RF Capture of Protons the IOTA Ring at Fermilab

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Abstract

This document discusses the RF Capture of 2.5 MeV proton beam into the IOTA ring at Fermilab. Two RF systems are considered: h=4 (2.19 MHz) and h=56 (30.62 MHz). It is found that we will be able to fully bunch the inejcted beam with 1 kV or less of voltage on the h=4 system. The 1kV limit of the h=56 system will not be enough to fully bunch them beam, but it will capture roughly 40% of it, which will be adequate for the operation of the BPM system. The option of a dual harmonic system is also discussed. Calculations and Synergia simulations are presented.

1 Introdution

Table 1: The proton parameters that are relevant to this study. Note that the required voltage increases as the square of the number of sigma in desired energy acceptance.

Parameter	**Value**	
	h=56	**h=4**
Particle Type	proton	
Kinetic Energy (MeV)	2.5	
Momentum (MeV/c)	68.556	
β	.0723	
dp/p	10^{-3}	
σ_E (keV)	5.0	
Current (mA)	8	
Circumference (m)	39.97	
Period (ns)	1829.13	
Q_{tot} (protons)	9.15×10^{10}	
f (MHz)	30.616	2.187
Q_{bunch} (protons)	1.63×10^9	2.23×10^{10}
$V_{min}, \Delta E = 1\sigma_E$ (V)	1005	71.8
ν_s, V=1 kV	.0407	.0109

The key parameters of the IOTA ring are shown in Table 1. The cavity which will be used for electron operation of the IOTA ring will operate at a harmonic of h=4, or approximately 30 MHz. This cavity can also be used in proton mode, but in that case it will of course be a much higher harmonic. The closest harmonic would be h=55; however, as we will discuss shortly, it is somewhat desireable for it to be a multiple of 4, so we have chosen h=56. Using this frequency will allow the same BPMs to be used for the protons as for electrons.

Unfortunately, as we will show, the 1kV limit of the 30MHz RF cavity is insufficient to fully bunch the beam, so a second RF cavity will be installed to operate at h=4, or approximately 2.2 MHz. At this lower frequency, 1kV is sufficient to fully bunch the beam, which will be useful for some of the proposed study.

It will also be possible to bunch the beam at h=4 and then modulate it at h=56 in order to use the BPMs. Because the higher harmonic is a multiple of 4, all h=4 bunches will be modulated in an identical way, which was the motivation for choosing the $h = 56$ harmonic for the high frequency component.

2 Required Bucket Area

The bucket are of a stationary RF bucket is given by[1]

$$\mathcal{A} = \frac{8\beta}{\pi f}\sqrt{\frac{eV_0 E_s}{2\pi h\,|\eta|}}$$

Because the proton beam is very non-relativistic, the 10^{-3} momentum spread will cause it to completely debunch within the first turn after injection into the IOTA ring. This means that the longitudinal phase space area represented by the beam will be

$$\mathcal{A} = 2\Delta E \tau_{\mathrm{rf}} = \frac{2\Delta E}{f}$$

To be consistent with the usual bucket height definition, we have used the convention $\Delta E \equiv n\sigma_E$, with n representing the standard deviations of energy acceptance desired for the injected beam.

Combining these, we find that the required voltage for a given *injected* energy spread is

$$V_0 = \left(\frac{1}{e}\right)\frac{(\Delta E)^2\,\pi^3 h\,|\eta|}{8\beta^2 E_s}$$

When we compare this to the equation for voltage vs. *bucket height*[2]

$$V_0 = \left(\frac{1}{e}\right)\frac{(\Delta E_b)^2\,\pi h\,|\eta|}{2\beta^2 E_s}$$

we see that there is a factor of $\pi^2/4 \approx 2.5$ increase in the required RF voltage, due to the fact that the energy spread of the beam will increase as it is bunched.

3 Simulation

Initial simulation is done using Synergia[3], which loads the MADX version 6 lattice for the IOTA ring. One of the drifts in the "E line" has been replaced with a 1m RF cavity. Because the beam is very non-relativistic, the slip factor η is very close to -1, and there is little dependence on the momentum compaction or any other specifics of the lattice.

