Project 2 - Written Assignment 1
priyananda (shenoy@cs.wisc.edu)

1.
$(4,-4)(4,-2)(6,-2)(6,-4)$

2.
translate 5 0
draw square
translate -2 0
scale 2 2
draw square
translate $-\frac{3}{2}$ 0
scale $\frac{3}{2}\frac{3}{2}$
draw square

3A.
translate 320 240
scale 320 -240

3B.

$$\begin{pmatrix} 320 & 0 & 320 \\ 0 & -240 & 240 \\ 0 & 0 & 1 \end{pmatrix}$$

4.

$$\begin{pmatrix} 2 & 2 & 1 \\ 2 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

5.
translate -3 -4
rotate 45
translate 3 4

6.
a. world-origin to base of the lamp: translate x y 0
b. align base on the table: rotate around vertical axis by $-\theta_0$
c. vertical(Z) to match first arm: rotate around y by θ_1
d. move origin to first joint: translate 0 0 5
e. vertical to match second arm: rotate around y by $-\theta_2$
f. move origin to second joint: translate 0 0 5
g. vertical to match third arm: rotate around y by $\theta_3 - 180$
h. move origin to bulb: translate 0 0 1

$T(x,y,0)R_z(-\theta_0)R_y(\theta_1)T(0,0,5)R_y(-\theta_2)T(0,0,5)R_y(\theta_3-180)T(0,0,1)$

7. The matrix M maps unit X vector $(1,0,0)$ to $(0,1,1)$
M maps unit Y vector $(0,1,0)$ to $(0,1,-1)$
This gives the first two columns of the matrix.

$$\begin{pmatrix} 0 & 0 & x & 0 \\ 1 & 1 & y & 0 \\ 1 & -1 & z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Assuming that this transformation was a result of a rotation and a scale. We can write
M = R . S
where R is some rotation matrix and S is the scale matrix. Since R is a rotation matrix, each column must be normalized and each pair of columns orthogonal. We split M into R so that R becomes normalized. This can be done using S =

$$\begin{pmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

To find the third column of R we can calculate the cross product of the two vectors.
$x = \frac{1}{\sqrt{2}}(-\frac{1}{\sqrt{2}}) - 1(1) = -1$
$y = 1(0) - 0(0) = 0$
$z = 0(1) - 0(1) = 0$

So the matrix R is

$$\begin{pmatrix} 0 & 0 & -1 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and matrix M = R.S is

$$\begin{pmatrix} 0 & 0 & -\sqrt{2} & 0 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

7.A The unit Z vector gets mapped to $(-\sqrt{2},0,0)$
7.B The uniform scale is $\sqrt{2}$