"IOTA Space Charge Matching Procedure and WARP Tests"

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This is a streamlined combination of Radiasoft Reports RAK9, RAK11 and RAK12, which address the topic of rms matching the IOTA lattice for space charge. **Note that the matching process and simulations described here are for the 50q lattice**. This process is currently being updated for the new lattice.

The matching procedure is outlined, followed by results of WARP tests using different distributions. An instability is observed when a KV beam is injected that is not present for other distributions.

Matching Procedure

A. Reduce the lattice of an insert to a toy model consisting of a drift and a thin lens. The inverse focal length of the thin lens can be determined from the phase advance as follows:

$$1/f = 1/s\,(1 - \cos\sigma_o),$$

where s is half the length of a drift and σ_o the phase advance. For a phase advance of 0.3 (corresponding to 108°), 1/f = 1.309 m^{-1}.

B. Matching into the thin lens implies

$$1/f = 2\alpha/\beta \quad ,$$

where α and β are the Twiss parameters at the entrance to the thin lens (plane nl). For the beam to be matched, this must be satisfied simultaneously with the waist condition, namely α_w = 0, where plane *w* is a distance *s* from the thin lens.

C. Unfortunately, the solvers in TRACE3D are not sophisticated enough to solve conditions at two different planes (they have the capability of a user-defined matching condition, but I do not deem that worth the effort). Furthermore, for 14.1 mA the beam is neither fully space-charge dominated nor fully emittance dominated, so an analytic formula cannot be used to determine the beam expansion.

D. I solve this by iteratively running TRACE starting from a waist and linearly interpolating the results. Run trace for 2 arbitrary starting values of β_w: β_{w1} and β_{w2}. The resulting Twiss parameters at nl are used to calculate two ratios: D1 = $2\alpha_1/\beta_1$ and D1 = $2\alpha_2/\beta_2$. The next guess for β_{w1} is:

$$\beta_{w1} = \beta_{w1} + C(1/f - D_1)(\beta_{w2} - \beta_{w1})/(D_2 - D_1)$$

Here, C is an arbitrary parameter between 1 and 2 to speed up the convergence. I found C = 1.5 gives good convergence.

For the above parameters, I get:

$$\beta_w = 0.86496 \text{ m},\ r_w = 2.94102 \text{ mm},\ \alpha = 1.5444,\ \beta = 2.3595 \text{ m},\ 2\alpha/\beta = 1.30909 \text{ m}^{-1}$$

The larger waist radius, compared to the zero-current solution, is needed to slow down the added expansion from space charge.

E. The matching procedure described in note RAK9 can be followed here. Namely, use stage 2 in **`gen_iota_trace.py`** to run TRACE through the insert, using MT=10 to match to the desired R-matrix (diagonal = 1, R21 = R43 = -1.309, R56 = 5.283, all else zero). This is repeated for each of the other 3 segments. The matched quadrupole settings, which will be different because of space charge, need to be manually copied from the trace3d output to the python file. There are several considerations in the optimization process here:

Choice of quadrupoles to vary. The maximum number of conditions for matching type 10 (R-matrix) is 6. Two segments (q601-606) have 12 quadrupoles each, in pairs of identical ones. The trace input files links each corresponding pair so that the values are adjusted simultaneously. The remaining two sections (q301-307 and q311-317) have 14 quads each. For best matching I found it best to omit adjusting q304 and q314.

Choice of R-matrix elements. Since we are limited to 6 conditions, I have to choose 6 elements of the R-matrix to shoot for. Which elements I choose determine if and how quickly the matching converges. I found it best to do a 2-step process.

Step 1: Rough matching by targeting {R11, R22, R21, R33, R44, and R43} or {R11, R12, R21, R33, R34, and R43}

Step 2: Fine tune with a dispersion match aiming at { R12, R21, R34, R43, R26, and R51}

Following, verify the match by running stage 4 in **`gen_iota_trace.py`** to run from waist to waist over one segment with the correct quadrupole settings found in Step 2. Use MT=1 to find the matched waist radius, which should agree with that found in step D.