Space charge effects are turned off in the initial studies, because they are not expected to affect the longitudinal dymanics much.

An optimized adiabtic capture curve between two voltages V_0 and V_1 is given by[4]

$$\begin{aligned} V(t) &= \frac{V_0}{\left(1-\left(1-\sqrt{\frac{V_0}{V_1}}\right)\frac{t}{t_1}\right)} && ;t < t_1 \\ &= V_1 && ;t \geq t_1 \end{aligned}$$

where

$$t_1 = \left(1 - \sqrt{\frac{V_0}{V_1}}\right)\frac{n_{ad}}{\omega_{s,0}}$$

Here, $\omega_{s,0}$ is the angular synchrotron frequency at $V = V_0$ and n_{ad} is an "adiabaticity number", which is typically of order 10. Time can be expressed in terms of turn number by substituting $2\pi\nu_{s,0}$ for $\omega_{s,0}$. The initial voltage should be chosen to be small compared to the appropriate V_{min} in Table 1.

3.1 RF Bunching at h=56

As shown in Table 1, V_{min} for h=56 is roughly equal to the maximum RF voltage, so we don't expect very efficent RF bunching. We also don't expect much dependence on adiabatic capture.

Figure 1 shows bunch formation for a maximum RF voltage of 1kV, with and without adiabatic capture. For adiabatic capture, the voltage was ramped adiabatically from 50V to 1000V, with an adiabaticity factor of 10. In the non-adiabtic case, the RF voltate was set to 1000V immediately at injection. Figure 6 and 7 show the z and dp/p distributions for each of these cases. As expected, there is very little difference. Because there is very little bunching, there is also very little increase in momentum spread, with only a 6% increase in the adiabatic case.

Figure 4 shows a fit of the h=56 longitudinal distribution to a function of the form (Offset) + (Amplitude) $*$ cos(). The fit corresponds to roughly 60% of the beam unbunched, with the rest bunched entirely at the h=56 harmonic.

It should be noted that the 40% beam capture is exactly what one would expect based on the fact that the 1kV maximum RF voltage corresponds to approximately a 1σ bucket area acceptance, as shown in Table 1.

3.2 RF Bunching at h=4

Figure 5 shows bunch formation for various values of maximum RF voltage, up to 1000V. In all cases, the initial value of the RF voltage is 10V and an adiabaticity number of 10 has been used. Snapshots are taken at least several synchrotron periods after voltage has stabilized. Also shown is the bunch shape if the beam is injected immediately into V=1000V with no capture ramp. Clearly, the latter results in unacceptable filamentation and effective emittance growth, in contrast to the h=56 case.

Figure 6 and 7 show the z and dp/p distributions for each of the cases described above. Figure 8 shows the dependence of dp/p on maximum RF voltage. For comparison, the RMS value for the case with $V = 1000V$ and no adiabatic capture is .0026.

It appears that operating the h=4 system at approximately 500V is a good compromise between bunching and minimizing the momentum spread.

3.3 Dual Harmonic Bunching

It might be useful to bunch the beam at h=4 and then modulate it at h=56 so that the BPMs will operate. To simulate this, the beam is initially bunched with an h=4 RF system, adiabatically ramping from 10V to 500V. Then, after 100 turns, a second h=56 RF system is adiabatically ramped from 50V to 1000V.

Figure 9 show the initial capture by h=4, followed by the h=56 modulation. Figure 10 shows the z distribution of the final bunches. Clearly the h=56 component represents only a small fraction of the beam. Figure 11 shows a Fourier transform of the z distribution. Only approximately 10% of the beam will contribute to the h=56 signal.

4 Conclusions

The proposed two component RF system[5] should be adequate for all anticipated proton needs. While the h=56 system will not have enough voltage to completely bunch the beam, it will bunch a significant fraction, allowing the BPM system to operate.

The h=4 system can fully capture the beam with roughly 500V. The h=56 system can be used to modulate the captured beam to operate the BPM system, but the h=56 component will only represent a small fraction of the beam.

