CS 540 Fall 2008 Homework 4

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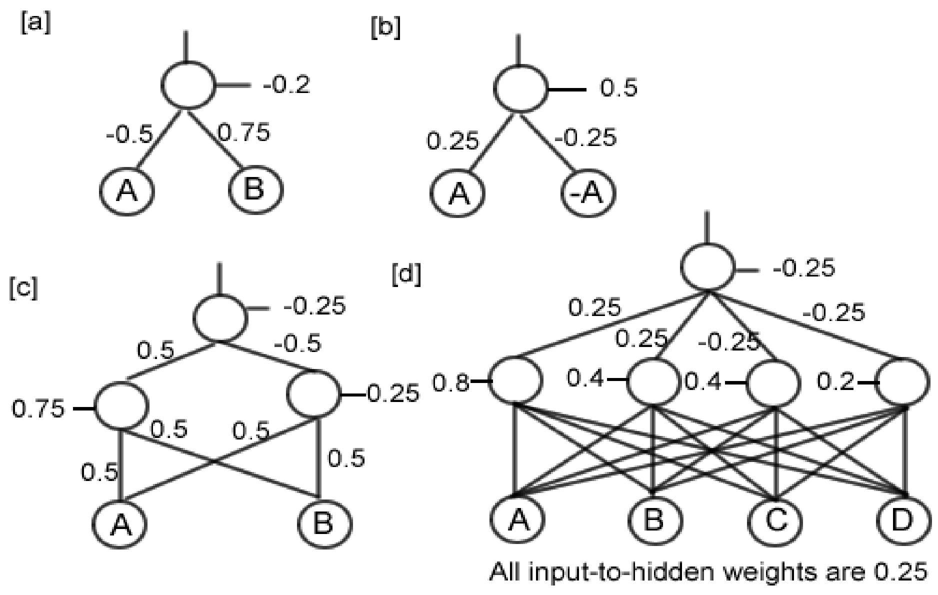
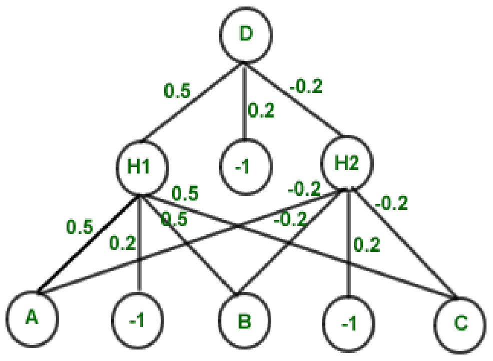
Late Days used: 0

1. [a] is linearly separable, hence is computable by a 1-layer perceptron. [b] is always false, hence is trivially linearly separable. As we saw in class, the XOR function is not linearly separable. Since the negation of a function is just flipping the labels, [c] is also not linearly separable. Parity is the same as XNOR, hence [d] is also not linearly separable. Hence [c] and [d] have no single-layer perceptrons.



2.
[a]



[b] The sigmoid function is defined as

$$\sigma(\mathbf{x}) = \frac{1}{1+e^{-\mathbf{w}\cdot\mathbf{x}}}.$$

For hidden gate H1,

$$\begin{aligned}\mathbf{w}\cdot\mathbf{x} &= (0.5)(A) + (0.5)(B) + (0.5)(C) + (0.2)(-1)\\ &= (0.5)(0.3) + (0.5)(0.8) + (0.5)(0.1) - 0.2\\ &= 0.4\\ h_1 = \sigma(0.4) &= \frac{1}{1+e^{-0.4}} = \frac{1}{1+0.67} = 0.5987.\end{aligned}$$

For hidden gate H2,

$$\begin{aligned}\mathbf{w}\cdot\mathbf{x} &= (-0.2)(A) + (-0.2)(B) + (-0.2)(C) + (0.2)(-1)\\ &= -(0.2)(0.3) - (0.2)(0.8) - (0.2)(0.1) - 0.2\\ &= -0.44\\ h_2 = \sigma(-0.44) &= \frac{1}{1+e^{0.44}} = \frac{1}{1+1.553} = 0.3917.\end{aligned}$$

