CS 540 Fall 2008 Homework 5

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Late Days used: 0

1.

[a]

$$
\begin{aligned}
P(Y|X) &= \frac{P(Y,X)}{P(X)} \\
&= \frac{P(X,Y,Z) + P(X,Y,\neg Z)}{P(X,Y,Z) + P(X,\neg Y,Z) + P(X,Y,\neg Z) + P(X,\neg Y,\neg Z)} \\
&= \frac{0.70 + 0.015}{0.70 + 0.015 + 0.10 + 0.02} \\
&= 0.8563
\end{aligned}
$$

[b]

$$
\begin{aligned}
P(Y|X,Z) &= \frac{P(X,Y,Z)}{P(X,Z)} \\
&= \frac{P(X,Y,Z)}{P(X,Y,Z) + P(X,\neg Y,Z)} \\
&= \frac{0.70}{0.70 + 0.10} \\
&= 0.875
\end{aligned}
$$

[c]

$$
\begin{aligned}
P(Y) &= P(X,Y,Z) + P(\neg X,Y,Z) + P(X,Y,\neg Z) + P(\neg X,Y,\neg Z) \\
&= 0.70 + 0.015 + 0.08 + 0.01 \\
&= 0.805
\end{aligned}
$$

[d]

$$
\begin{aligned}
P(X,Z) &= P(X,Y,Z) + P(X,\neg Y,Z) \\
&= 0.70 + 0.10 \\
&= 0.80
\end{aligned}
$$

[e] If X and Z are independent, then the following should hold

$$P(X,Z) = P(X)P(Z).$$

We can calculate P(X) and P(Z) as follows:

$$\begin{aligned}
P(X) &= P(X, Y, Z) + P(X, \neg Y, Z) + P(X, Y, \neg Z) + P(X, \neg Y, \neg Z) \\
&= 0.70 + 0.015 + 0.10 + 0.02 = 0.835 \\
P(Z) &= P(X, Y, Z) + P(\neg X, Y, Z) + P(X, \neg Y, Z) + P(\neg X, \neg Y, Z) \\
&= 0.70 + 0.08 + 0.10 + 0.07 = 0.95 \\
P(X)P(Z) &= (0.835)(0.95) = 0.7933
\end{aligned}$$

Since $P(X, Z) \neq P(X)P(Z)$, X and Z are not independent.

2. Let the boolean random variables $\{HW, HANDIN, TP\}$ be defined as:

- $HW = true$ iff Jack finished the HW.
- $HANDIN = true$ iff Jack handed in the HW.
- $TP = true$ if Jack tells the professor that he forgot to handin the HW.

We know the following probabilities:

- $P(TP|HW, \neg HANDIN) = 0.01$.
- $P(TP|\neg HW) = 0.5$.
- $P(HW) = 0.9$.

We can make a few simplifying assumptions. Since Jack may be a liar but not a cheat, we can take $P(HANDIN|\neg HW) = 0$. People who did submit will of course not lie to the professor, so $P(TP|HANDIN) = 0$. We are interested in finding if Jack did complete his homework, i.e. $P(HW|TP)$.

$$\begin{aligned}
P(HW|TP) &= \frac{P(TP|HW)P(HW)}{P(TP)} \text{By Bayes' Rule} \\
P(TP) &= P(TP, HW, HANDIN) + P(TP, HW, \neg HANDIN) \\
&+ P(TP, \neg HW, HANDIN) + P(TP, \neg HW, \neg HANDIN) \\
&= 0 + 0.01 + 0 + 0.5 = 0.501 \\
P(TP|HW) &= P(TP, HW)P(HW) \\
&= (P(TP, HW, HANDIN) + P(TP, HW, \neg HANDIN))P(HW) \\
&= (0 + 0.01)(0.5) = 0.005 \\
P(HW|TP) &= \frac{0.005 \cdot 0.5}{0.501} = 0.0049
\end{aligned}$$

So Jack is telling the truth with 0.49% probability. He shouldn't get any points for this assignment.

