Week 7: Systems Applications & Inequalities

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Session 7.2
Systems of Inequalities

Quick Reference: Graphing Inequalities

Single Linear Inequality

Steps to Graph:

1. Write in slope-intercept form if possible: $y = mx + b$
2. Graph the boundary line:
   - Use **solid line** for $\leq$ or $\geq$ (boundary included)
   - Use **dashed line** for $<$ or $>$ (boundary not included)
3. Choose a test point (often $(0, 0)$ if not on the line)
4. Shade the region where the inequality is true

System of Linear Inequalities

Steps to Graph:

1. Graph each inequality on the same coordinate plane
2. The **solution region** is where ALL shaded areas overlap
3. The solution region is called the **feasible region**
4. Any point in this region satisfies all inequalities

Key Vocabulary:

- **Feasible region**: the solution set (overlapping shaded area)
- **Vertex**: corner point where boundary lines intersect
- **Bounded**: the feasible region is enclosed (finite area)
- **Unbounded**: the feasible region extends infinitely

Homework 7.2: Systems of Inequalities

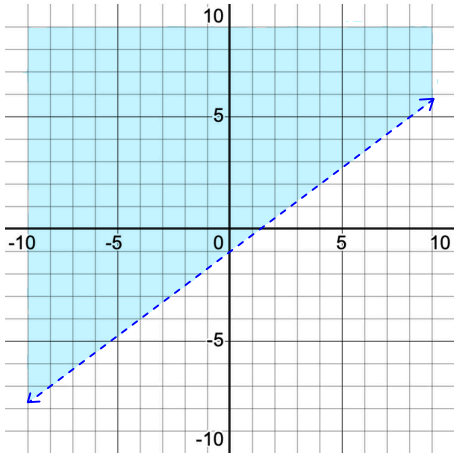
Instructions: For each problem,

1. Graph each inequality carefully (solid vs. dashed lines)
2. Identify and shade the feasible region
3. Verify with a test point
4. Answer interpretation questions

Homework Problem 1: Basic Inequalities

Graph the inequality:

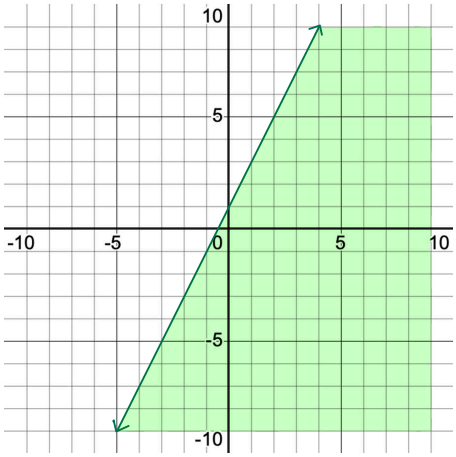
$y > \frac{3}{4}x - 1$



- Slope: $\frac{3}{4}$
- y-intercept: -1
- Solid or dashed line? dashed
- Shade above or below the line? above

Graph the inequality:

$y \leq -2x + 1$

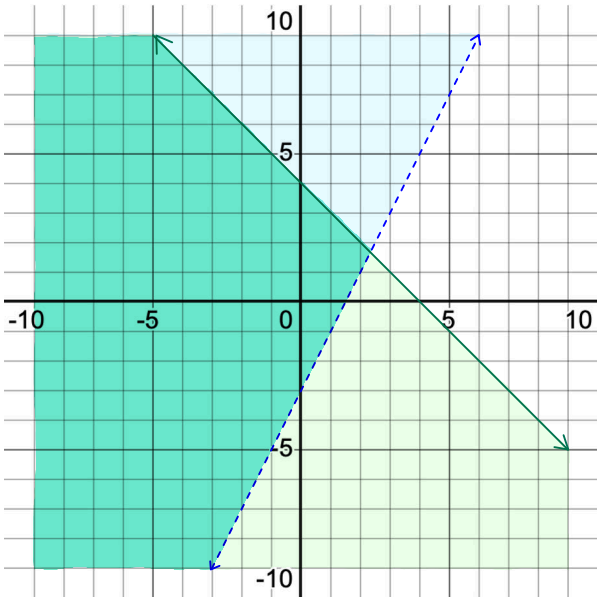


- Slope: -2
- y-intercept: 1
- Solid or dashed line? solid
- Shade above or below the line? below

Homework Problem 2: Basic System

Graph the system of inequalities:

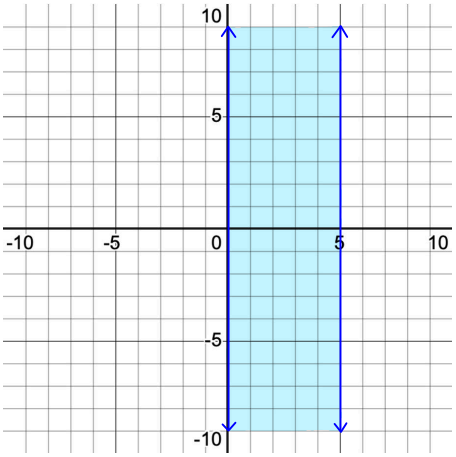
$y > 2x - 3$ (blue)
$y \leq -x + 4$ (green)



Homework Problem 3: Bounded Inequalities

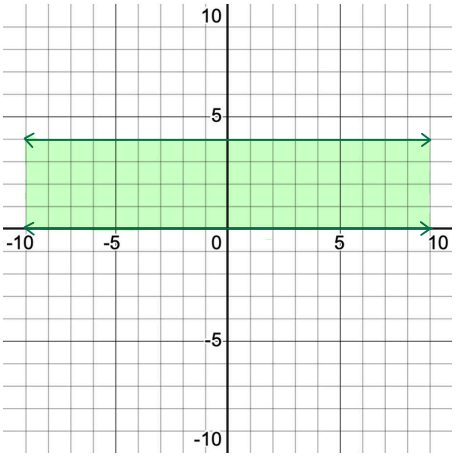
Graph the compound inequality:

$0 \leq x \leq 5$



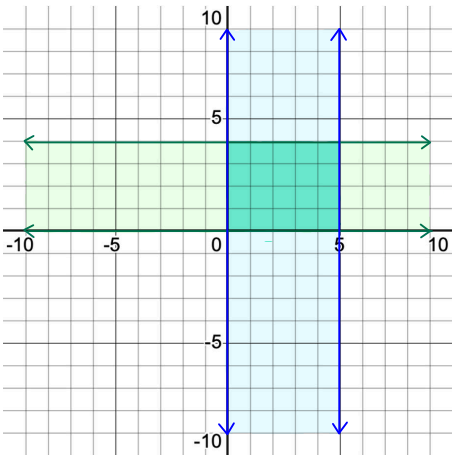
Graph the compound inequality:

$0 \leq y \leq 4$



Graph the system of inequalities:

$0 \leq x \leq 5$ (blue)
$0 \leq y \leq 4$ (green)



Understanding Check:

What shape will the feasible region be? rectangle

Is it bounded or unbounded? bounded

Vertices of Feasible Region:

List all four corner points:

(0,0), (5,0), (0,4), (5,4)

Check:
Pick a test point in the feasible region and show that it satisfies both inequalities.

(3, 2)

$0 \leq 3 \leq 5$ ✓

$0 \leq 2 \leq 4$ ✓

Pick a test point NOT in the feasible region
and show that it does NOT satisfy at least one of the inequalities.

(3, -2)

$0 \leq 3 \leq 5$ ✓

$0 \leq -2 \leq 4$ ✗

Homework Problem 4: Application - Manufacturing

A factory makes tables and chairs.

Each table requires 4 hours of labor.
Each chair requires 2 hours of labor.

The factory has <u>at most</u> 20 hours of labor per day.

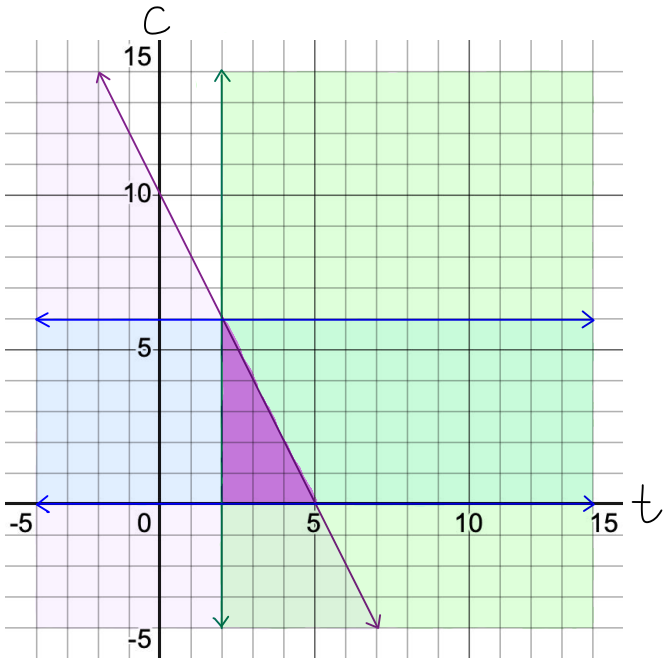
They must make <u>at least</u> 2 tables per day.
They can make at most 6 chairs per day.

Write and graph a system of inequalities.

Variables:
t = number of tables
c = number of chairs

System of Inequalities:

1. Labor constraint: $4t+2c\leq 20 \Rightarrow 2c\leq 20-4t \Rightarrow c\leq -2t+10$
2. Minimum tables: $t\geq 2$
3. Maximum chairs: $c\leq 6$
4. Non-negative: $t \geq 0$ and $c \geq 0$ (already implied by context)



$\begin{cases} c\leq -2t+10 \text{ (purple)} \\ t\geq 2 \text{ (green)} \\ 0\leq c\leq 6 \text{ (blue)} \end{cases}$

Interpretation Questions:

Can they make 3 tables and 4 chairs?

Yes

Why or why not?

$$\begin{cases} 4t+2c \leq 20 \\ t \geq 2 \\ 0 \leq c \leq 6 \end{cases}$$

$$\begin{cases} 4(3)+2(4) \leq 20 \Rightarrow 12+8 \leq 20 \Rightarrow 20 \leq 20 \checkmark \\ 3 \geq 2 \checkmark \\ 0 \leq 4 \leq 6 \checkmark \end{cases}$$

because it satisfies all 3 inequalities.

Can they make 4 tables and 2 chairs?

Yes

Why or why not?

$$\begin{cases} 4t+2c \leq 20 \\ t \geq 2 \\ 0 \leq c \leq 6 \end{cases}$$

$$\begin{cases} 4(4)+2(2) \leq 20 \Rightarrow 16+4 \leq 20 \Rightarrow 20 \leq 20 \checkmark \\ 4 \geq 2 \checkmark \\ 0 \leq 2 \leq 6 \checkmark \end{cases}$$

because it satisfies all 3 inequalities.

What is the maximum number of chairs if they make exactly 2 tables?

$t=2$

$4t+2c \leq 20$

$4(2)+2c \leq 20$

$8+2c \leq 20$

$-8 \quad -8$

$2c \leq 12$

$c \leq 6$ and $0 \leq c \leq 6$

6 chairs