References

[1] D.A. Edwards & M.J. Syphers, "An Introduction to the Physics of High Energy Accelerators", eq. 2.75, p. 45 (1993)

[2] Chao &Tigner, "Accelerator Physics", sec. 2.1.2, eq. 12 (plug in $\phi = \phi_s = 0$ and solve)

[3] `http://compacc.fnal.gov/~amundson/html/`

[4] K.Y. Ng, "Adiabatic capture and debunching", FERMILAB-FN-0943-APC (2012)

[5] K. Carson, *private communication.*

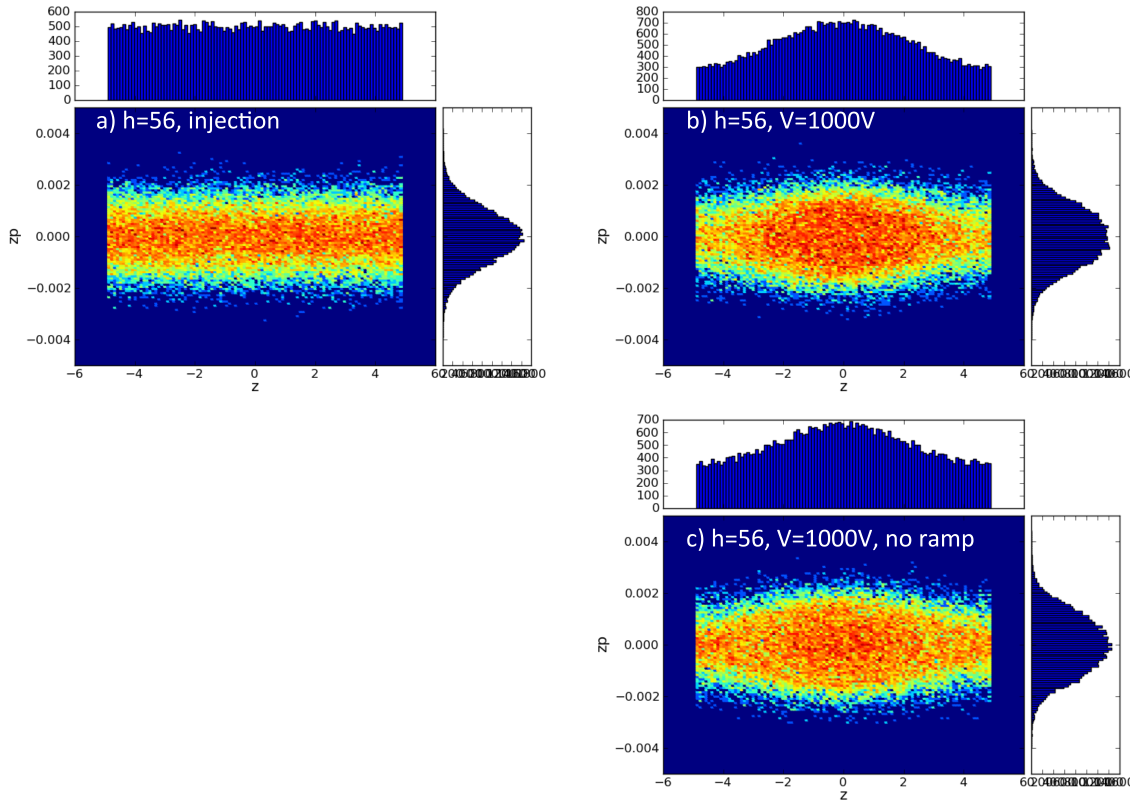

Figure 1: Bunches formation for h=56. Note, "z" represents cdt and zp is dp/p. (a) shows the injection distribution. (b) shows the bunches after adiabatic capture from 10V to 1000V. cf) show the bunch resulting from injecting into an RF voltage of V=1000V with no adiabatic capture ramp.

