Week 7: Systems Applications & Inequalities

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Session 7.3
Linear Modeling & Fit-by-Eye

Quick Reference: Linear Modeling

What is a Line of Best Fit?

A **line of best fit** (or trend line) is a straight line that best represents the data on a scatter plot.

Key Ideas:

- Not all points will be exactly on the line
- The line shows the general trend or pattern
- We use the line to make predictions
- The line minimizes the distance from all the data points

Steps to Create a Line of Best Fit

1. Plot the Data

- Create a scatter plot of all data points
- Label axes with variable names and units
- Choose an appropriate scale

2. Draw the Line

- Use a ruler or straightedge
- Try to balance points above and below the line
- The line should follow the general trend
- Extend the line across the entire graph

3. Find the Equation

- Pick two points ON your line (they don't have to be data points)
- Calculate the slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Find the y-intercept using point-slope form
- Write in slope-intercept form: $y = mx + b$

Residuals

A **residual** measures how far each data point is from the line of best fit.

Formula:

$$\text{Residual} = \text{Actual value} - \text{Predicted value}$$

Interpretation:

- **Positive residual**: data point is above the line
- **Negative residual**: data point is below the line
- **Zero residual**: data point is exactly on the line
- Smaller residuals mean better fit

Interpreting Slope and y-Intercept

Slope (m):

- Rate of change
- How much y changes for each 1-unit increase in x
- Include units: "For every [unit of x], [variable y] changes by [slope] [unit of y]"

y-Intercept (b):

- Starting value when $x = 0$
- May or may not make sense in context
- Include units: "When [variable x] is 0, [variable y] is [y-intercept] [unit of y]"

Worksheet 7.3: Linear Modeling

Instructions

:

For each problem,

1. Plot the data points on graph paper
2. Draw a line of best fit by eye
3. Find the equation using two points on your line
4. Calculate residuals for specified data points
5. Interpret the slope and y-intercept in context

Worksheet Problem 1: Temperature and Hot Chocolate Sales

A café tracks hot chocolate sales at different temperatures.

Temperature (°F)	Hot Chocolates Sold
30	45
35	42
40	38
45	35
50	30
55	28

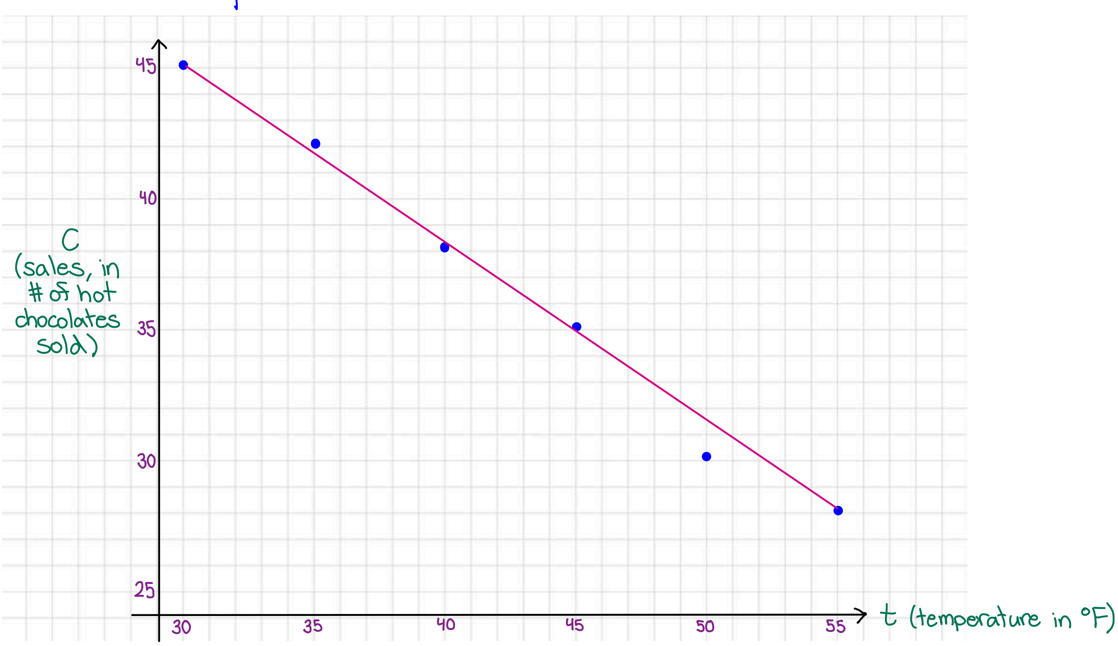
Part A

Choose your scale:

- x-axis: Temperature from 30 °F to 55 °F (include units)
- y-axis: Sales from 28 hc to 45 hc (include units)

Add your scale and label axes. Scale is in purple
axes labels are in green

Plot the points. points are in blue



Part B

Draw a line of best fit (using a different color). line of best fit is in pink

Part C

Choose two points ON your line and find the equation.

