Synthetic Control with Time Varying Coefficients

A State Space Approach with Bayesian Shrinkage

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A Supplemental Appendix - For Online Publication

A.1 Additional Distributions

The centered distributions for β_j and θ_j are (Bitto and Frühwirth-Schnatter 2019):

$$\theta_j \sim GIG\left(\frac{-T_0-1}{2}, \frac{1}{\xi_j^2}, \sum_{t=1}^{T_0-1}(\beta_{jt,}-\beta_{j,t-1})^2 + \frac{(\beta_{j,0}-\beta_j)^2}{P_{0,jj}}\right) \tag{A0}$$

$$\beta_j \propto \mathcal{N}\left(\frac{\beta_{j,0}\alpha_j^2}{\alpha_j^2+\theta_j P_{0,jj}}, \frac{\alpha_j^2\theta_j P_{0,jj}}{\alpha_j^2+\theta_j P_{0,jj}}\right) \tag{A0}$$

where *GIG* is the generalized inverse Gaussian distribution.

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A.2 Additional Figures for TVP simulation

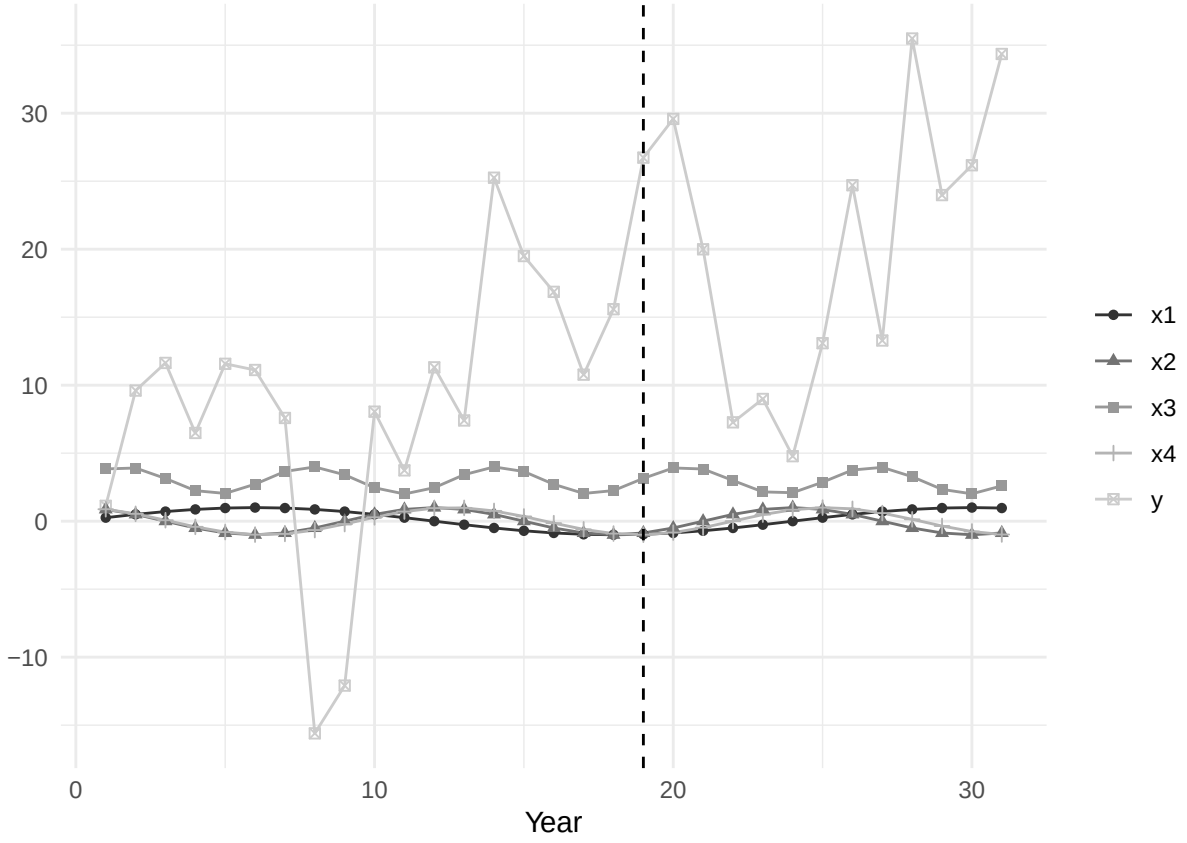

Figure A1: Example Simulation data. Period 19 (dotted vertical line) is the hypothetical treatment period. The treatment effect is 0 for all simulations.

