Residual Analysis Techniques for Assessing Space-Time Earthquake Forecasts

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Numerical summary methods (Schorlemmer et al. 2007)

- useful for providing a score and comparing models
- Likelihood-Test (L-Test)
    - one compares the likelihood of observed data with the likelihood from simulations of the model
- Number of Events-Test (N-Test)
    - one compares the number of observed events with the number of simulated events
- Problem:
    - low power tests

Other commonly used numerical summaries

- log likelihood

$$\log L = \sum_{i=1}^{n} \log \lambda(x_i, y_i, m_i, t_i) - \int_S \lambda(u)du$$

- $AIC = -2 \log L + 2k$
- $BIC = -2 \log L + 2k \log n$
- likelihood ratio $LR = \frac{sup\{L_0\}}{sup\{L_1\}}$

Ripley's K-function (Ripley, 1977)

- test for detecting clustering or inhibition
- null hypothesis H_0: homogeneous Poisson
- $\boxed{K(h) = \frac{1}{\lambda} E[\# \text{ of other points within } h \text{ of any given point}]}$
- for homogeneous Poisson $K(h) = \frac{1}{\lambda}\lambda\pi h^2 = \pi h^2$
- estimated by

$$\hat{K}(h) = \frac{|S|}{N^2} \sum_{i \neq j} \frac{1}{v(s_i, s_j)} 1_{\{|s_i - s_j| \leq h\}}$$

where $v(s_i, s_j)$ = the proportion of the circle centered at s_i and passing through s_j which is inside S

Weighted K-function (Veen & Schoenberg, 2005)

- let $\lambda_0(p_r)$ = the conditional intensity under the null hypothesis at point p_r
- use

$$K_w(h) = \frac{1}{b\hat{E}_{H_0}(N)} \sum_r w_r \sum_{s \neq r} w_s I(|p_r - p_s| \leq h)$$

where $b = min\{\lambda(x_i, y_i, m_i, t_i)\}$, $\hat{E}_{H_0}(N)$ = expected number of points in S, and $w_r = \frac{b}{\lambda_0(p_r)}$

- in 2-dimensions

$$K_w(h) \sim N\left(\pi h^2, \frac{2\pi h^2 S}{\left(\hat{E}_{H_0}(N)\right)^2}\right)$$

Problems with numerical summary methods

- provides no information about where the model fits poorly or where the model fits well
- mainly useful for comparing models
- further tests usually necessary to draw conclusions
- homogeneous Poisson may be a poor null hypothesis (Stark 1997)

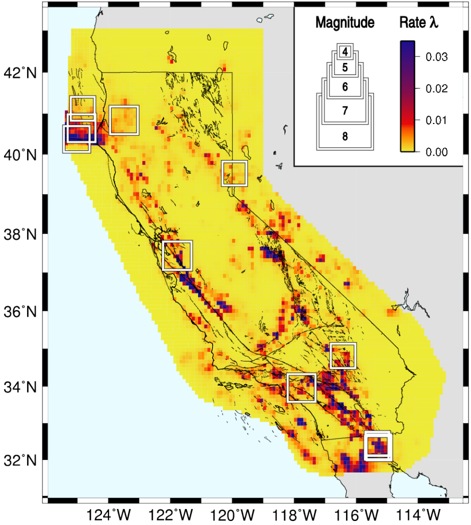
Thinning (Schoenberg, 2003)

- keep each observation in the space S with probability

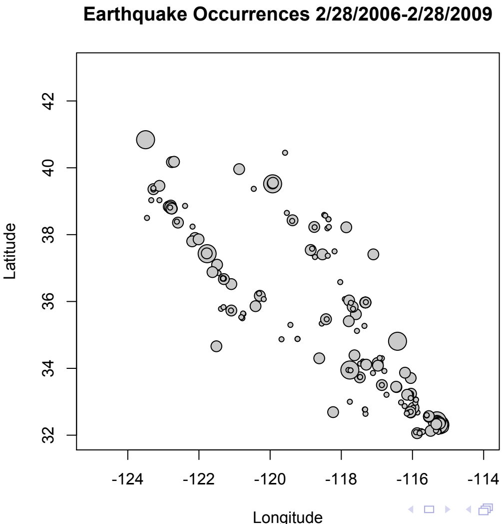
$$\frac{b}{\hat{\lambda}(x_i, y_i, m_i, t_i)} \text{ where } b = min\{\hat{\lambda}(x_i, y_i, m_i, t_i)\} \text{ over } S$$

- remaining points, called *thinned residuals*, are homogeneous Poisson if model is correct
- use Ripley's K-function to test for uniformity
- repeat several times

