\epsilon_0}\left[\frac{x^2-a^2-\sigma^2}{r_-^2r_+^2}\right]$$

$$E_\sigma(x,\sigma)=-\frac{\partial V}{\partial\sigma}=-\frac{aq}{\pi\epsilon_0}\left[\frac{2x\sigma}{r_-^2r_+^2}\right]$$

Lorentz's force is given by $\mathrm{d}\mathbf{p}/\mathrm{d}t=\frac{1}{p_0}q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$. Combined with $\mathbf{v}=\beta c\hat{\sigma}$ we obtain the equations of motion of a particle with longitudinal distance z from the synchronous particle as

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} = \frac{q}{p_0} E_x(x, \beta ct + z) \cos(ky) \sin\left(\omega\left(t - \frac{z}{\beta c}\right)\right)$$
$$\frac{\mathrm{d}p_y}{\mathrm{d}t} = -\frac{q}{p_0} \frac{\beta c}{Z_0} E_\sigma(x, \beta ct + z) \sin\left(\frac{2\pi y}{\lambda}\right) \cos\left(\omega\left(t - \frac{z}{\beta c}\right)\right)$$
$$\frac{\mathrm{d}p_z}{\mathrm{d}t} = -\frac{q}{p_0} E_\sigma(x, \beta ct + z) \cos(ky) sin\left(\omega\left(t - \frac{z}{\beta c}\right)\right).$$

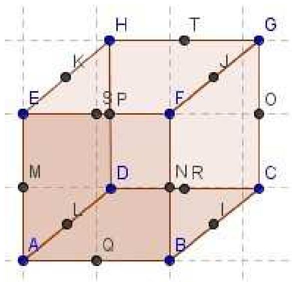
The reference particle passes through the cavity gap in time $t \in nT_0 + (-L_\sigma/2\beta c, L_\sigma/2\beta c)$, where L_σ is the cavity length along the σ direction.

No analytical formula for crab cavity kicks is available for this ODU-JLab model design. We have to obtain it via numerical integration.

In the actual problem, the fields are not identical to the prediction of this analytical model. However, the strength of field components are obtained via numerical simulations by an CSD Microwave Studio design. Along the σ-direction (the direction of propagation), E_x, E_y and H_z are symmetric about the origin while E_z, H_x and H_y are antisymmetric. The symmetric fields will bend the particles transversely and also cause helical motion along the σ axis. The net effect is small along the longitudinal direction of the anti-symmetric fields.

3.2. **Interpolation.** From experience, quadratic interpolation is usually sufficient for EM field data. At the same time, a lower order interpolation minimizes the side effect of oscillations in the function value between tabulated points. Therefore we are adopting an algorithm that is a slight variation of quadratic interpolation. A description of the algorithm can be found in Dhatt and Touzot, *The Finite Element Method Displayed*.

We cover the entire domain with cubes that contains 3 grid points along each direction, then choose 20 grid points in each cube (discarding the grid points at the center of each face and in the center of the cube). The grid spacings are normalized to 1, therefore with the coordinate origin at the center, each axis is in the range $-1 < x, y, z < 1$.



At each point of interpolation,

$$f(x, y, z) = \sum_{i=1}^{20} c_i N_i(x, y, z, \xi_i, \eta_i, \zeta_i)$$

where the constants c_i are found from $f(x_i, y_i, z_i) = c_i N_i(x_i, y_i, z_i, \xi_i, \eta_i, \zeta_i)$ and N_i's are polynomial functions which change from site to site.

The functions N_i are found in the following way.

- Nodes at the vertices:

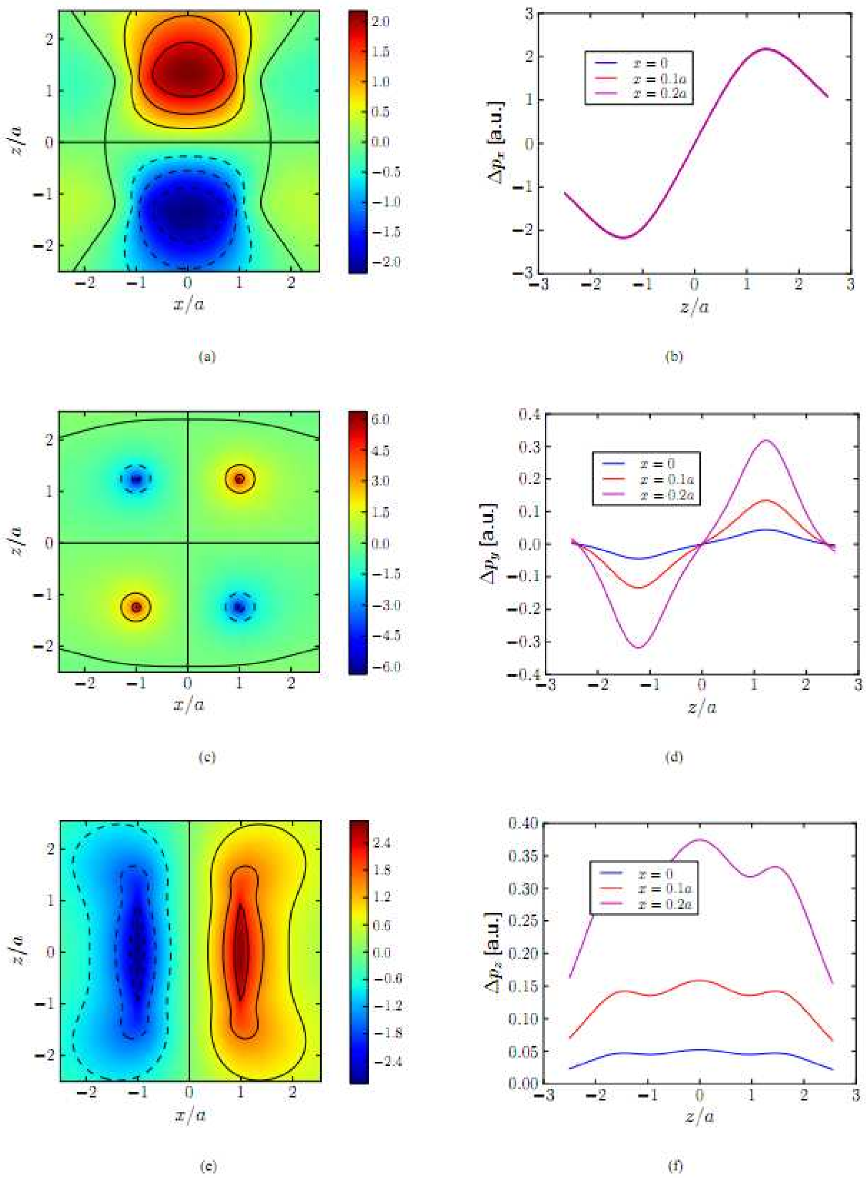


FIGURE 4. Plot of the cavity kicks: (a)-(b) horizontal kicks, (c)-(d) vertical kicks, and (e)-(f) longitudinal kicks. In this plot, $\beta = 1, \omega = 400$ MHz, the cavity gap $L_\sigma = 0.5$m, the bar position $a = 0.1$m, and $y = 0$ are applied ([4]).

Node i	A	B	C	D	E	F	G	H
ξ_i	-1	1	1	-1	-1	1	1	-1
η_i	-1	-1	1	1	-1	-1	1	1
ζ_i	-1	-1	-1	-1	1	1	1	1

$$N_i = \frac{1}{8}(1+\xi_i x)(1+\eta_i y)(1+\zeta_i z)(-2+\xi_i x+\eta_i y+\zeta_i z)$$

-Nodes on the yz-plane:

Node i	Q	R	T	S
ξ_i	0	0	0	0
η_i	-1	1	-1	1
ζ_i	-1	-1	1	1

$$N_i = \frac{1}{4}(1-x^2)(1+\eta_i y)(1+\zeta_i z)$$

-Nodes on the xy-plane:

Node i	I	J	K	L
ξ_i	1	-1	1	-1
η_i	0	0	0	0
ζ_i	-1	-1	1	1

$$N_i = \frac{1}{4}(1+\xi_i x)(1-y^2)(1+\zeta_i z)$$

-Nodes on the xz-plane:

Node i	M	N	O	P
ξ_i	-1	1	1	-1
η_i	-1	-1	1	1
ζ_i	0	0	0	0

$$N_i = \frac{1}{4}(1+\xi_i x)(1+\eta_i y)(1-z^2)$$

4. Results

4.1. **Interpolation.** The interpolation results are compared to Mathematica 3D plot as a benchmark test. The comparisons are shown below (Mathematica interpolation is labeled "Field" and plots of interpolated fields are labeled "Interpolation").

