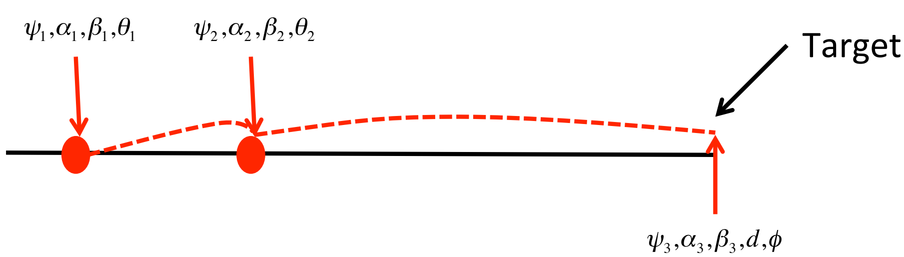
Accelerator Physics Homework #4

1. In a previous homework, you derived the general matrix for the propagation of the lattice functions α , β ,and γ

$$\begin{pmatrix} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} (m_{11}m_{22} + m_{12}m_{21}) & (-m_{11}m_{21}) & (-m_{12}m_{22}) \\ (-2m_{11}m_{12}) & (m_{11}^2) & (m_{12}^2) \\ (-2m_{21}m_{22}) & (m_{21}^2) & (m_{22}^2) \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{pmatrix}$$

   a. (5 points) Write explicit transformations for the three lattice function for the specific cases of
      i. A thin quadrupole of focal length *f*
      ii. A drift of length *s*
   b. (5 points) Prove that the β and γ functions can ever go negative in these cases.

2. When focusing a beam on a target, it is useful to be able to use corrector magnets to *independently* correct the position and angle at the target. For the figure described below, write expressions for the values of θ_1 and θ_2 which will give
   a. (10 points) $\theta_3 = \phi, x_3 = 0$. (hint: you can get this very easily from the three-bump equation)
   b. (10 points) $\theta_3 = 0., x_3 = d$ (might be a bit messy)



3. (10 points) Recall that the expansion of a normal sextupole is given by

$$\begin{aligned} B_y &= \frac{B''}{2}\left(x^2 - y^2\right) \\ B_x &= B''xy \end{aligned}$$

   so an offset in *x* will produce a linear slope in B_x vs *y*. Use this to calculate the chromaticity in both planes (ξ_x, ξ_y), in terms of $(B\rho), \beta_x, \beta_y, D_x$, and sextupole strength $S \equiv B''l$. You may assume that the motion in the bend plane due to betatron oscillations is *small* compared to the motion due to dispersion.