Point 1: (30, 45)

Point 2: (55, 28)

Calculate the slope:

$m=\frac{28-45}{55-30}=\frac{-17}{25}$ $\quad -\frac{17}{25}=-\frac{17\cdot 4}{25\cdot 4}=-\frac{68}{100}=-0.68$

Slope $=-0.68\ \frac{\text{hc sold}}{°F}$

Find the y-intercept using point-slope form:

$45=-\frac{17}{25}(30)+b$ $\quad -\frac{17}{25}(30)=-\frac{17\cdot 30}{25}=-\frac{17\cdot 5\cdot 6}{5\cdot 5}=-\frac{17\cdot 6}{5}=-\frac{102}{5}$

$45=-\frac{102}{5}+b$

$b=45+\frac{102}{5}=\frac{225}{5}+\frac{102}{5}=\frac{225+102}{5}=\frac{327}{5}$ $\quad \frac{327}{5}=\frac{327\cdot 2}{5\cdot 2}=\frac{654}{10}=65.4$

y-intercept $=65.4$ hc sold

Equation: $C=-\frac{17}{25}t+65.4$

Part D

Calculate residuals for $x=30$, $x=40$, and $x=50$:

For $x=30$:

- Actual value: 45
- Predicted value (from equation): $-0.68(30)+65.4=65.4-(6.8\cdot 3)=65.4-20.4=45$
- Residual = Actual − Predicted = $45-45=0$

$$\begin{array}{r} 65.4 \\ -\,20.4 \\ \hline 45.0 \end{array}$$

For $x=40$:

- Actual value: 38
- Predicted value: $-0.68(40)+65.4=65.4-(6.8\cdot 4)=65.4-27.2=38.2$
- Residual = $38-38.2=-0.2$

$$\begin{array}{r} {}^{5}\cancel{6}{}^{1}5.4 \\ -\,27.2 \\ \hline 38.2 \end{array}$$

For $x=50$:

- Actual value: 30
- Predicted value: $-0.68(50)+65.4=65.4-(6.8\cdot 5)=65.4-34=31.4$
- Residual = $30-31.4=-1.4$

$$\begin{array}{r} 65.4 \\ -\,34.0 \\ \hline 31.4 \end{array}$$

Part E

Interpret the slope in context:

For every 25 °F (include units) increase in temperature,

the number of hot chocolates sold

decreases (increases/decreases) by about 17 hc sold (include units).

Part F

Interpret the y-intercept in context:

When the temperature is 0°F, the model predicts 65.4 hot chocolates sold.

* if we decide we cannot sell part of a hc,
then this should be rounded to: ~65 hc sold

this means approximately

Part G

Does the y-intercept make sense in this context? Why or why not?

Yes,

because you can sell ~65 hot chocolates (in the range)

and the temperature can be 0 °F (in the domain).

Part H

Use your model to predict sales at 60°F:

$C = -0.68t + 65.4$

$C = -0.68(60) + 65.4 = 65.4 - (6.8 \cdot 6) = 65.4 - 40.8 = 24.6$ hc sold

$$\begin{array}{r} 65.4 \\ -\ 40.8 \\ \hline 24.6 \end{array}$$

* if we decide we cannot sell part of a hc,
then this should be rounded to: ~25 hc sold

this means approximately

Is this prediction reasonable? Explain.

Yes,

because you can sell ~25 hot chocolates (in the range)

Worksheet Problem 2: Study Time and Quiz Score

A teacher tracks student study time and quiz scores.

