On Proton Experiment at IOTA

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Due to significantly lower velocity protons experience much harder scattering both intra-beam and on residual gas compared to electrons of the same momentum. That may lead to beam lifetime of as low as seconds in the IOTA ring. Here I present the estimates of IBS and residual gas scattering, and propose several parameters for proton experiment. For the sake of saving the space I will omit many formulas and calculations.

Beam from RFQ

Parameters of the beam		**Parameters of the ring**	
Energy	2.5 MeV	Physical aperture	24 mm
Momentum	68.5 MeV/c	Max beta, x y	8.9, 4.1 m
Velocity	0.073 c	Acceptance, x y	65, 140 mm-mrad
Frequency	325 MHz	Max dispersion	2.5 m
Transverse RMS emittance	3.4 mm-mrad	Max momentum spread	$9.6*10^{-3}$
Bunch length, RMS	1.3 mm	Revolution period	1.8 µs
Distance between bunches	67 mm	Slip factor	-0.927
RMS momentum spread	$5.5*10^{-3}$		

RMS momentum spread can be reduced to 10^{-3} using a debunching cavity. Assuming the first harmonic and RMS bunch length after the cavity 7 mm – angle $\Delta\varphi = 37.5\mathrm{deg}$, we obtain that the cavity shall be placed approximately $\sigma_s / \sigma_p \sim 120\mathrm{cm}$ away from the RFQ, and the required voltage is

$$V_{cav} \sim \frac{W \cdot \delta W}{q_e \cdot \sin(\varphi_s + \Delta\varphi)} \sim 50\mathrm{kV}$$

Note that the above numbers are just rough estimates, obtained under assumption that one can neglect bunch length at the exit of RFQ as well as space charge effects. Nevertheless, dp/p ~ 10^{-3} can be achieved and I will use this value in the following estimates.

Residual gas scattering

For the sake of current work one can use the following description of vacuum: H_2 – 48%, CO – 48%, Ar – 2%. Then we introduce effective gas pressure as

$$p_{eff} = \sum_j p_j Z_j (Z_j + 1),$$

Where p_j and Z_j are pressure and number of electrons of *j*-th compound.

Lifetime due to single scattering

Consider scattering on angles greater than $\theta_{\max} = \sqrt{\varepsilon_{\max} / \beta}$; all such particles will be lost. The lifetime than is given by

$$\tau_{\text{single\_loss}} = \left[\frac{2\pi \cdot n_{eff} c r_p^{\ 2}}{\gamma^2 v^3} \left(\frac{\beta_{avg\_x}}{\varepsilon_{\max\_x}} + \frac{\beta_{avg\_y}}{\varepsilon_{\max\_y}} \right) \right]^{-1},$$

where n_{eff} is the effective concentration of gas, r_p - classical proton radius, and *v* is used instead of relativistic β to avoid confusion with beta-function. The results for several different vacuums are summarized in table below. This formula gives only a rough estimate, to get the exact number one needs to take into account beam size, know exact chemical composition of residual gas and distribution of pressure throughout the ring.

Multiple scattering

Small angle scattering does not lead to loss of particles but instead to growth of beam emittance. The growth rate is described by the following formula:

$$\frac{d\varepsilon}{dt} \sim \frac{2\pi \cdot n_{eff} c r_p^{\ 2}}{\gamma^2 v^3} \cdot \beta \sum_j n_{eff_j} \cdot L_{c_j},$$

where the sum is through all compound gazes, n_{eff_j} is the effective pressure of gas *j* and L_c is the corresponding coulomb log.

Knowing the growth rate one can estimate characteristic time and beam lifetime due to multiple scattering as

$$\tau_{mult} \sim \frac{\varepsilon_x}{\mathrm{d}\varepsilon / \mathrm{dt}}, \quad \tau_{mult\_loss} \sim \frac{\varepsilon_{x\_\max}}{\mathrm{d}\varepsilon / \mathrm{dt}}$$

It is reasonable to neglect the initial emittance coming from RFQ, since it is ~ 20 times smaller than $\varepsilon_{\max}$.

Pressure	$\tau_{\text{single\_loss}}$	τ_{mult}	τ_{mult_loss}
10^{-8} torr	30 s	0.4 s	10 s
10^{-9} torr	5 min	4 s	1 min
10^{-10} torr	40 min	41 s	10 min

Intra-beam scattering

Intra-beam scattering leads to the growth of emittance and, eventually, particle loss. This effect sets limits on maximum number of particles we can have in the ring, and thus on maximum space-charge tune shift we can achieve in experiment. The effect was calculated using an approach described in “Accelerator Physics at the Tevatron Collider”.

