Week 7: Systems Applications & Inequalities

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Session 7.1
Systems Word Problems Mixture, Motion, Break-Even

Quick Reference: Problem Type Formulas

Mixture Problems

Key Formula: $(\text{concentration}_1)(\text{volume}_1) + (\text{concentration}_2)(\text{volume}_2) = (\text{concentration}_{\text{final}})(\text{volume}_{\text{final}})$

Two Equations Needed:

1. Total volume equation

2. Total substance (acid, alcohol, etc.) equation

Motion Problems

Key Formula: $\text{distance} = \text{rate} \times \text{time} \quad (d = rt)$

Two Equations Needed:

1. Total distance equation (add if opposite directions, subtract if same direction)

2. Distance equations for each object: $d_1 = r_1 t$ and $d_2 = r_2 t$

Break-Even Problems

Key Formula: $\text{Total Cost} = \text{Total Revenue}$

Two Equations Needed:

1. Cost: $C = \text{Fixed Costs} + (\text{variable cost per item})(x)$

2. Revenue: $R = (\text{price per item})(x)$

3. Set them equal: $C = R$

Homework 7.1: Systems Word Problems

Instructions

: For each problem,

1. Define your variables with units

2. Write your system of equations

3. Solve using any method

4. Check your answer for reasonableness

Homework Problem 1: Mixture Problem

A pharmacist needs to prepare 200 mL of a 15 mg/mL alcohol solution.

She has solution "A", which is a 10 mg/mL solution.

She has solution "B", which is a 25 mg/mL solution.

How much of each should she mix?

Variables:
A = number of mL of 10 mg/mL solution
B = number of mL of 25 mg/mL solution

Understanding Check:

Total alcohol needed = 200 mL $\times$ 15 mg/mL = _____ mg

System:

1. Equation for total mL: $A + B = 200$

2. Expression for mg from solution A: $A \text{ mL} \cdot 10 \frac{mg}{mL} = 10A$

3. Expression for mg from solution B: $B \text{ mL} \cdot 25 \frac{mg}{mL} = 25B$

4. Equation for total mg: $10A + 25B = 3000$

$$\begin{cases} A = 200 - B \\ 2A + 5B = 600 \end{cases}$$

$$2(200 - B) + 5B = 600$$

$$400 - 2B + 5B = 600$$

$$\underline{-400} \qquad\qquad \underline{-400}$$

$$3B = 200$$

$$B = \frac{200}{3}$$

$$A = 200 - \frac{200}{3} = \frac{600}{3} - \frac{200}{3} = \frac{400}{3}$$

Solution:

$A = \dfrac{\frac{400}{3}}{}$ mL

$B = \dfrac{\frac{200}{3}}{}$ mL

Check Your Answer:

Does $A + B = 200$? $\dfrac{400}{3} + \dfrac{200}{3} = \dfrac{600}{3} = 200$ ✓

Does the total alcohol equal 3000 mg? $10\left(\dfrac{400}{3}\right) + 25\left(\dfrac{200}{3}\right) = \dfrac{4000}{3} + \dfrac{5000}{3} = \dfrac{9000}{3} = 3000$ ✓

Homework Problem 2: Motion Problem

Two trains leave the same station at the same time.

They travel in opposite directions.

Train A travels at 75 mph.

Train B travels at 65 mph.

After how many hours will they be 420 miles apart?

Variables:
t = time in hours
A = distance traveled by train A in miles
B = distance traveled by train B in miles

Understanding Check:

Combined speed = ___75___ + ___65___ = ___140___ mph

System:

1. Equation for total distance: $A + B = 420$

2. Equation for train A distance: $75 \frac{miles}{hr} \cdot t\, hr = 75t = A$

3. Equation for train B distance: $65 \frac{miles}{hr} \cdot t\, hr = 65t = B$

Solution:

t = ___3___ hours

$$A + B = 420$$
$$75t + 65t = 420$$
$$140t = 420$$
$$t = \frac{420}{140} = \frac{10 \cdot 42}{10 \cdot 14} = \frac{42}{14} = \frac{2 \cdot 21}{2 \cdot 7} = \frac{21}{7} = \frac{7 \cdot 3}{7} = 3$$

Check Your Answer:

Train A distance: $75(3) = 225$ miles

Train B distance: $65(3) = 195$ miles

Total: $225 + 195 = 420$ ✓ miles (should equal 420)

Homework Problem 3: Break-Even Problem

A bakery makes cakes.

Fixed costs are $1500 per month,
and each cake costs $20 to produce.

Each cake sells for $45.

How many cakes must be sold to break even?

Variables:
x = number of cakes
C = total cost in dollars
R = total revenue in dollars

Understanding Check:

Profit per cake = $\underline{\quad 45 \quad}$ - $\underline{\quad 20 \quad}$ = $\underline{\quad \$25 \quad}$

System:

1. Equation for total cost: $C = \underline{\quad 20x + 1500 \quad}$

2. Equation for total revenue: $R = \underline{\quad 45x \quad}$

$$45x = 20x + 1500$$
$$-20x \qquad -20x$$
$$25x = 1500$$
$$x = \frac{1500}{25} = 60$$

3. Break even means: $\underline{\quad C = R \quad}$

Solution:

$x = \underline{\quad 60 \quad}$ cakes

$$25\overline{)1500} \quad \begin{array}{r} 60 \\ \end{array}$$
$$-150$$
$$00$$
$$-0$$
$$0$$

Check Your Answer:

Total cost: $\underline{\quad 20(60) + 1500 = 1200 + 1500 = 2700 \ \$ \quad}$

Total revenue: $\underline{\quad 45(60) = 2700 \ \$ \quad}$

Are they equal? $\underline{\quad \text{Yes} \quad}$

Homework Problem 4: Mixture Problem Challenge

A coffee shop mixes two types of beans.

Premium beans cost $12 per pound.
Regular beans cost $8 per pound.

The shop wants to make 50 pounds of a blend.

The blend should cost $9.60 per pound.

How many pounds of each type should be used?

Variables:
p = pounds of premium beans
r = pounds of regular beans

Understanding Check:
Total cost of blend = 50 pounds × $9.60 per pound = $\underline{\ \$480\ }$
This problem is like a mixture problem, but with **cost** instead of concentration!

System:
1. Equation for total pounds: $\underline{p + r = 50}$

2. Expression for cost from premium beans: $\underline{12\frac{\$}{lb} \cdot p\ lb = 12p}$

3. Expression for cost from regular beans: $\underline{8\frac{\$}{lb} \cdot r\ lb = 8r}$

4. Equation for total cost: $\underline{12p + 8r = 480}$

Solution:

$p = \underline{\ \ 20\ \ }$ pounds

$r = \underline{\ \ 30\ \ }$ pounds

$$\begin{cases} r = 50 - p \\ 12p + 8(50-p) = 480 \end{cases}$$

$$12p + 400 - 8p = 480$$
$$ -400 -400$$
$$4p = 80$$
$$p = \frac{80}{4} = 20$$
$$r = 50 - 20 = 30$$

Check Your Answer:
Does $p + r = 50$? $\underline{20 + 30 = 50\ \checkmark}$

Does the total cost equal 480? $\underline{20(12) + 30(8) = 240 + 240 = 480\ \checkmark}$

Makes sense: Should use more regular beans (cheaper) than premium beans? $\underline{\text{Yes, } 9.6\frac{\$}{lb} \text{ closer to } 8\frac{\$}{lb} \text{ than } 12\frac{\$}{lb}}$

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