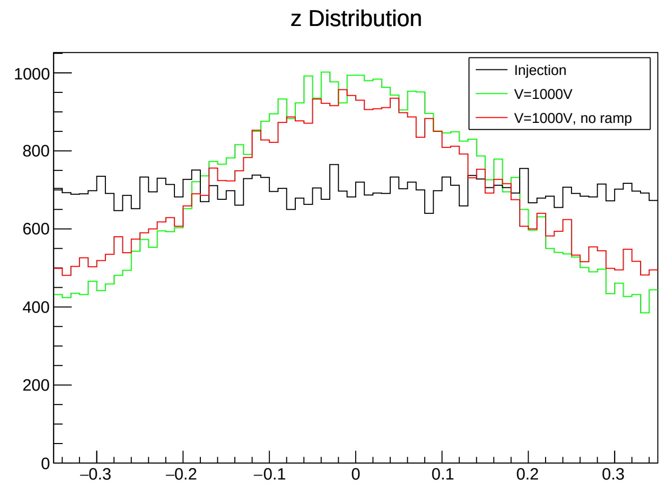

Figure 2: z distributions for h=56 bunches. All distributions except the final one have an adiabatic capture ramp with V0=10v and an adiabaticity of number of 10.

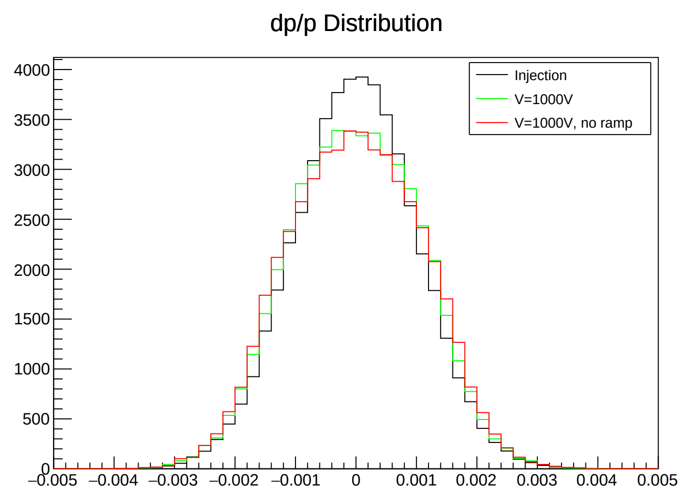

Figure 3: dp/p distributions for h=56 bunches. All distributions except the final one have an adiabatic capture ramp with V0=10v and an adiabaticity of number of 10.

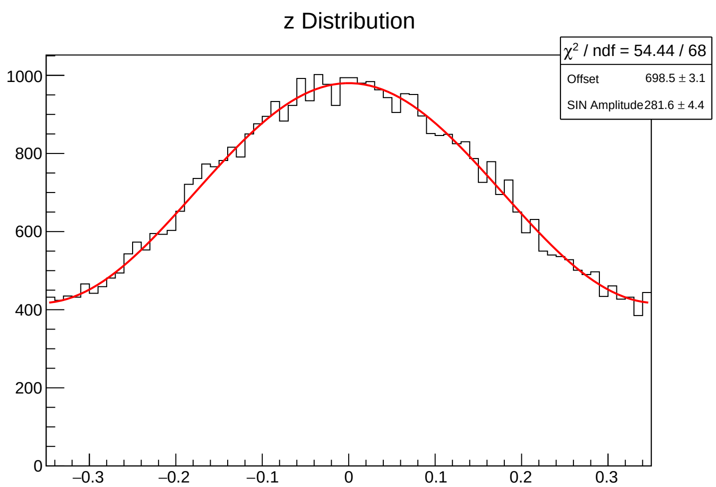

Figure 4: Fit to offset sine wave for h=56.

