CS 430 Homework 2

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1. **Even-odd Algorithm**

Citation: `http://idav.ucdavis.edu/~okreylos/TAship/Spring2000/PointInPolygon.html`
I am sending rays in the positive x-direction only.
R1 containing all the points (p_x, p_y) such that $p_y < P_y$
R2 containing all the points (p_x, p_y) such that $p_y >= P_y$

$P = (2, 3)$
$Ray = (2, 3) \rightarrow (5, 3)$

`Edge 1` $= (1, 2)(p1) \rightarrow (4, 2)(p2)$

Number of intersections = 0

If $p1_y < P_y$ and $p2_y < P_y$, do nothing (both points are in region R1).
Because $2 < 3$ and $2 < 3$, both points are in R1.

`Edge 2` $= (4, 2) \rightarrow (2, 4)$

If $p1_y < P_y$ and $p2_y < P_y$, do nothing (both points are in region R1).
Because $2 < 3$ and $4 > 3$, the points are not both in R1.
If $p1_y >= P_y$ and $p2_y >= P_y$, do nothing (both points are in region

R2).
Because $2 < 3$ and $4 >= 3$, both points are not in R2.

Find intersection point:
$x_0 = 4$
$y_0 = 2$

$P(t) = P_0 + t_0(P_1 - P_0)$
$P(t) = (4,2) + t_0((2,4) - (4,2))$
$P(t) = (4,2) + t_0((-2,2))$
$P(t) = P_2 + t_2(P_3 - P_2)$
$P(t) = (2,3) + t_2((5,3) - (2,3))$
$P(t) = (2,3) + t_2(3,0)$

$x_2 = 2$
$y_2 = 3$

$D_0 = P_1 - P_0$
$D_0 = (2,4) - (4,2) = (-2,2)$

$D_2 = P_3 - P_2$
$D_2 = (2,5) - (2,3) = (3,0)$

$t_0 = \frac{(x_0 - x_2)D_{y2} + (y_2 - y_0)D_{x2}}{D_{y0}D_{x2} - D_{x0}D_{y2}}$
$t_0 = \frac{(4-2)0 + (3-2)3}{2*3 - (-2*0)} = \frac{3}{6} = \frac{1}{2} \geq 0$

$t_2 = \frac{(x_2 - x_0)D_{y0} + (y_0 - y_2)D_{x0}}{D_{y2}D_{x0} - D_{x2}D_{y0}}$
$t_2 = \frac{(2-4)2 + (2-3)-2}{0*-2-3*-2} = \frac{-2}{6} = \frac{-1}{3} \leq 1$

Intersection point: $P_i = P_0 + t_0(P_1 - P_0)$
$P_i = (4,2) + \frac{1}{2}((2,4) - (4,2)) = (4,2) + (-1,1) = (3,3)$

`Edge 3` $= (2,4) \rightarrow (1,2)$

If $p1_x <= P_x$ and $p2_x <= P_x$, do nothing (both points are to the left of the point in question).
Because 2 <= 2 and 1 <= 2, this edge is to the left of the point.

Number of intersections = 1
This is an odd number, so the point P is inside the polygon.

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2. **Sutherland-Hodgman Polygon Clipping**
   Input: <(-1,-5) , (5,5), (2,7)>
   Output: < >

   `Edge 1 (left edge)`: (0,0) → (0,10)
   First vertex is outside, so don't add to output.
   v_1 = (-1,-5), v_2 = (5,5) → outside-inside
   Find intersection point:
   $x_0 = -1$
   $y_0 = -5$

   $P(t) = P_0 + t_0(P_1 - P_0)$
   $P(t) = (-1,-5) + t_0((5,5) - (-1,-5))$
   $P(t) = (-1,-5) + t_0((6,10))$
   $P(t) = P_2 + t_2(P_3 - P_2)$
   $P(t) = (0,0) + t_2((0,10) - (0,0))$
   $P(t) = (0,0) + t_2((0,10))$

   $x_2 = 0$
   $y_2 = 10$

   $D_0 = P_1 - P_0$
   $D_0 = (5,5) - (-1,-5) = (6,10)$

$D_2 = P_3 - P_2$
$D_2 = (0, 10) - (0, 0) = (0, 10)$

$t_0 = \frac{(x_0 - x_2)D_{y2} + (y_2 - y_0)D_{x2}}{D_{y0}D_{x2} - D_{x0}D_{y2}}$
$t_0 = \frac{(-1-0)10+(10--5)0}{10*0-6*10} = \frac{-10}{-60} = \frac{1}{6} \geq 0$

$t_2 = \frac{(x_2 - x_0)D_{y0} + (y_0 - y_2)D_{x0}}{D_{y2}D_{x0} - D_{x2}D_{y0}}$
$t_2 = \frac{(0--1)10+(-5-10)6}{10*6-0*10} = \frac{-100}{60} = \frac{-10}{6} \leq 1$

Intersection point: $P_i = P_0 + t_0(P_1 - P_0)$
$P_i = (-1, -5) + \frac{1}{6}((5, 5) - (-1, -5)) = (-1, -5) + (\frac{6}{6}, \frac{10}{6}) = (0, \frac{-20}{6}) =$
$(0, -3.33)$

