sequence 1

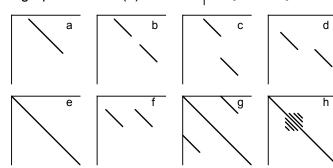
CACGATC

<u>Homology may indicate function</u>: One of the most important inferences we can make in bioinformatics is if two sequences are related to each other. If so, they may function similarly.

This is an old, classic question in the field and still relevant today. In order to determine if two sequences are related to each other, we need (1) some way to align sequences (2) some way to determine if the alignments are statistically significant.

Dot plots: A simple way to look at the relationship between two sequences is a dot plot (or dot matrix). This is a 2D matrix with a sequence along each axis. Each point in the matrix corresponds to a specific letter in each sequence. Regions of similarity appear as diagonals in the matrix. Rather than draw dots, it easier to draw lines showing just the similar regions. In the 8 graphs shown: (a) an

alignment showing a regional similarity between two sequences (b) a section in the middle does not align as well (c, d) the similar regions are separated by a gap (e) a sequence aligned to itself (f) sequence 1 has a duplication (g) a sequence with a repeat aligned to itself (h) a sequence with an SSR aligned to itself. Note that in (c, d) there is either an insertion in one sequence or a deletion in the other. Gaps are therefore often called **indels**.



A C

G

Т

Τ

С

Α

Pairwise alignment: There are two "flavors" of pairwise alignment: **global** and **local**. In global alignment, the goal is to align every letter of the two sequences. Consider aligning the letters in these two sequences: (1) ACTTTGA (2) TTT. One possible alignment between these is shown as "align 1". Every letter in each sequence is either aligned to another letter or a gap (-) symbol. If the sequences are identical, it is typical to use a pipe A

or a gap (-) symbol. If the sequences are identical, it is typical to use a pipe symbol between the sequences to indicate this. Another common convention is to use the letter. Whenever sequences have unequal lengths, there will be gaps. The gaps can occur anywhere. For example, an alignment between (1) and the thorough the sequences (2) ACTCA is shown as "align 2". In least alignment apply the least alignment.

Align 3
ACT TGA
III or III
ACT TGA

Align 2

ACTTTGA

IIIIIII

AC-T-GA

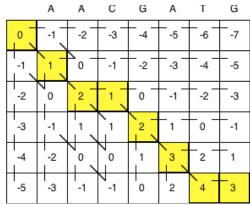
another sequence (3) ACTGA is shown as "align 2". In local alignment, only the best region is kept. "Align 3" shows two possible local alignments of sequences (1) and (3). Both are equally good. There are also a large number of really poor alignments one could make.

<u>Alignment scoring</u>: In order to compare alignments to each other, we can give them a **score**. A simple scoring scheme is to give every matching letter a score of +1 and every mismatch or gap a score of -1. Under such a scheme, the scores for alignments 1-3 are: -1, 3, and 3.

Needleman-Wunsch algorithm: To find the best global alignment one uses the N-W algorithm (or some variant of it). The number of possible alignments between two sequences is huge. You can put gaps in either sequence anywhere you like (but not across from each other). A naive alignment algorithm would enumerate all possible gaps and then choose the alignment with the best score. Even with short sequences this quickly becomes unwieldy and in biological sequences, which can be huge in the case of chromosomes, the number of alignments becomes astronomical. N-W uses dynamic programming (DP) to efficiently find a single highest scoring alignment. There may be more than one alignment with the maximum score, but the algorithm returns only one of these. To begin the N-W algorithm, the sequences are entered into a matrix (like a dot plot) with an extra 1st column and row. There are 3 steps to the algorithm: (1) initialization (2) fill (3) trace back. In the initialization, the first row and column are set to gap scores. In the fill, a recursive operation is used to update the maximum score of every cell. In

the trace back, the alignment is recovered by following the maximum alignment from the bottom right of the matrix through the top left.