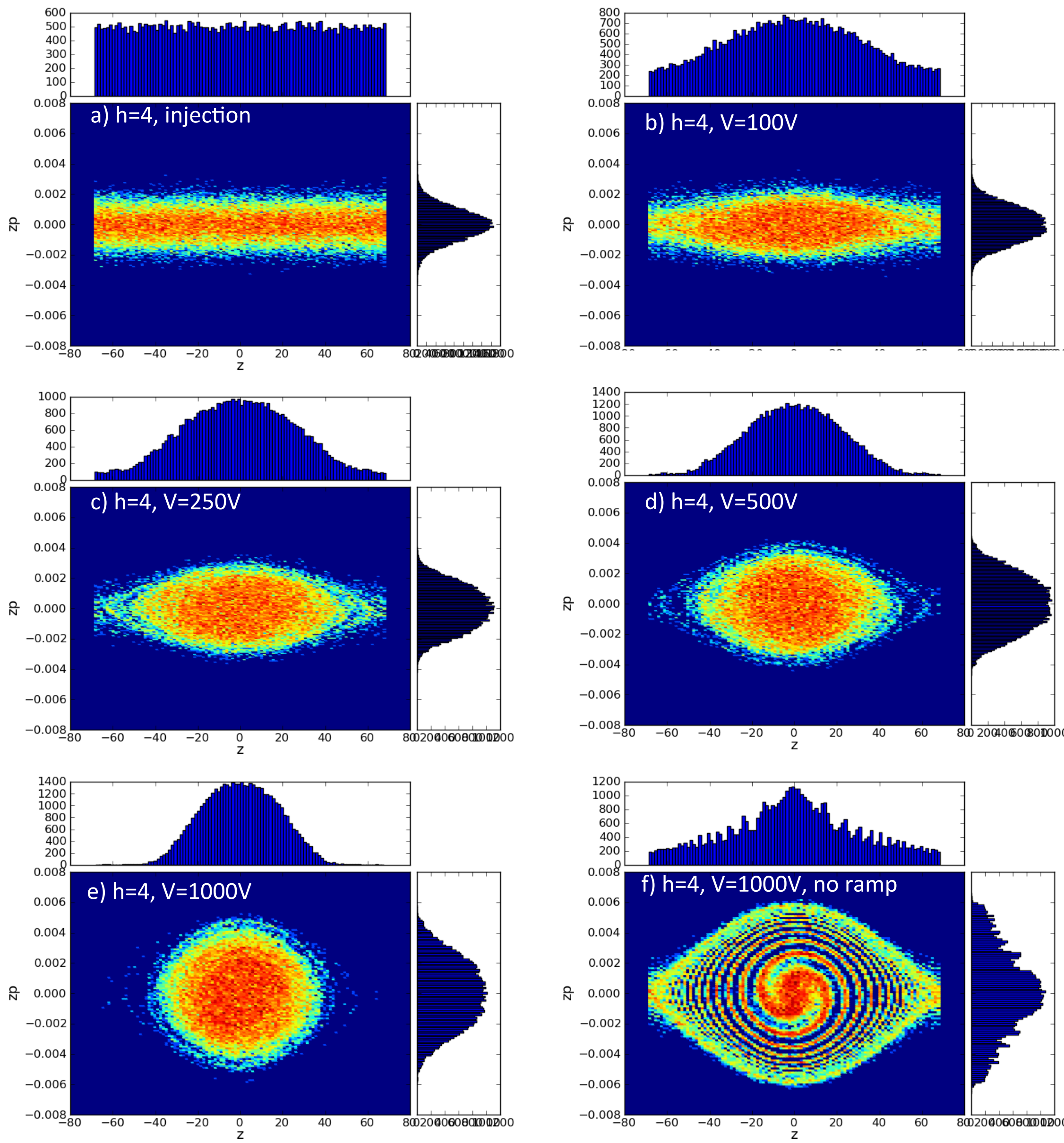

Figure 5: Bunches formation for h=4. Note, "z" represents cdt and zp is dp/p. (a) shows the injection distribution. (b)-(e) show the bunches after adiabatic capture from 10V to the indicated value. (f) show the bunch resulting from injecting into an RF voltage of V=1000V with no adiabatic capture ramp.

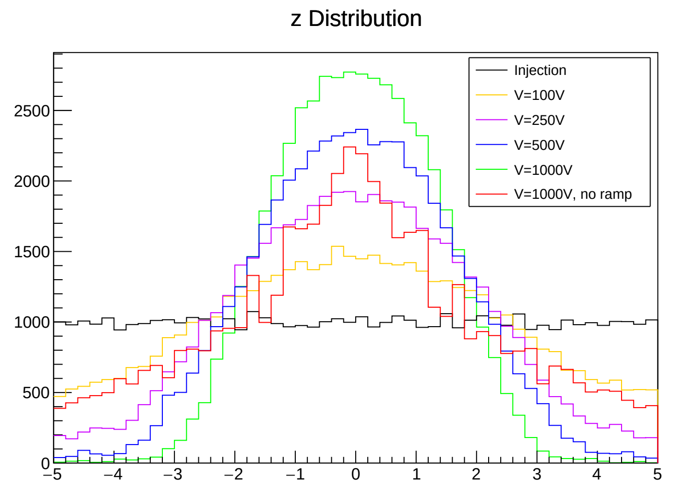

Figure 6: z distributions for h=4 bunches. All distributions except the final one have an adiabatic capture ramp with V0=10V and an adiabaticity of number of 10.