3. We will precalculate the following priors:

$$
\begin{aligned}
P(B) &= P(B|A)P(A) + P(B|\neg A)P(\neg A) = (0.2)(0.8) + (0.9)(1-0.8) = 0.34 \\
P(D) &= P(D|A,B)P(A)P(B) + P(D|\neg A,B)P(\neg A)P(B) \\
&\quad + P(D|A,\neg B)P(A)P(\neg B) + P(D|\neg A,\neg B)P(\neg A)P(\neg B) \\
&= (0.85)(0.8)(0.34) + (0.25)(0.8)(0.66) + (0.15)(0.2)(0.34) + (0.05)(0.2)(0.66) \\
&= 0.38
\end{aligned}
$$

[a]

$$
\begin{aligned}
P(E|B,C) &= \frac{P(E)P(B|E)P(C|E,B)}{P(B)P(C|B)} \text{By conditional chain rule} \\
P(E) &= P(D)P(E|D) + P(\neg D)P(E|\neg D) \\
&= (0.38)(0.6) + (0.62)(0.5) = 0.538 \\
P(C|E,B) &= P(C|B) \text{since C and E are independent given B} \\
&= 0.95 \\
P(B|E) &= \frac{P(E|B)P(B)}{P(E)} = \frac{P(E|D)P(D|B)P(B)}{P(E)} \\
&= \frac{P(E|D)(P(D|A,B) + P(D|\neg A,B))P(B)}{P(E)} \\
&= \frac{(0.6)(0.85+0.15)(0.34)}{0.538} = 0.3792 \\
P(E|B,C) &= \frac{(0.538)(0.3792)(0.95)}{(0.34)(0.95)} = 0.6
\end{aligned}
$$

[b]

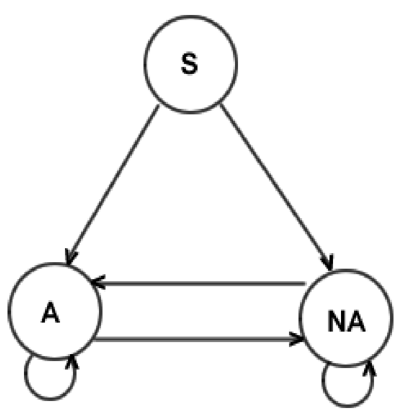
$$
\begin{aligned}
P(D|E,A) &= \frac{P(D)P(A|D)P(E|D)}{P(A)P(E|A)} \text{By conditional chain rule} \\
P(E|A) &= P(E|D)P(D|A) \\
&= P(E|D)(P(D|A,B)P(B|A) + P(D|A,\neg B)P(\neg|A)) \\
&= (0.6)((0.85)(0.2) + (0.25)(0.9)) \\
&= 0.237 \\
P(A|D) &= \frac{P(D|A)P(A)}{P(D)} \\
&= \frac{((0.85)(0.2) + (0.25)(0.9))(0.8)}{0.38} \\
&= 0.8316 \\
P(D|E,A) &= \frac{(0.38)(0.8316)(0.60)}{(0.8)(0.237)} = 1
\end{aligned}
$$

4.

[a] The Markov model has three states $\{S, A, NA\}$. S corresponds to a start state, A corresponds to the lever being 'on', and NA corresponds to the lever being 'off'. The state transition matrix $\mathscr{A}$ is given as follows:

$$
\mathscr{A} = \begin{array}{rc} & \begin{array}{ccc} S & A & NA \end{array} \\ \begin{array}{r} S: \\ A: \\ NA: \end{array} & \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0 & 0.7 & 0.3 \\ 0 & 0.3 & 0.7 \end{pmatrix} \end{array}
$$

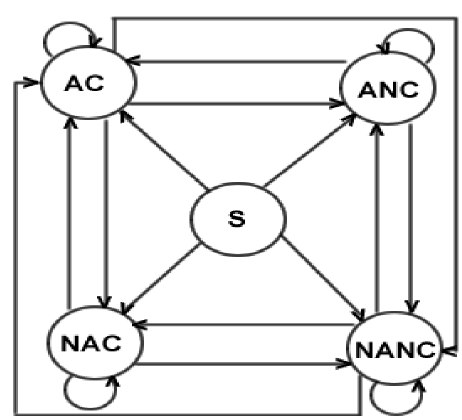
The initial state vector is given by $\pi = \begin{pmatrix} 1.0 & 0 & 0 \end{pmatrix}$.



[b] The Markov model now has five states $\{S, AC, ANC, NAC, NANC\}$, where S is the start state, 'AC' corresponds to both almond and cononut levers being on, 'ANC' corresponds to almond lever being on and cocunut lever being off, 'NAC' corresponds to almond lever being off and coconut lever of, 'NANC' corresponds to all levers being off. The state transition matrix is given by:

$$\mathscr{A} = \left(\begin{array}{rccccc} & S & AC & ANC & NAC & NANC \\ S: & 0 & 0.25 & 0.25 & 0.25 & 0.25 \\ AC: & 0 & 0.49 & 0.21 & 0.21 & 0.09 \\ ANC: & 0 & 0.21 & 0.49 & 0.09 & 0.21 \\ NAC: & 0 & 0.21 & 0.09 & 0.49 & 0.21 \\ NANC: & 0 & 0.09 & 0.21 & 0.21 & 0.49 \end{array} \right)$$

For the model to remain in the same state, neither of the levers must be changed which can happen with probability $(1 - 0.3)(1 - 0.3) = 0.49$. Similarly to go to a state where both the lever have changed, the probability is $(0.3)(0.3) = 0.09$. To go to a state where only one lever has changed, the probability is $(0.3)(1 - 0.3) = 0.21$. The initial state vector is given by $\pi = \begin{pmatrix} 1.0 & 0 & 0 & 0 & 0 \end{pmatrix}$.