Let's take a close look at the fill. In order to fill a cell, A you must have 3 neighboring cells located above, to the left, and diagonally above and left. At the G beginning, there is only one cell that can be filled. This is the one that aligns the first A and A in the example. A To fill this cell, you must determine the maximum score of 3 possible directions (diagonal, up, and left).



Diagonal score = score of diagonal cell + match or mismatch score (either +1 or -1)

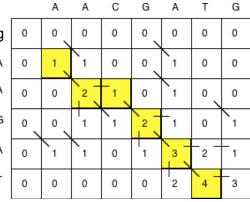
Left score = score of left cell + gap score (-1)

Up score = score of up cell + gap score (-1)

When you move horizontally or vertically, you do not consider wether the nucleotides match or not because this operation introduces a gap character. The power of DP is that we do this same operation of looking at 3 possible alignments at every position in the matrix. But we are not enumerating all possible alignments, we are always extending the previous maximum alignment.

Smith-Waterman algorithm: To find the maximum scoring local alignment, you can use the exact same procedure as N-W except that (a) any score below 0 is given the A score of 0. At the end, the trace back is performed from the highest score in the matrix rather than the last cell of the matrix.

<u>Scoring system</u>: Different match, mismatch, and gap $_{\rm A}$ scores will result in different alignments. Try the same sequences with a gap score of -2 and you will get a $_{\rm T}$ slightly different alignment.



Computational considerations: The N-W and S-W algorithms as described are not used for aligning long sequences. One reason is that the amount of memory to hold the DP matrix becomes excessive. Each cell in the matrix must hold a score and a directional pointer. This might be 5 bytes of RAM per cell. In order to align two BACs of 100 kb each, you would need about 50 GB of RAM (1e5 x 1e5 x 5). What if you wanted to align some genomes? No computer on the planet has enough RAM. Another reason not to use N-W and S-W is that most of the the space in a DP matrix has a low score. Why align everything rather than just the best parts? Sequence alignment is one of the oldest areas of bioinformatics research, but it is still very active. There are a lot of clever programs that perform alignments very quickly without using much memory. At the root of all these programs is some variant of the S-W algorithm.

What is information?: We all have an intuitive idea about what information is. If we ask a kid what their favorite food is and they say 'chocolate' or 'ice cream', it's not very informative. We expected an answer like that. If they say 'broccoli and cheese' we would probably remember that weird kid. Information is a degree of surprise. The more surprising the **message**, the informative the message. The mathematical definition of information assumes there is an information **source** that emits messages. The information of any particular message is given in equation (1). Information is almost always calculated in base 2 and therefore given the unit **bits** (binary digits). Sometimes you may see **nats**, which corresponds to the log base e. As an

example, let's say that the answer to an average kid's favorite food is 'chocolate' 50% of the time. The information of 'chocolate' is simply the $-\log(0.5)$ or 1.0. Similarly, if the probability of 'ice cream' is 0.25, this is 2.0 bits. Any easy way to think about this is as powers of two: 0.5 is 2^{-1} and 0.25 is 2^{-2} . A message occurring $\sim 1/1000$ times has 10 bits of information ($2^{10} = 1024$).

$$I(m) = -\log_2 P(m)$$

Information content: Generally, we are more interested in the information content of a source rather than the information of any particular message. A rich source of information provides you with a lot of surprise on average. Information content is simply the average information per message (equation 2). Information content is also called **entropy** or Shannon's Entropy because it was invented by Claude Shannon (the "Father of Information Theory"). An information source can be thought of as a frequency distribution (histogram). Some histograms are more *predictable* than others. For example, consider two coins, one is fair, the other a trick coin. The fair coin comes up heads 50% of the time. The trick coin is heads 90% of the time. Which one is more predictable? As an information source, which one has higher information content? What about a fair vs loaded die? What about DNA? What about DNA with highly biased composition? Try working some examples.

$$(2) H = -\sum P_i \log_2 P_i$$

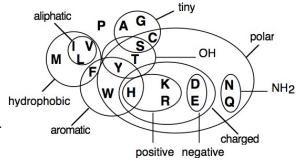
<u>Relative entropy:</u> Let's say you have nucleotide frequencies from several different genomes and you want to know which ones are the most similar to each other. How might you compare them? If you consider the sequences to be information sources and nucleotides to be messages, then you can use relative entropy to measure the similarity. This is also called **Kullback-Leibler distance** (equation 3). Strictly speaking, D(P||Q) is not always D(Q||P) but it's usually close. Not that if P or Q contains any zero probability values, you may get numerical errors (divide by zero or log zero).