After calculating the match in all 4 segments, verify it by running stage 5, which generates a trace input file for a whole turn, and saves the matched settings to WARP and mad formats.

F. The main alteration I make here is for Segments 2 and 4 (311-317 and q301-307, respectively), each of which has 14 main quadrupoles. Since TRACE uses a maximum of 6 matching conditions, a maximum of 6 linked pairs of quadrupoles can be varied. In the previous note, I kept q304 and q314 set to their zero-current setting of 6.48 T/m. This resulted in an imperfect match for those two segments. In particular the dispersive elements of the R-matrix could not converge to zero. To better achieve the achromat condition, I adjusted the values for those two pairs of quads by hand, matching with TRACE, then checking the values of the R-matrix. This process can be semi-automated with an interpolation algorithm similar to the one in section D above, but I did not attempt that.

This resulted in two designs:

Design A: q314 = q304 = 6.6 T/m
Design C: q314 = 6.75 T/m; q304 = 6.8 T/m

The latter design has R_{26} down to the levels of 0.001-0.002, and hence is closer to an ideal achromat. However, the WARP test over one turn showed little difference from Design A.

WARP Test Results

The following figures illustrate the WARP test results. Fig. 1 indicates design A shows a reasonable match over 1 turn, though not as good as the zero-space charge case.

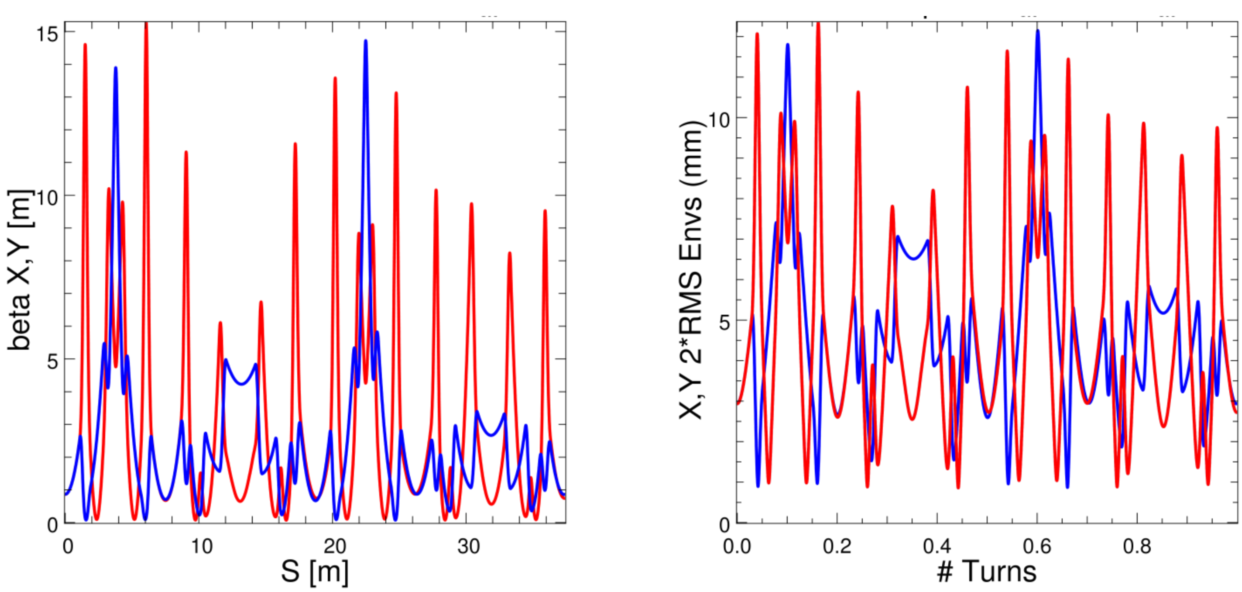

Fig. 1. WARP test of the 14.1 mA beam over 1 turn of IOTA (Design A): (left) beta functions; (right) 2*rms x,y beam envelopes.