Each 3D graph is obtained by fixing one of the variables x, y or z to -0.01, and plot the respective field component against the other two axes. The value -0.01 is chosen such that it is the nearest grid to the origin, where we are most interested in since the particles travel near the origin (reference orbit).

Note that the graphs labeled with "Interpolation" cover only an octant of the given domain. The known data has a symmetric domain and is mostly symmetric or antisymmetric with all axes, with x and y ranging from -0.05m to 0.05m and z ranging from -0.23m to 0.23m. The grid spacing along each direction is 0.01m apart.

Since all field components have obvious symmetry or antisymmetry about the origin and along axes due to the symmetric geometry of the cavity, it suffices to check one octant of the domain. In fact, in order to generate a fine enough sample of interpolation values, checking the whole domain is not even practical.

We take the octant where x, y and z are all negative (ranging from the minimum value to 0) and take the step size as 1/8 of the grid spacing. Comparing the interpolation to the proper quadrant to the Mathematica graph, we see that they match up quite well. When there is a discrepancy between the two algorithms, for example E_x along the x-direction, it seems that our algorithm presents a smoother field in this particular case.

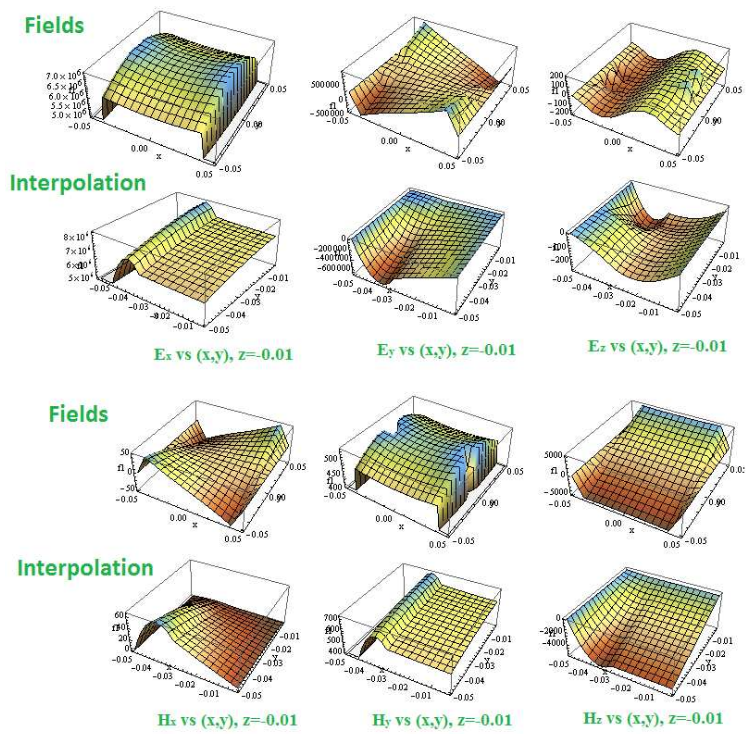


FIGURE 5. Comparison between Mathematica interpolation (labeled as "fields") and results of our interpolation scheme (labeled as "interpolation"). Note that only one quadrant of the interpolated fields are plotted.

4.2. **Simulations.** We use a beam-beam simulation program BBSIM to track particles through a model of SPS with all linear focusing fields and nonlinear fields, and look for the change in beam parameters such as tune footprint, dynamic aperture and emittance growth. We start with SPS simulation since a crab cavity will first be tested at SPS. While the actual cavity has a TEM mode, the simulation were done with a simpler TM model of the crab cavity. We expect to update the program to accommodate simulations of a TEM cavity model in the future. The crab cavity kicks are calculated analytically for a TM mode cavity, and the kicks of a TEM cavity has to be calculated numerically in aforementioned ways.

The crab cavity parameters are

energy(GeV)	voltage(GV)	frequency(MHz)	radius(m)
26	13×10^{-4}	400	0.433

A comparison of tune footprint, dynamic aperture and emittance is shown.