The expected output with these set of weights is,

$$\begin{aligned}\mathbf{w}\cdot\mathbf{x} &= (0.5)(h_1) + (-0.2)(h_2) + (0.2)(-1)\\ &= (0.5)(.5987) - (0.2)(.3917) - 0.2\\ &= 0.021\\ d = \sigma(0.021) &= \frac{1}{1+e^{-0.021}} = \frac{1}{1+1.9792} = 0.5052.\end{aligned}$$

The error at the output is

$$\begin{aligned}\Delta &= \sigma'(\{h_1, h_2, -1\}) \cdot (D - d)\\ \sigma'(\mathbf{x}) &= \sigma(\mathbf{x})(1 - \sigma(\mathbf{x}))\\ &= (0.5052)(1 - 0.5052) = 0.25\\ \Delta &= (0.25)(1 - 0.5052) = 0.1237\end{aligned}$$

Backpropogating error to H1,

$$\begin{aligned}\Delta_{h1} &= \sigma'(h_1)W_{H1D}\Delta\\ \sigma'(h_1) &= 0.5987(1 - 0.5987) = 0.2402\\ \Delta_{h1} &= 0.2402 \cdot 0.5 \cdot 0.1237 = 0.0149\\ W_{H1D} &= W_{H1D} + \alpha h_1 \Delta_{h1}\\ &= 0.5 + 0.3 \cdot 0.5987 \cdot 0.0149\\ &= 0.5027\end{aligned}$$

Backpropogating error to bias node,

$$\begin{aligned}
\Delta_{d'} &= \sigma'(-1) W_{d'D} \Delta \\
\sigma'(-1) &= .2689(1 - .2689) = 0.1966 \\
\Delta_{d'} &= 0.1966 \cdot 0.2 \cdot 0.1237 = 0.0049 \\
W_{d'D} &= 0.2 + 0.3 \cdot 0.0049 \cdot -1 = 0.1985
\end{aligned}$$

Backpropogating error to H2,

$$\begin{aligned}
\Delta_{h2} &= \sigma'(h_2) W_{H2D} \Delta \\
\sigma'(h_2) &= 0.3917(1 - 0.3917) = 0.2383 \\
\Delta_{h2} &= 0.2383 \cdot -0.2 \cdot 0.1237 = -0.0059 \\
W_{H2D} &= W_{H2D} + \alpha h_2 \Delta_{h2} \\
&= -0.2 + 0.3 \cdot 0.3917 \cdot -0.0059 = -2.0007 \\
&= 0.5027
\end{aligned}$$

Backpropogating error to A,

$$\begin{aligned}
\Delta_A &= \sigma'(A)(W_{AH1}\Delta_{h1} + W_{AH2}\Delta_{h2}) \\
&= 0.2445(0.5 \cdot 0.0149 - 0.2 \cdot 0.0049) \\
&= 0.00158 \\
W_{AH1} &= 0.5 + 0.3 \cdot 0.3 \cdot 0.00158 \\
&= 0.5001 \\
W_{AH2} &= -0.2 + .3 \cdot 0.3 \cdot 0.00158 \\
&= -0.1999
\end{aligned}$$

Backpropogating error to B,

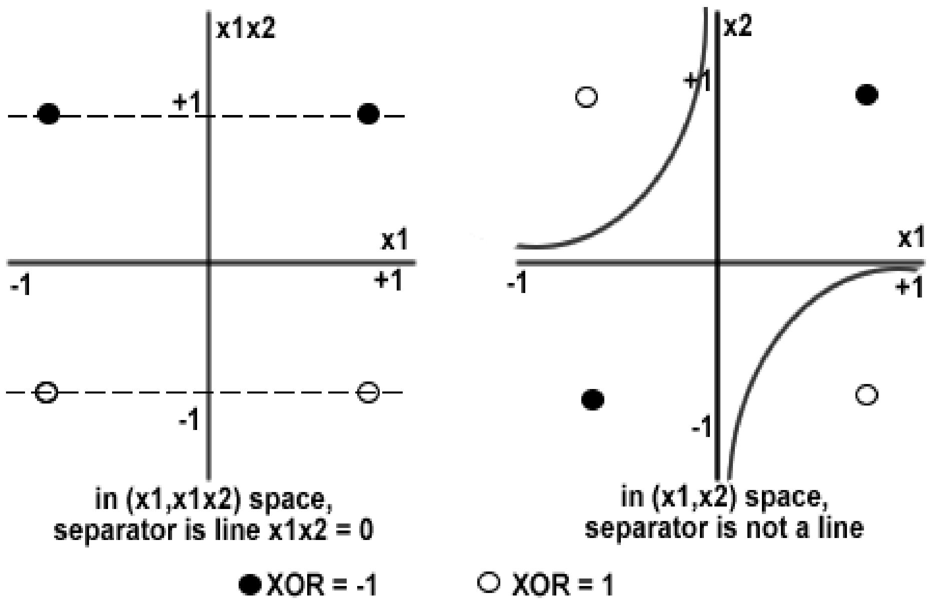
$$\begin{aligned}
\Delta_B &= \sigma'(B)(W_{BH1}\Delta_{h1} + W_{BH2}\Delta_{h2}) \\
&= 0.2139(0.5 \cdot 0.0149 - 0.2 \cdot 0.0049) \\
&= 0.00138 \\
W_{BH1} &= 0.5 + 0.3 \cdot 0.3 \cdot 0.00138 \\
&= 0.5001 \\
W_{BH2} &= -0.2 + .3 \cdot 0.3 \cdot 0.00138 \\
&= 0.1999
\end{aligned}$$