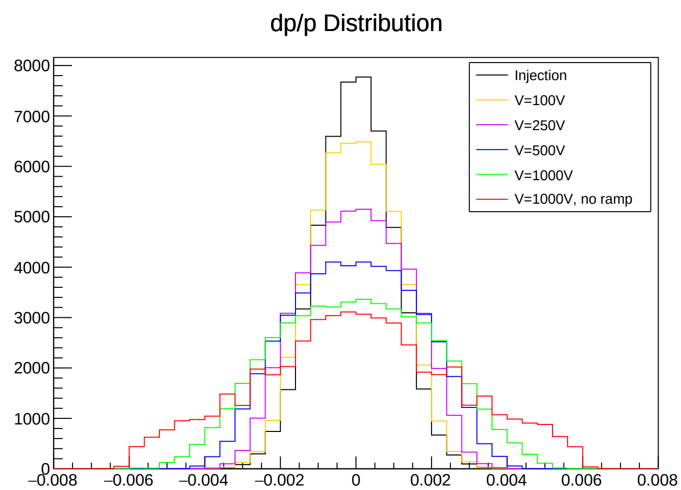

Figure 7: dp/p distributions for h=4 bunches. All distributions except the final one have an adiabatic capture ramp with V0=10V and an adiabaticity of number of 10.

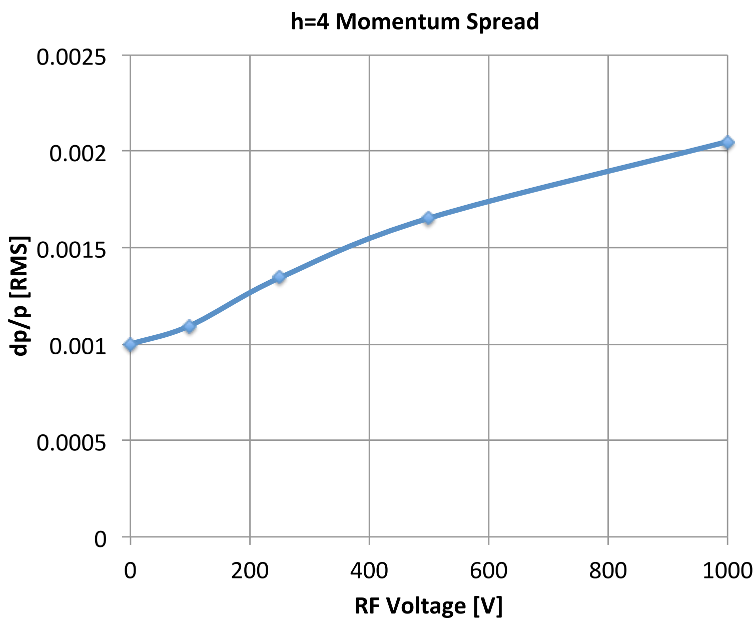

Figure 8: dp/p RMS values as a function of maximum RF voltage, assuming V0=10V and an adiabaticity number of 10.

