Week 7: Systems Applications & Inequalities

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Session 7.3
Linear Modeling & Fit-by-Eye

Quick Reference: Linear Modeling

What is a Line of Best Fit?

A **line of best fit** (or trend line) is a straight line that best represents the data on a scatter plot.

Key Ideas:

- Not all points will be exactly on the line

- The line shows the general trend or pattern

- We use the line to make predictions

- The line minimizes the distance from all the data points

Steps to Create a Line of Best Fit

1. Plot the Data

- Create a scatter plot of all data points

- Label axes with variable names and units

- Choose an appropriate scale

2. Draw the Line

- Use a ruler or straightedge

- Try to balance points above and below the line

- The line should follow the general trend

- Extend the line across the entire graph

3. Find the Equation

- Pick two points ON your line (they don't have to be data points)

- Calculate the slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

- Find the y-intercept using point-slope form

- Write in slope-intercept form: $y = mx + b$

Residuals

A **residual** measures how far each data point is from the line of best fit.

Formula:
$$\text{Residual} = \text{Actual value} - \text{Predicted value}$$

Interpretation:

- **Positive residual**: data point is above the line

- **Negative residual**: data point is below the line

- **Zero residual**: data point is exactly on the line

- Smaller residuals mean better fit

Interpreting Slope and y-Intercept

Slope (m):

- Rate of change

- How much y changes for each 1-unit increase in x

- Include units: "For every [unit of x], [variable y] changes by [slope] [unit of y]"

y-Intercept (b):

- Starting value when $x = 0$

- May or may not make sense in context

- Include units: "When [variable x] is 0, [variable y] is [y-intercept] [unit of y]"

Homework 7.3: Linear Modeling

Instructions

:

For each problem,

1. Plot the data points on graph paper

2. Draw a line of best fit by eye

3. Find the equation using two points on your line

4. Calculate residuals for specified data points

5. Interpret the slope and y-intercept in context

Homework Problem 1: Temperature and Hot Chocolate Sales

A café tracks hot chocolate sales at different temperatures.

Temperature (°F)	Hot Chocolates Sold
20	60
30	49
40	40
50	28
60	60
70	10

Part A

Choose your scale:

- x-axis: Temperature from _____ to _____ (include units)

- y-axis: Sales from _____ to _____ (include units)

Add your scale and label axes.

Plot the points.

Part B

Draw a line of best fit (using a different color).

Part C

Choose two points ON your line and find the equation.

Point 1: (_____, _____)

Point 2: (_____, _____)

Calculate the slope:

Find the y-intercept using point-slope form:

Equation: _____

Part D

Calculate residuals for $x = 30$, $x = 40$, and $x = 50$:

For $x = 30$:

- Actual value: _____
- Predicted value (from equation): _____
- Residual $=$ Actual $-$ Predicted $=$ _____

For $x = 40$:

- Actual value: _____
- Predicted value: _____
- Residual $=$ _____

For $x = 50$:

- Actual value: _____
- Predicted value: _____
- Residual $=$ _____

Part E

Interpret the slope in context:

For every _____ (include units) increase in temperature,

the number of hot chocolates sold

_____ (increases/decreases) by about _____ (include units).

Part F

Interpret the y-intercept in context:

When the temperature is 0°F, the model predicts _____ hot chocolates sold.

Part G

Does the y-intercept make sense in this context? Why or why not?

Part H

Use your model to predict sales at 60°F:

Is this prediction reasonable? Explain.

Homework Problem 2: Study Time and Quiz Score

A teacher tracks student study time and quiz scores.

Study Time (hours)	Quiz Score (%)
0.5	62
1.0	68
1.5	74
2.0	78
2.5	83
3.0	87

Part A

Add a scale and label axes.

Plot the points.

Part B

Draw a line of best fit (using a different color).

Part C

Find the equation of the line of best fit using two points on your line.

Part D

Calculate residuals for $x = 1.0$, $x = 2.0$, and $x = 3.0$:

Part E

Use your model to predict the quiz score for someone who studies 4 hours:

Part F

Is this prediction reasonable? Why or why not?

(Hint: Can quiz scores go above 100%?)

Homework Problem 3: Car Value Over Time

A used car dealer tracks how a car's value changes with age.

Car Age (years)	Value ($1000s)
1	22
2	19
3	17
4	14
5	12
6	10

Part A

Add a scale and label axes.

Plot the points.

Part B

Draw a line of best fit (using a different color).

Part C

Find the equation of the line of best fit.

Part D

What does the y-intercept represent?

Does this make sense? (What was the car worth when new?)

Part E

According to your model, when will the car be worth $0?

Part F

Is this prediction realistic? Why or why not?

Homework Problem 4: Plant Growth

A biology student measures plant height over time.

Days	Height (cm)
0	2
5	5
10	8
15	11
20	13
25	16

Part A

Add a scale and label axes.

Plot the points.

Draw a line of best fit (using a different color).

Part B

Find the equation of the line of best fit.

Part C

Interpret the y-intercept:

Part D

Calculate the residual for day 10:

Part E

Predict the height after 30 days:

Part F

If the plant can only grow to a maximum of 25 cm,
when will it reach this height according to your model?