(3)
$$D(P \parallel Q) = \sum P_i \log_2 \left(\frac{P_i}{Q_i}\right)$$

<u>Codon bias example</u>: In the genetic code, some triplets code for the same amino acid, but not all codons are used with the same frequency. In different genomes the biased codons may not be the same. Codon bias probably exists because of translational efficiency. Not all tRNAs are expressed at the same level. As a result, highly expressed genes are optimized to use abundant tRNAs so they don't have to "wait" for rare tRNAs. Given that the codon bias of an organism is a kind of signature, if you found a gene with very different codon usage, you might expect horizontal gene transfer. You could use K-L distance to find outliers in a genome. In such an experiment, the source is the codon usage, and each message is a triplet.

<u>Similarity</u>: One of the most fundamental concepts in biology is that similarity is an indicator of common evolutionary history. Many phylogenetic trees are based on proteins. Before we can

compare proteins, we need to be able to compare amino acids with something more biological than +1 match, -1 mismatch. How similar are any two amino acids? One could look at size, shape, charge, hydrophobicity, functional groups, etc. You can imagine a lot of different ways. The image at the right summarizes some of these ideas. Another way to determine similarity is by asking how often can one amino acid substitute for another in a protein? How would you design such



an experiment? Replace an amino acid with another and perform some kind of assay for protein function? That would be great but it's a lot of work. Luckily, these experiments have already been performed billions and billions of times... in nature.

Amino Acid Similarity: Margaret Dayhoff, the "mother of bioinformatics", aligned orthologous proteins by hand and published the multiple alignments in the Atlas of Protein Sequence and Structure. She and her colleagues produced multiple volumes of the atlas. Using known phylogenetic relationships, she was able to observe the rate at which one amino acid changes to another, which is called the **substitution frequency**. These changes are not symmetrical. That is, changing from $G \rightarrow V \neq V \rightarrow G$. This is the truth, but we generally ignore this and average them. Today, there are too many proteins for a print publication. Databases like SwissProt, GenBank, and TrEMBL take place of the Atlas. Similarly, aligning all the sequences by hand would not be possible. There are several computer programs for creating multiple alignments (ClustalW, Dialign, T-Coffee).

```
HM40_CAEEL/188-247
                                       RRKRRNFSKTSTEILNEYFLANIN..HPYPSEEVKQALAMQC.....NISVAQVSNWFGNKRIRYKK
PBX1_HUMAN/234-293
                                       RRKRRNFNKQATEILNEYFYSHLS..NPYPSEEAKEELAKKC.....GITVSQVSNWFGNKRIRYKK
CUT DROME/1746-1802
                                       KKQRVLFSEEQKEALRLAFA...L..DPYPNVGTIEFLANEL....GLATRTITNWFHNHRMRLKQ
CUTL2_MOUSE/1114-1170
CUTL1_MOUSE/1240-1296
                                       KKPRVVLAPAEKEALRKAYQ...L..EPYPSQQTIELLSFQL....NLKTNTVINWFHNYRSRMRR
                                       KKPRVVLAPEEKEALKRAYQ...Q..KPYPSPKTIEELATQL....NLKTSTVINWFHNYRSRIRR
                                       KKTKSPFTEHEIAVMMALFE...I..NKSPNHEEVQKLAVQL....NLGYRSVANFFMNKRAKERK
Q22810_CAEEL/212-268
                                       PKPEWKPNQHQAQILEELFI...G.GTVNPSLTSIKQITIKLQSYGEEVDDADVYKWFHNRKYSRKP
O9FNM1 ARATH/51-113
WOX9_ARATH/52-113
                                       PKPRWNPKPEQIRILEAIFN...S.GMVNPPREEIRRIRAQLQE.YGQVGDANVFYWFQNRKSRSKH
WUS PETHY/44-106
                                       NSTRWTPTTDQIRILKDLYY....NNGVRSPTAEQIQRISAKLRQ.YGKIEGKNVFYWFQNHKARERQ
WOX6_ARATH/58-119
                                       ATLRWNPTPEQITTLEELYR...S.GTRTPTTEQIQQIASKLRK.YGRIEGKNVFYWFQNHKARERL
                                      VSSRWNPTPDQLRVLEELYR...Q.GTRTPSADHIQQITAQLRR.YGKIEGKNVFYWFQNHKARERQ
SSSRWNPTKDQITLLENLYK...E.GIRTPSADQIQQITGRLRA.YGHIEGKNVFYWFQNHKARQRQ
WOX1 ARATH/73-134
WOX2 ARATH/11-72
                                      ANARWTPTKEQIAVLEGLYR...Q.GLRTPTAEQIQQITARLRE.HGHIEGKNVFYWFQNHKARQRQ
STTRWCPTPEQLMMLEEMYR...G.GLRTPNAAQIQQITAHLST.YGRIEGKNVFYWFQNHKARDRQ
Q8LR86_ORYSA/41-102
Q9LIX7_ORYSA/24-85
                                       GGTRWNPTQEQIGILEMLYK...G.GMRTPNAQQIEHITLQLGK.YGKIEGKNVFYWFQNHKARERQ
WOX4_ARATH/87-148
WOX5_ARATH/21-82
                                       KCGRWNPTVEQLKILTDLFR...A.GLRTPTTDQIQKISTELSF.YGKIESKNVFYWFQNHKARERQ
WOX5 ORYSA/11-72
                                       KCGRWNPTAEQVKVLTELFR...A.GLRTPSTEQIQRISTHLSA.FGKVESKNVFYWFQNHKARERH
```