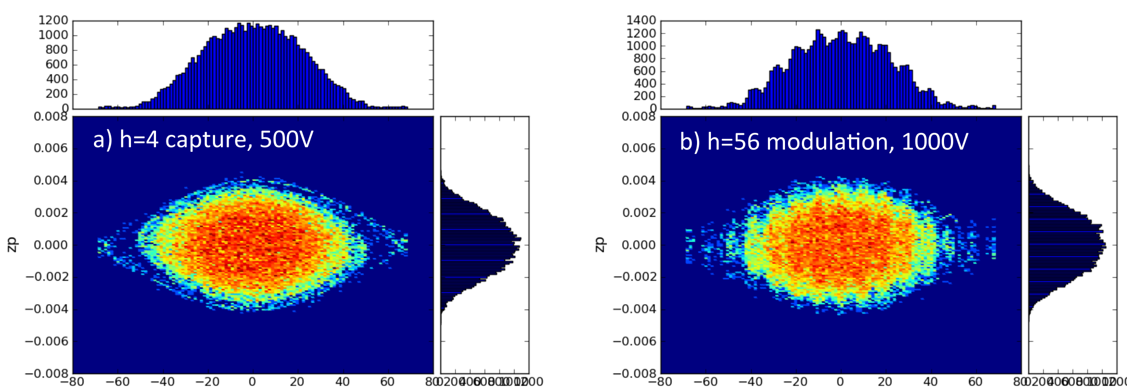

Figure 9: Bunches formation for dual harmonic operation. Note, "z" represents *cdt* and *zp* is dp/p. (a) shows the buncher after capture by h=4. (b) shows modulation at h=56.

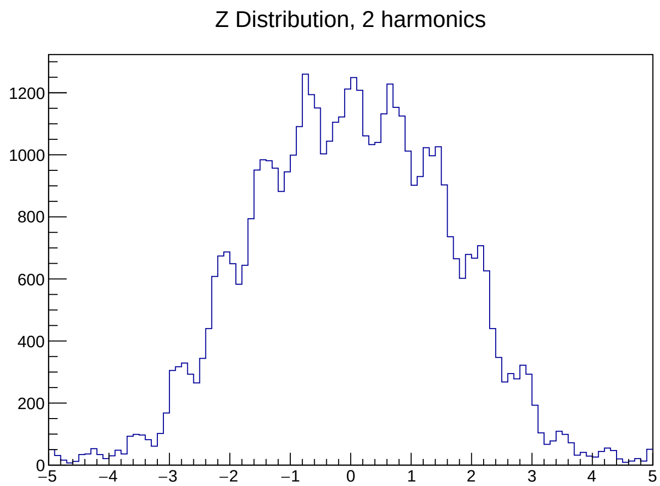

Figure 10: z distribution for for dual harmonic operation. Beam is adiabatically captured with 500V at h=4 and then modulated with 1000V at h=56.

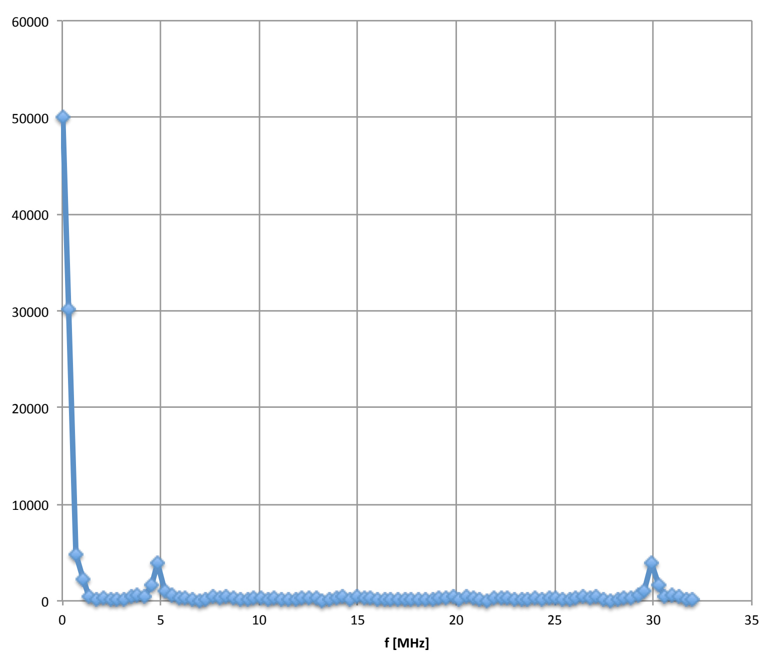

Figure 11: Fourier transform of the 2 harmonic (h=4 and h=56) case.