[c]

$$\begin{aligned} & P(\mathit{Plain}, \mathit{Almond}, \mathit{Almond}, \mathit{Almond} + \mathit{Coconut}) \\ & = P(NCNC|S)P(ANC|NCNC)P(ANC|ANC)P(AC|ANC) \\ & = (0.25)(0.21)(0.49)(0.21) \\ & = 0.005402 \end{aligned}$$

5.

[a] Empty documents are classified on the basis of prior probabilities

[b] For the given training set, the prior probabilities are given below:

$$P(\text{English}) = 0.272727$$
$$P(\text{Spanish}) = 0.389610$$
$$P(\text{Japanese}) = 0.337662$$

The conditional probabilities for each character given a languages is as follows:

Char(c)	$P(c\|\text{English})$	$P(c\|\text{Spanish})$	$P(c\|\text{Japanese})$
a	0.061609	0.107405	0.131701
b	0.011893	0.010668	0.010015
c	0.021727	0.037540	0.005156
d	0.022358	0.039021	0.016358
e	0.106133	0.110281	0.059925
f	0.019941	0.007105	0.003374
g	0.016178	0.008199	0.014792
h	0.046502	0.005366	0.030935
i	0.054742	0.049088	0.098364
j	0.000824	0.007147	0.002186
k	0.003955	0.000300	0.057037
l	0.030269	0.052350	0.001026
m	0.022166	0.023868	0.040814
n	0.057599	0.054583	0.056929
o	0.065290	0.072569	0.091076
p	0.016343	0.023997	0.000540
q	0.000659	0.007147	0.000027
r	0.050567	0.060034	0.041948
s	0.063367	0.065894	0.043298
t	0.083061	0.034707	0.058414
u	0.025517	0.034535	0.070507
v	0.009311	0.005988	0.000135
w	0.015437	0.000279	0.020245
x	0.001181	0.002382	0.000000
y	0.013349	0.007384	0.014711
z	0.000549	0.003670	0.007774
space	0.179471	0.168491	0.122712

[c] For the given test set, the confusion matrix is given below

	English	Spanish	Japanese
English	15	0	0
Spanish	15	30	0
Japanese	0	0	30

[d] Nowhere in the conditional probability calculation is the position of a character important. The classifier only looks at relative frequencies of characters in a language, and is therefore independent of the ordering of the characters. Hence, our classifier would output the same value it used to before the letters were scrambled.

[e] All natural languages possess much more structure than what the naive classifier looks for. For example, the values of preceding character(s) often narrows down the probable value of the next character. This can be extended to arbitrarily large syntactic structures(words, phrases, sentences etc). In practice, the relative frequencies of digrams(groups of two characters) and trigrams(groups of three characters) are used for classification.

Most languages also treat white space with much less importance than what the classifier does. For example, groups of adjacent white space can be counted only once to avoid giving too much importance to white space.

6.

```
[shenoy@cs]$ forget --help
    forget(8) is a tool developed by Univ of Wisconsin,
    Madison. Using advanced techniques like Bayesian
    Indifference, Pedantically arbitrarily complicated
    (PAC) unlearning and Statistical Senility, forget
    forgets even the most memorable things.
    Report bugs to shenoy@cs.wisc.edu.

[shenoy@cs]$ forget todays-date

[shenoy@cs]$ date
    0:0:0 0:0:0 some day, some time zone

[shenoy@cs]$ su
    password:
    incorrect password
```

```
[shenoy@cs]$ forget root-password

[shenoy@cs]$ su

[root@cs]$ forget ex-girlfriend
    deleting 1038 pictures             [DONE]
    deleting 227 emails                [DONE]
    removing her from facebook profile[DONE]
    reactivating match.com profile     [DONE]

[root@cs]$ forget pennstate-beating-badgers
    forgetting complete with 1 warning(s)
    warning: Reality not changed: we still lost.

[root@cs]$ forget cs540-hw5
    forgetting failed: cs540-hw5 marked as unforgettable.

[root@cs]$ forget everything
    Are you sure you want to forget everything[Y/n]:y

[?@?]$ ls
    huh?

[?@?]$ reboot
    huh?

[?@?]$ help
    huh?
```