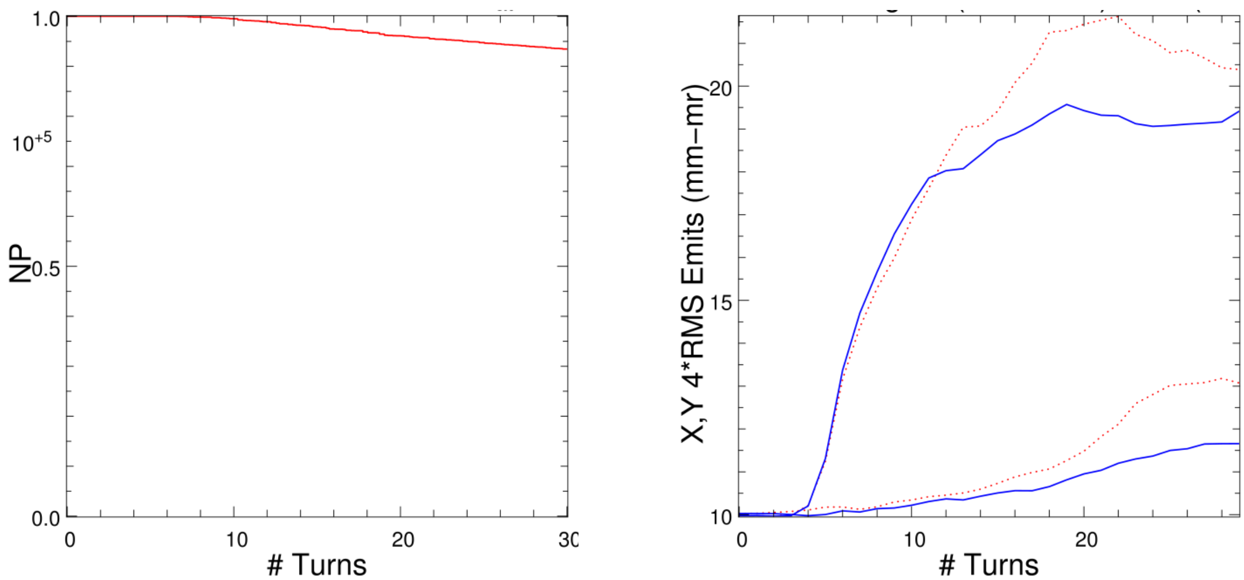

Fig. 2. WARP test the 14.1 mA beam over 30 turns of IOTA: (left) number of live particles in simulation in Design C; (right) 4*rms beam emittances comparing Design C (solid blue) with Design A (dotted red).

Running either design with a KV beam showed an instability developing within a handful of turns, resulting in growth of beam radius and beam loss (Figs. 2). The phase space pictures showed the development of a 3-spoked sextupolar pattern (Fig. 3), reminiscent of Fig 6.16 in Reiser's book. Design C fared only slightly better in terms of particle loss and emittance growth, but the instability was still there. I note here that the zero-current phase advance > 90°, which is generally not recommended when transporting extreme space charge in a linear lattice. It is likely that the beam is encountering an envelope instability or parametric resonance.