The **score for pairing amino acids** is shown in Equation 4. The score, S, for any two amino acids i and j is the log of the observed substitution frequency (Q_{ij}) divided by the expected substitution frequency. The observed frequency comes from counting occurrences in multiple alignments. The expected frequency is simply the chance that any two amino acids would be selected at random, so this is the product of the probabilities of the individual amino acid frequencies P_i and P_i .

$$S_{ij} = \log \left(\frac{Q_{ij}}{P_i P_j} \right)$$

Try working some examples to make sure you understand the details. In the box at the right, the probabilities of individual amino acids and pairs is given. These are fictitious examples.

```
Amino acid score examples
Given: P_M = 0.02, P_L = 0.1, P_E = 0.04
Q_{ML} = 0.004, Q_{ME} = 0.001, Q_{LE} = 0.002
Calculate: S_{ML}, S_{LE}, S_{ME}

S_{ML} = \log(0.004 / (0.02)(0.1)) = 1.0 \text{ bit}
S_{LE} = \log(0.002 / (0.04)(0.1)) = -1.0 \text{ bit}
S_{ME} = \log(0.001 / (0.02)(0.04)) = 0.32 \text{ bits}
```

<u>Scoring matrices</u>: A **scoring matrix** is simply a table of all pairwise scores. The matrix produced by Dayhoff is called the PAM matrix (a rearrangement of <u>acceptable point mutations</u>). If you look at the scores in a matrix, you will note that they are all integers. What happened to values like 0.32 bits? They were scaled and rounded off. For example, one might scale 0.32 by a factor of 2 and then round off 0.64 to +1. Why? Historically, computers were slow and had little memory, so people used integers. There is no reason to do this now (floating point calculations are actually faster than integer today), but the practice of using integers for scoring matrices continues. Once the scores in a matrix are scaled and rounded off, the units are no longer bits.