Backpropogating error to C,

$$\begin{aligned}
\Delta_C &= \sigma'(C)(W_{CH1}\Delta_{h1} + W_{CH2}\Delta_{h2}) \\
&= 0.2494(0.5 \cdot 0.0149 - 0.2 \cdot 0.0049) \\
&= 0.00161 \\
W_{CH1} &= 0.5 + 0.3 \cdot 0.3 \cdot 0.00161 \\
&= 0.5001 \\
W_{CH2} &= -0.2 + .3 \cdot 0.3 \cdot 0.00161 \\
&= 0.1986
\end{aligned}$$

3. The XOR function over the $\{1, -1\}$ domain is defined as follows. We will use the map $(x_1, x_2) \mapsto (x_1, x_1x_2)$ to linearly separate the XOR function.

x_1	x_2	x_1x_2	$x_1\ XOR\ x_2$
-1	-1	1	-1
-1	1	-1	1
1	-1	-1	1
1	1	1	-1



Any line $x_1x_2 = c, -1 \le c \le +1$ is a separator in the transformed space. The maximal margin separator is the line $x_1x_2 = 0$, which has margin (1+1)

= 2. The separator doesn't remain a line in (x_1, x_2) space. The separator $x_1 x_2 = c$ becomes the hyperbola $x_2 = \frac{c}{x_1}$ in euclidean space, with the degenerate hyperbola $x_1 x_2 = 0$ being the maximal margin separator.
Question 4. Part 1

1. [a] 1.19 seconds.

   [b] 63.35% of examples.

   [c] Confusion matrix

a	b	classified as
81	48	a = face
44	78	b = non-face

   [d] The SVM had errors in both directions(i.e. classifying faces as non-faces and vice versa).

2. [a] 2.48 seconds.

   [b] 65.74% of examples.

   [c] The results improved by $\approx$ 2%. The 'exponent' option controls the exponent of the polynomial kernel. The improvement shows that while the data is not linearly separable, the data when mapped to some space through a quadratic transformation, the separation increases.

3. [a] 73.31% of examples.

   [b] There was a significant difference in the accuracy using RBF instead of polynomial kernel. This shows that the data is not quadratically separable, but separation increases if a radial basis function is used to transform the feature space.

Part 2

1. [a]

Learning Rate	Hidden Layers	Training Time	Success Percentage
0.1	5	7.09s	64.54%
0.1	10	12.56s	63.74%
0.1	20	23.81s	64.14%
0.3	5	6.95s	67.93%
0.3	10	11.91s	66.93%
0.3	20	22.39s	68.13%

   [b] The most successful network for this data was learnt using learning rate = 0.3 and 20 hidden layers. For a given learning rate, increasing the number of hidden layers beyond a threshold results in a network which overfits the training data. For a fixed time

bound on training time, a faster learning rate results in more accurate networks.

2. [a]

Learning rate	Time to train	Percentage correct
0.1	38.12s	64.94%
0.3	36.69s	67.33%

[b] With the same learning rate (0.3) and hidden layers(10), increasing the training time from 100 to 300 increased the success percentage from 66.93% to 67.33%. This is not always guaranteed to increase, since past a certain threshold the network will begin to overfit the examples, and might do worse on the testing data.