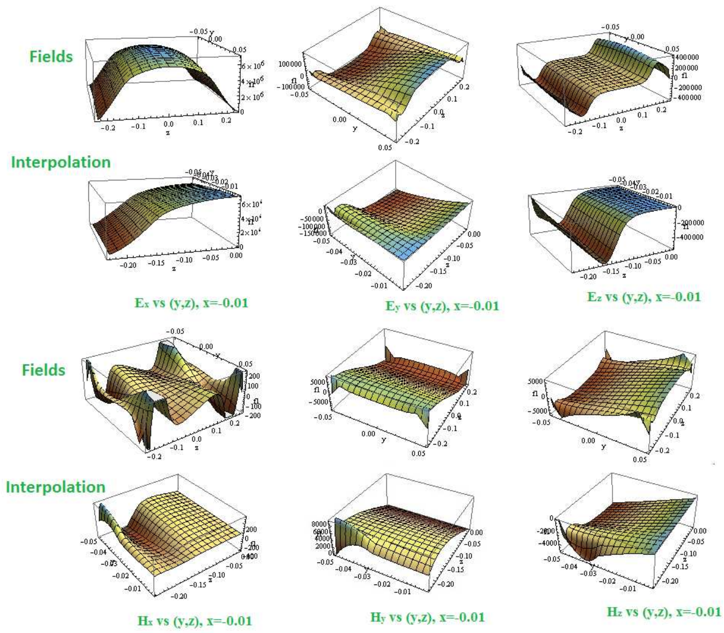


FIGURE 6. Comparison between Mathematica interpolation (labeled as "fields") and results of our interpolation scheme (labeled as "interpolation") II.

The footprint is changed slightly when the crab cavity is turned on, but the difference is almost negligible.

Dynamic aperture specifies the maximal range under which particles are stable. Particles outside of the dynamic aperture will be lost.

No difference is seen in the dynamic aperture when the crab cavity is turned on or off. Also note that no particle loss is observed up to the radius of 60σ, where particularly, the radius of the beam pipe is around 20σ. Therefore there is basically no particle loss with or without the crab cavity. This is possible since the sextuple fields are weak and the beam is fine tuned to avoid resonances.

The emittance is tracked up to 10^6 turns. There is some difference with or without the crab cavity, but it is bounded in the same vicinity.

5. Conclusion

We have implemented a quadratic interpolation scheme and compared it to Mathematica results. The interpolated value is mostly smooth over the domain, and the algorithm is validated by comparison with Mathematica interpolation. This interpolation will be implemented in the tracking code BBSIM and should be valid for any model of a crab cavity.

In the simulation of TM mode cavity at 26 GeV in the SPS, we see precisely the same (and very large) dynamic aperture, very similar tune footprints and similar transverse emittance. No significant change of tune footprints or emittance growth is found with the implementation of the crab cavity.

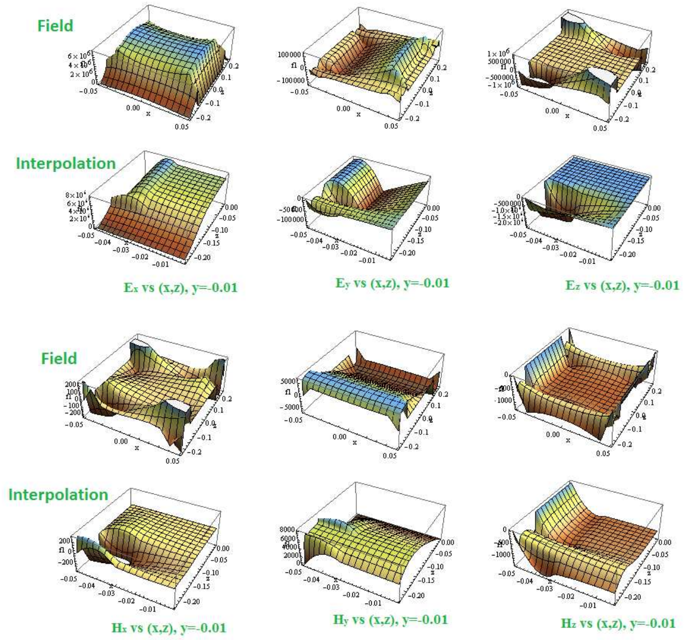


FIGURE 7. Comparison between Mathematica interpolation (labeled as "fields") and results of our interpolation scheme (labeled as "interpolation") III.