Expected score: An important property of a matrix is its expected score (equation 5). To calculate this, one sums up the score contribution of each pairing (the contribution depends on the score and the expected frequencies of the individual amino acids). In general, the expected score of a matrix is negative.

$$Exp = \sum_{i} \sum_{j} P_{i} P_{j} S_{ij}$$

Relative entropy: The most important property of a scoring matrix is its relative entropy (equation 6). This is the bits per *aligned* pair of amino acids. To gain some intuition for this, imagine if the observed pairing (Q_{ij}) is equal to expected (P_iP_j) . In this case, H = 0. That is, the scoring system reflects the random expectation. This is not so different from K-L distance if you compare to identical histograms. The distance is zero. H is maximum when what is observed is very different from what is expected. When does this happen? Continuing from the previous example where $P_M = 0.04$ and $P_L = 0.1$, the expectation is 0.004. If M is rarely observed to align with L, then Q_{ML} will be different from P_MP_L . If you create a scoring matrix from proteins that are all very similar to each other, there will be few substitutions, and Q_{ij} will be very different from P_iP_j . In biological terms, a scoring matrix from highly conserved orthologous proteins will result in a matrix with high H whereas a matrix derived from less similar proteins will have low H. If the alignments are random sequences with no real relationship, H will be zero.

(6)
$$H = \sum_{i} \sum_{j} Q_{ij} \log \left(\frac{Q_{ij}}{P_i P_j} \right)$$

<u>Sequence similarity</u>: To determine protein similarity we simply align two proteins and sum up the amino acid scores. In principle, we could determine similarity scores from local or global alignments. In practice, we use local alignment only. One reason for this is that there is no established procedure for determining global alignment significance.

<u>Alignment scores</u>: What does an alignment score mean? Is a score of 30 *good*? Does 30 mean the proteins are homologous or functionally related? What if the scores in the matrix were scaled by 10 vs 5? Is a score of 100 necessarily better than 50?

Significance: In typical frequentist statistics, one accepts or rejects an hypothesis based on some random model. For local alignments, we use the same idea. Given an alignment score, we would like to know how often such a score would be expected to occur at random. If the score is easily attained at random, then it is probably not very significant.

Karlin-Altschul statistics: Local alignment statistics were formalized by Karlin & Altschul using information theoretic methods. Given certain assumptions (see box) the K-A equation (equation 7) tells you how often such a score (or higher) is expected at random. For some intuition in this, imagine comparing two books to see if they have similar sentences. If the books are very short, you don't expect may similar sentences. Conversely, if the books were gigantic, you would expect to find many more similar sentences. The product MN is called the search space, and the number of expected alignments varies linearly with the size of the space. Now imagine that

Karlin-Altschul Assumptions

- 1.A positive score must be possible
- 2. Expected score of matrix must be negative
- 3. Sequences are infinitely long
- 4.Letters are independent and identically distributed
- 5. Alignments do not contain gaps

you have a threshold score for what you accept as similar sentences. If you ask for a higher score, you will find fewer sentences. The K-A equation shows that this is an inverse exponential relationship. In other words, a small change increase in score can lead to a large reduction in the number of alignments expected at random.

The fact that λ is in the exponent indicates that E is also highly dependent on its value. λ is effectively the inverse of the scaling factor used to create the matrix (but not exactly due to rounding). In other words, λ turns the matrix score into a log-odds score. Now we can begin to answer the questions we previously posed. Is a score of 30 good? It depends on the search space. In a large search space, 30 may be expected at random, but it might be highly significant in a small search space. Is a score of 100 better than 50? If the only difference is the scaling factor, then the significance is the same because λ will normalize them to the same bit score.

$$(7) E = kMNe^{-\lambda S}$$

K-A issues: Let's take a closer look at the K-A assumptions. #1 and #2 are true of any scoring matrix derived from multiple alignments. But we can also make up an arbitrary scoring scheme such as our original +1/-1 match/mismatch scheme. Is this legal? What would happen if the scheme was +2/0? What about -1/-2? What about +10/-1? When might +1/-1 be illegal? #3 is only a problem when sequences are very short. To deal with this problem, people consider the search space to be smaller in each dimension by log(kMN)/H, which is the length of the expected random alignment. #4 states that letters are independent and identically distributed. In other words, the probability of finding a sequence such as AAA is simply the product of finding A cubed. Does this make sense? Not really considering that genomes and proteins contain a lot of repeats. #5 disallows gaps. But we know S-W alignments can contain gaps. We will return to the gap problem in a bit.