A.3 Empirical Monte Carlo Simulation Example - Tennessee

To further explain the Monte Carlo simulation, I provide an example using a randomly drawn state - Tennessee. Tennessee would be fit from 1970-2000 using the 37 other untreated units. For the time invariant data generating process, the parameter estimates from Makalic and Schmidt (2016) are used to generate 1000 simulated treated units. This is done to create simulations that still have the underlying relationship between treated and control units. By using a Bayesian data generating processes, each simulation will have different parameter values but still maintain the underlying relationship. The approach creates similar estimates to the actual data. Figure A2 displays 50 of the simulations for Tennessee:

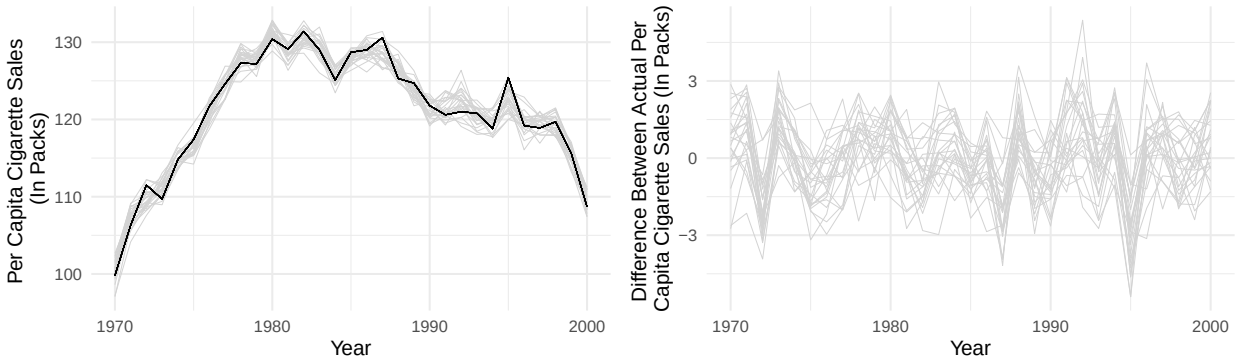

Figure A2: Example Empirical Monte Carlo: Actual Tennessee (black) versus 50 simulated Tennessee (grey).

Each simulated unit is fitted with the synthetic control approaches in the pre-period using the untreated units as controls, then predicted into the post period. The process is performed for every control unit. Modeling the data generating process with time invariant coefficients tests the ability of BL-TVP to shrink irrelevant coefficients. If BL-TVP isn't able to shrink irrelevant coefficients, the credibility interval in the post-period should be inflated, similarly to CI-TVP. If it does shrink, then the credibility interval should be of similar size to CI.

A.4 Simulation Tables - Empirical Monte Carlo

Table A1: Empirical MCMC Results for Each State in the Donor Pool

	Average Treatment Effect Credibility Spread						
State	ArCo	BL-TVP	BSCM-Horseshoe	CI	CI-TVP	DM-LFM	SC
Alabama	18.10	74.98	23.67	40.01	212.28	15.16	NA
Arkansas	24.00	77.15	21.39	46.47	229.54	15.67	NA
Colorado	45.48	79.16	28.61	37.34	284.22	29.24	NA
Connecticut	23.79	82.15	26.06	29.36	89.99	28.12	NA
Delaware	34.04	86.97	31.35	39.79	134.74	28.88	NA
Georgia	12.34	77.29	17.45	17.86	155.39	27.97	NA
Idaho	26.44	83.10	39.38	45.64	293.15	33.98	NA
Illinois	27.99	76.42	24.20	32.97	196.56	31.94	NA
Indiana	20.89	80.40	30.06	53.36	257.89	33.54	NA
Iowa	13.11	72.03	15.85	30.79	184.90	32.53	NA
Kansas	16.96	74.55	20.71	31.92	218.40	33.70	NA
Kentucky	39.89	100.42	52.37	58.39	518.20	44.71	NA
Louisiana	17.38	74.48	20.31	36.75	249.26	33.72	NA
Maine	20.73	78.12	24.61	29.86	166.92	34.32	NA
Minnesota	18.94	76.08	21.05	29.11	158.48	33.35	NA
Mississippi	16.56	73.07	21.47	36.40	191.12	33.98	NA
Missouri	10.54	73.21	12.24	23.56	133.98	32.65	NA
Montana	27.47	74.66	25.23	41.87	269.97	33.26	NA
Nebraska	15.04	72.97	13.91	29.63	147.90	33.48	NA
Nevada	49.00	100.07	65.48	78.45	548.56	42.23	NA