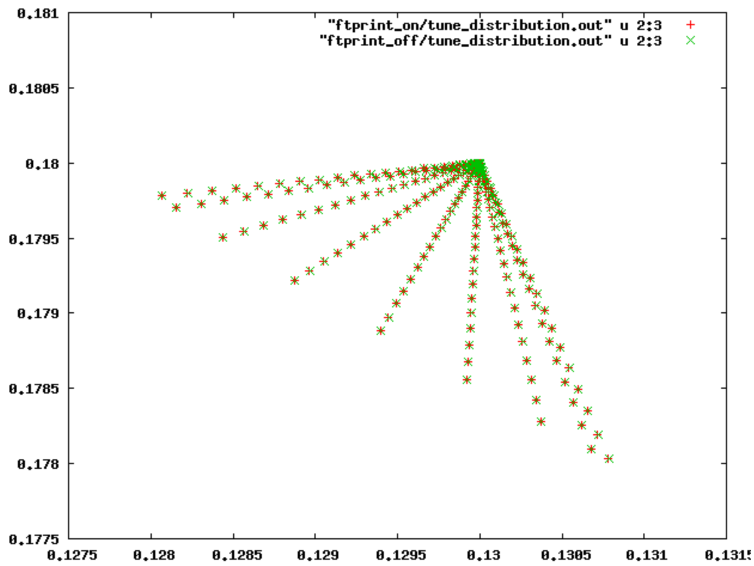


FIGURE 8. Tune footprint with crab cavity on (red) and off (green).

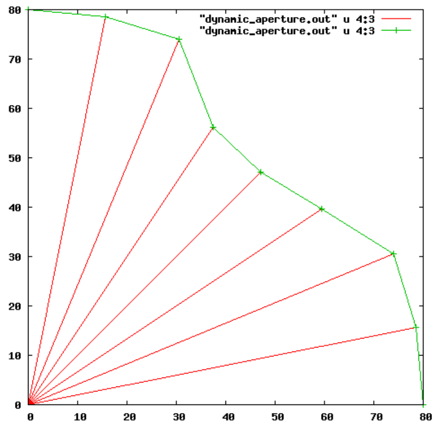


FIGURE 9. Dynamic aperture under TM mode (identical with or without crab cavity).

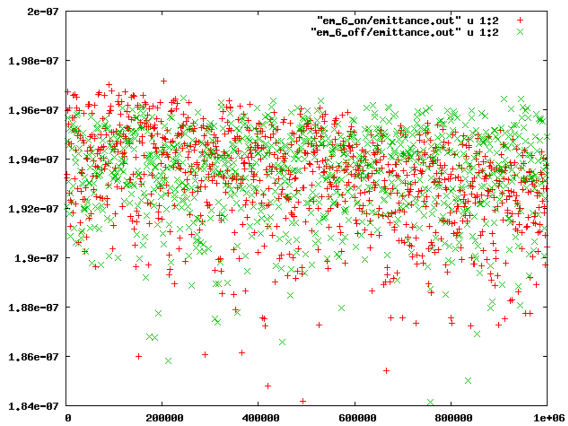


FIGURE 10. Emittance along x-axis up to 10^6 turns with crab cavity on (red) and off (green).

In future we plan to continue simulation with the SPS accelerator using our interpolation scheme with a new TEM mode cavity at various energies. We might also do simulations for the LHC if time allows.

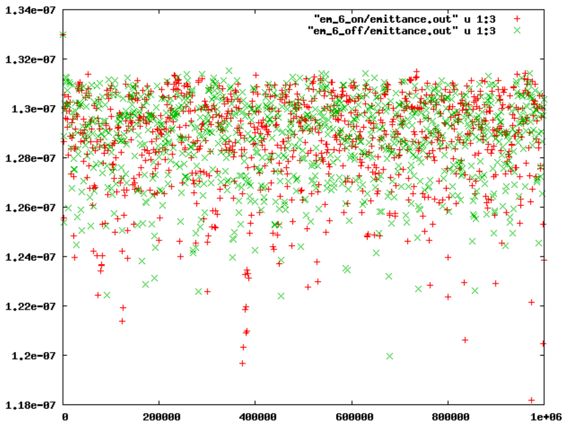


FIGURE 11. Emittance along y-axis up to 10^6 turns with crab cavity on (red) and off (green).

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