Output: $< (0, -3.33), (5, 5) >$

v_2 = (5,5), v_3 = (2,7) $\rightarrow$ inside-inside
Output: $< (0, -3.33), (5, 5), (2, 7) >$

v_3 = (2,7), v_1 = (-1,-5) $\rightarrow$ inside-outside
Find intersection point:
$x_0 = 2$
$y_0 = 7$

$P(t) = P_0 + t_0(P_1 - P_0)$
$P(t) = (2, 7) + t_0((-1, -5) - (2, 7))$
$P(t) = (2, 7) + t_0((-3, -12))$
$P(t) = P_2 + t_2(P_3 - P_2)$
$P(t) = (0, 0) + t_2((0, 10) - (0, 0))$
$P(t) = (0, 0) + t_2((0, 10))$

$x_2 = 0$
$y_2 = 10$

$D_0 = P_1 - P_0$

$D_0 = (-1, -5) - (2, 7) = (-3, -12)$

$D_2 = P_3 - P_2$
$D_2 = (0, 10) - (0, 0) = (0, 10)$

$t_0 = \frac{(x_0 - x_2)D_{y2} + (y_2 - y_0)D_{x2}}{D_{y0}D_{x2} - D_{x0}D_{y2}}$
$t_0 = \frac{(2-0)10 + (10-7)0}{-12*0-(-3*10)} = \frac{20}{30} = \frac{2}{3} \geq 0$

$t_2 = \frac{(x_2 - x_0)D_{y0} + (y_0 - y_2)D_{x0}}{D_{y2}D_{x0} - D_{x2}D_{y0}}$
$t_2 = \frac{(0-2)-12+(7-10)-3}{10*-3-0*-12} = \frac{33}{-30} = \frac{-11}{10} \leq 1$

Intersection point: $P_i = P_0 + t_0(P_1 - P_0)$
$P_i = (2, 7) + \frac{2}{3}((-1, -5) - (2, 7)) = (2, 7) + (\frac{-6}{3}, \frac{-24}{3}) = (0, -1)$

Output: $< (0, -3.33, (5, 5), (2, 7), (0, -1) >$

`Edge 2 (right edge)`: (10,10) → (10,0)
Input: $< (0, -3.33), (5, 5), (2, 7), (0, -1) >$
First vertex is inside.
v_1 = (0,-3.33), v_2 = (5,5) → inside-inside
v_2 = (5,5), v_3 = (2,7) → inside-inside
v_3 = (2,7), v_1 = (0,-1) → inside-inside
Output: $< (0, -3.33, (5, 5), (2, 7), (0, -1) >$

`Edge 3 (top edge)`: (0,10) → (10,10)
Input: $< (0, -3.33), (5, 5), (2, 7), (0, -1) >$
First vertex is inside.
v_1 = (0,-3.33), v_2 = (5,5) → inside-inside
v_2 = (5,5), v_3 = (2,7) → inside-inside
v_3 = (2,7), v_1 = (0,-1) → inside-inside
Output: $< (0, -3.33, (5, 5), (2, 7), (0, -1) >$

`Edge 4 (bottom edge)`: (10,0) $\rightarrow$ (0,0)
Input: $< (0, -3.33), (5, 5), (2, 7), (0, -1) >$
First vertex is outside, so don't add to output.
v_1 = (0,-3.33), v_2 = (5,5) $\rightarrow$ outside-inside
Find intersection point:
$x_0 = 0$
$y_0 = -3.33$

$x_2 = 0$
$y_2 = 0$

$D_0 = (0, -3.33) - (5, 5) = (-5, -8.33)$
$D_2 = (10, 0) - (0, 0) = (10, 0)$

$t_0 = 0.8125 \geq 0$

Intersection point: $P_i = (1.875, 0)$

Output: $< (1.875, 0), (5, 5) >$

v_2 = (5,5), v_3 = (2,7) $\rightarrow$ inside-inside
Output : $< (1.875, 0), (5, 5), (2, 7) >$
v_3 = (2,7), v_1 = (0,-1) $\rightarrow$ inside-outside
Find intersection point:
$x_0 = 2$
$y_0 = 7$

$x_2 = 0$
$y_2 = 0$

$D_0 = (2, 7) - (0, -1) = (2, 8)$
$D_2 = (10, 0) - (0, 0) = (10, 0)$

$t_0 = 0.975 \geq 0$

Intersection point: $P_i = (0.25, 0)$

Output (**final vertices**): $< (1.875, 0), (5, 5), (2, 7), (0.25, 0) >$

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3. **Polygon Filling**

- `Extrema points for scan-line 6`: 4, 11
- Scan-line 6 will draw as it goes from x=0 to x=13. When the parity bit is 0, it does not draw; when it's 1, it does. When it hits an extrema point, the bit will flip. From x=0, 2, 3 ..., 13 scan-line 6 will draw according to the parity bit's value.

x	Parity bit
0	0
1	0
2	0
3	0
4	1
5	1
6	1
7	1
8	1
9	1
10	1
11	1
12	0
13	0