First, for each point of the ring we write down the matrix

$$\Sigma = (\gamma v)^2 \begin{pmatrix} \theta_x^2\left(1+\sigma_p^2\dfrac{(\beta_x\Phi_x)^2}{\sigma_x^2}\right) & 0 & \sigma_p^2\dfrac{\beta_x\Phi_x\varepsilon_x}{\gamma\sigma_x^2} \\ 0 & \theta_y^2 & 0 \\ \sigma_p^2\dfrac{\beta_x\Phi_x\varepsilon_x}{\gamma\sigma_x^2} & 0 & \theta_p^2\dfrac{\beta_x\varepsilon_x}{\sigma_x^2} \end{pmatrix}, \qquad \begin{matrix} \theta_{x,y} = \sqrt{\dfrac{\varepsilon_{x,y}}{\beta_{x,y}}} \\ \theta_p = \sigma_p/\gamma \\ \Phi_x = D'_x + \dfrac{\alpha_x D_x}{\beta_x} \end{matrix}$$

It has x-s coupling terms due to nonzero dispersion in x-plane, so the next step is to perform a rotation of coordinate system to reduce Σ to its diagonal form. Then then distribution function in velocity space is a Gaussian with the diagonal elements being the squares of the corresponding RMS values $\sigma_{vx,y,s}$. After that we calculate the rates of exchange of energy between degrees of freedom:

$$\frac{d\Sigma}{dt} = \frac{(2\pi)^{3/2} n r_p^{\,2} L_c c}{\mathrm{Tr}\Sigma} \begin{pmatrix} \Psi(\sigma_{vx},\sigma_{vy},\sigma_{vs}) & 0 & 0 \\ 0 & \Psi(\sigma_{vy},\sigma_{vs},\sigma_{vx}) & 0 \\ 0 & 0 & \Psi(\sigma_{vz},\sigma_{vx},\sigma_{vy}) \end{pmatrix},$$

$$\Psi(a,\mathrm{b},\mathrm{c}) = \frac{\sqrt{2}\sqrt{a^2+b^2+c^2}}{3\pi}(\mathrm{b}^2\mathrm{R}_D(\mathrm{c}^2,\mathrm{a}^2,\mathrm{b}^2) + c^2 R_D(\mathrm{a}^2,\mathrm{b}^2,c^2) - 2a^2 R_D(b^2,c^2,\mathrm{a}^2)),$$

$$R_D(u,v,w) = \frac{3}{2}\int_0^\infty \frac{dt}{\sqrt{(t+u)(t+v)(t+w)^3}},$$

Where n denotes the concentration of particles, L_c – Coulomb log. And then transform back to the physical coordinates: $\mathbf{\Psi}_{\mathbf{IBS}} \to \mathbf{V}_{\mathbf{IBS}}$, and obtain the rates of emittance growth at every point of the ring. In short, the rate can be written as:

$$\mathbf{r}_{\mathbf{IBS}}(s) = \frac{1}{4\sqrt{2}} \frac{N r_p^{\,2} L_c c}{\mathrm{v}^2\gamma^4 \mathrm{Tr}\Sigma \cdot \sigma_x(s)\sigma_y(s)\sigma_s} \mathbf{V}_{\mathbf{IBS}}(s)$$

And finally, we integrate over the machine circumference to get the average values. Also, one can estimate space-charge tune shifts for the particular current and emittances as:

$$\delta Q_{SC_{x,y}} = \frac{N r_p C}{(2\pi)^{3/2} \mathrm{v}^2\gamma^3\sigma_s} \left\langle \frac{\beta_{x,y}}{(\sigma_x+\sigma_y)\sigma_{x,y}} \right\rangle$$

All the previous formulas assume Gaussian distribution of particles in a bunch. For a coasting beam, one needs to replace σ_s with the ring circumference and add a factor of $\sqrt{2\pi}$ to get the same peak density.

In the table below you can see a comparison of IBS emittance growth rates for 3 different RFQ currents, in the coasting beam operation regime of IOTA, and assuming that all 100% of particles are accepted and transverse emittances for different currents differ insignificantly.

RFQ Current	45 mA	20 mA	10 mA
τ_x	0.5 s	1 s	2.5 s
τ_y	-20 s	-40 s	-83 s
τ_s	0.5 s	1 s	2 s
SC tune shift, x y	3.2 2.8	1.4, 1.2	0.7, 0.6

Negative values in the table correspond to transfer of energy from that degree of freedom to the others (emittance decreases). Note that only maximum tune shifts are shown in the table, i.e. as emittance grows due to IBS they will decrease. To sum it up, IBS significantly reduces the amount of time available to perform an experiment. But we can still achieve reasonably high SC tune shifts and slow emittance growth rates (~ 10^6 revolution periods) at 10 mA

Bunched Beam Operation

Valeri Lebedev proposes to work not with a coasting beam, but instead with a bunched beam. The main advantage is that this regime of operation is generally more interesting for the studies of instabilities, and the synchrotron frequency is one of the significant factors there.

Assuming RF voltage of about 500 V and the first harmonic, we obtain a synchrotron tune

$$Q_s = \sqrt{\frac{h\eta q_e V_{rf} \cos(\phi_s)}{2\pi \cdot v^2 W}} \sim 2.4 \cdot 10^{-3}$$

The table below summarizes the parameters of proton bunch after the thermalization of energy through degrees of freedom due to IBS for a sufficiently low average current from the RFQ.

Parameter	**Value**
Current	3 mA
Emittances: x, y; σ_p	14, 7 mm-mrad; $2 \cdot 10^{-3}$
Average bunch size, RMS: x, y; s	5.6, 3.5 mm; 3 m
IBS time (same for all degrees of freedom)	35 s
Space charge tune shifts: x, y	0.35, 0.45

Once again, I shall note, that the numbers above are just estimates and are subject to optimization and changes. Still, one can get an idea of the orders of magnitude. As one can see, space charge tune shifts in this case are reasonably high. The down side is the low ratio of emittance to acceptance ~ 4, so we have only 2 sigma clearance for the beam on each turn, and the situation is going to get worse as the beam size increases because of IBS. This also places strict limits on the amount of emittance dilution at injection, which we can tolerate.