Table A1: Empirical MCMC Results for Each State in the Donor Pool *(continued)*

	Average Treatment Effect Credibility Spread						
State	ArCo	BL-TVP	BSCM-Horseshoe	CI	CI-TVP	DM-LFM	SC
New Hampshire	77.43	124.93	83.31	156.98	821.77	51.94	NA
New Mexico	16.79	70.41	15.36	31.20	186.12	33.62	NA
North Carolina	43.55	112.16	71.82	122.82	675.43	45.24	NA
North Dakota	18.44	76.38	23.79	41.52	300.04	34.38	NA
Ohio	18.17	77.86	17.37	27.31	96.52	33.60	NA
Oklahoma	30.90	85.31	36.32	42.83	322.50	36.83	NA
Pennsylvania	12.70	72.84	15.58	29.91	150.69	33.41	NA
Rhode Island	23.73	78.69	27.90	37.55	181.22	31.72	NA
South Carolina	17.85	76.14	20.44	39.91	207.44	34.03	NA
South Dakota	16.73	71.60	20.33	29.85	217.11	33.09	NA
Tennessee	24.31	77.96	24.71	44.98	219.61	34.94	NA
Texas	22.48	78.13	27.03	38.99	229.86	32.55	NA
Utah	12.59	67.96	13.34	24.62	146.73	33.38	NA
Vermont	28.72	83.55	33.62	53.39	355.51	34.84	NA
Virginia	16.73	75.74	18.27	29.33	239.41	32.45	NA
West Virginia	14.51	73.77	20.10	29.35	163.98	32.64	NA
Wisconsin	11.04	72.14	13.78	18.48	122.49	33.36	NA
Wyoming	29.33	84.93	36.66	68.26	371.61	38.05	NA
Average	**24.07**	**80.20**	**27.77**	**43.07**	**253.93**	**33.49**	**NA**
Median	**19.83**	**76.78**	**23.73**	**37.05**	**214.69**	**33.45**	**NA**

Artificial Counterfactual (ArCo), Bayesian Lasso with Time Varying Parameters (BL-TVP), Bayesian Synthetic Control with Horseshoe (BSCM-Horseshoe), CausalImpact (CI), CausalImpact with time varying parameters (CI-TVP), DM-LFM, and Synthetic Control (SC) are compared using two metrics. The Average Treatment Effect Credibility Spread is defined as the length of the 95% credibility interval for the average treatment effect averaged over the 1000 simulations.

Table A2: Empirical MCMC Results for Each State in the Donor Pool

	Mean Squared Forecast Error						
State	ArCo	BL-TVP	BSCM-Horseshoe	CI	CI-TVP	DM-LFM	SC
Alabama	59.26	33.43	40.78	404.87	10.86	23.27	14.24
Arkansas	29.74	22.64	18.28	100.28	24.66	35.88	14.35
Colorado	61.58	14.22	35.64	134.54	41.29	212.76	59.69
Connecticut	455.16	197.36	312.60	553.18	197.19	228.64	226.86
Delaware	77.33	181.63	46.94	102.31	50.78	278.96	201.01
Georgia	216.51	42.57	148.71	257.02	87.52	213.30	94.80
Idaho	34.92	59.26	44.77	40.49	56.32	64.94	34.39
Illinois	227.06	149.76	94.11	292.43	27.45	134.07	78.40
Indiana	64.80	218.85	40.64	113.05	54.63	301.93	343.44
Iowa	13.67	24.95	19.11	9.34	43.85	29.93	26.50
Kansas	46.10	11.93	20.55	67.27	18.83	32.67	15.09
Kentucky	1479.98	292.95	1089.62	2271.76	388.68	152.16	1599.07
Louisiana	22.24	39.57	46.63	10.20	96.77	15.26	22.81
Maine	285.66	93.57	172.41	214.88	54.09	76.29	58.84
Minnesota	41.10	20.66	27.30	61.57	18.92	51.66	17.01
Mississippi	23.38	62.12	16.28	30.79	69.52	28.26	36.26
Missouri	4.57	88.51	7.66	4.44	14.86	46.62	119.36
Montana	153.67	49.58	59.69	152.20	112.31	94.56	45.19
Nebraska	10.92	8.54	7.73	16.82	10.75	13.39	32.11
Nevada	667.24	419.95	412.16	588.17	266.53	199.48	195.64