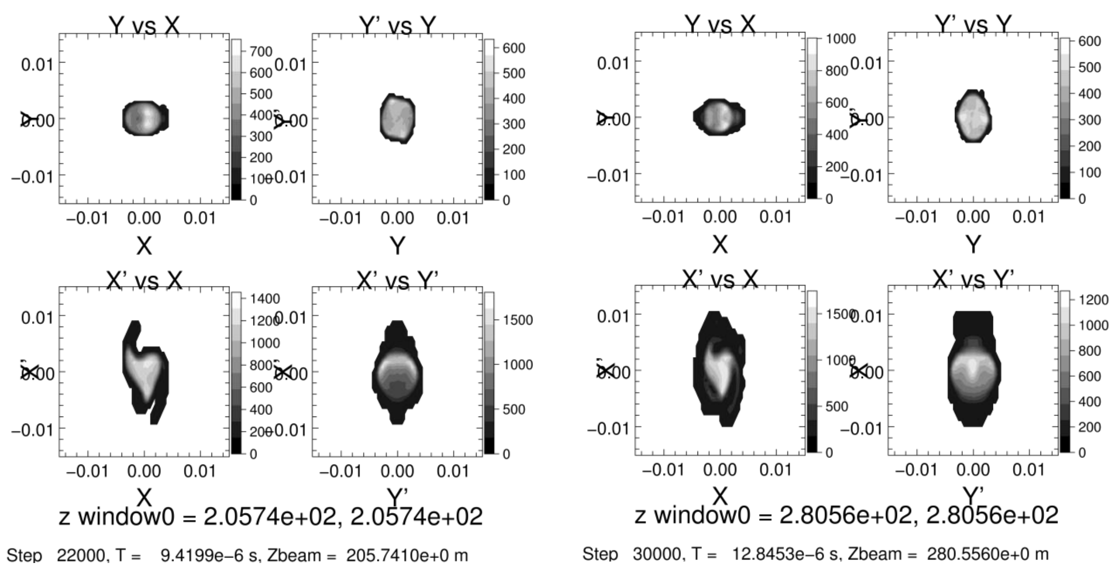

Fig. 3. Trace space plots for Design A after (left) 5.5 turns; (right) 7.5 turns.

Effect of Initial Distribution

The following discussion is for the match described as Design C (based on the 50q lattice) in Note RAK11.

The change to the semi-Gaussian (SG) distribution eliminated the instability entirely. In fact, the linear IOTA lattice performed well with space-charge (when matched properly), provided this more realistic distribution is used.

Figures 4 and 5 below illustrate the effect on the rms quantities from the change of distribution. Note that the rms emittance of the semi-Gaussian beam experiences a modest increase in the first turn, but it remains stable thereafter. This behavior is in stark contrast with the rms beam size and emittance of the K-V beam, which blows up in a handful of turns. The phase space of the semi-Gaussian beam is well-behaved throughout transport (*e.g.,* Fig. 6, compared with that of the KV beam. Despite that, the particle leakage (a steady loss of about 7.5% in 30 turns) observed in the KV case (Fig. 2) is present also here. I suspect this particle loss is an indication of the poor quality of the match.

