CS 430 Homework 3

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1. $\mathbf{C}^0$
   Two curves are C^i continuous at point p iff the i^{th} derivative of the curves are equal at p. When two curves are C^0 continuous, there is a discontinuity in the slope. Their first derivatives are not equal.

   (Note: I made the drawings with Google Drawings.)

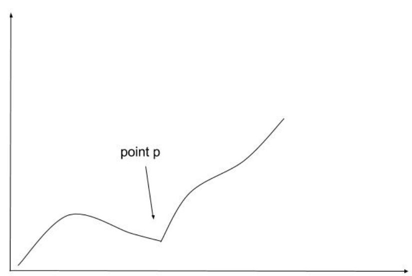

Figure 1: C^0 continuity.

2. $\mathbf{C}^1$
   When two curves are C^1 continuous, there is a discontinuity in the curvature. Their first derivatives are equal, but their second derivatives are not.

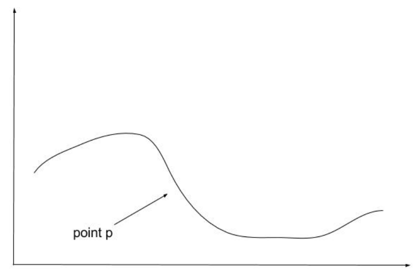

Figure 2: C^1 continuity.

3. **$\mathbf{C}^2$**
   When two curves are C^2 continuous, their first and second derivatives are both equal.

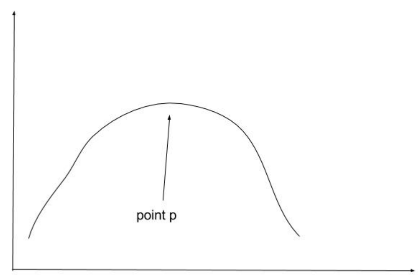

Figure 3: C^2 continuity.

4. **True or False**

   (a) False. The number of control points is independent of the degree of the B-spline curve.

   (b) True. B-splines have a local control property which means each control point affects its local curve.

   (c) True. Using non-uniform B-splines, they can be forced to interpolate their control points.

   (d) False. B-splines are single piecewise curves.

5. **Bernstein Polynomials**
The basis matrix of a Bézier curve with 3 control points and using polynomials of degree 2

$b_{02} = \binom{2}{0}(1-t)^2t^0 = (1-t)^2 = t^2 - 2t + 1$
$b_{12} = \binom{2}{1}(1-t)^1t^1 = 2(1-t)t = -2t^2 + 2t$
$b_{22} = \binom{2}{2}(1-t)^0t^2 = (1-t)^0t^2 = t^2$

$$\begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

6. **Bézier Curve**

(a) $G = \begin{bmatrix} 2 & 4 & 5 & 0 \\ 4 & 2 & 7 & 0 \end{bmatrix}$

(b) $M = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

(c) Q = GMT

$GM = \begin{bmatrix} -5 & -3 & 6 & 2 \\ -19 & 21 & -6 & 4 \end{bmatrix}$

$T = \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}$

$Q = \begin{bmatrix} -5t^3 - 3t^2 + 6t + 2 \\ -19t^3 + 21t^2 - 6t + 4 \end{bmatrix}$

(d) Q(0.5) = (3.625, 3.875)

7. **de Casteljau**

$p_{00} = (2, 4)$
$p_{10} = (4, 2)$
$p_{20} = (5, 7)$
$p_{30} = (0, 0)$

r = 1
$p_{01}(u)$
$= (1 - u)(2, 4) + u(4, 2) = (2 - 2u, 4 - 4u) + (4u, 2u)$
$= (2 + 2u, 4 - 2u)$

$p_{11}(u)$
$= (1 - u)(4, 2) + u(5, 7) = (4 - 4u, 2 - 2u) + (5u, 7u)$
$= (4 + u, 2 - 5u)$

$p_{21}(u) = (1 - u)(5, 7) + u(0, 0) = (5 - 5u, 7 - 7u)$

r = 2
$p_{02}(u)$
$= (1 - u)(2 + 2u, 4 - 2u) + u(4 + u, 2 - 5u)$
$= (-2u^2 + 2, 2u^2 - 6u + 4) + (4u + u^2, 2u + 5u^2)$
$= (-u^2 + 4u + 2, 7u^2 - 4u + 4)$

$p_{12}(u)$
$= (1 - u)(4 + u, 2 - 5u) + u(5 - 5u, 7 - 7u)$
$= (4 - 3u - u^2, 2 + 3u - 5u^2) + (5u - 5^2, 7u - 7u^2)$
$= (-6u^2 + 2u + 4, -12u^2 - 10u + 2)$

r = 3
$p_{03}(u)$
$= (1-u)(-u^2+4u+2, 7u^2-4u+4)+u(-6u^2+2u+4, -12u^2-10u+2)$
$= (u^3-5u^2+2u+2, -7u^3+11u^2-8u+4)+(-6u^3+2u^2+4u, -12u^3+ 10u^2 + 2u)$
$= (-5u^3 - 3u^2 + 6u + 2, -19u^3 + 21u^2 - 6u + 4)$

$u = 0.5$

(3.625, 3.875)