Table A2: Empirical MCMC Results for Each State in the Donor Pool *(continued)*

	Mean Squared Forecast Error						
State	ArCo	BL-TVP	BSCM-Horseshoe	CI	CI-TVP	DM-LFM	SC
New Hampshire	200.57	156.70	207.86	280.82	320.70	391.96	144.03
New Mexico	54.96	27.74	15.04	48.03	8.47	10.45	19.15
North Carolina	106.82	109.81	241.03	123.04	204.08	82.75	54.03
North Dakota	28.62	7.05	31.54	25.38	140.47	19.11	65.85
Ohio	12.68	6.33	35.26	42.26	22.49	7.57	9.24
Oklahoma	225.36	207.96	270.90	215.20	175.59	129.92	259.71
Pennsylvania	14.00	4.69	20.84	28.87	15.23	12.73	5.24
Rhode Island	559.20	368.82	343.44	461.99	204.08	478.72	443.23
South Carolina	23.77	28.38	19.76	21.08	22.87	19.53	19.28
South Dakota	33.20	76.89	52.14	34.71	59.37	43.98	52.71
Tennessee	49.66	51.64	21.63	120.75	18.31	12.47	111.39
Texas	168.62	170.40	152.79	332.57	24.73	382.46	279.88
Utah	13.33	10.91	7.98	12.22	17.16	20.09	204.70
Vermont	122.31	92.10	63.53	98.36	63.28	297.03	332.58
Virginia	406.07	86.81	302.50	520.23	75.39	228.98	199.74
West Virginia	87.68	227.94	80.49	126.16	80.20	278.16	260.80
Wisconsin	59.22	5.83	18.45	58.72	9.34	10.84	52.32
Wyoming	229.23	59.89	229.34	267.10	323.25	109.20	38.78
Average	**167.64**	**98.21**	**125.69**	**216.92**	**90.28**	**125.53**	**152.31**
Median	**60.42**	**59.58**	**45.70**	**107.68**	**54.36**	**70.62**	**59.26**

Artificial Counterfactual (ArCo), Bayesian Lasso with Time Varying Parameters (BL-TVP), Bayesian Synthetic Control with Horseshoe (BSCM-Horseshoe), CausalImpact (CI), CausalImpact with time varying parameters (CI-TVP), DM-LFM, and Synthetic Control (SC) are compared using two metrics. The Mean Squared Forecast Error is defined as the average error in the post-treatment period averaged over the 1000 simulations. Each state was simulated 1000 times using BL-TVP with time invariant coefficients.

Table A3: Empirical MCMC Results for Each State in the Donor Pool

	Number Simulations Reject Null SATT=0 Out of 1000					
State	ArCo	BL-TVP	BSCM-Horseshoe	CI	CI-TVP	DM-LFM
Alabama	719	0	11	1	0	6
Arkansas	36	0	12	0	0	61
Colorado	18	0	16	1	0	112
Connecticut	1000	0	875	957	0	306
Delaware	12	0	4	3	0	270
Georgia	855	0	774	929	0	353
Idaho	1	0	4	0	0	0
Illinois	746	0	141	234	0	1
Indiana	325	0	21	0	0	333
Iowa	8	0	35	0	0	0
Kansas	166	0	20	0	0	0
Kentucky	948	0	465	910	0	60
Louisiana	0	0	66	0	0	0
Maine	599	0	282	140	0	0
Minnesota	129	0	53	16	0	0
Mississippi	48	0	13	0	0	0
Missouri	0	0	5	0	0	0
Montana	15	0	50	0	0	0
Nebraska	86	0	10	0	0	0
Nevada	471	0	55	5	0	12

Table A3: Empirical MCMC Results for Each State in the Donor Pool *(continued)*

	Number Simulations Reject Null SATT=0 Out of 1000					
State	ArCo	BL-TVP	BSCM-Horseshoe	CI	CI-TVP	DM-LFM
New Hampshire	0	0	5	0	0	126
New Mexico	340	0	17	0	0	0
North Carolina	5	0	4	0	0	1
North Dakota	9	0	21	0	0	0
Ohio	27	0	85	7	0	0
Oklahoma	38	0	74	0	0	0
Pennsylvania	22	0	47	0	0	0
Rhode Island	857	0	582	494	0	803
South Carolina	8	0	14	0	0	0
South Dakota	44	0	96	0	0	0
Tennessee	159	0	11	0	0	0
Texas	593	0	216	71	0	857
Utah	50	0	10	0	0	0
Vermont	21	0	16	0	0	217
Virginia	920	0	883	851	0	116
West Virginia	642	0	218	6	0	306
Wisconsin	458	0	67	26	0	0
Wyoming	88	0	172	0	0	37
Average	**275.34**	**0**	**143.42**	**122.39**	**0**	**104.66**

Median	**68**	**0**	**41**	**0**	**0**	**0**

* All approaches use 95% confidence/credibility intervals.

Work Cited

Bitto, Angela, and Sylvia Frühwirth-Schnatter. 2019. "Achieving Shrinkage in a Time-Varying Parameter Model Framework." *Journal of Econometrics* 210 (1): 75–97. https://doi.org/10.1016/j.jeconom.2018.11.006.

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