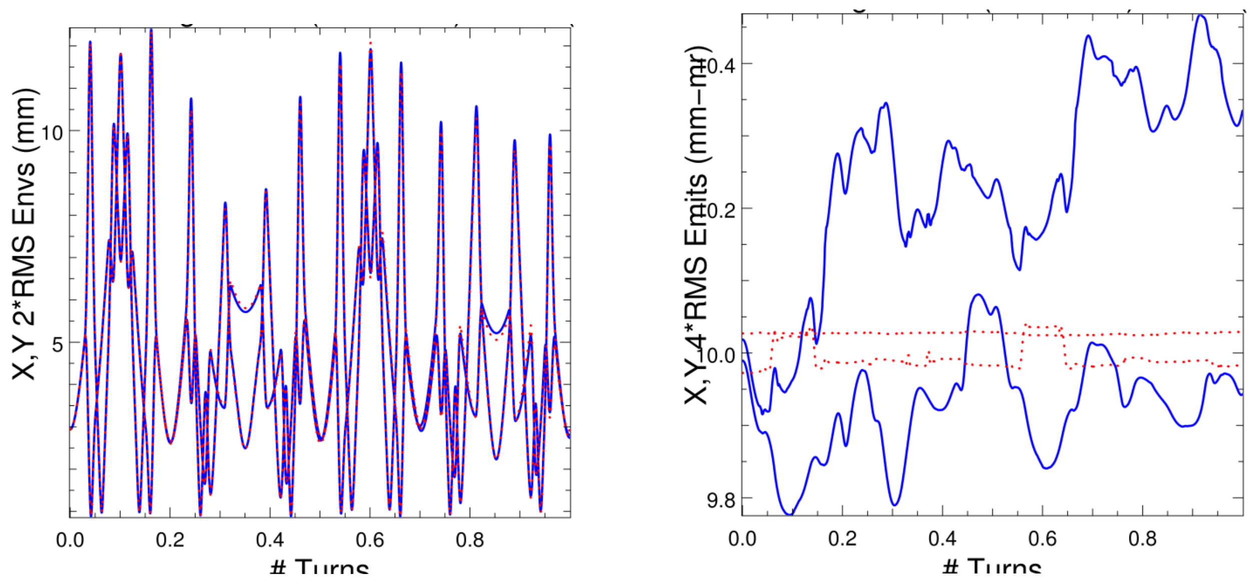

Fig. 4. X and Y beam envelopes (left) and emittances (right) over the first turn of IOTA with 14.1 mA: (blue) SG; and (red) KV distributions.

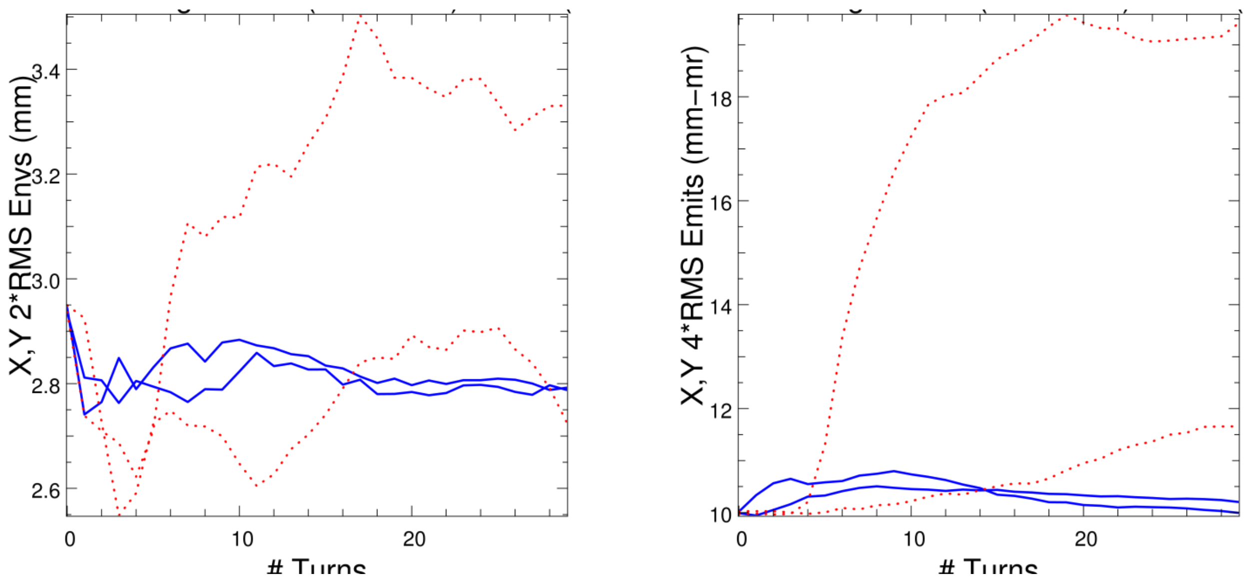

Fig. 5. X and Y beam envelopes (left) and emittances (right) over 30 Turns of IOTA with 14.1 mA: (blue) SG; and (red) KV distributions.

Since the SG distribution is not an equilibrium one, I have also tested a thermal equilibrium (TE) distribution. The behavior of that beam is similar to the SG, except the initial emittance growth in turn 1 is somewhat larger (Fig. 7). Note that in both cases the initial emittance growth takes place at a position in the lattice where the beam is extremely elliptical (1 mm x 12 mm 2*rms radius). This is something observed in the design of UMER: namely, for high space charge, such extreme manipulations need to be avoided.