Estimating Lambda: In order to compute E, we need λ for our scoring scheme. We might know this value ahead of time if we created our own scoring matrix, but if someone else created it, or we used a system like +1/-1, we need to be able to derive λ somehow. λ cannot be solved for algebraically, but we can estimate its value to arbitrary precision by quessing at its value and repeatedly refining our guesses (see below).

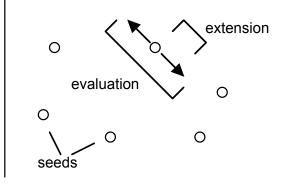
Gaps revisited: So what do we do about gaps? Gaps make it easier to align two sequences. Therefore, gaps effectively reduce H. To account for this in the K-A equation, we can simply decrease λ , and this will decrease the bit score of the alignment and therefore increase the E value. How much we reduce λ depends on the specific match, mismatch, and gap penalties. It is not possible to compute these adjustments algebraically, so they are computed via simulation.

<u>BLAST</u>: One of the most famous and popular bioinformatics applications is BLAST (Basic Local Alignment Search Tool). This combines sequence alignment and statistical evaluation in a single, efficient program. BLAST is similar to S-W in principle: both are local alignment algorithms. But BLAST is much faster because it does not explore the entire search space.

There are 3 steps to the BLAST algorithm: (1) seeding (2) extension (3) evaluation. In the seeding phase, regions containing identical (or highly similar) strings are identified. These points in the space are expected to contain the good local alignments. In the extension phase, each seed undergoes a S-W-like alignment, but the extension stops if the alignment quality degrades too much. In the evaluation phase, the alignment is subjected to the K-A equation to determine how often the alignment is expected by chance. If the E value is less than some user-defined threshold, then the alignment is reported. The simple K-A equation is adjusted by several factors including the lengths of the sequences and the number of alignments.

BLAST can be used to compare proteins to proteins, nucleotides to nucleotides, or proteins to nucleotides (by first translating nucleotides into peptides). In addition to the 5 common BLAST algorithms, there are several variants. To control how BLAST runs, you can change the seeding, extension, and evaluation parameters. Each one has an effect on the speed and sensitivity of the search. Simply using the default parameters for every search is sort of like doing PCR under the exact same cycling conditions (if you're a bench biologist, you're cringing now).

$S_{ij} = \log \left(\frac{Q_{ij}}{P_i P_j} \right)$	The usual equation for the score of any amino acid pair.			
$\lambda S_{ij} = \log \left(\frac{Q_{ij}}{P_i P_j} \right)$	λ is the inverse of the scaling factor used when the matrix was scaled and rounded off. When scores are in bits, λ = 1.			
$e^{\lambda S_{ij}} = \frac{Q_{ij}}{P_i P_j}$	Exponentiate each side of the equation.			
$Q_{ij} = P_i P_j e^{\lambda S_{ij}}$	This is the most important part. It shows that an observed pairing frequency is <i>implied</i> given the marginal compositions and a scoring scheme.			
$\sum \sum Q_{ij} = 1$	By definition all observed pairing frequencies sum to 1.0			
$\sum \sum = P_i P_j e^{\lambda S_{ij}}$	We can solve for λ by making refined guesses at its value. If our guess is too high, the sum will be > 1. If it is too low, the sum will be < 1.			



Program	Database	Query	Example
BLASTN	DNA	DNA	Align mRNA to genome
BLASTP	AA	AA	Search for proteins related to
BLASTX	AA	DNA	Find coding exons in a BAC
TBLASTN	DNA	AA	Search for transcripts similar to
TBLASTX	DNA	DNA	Find orthologous coding exons