[c] With training time set to 300, hidden layers to 5, learning rate to 0.3, we get a success rate of 66.14% instead of 67.93% when the training time was 100. So in this case extra training time made it worse.

3. [a] Time: 28.27s, Success percentage: 67.73%

[b] The epochs required by each fold were approximately {80,52,500,92,77}. The third fold reached the limit and stopped. The number of epochs are distributed widely, as they depend on the training set.

[c] Keeping hidden layers 10 and learning rate 0.3, the accuracies for different training times are { 100 = 66.93% , 300 = 67.33% , 500 = 67.73% }. So in this case training time did help in improving accuracy.

[d] NO ANSWER

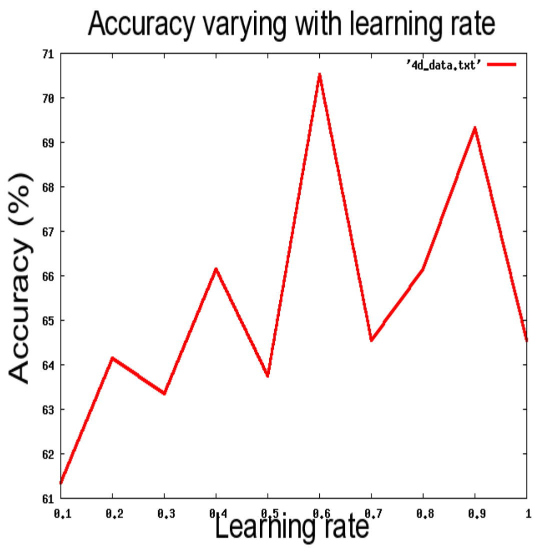
4. [a] The following parameters were kept constant: hidden layers = 5, training time = 100, validation set size = 10, validation threshold = 20. The parameter 'Learning rate' varied in the range $\{0.1, 0.2, \cdots, 1.0\}$.

[b] For a fixed training time, the learning rate can affect the success percentage in two ways. If the learning rate is too small, the learning algorithm makes slow progress and doesn't reach the desired accuracy within the training time. With a large learning rate, the weights fluctuate a lot, and may not converge to a fixed value. There is an "optimal" learning rate at which the two factors give the best result.

[c]

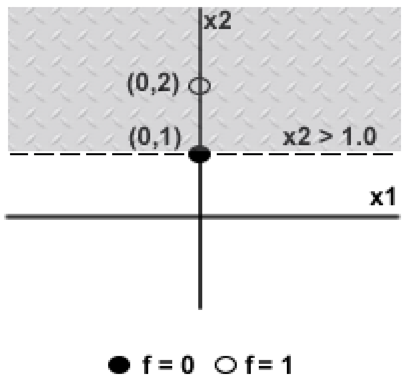
α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
%	61.35	64.14	63.34	66.14	63.75	70.52	64.54	66.14	69.32	64.54

[d] Graph

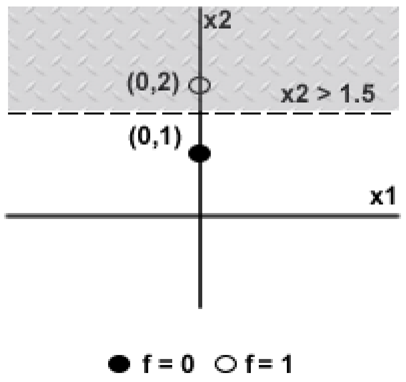


[e] For the given training set, the best learning rate is 0.6.

Question 5.
[a] The weights were $w_0 = 1.0, w_1 = 0.0, w_2 = 1.0$, where w_0 is the bias weight.
[b] The line $x_2 = 1.0$ is the decision boundary. The half-space defined by the inequality $x_2 > 1.0$ gets the label '1' and the other half-space gets the label '0'.



[c] With $\alpha = 0$, the algorithm never terminates. With any other learning rate, the algorithm will eventually find the same result, but will differ in number of iterations. In this small example, changing α didn't change the number of interations. Also different α values give rise to different weights all of which represent the same line.
[d] Weka classifies the data using the line $1 * x_2 - 1.5 > 0$. This is parallel to the line which was found using our program.



[e] The example $(x_1 = 0, x_2 = 0, \mathit{output} = 1)$ makes the learning algorithm run indefinitely since the data is now not linearly separable.