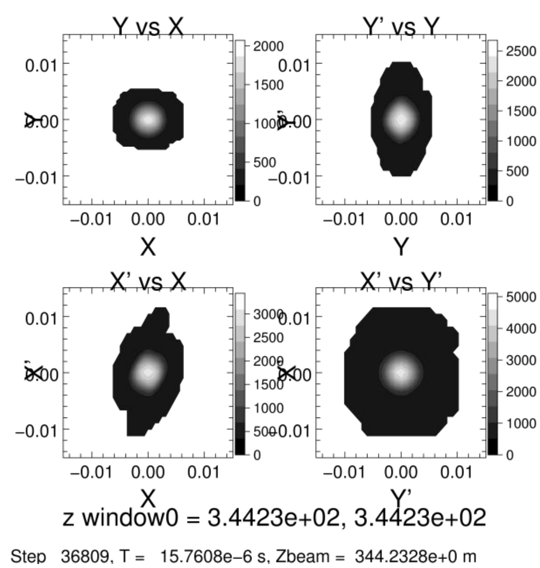

Fig. 6. Trace space plots for the SG beam after 9.2 turns (cf. Fig. 3).

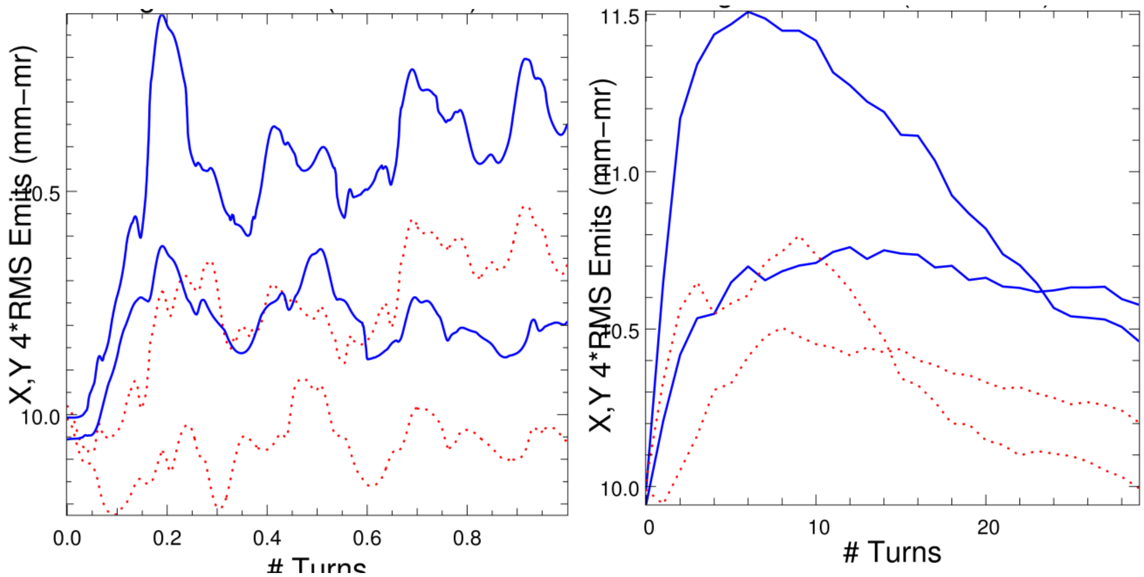

Fig. 7. X and Y beam emittances over the first turn of IOTA with 14.1 mA: (blue) TE; and (red) SG distributions: (left) over Turn 1; (right) over 30 turns.

Conclusions

1. The instability of the KV distribution poses a problem for space-charge simulations in IOTA. **I recommend using the SG distribution for design purposes** with an understanding it is not an equilibrium distribution. It is an open physics question as to what is a reasonable equilibrium distribution when both space charge and nonlinear magnets are present.
2. **The matching procedure needs to be improved** so there is far less particle loss and less emittance growth. At this point, I am moving towards simulations with **the latest iota lattice**, so will have to redevelop some aspects of the matching technique anyway.