Helmstetter et al. (2005) forecast

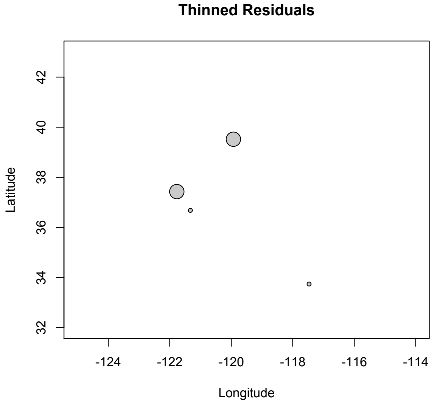


Example


Earthquake Occurrences 2/28/2006-2/28/2009
Latitude
42
40
38
36
34
32
-124
-122
-120
-118
-116
-114
Longitude

Example

Thinned using the Helmstetter et al. (2005) forecast model



Problems with thinning

- some information may be lost when removing points resulting in lower power
- if b is very small, we end up with few points
  - can instead use probability $\frac{k}{\hat{\lambda}(x_i,y_i,m_i,t_i)\sum_{i=1}^{N(S)}\frac{1}{\hat{\lambda}(x_i,y_i,m_i,t_i)}}$ where $k = \#$ of points to keep
- optimal choice of k is an open research question

Superposition (Brémaud, 1981)

- similar to thinning, but points are added instead of removed
- points are simulated using the rate

$$c - \hat{\lambda}(x, y, m, t) \text{ where } c = max\{\hat{\lambda}(x_i, y_i, m_i, t_i)\}$$

- again, the result should be homogeneous Poisson
- Problem:
  - if c is very large, may end up with too many simulated points

Rescaling (Meyer, 1971)

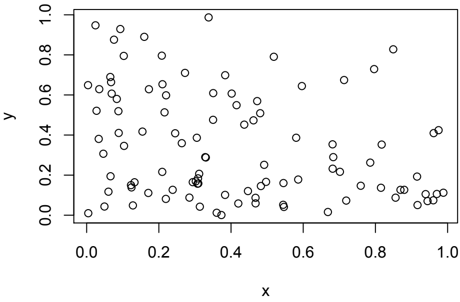
- useful for analyzing two dimensions
- any dimension can be rescaled
- for example, to rescale in the time dimension each point (x_i, y_i, m_i, t_i) is moved to

$$\left(x_i, y_i, m_i, \int_0^{t_i} \hat{\lambda}(x_i, y_i, m_i, t)dt\right)$$

- the result should be homogeneous Poisson if model is correct

Example

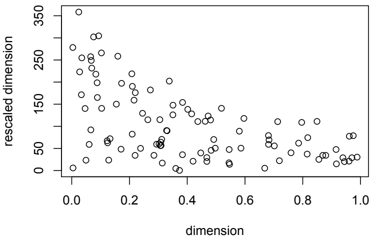
Inhomogeneous Poisson process
simulated using $\lambda(x, y) = 300e^{-3x} + 300e^{-3y}$



Example

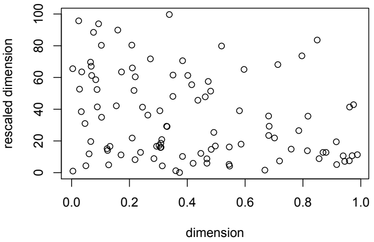
rescaled using the inhomogeneous Poisson model

$$\lambda(x, y) = 300e^{-3x} + 300e^{-3y}$$



rescaled using the homogeneous Poisson model

$$\lambda(x, y) = 101$$



Problems with rescaling

- rescaled points may form an irregular boundary which is difficult to analyze
    - can try rescaling in different dimensions
- hard to interpret rescaled space
- boundary effects if λ is irregular or highly clustered

Pixel-based methods (Baddeley et al. 2005)

- λ is the Papangelou conditional intensity (Papangelou 1974)
- space S is divided into equally spaced bins using a grid
- residuals are calculated in each bin, B

$$R_{\hat{\theta}}(B) = N(B) - \int_B \hat{\lambda}(u)du$$

  where $N(B) = \#$ of points in the bin, B
- diagnostic plots are produced

Problems with pixel-based methods

- one large residual may be all that is noticed, even after rescaling
- usefulness is dependent on the choice of grid, which is arbitrary
- plots are not always helpful in identifying where the model is fitting poorly

Summary

- Numerical summaries
  - low power
- Weighted 2nd-order statistics (Ripley's K-function)
  - only focuses on clustering or inhibition
- Thinning
  - too few points
- Superposition
  - too many points
- Rescaling
  - hard to interpret
- Pixel-based methods (diagnostic plots)
  - large residuals

Thank you for listening