

From Syphers and Edwards, eq. 2.15, the contours of particle motion are given by

$$\Delta E^2 + \frac{2\beta^2 E_s e V}{\eta \omega_{RF} \tau} (\cos \phi + \phi \sin \phi_s) = \Delta E^2 + \frac{\beta^2 E_s e V}{\eta \pi h} (\cos \phi + \phi \sin \phi_s) = \text{constant}$$

For coasting beam below transition, $\eta < 0$, $\phi_s = 0$, and the endpoints of stable motion are $\phi = \pm \pi / 2$. To find the constant for the separatrix, we plug in $\phi = \pi / 2, \Delta E = 0$ and get

$$\text{constant} = - \frac{\beta^2 E_s e V}{\eta \pi h}$$

We can then find the bucket height by plugging in $\phi = 0$ to obtain

$$\begin{aligned} (\Delta E_b)^2 &= - \frac{2\beta^2 E_s e V}{\eta \pi h} \\ \rightarrow V &= \frac{1}{e} \frac{|\eta| \pi h (\Delta E_b)^2}{2 E_s \beta^2} \end{aligned}$$