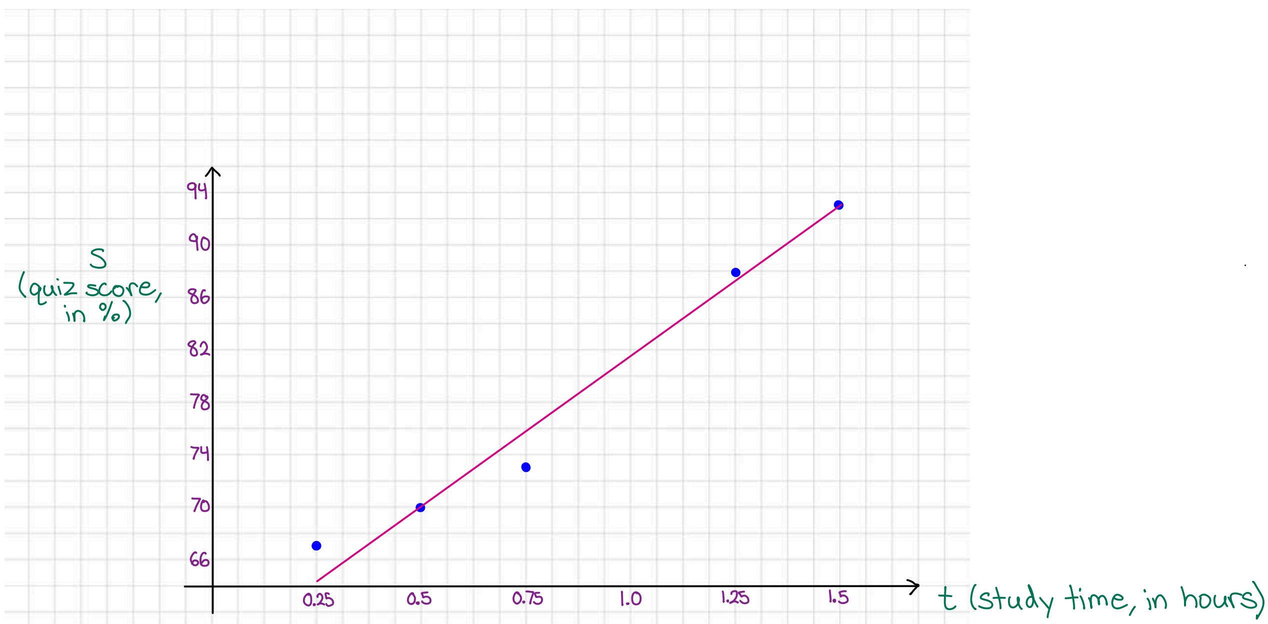
Study Time (hours)	Quiz Score (%)
0.25	67
0.5	70
0.75	73
1.0	79
1.25	88
1.5	93

S t

Part A

Add a scale and label axes. scale is in purple
axes labels are in green

Plot the points. points are in blue



Part B

Draw a line of best fit (using a different color). line of best fit is in pink

Part C

Find the equation of the line of best fit using two points on your line.

(0.5, 70) and (1.5, 93)

$m = \frac{93-70}{1.5-0.5} = \frac{23}{1} = 23$ $m = 23 \frac{\%}{\text{hrs study}}$

$70 = 23(0.5) + b$

$70 = 11.5 + b$

$b = 70 - 11.5 = 58.5$ $b = 58.5\ \%$

$$\begin{array}{r} 70.0 \\ -11.5 \\ \hline 58.5 \end{array}$$

$S = 23t + 58.5$

Part D

Calculate residuals for $x = 0.5$, $x = 1.0$, and $x = 1.5$:

$t = 0.5$

$S = 23(0.5) + 58.5 = 11.5 + 58.5 = 70$

residual $= 70 - 70 = 0$

$t = 1.0$

$S = 23(1.0) + 58.5 = 23 + 58.5 = 81.5$

residual $= 79 - 81.5 = -2.5$

$t = 1.5$

$S = 23(1.5) + 58.5 = 34.5 + 58.5 = 93$

residual $= 93 - 93 = 0$

Part E

Use your model to predict the quiz score for someone who studies 4 hours:

$S = 23t + 58.5$

$S = 23(4) + 58.5 = 92 + 58.5 = 150.5\ \%$

Part F

Is this prediction reasonable? Why or why not?

(Hint: Can quiz scores go above 100%?)

No,

because quiz scores cannot (usually) go above 100% (not in range).

Worksheet Problem 3: Car Value Over Time

A used car dealer tracks how a car's value changes with age.

Car Age (years)	Value ($1000s)
1	23
2	20
3	17
4	15
5	12
6	9

V t

Part A

Add a scale and label axes. scale is in purple
axes labels are in green

Plot the points. points are in blue

Part B

Draw a line of best fit (using a different color). line of best fit is in pink

Part C

Find the equation of the line of best fit.

$m = \frac{9-23}{6-1} = \frac{-14}{5}$ $\quad -\frac{14}{5} = -\frac{14\cdot 2}{5\cdot 2} = -\frac{28}{10} = -2.8$ $\quad$ slope $= -2.8 \frac{\$1000}{\text{year}}$

$23 = -2.8(1) + b$

$23 = -2.8 + b$

$b = 23 + 2.8 = 25.8$ $\quad$ y-intercept $= 25.8$ $1000 value

$V = -2.8t + 25.8$

Part D

What does the y-intercept represent?

Value of car when new (0 years old)

Does this make sense? (What was the car worth when new?)

Yes,

a new car could be worth 25.8 thousand $.

Part E

According to your model, when will the car be worth $0?

$0 = -2.8t + 25.8$

$+2.8t \quad +2.8t$

$2.8t = 25.8$

$t = \frac{25.8}{2.8} = \frac{25.8\cdot 10}{2.8\cdot 10} = \frac{258}{28} = \frac{129}{14} = 9\frac{3}{14}$

$$\begin{array}{r} 9 \\ 14\overline{)129} \\ -126 \\ \hline 3 \end{array}$$

x-intercept $= 9\frac{3}{14}$ years

Part F

Is this prediction realistic? Why or why not?

No,

because a car does not become worth $0

when it is between 9 and 10 years old.

Worksheet Problem 4: Plant Growth

A biology student measures plant height over time.

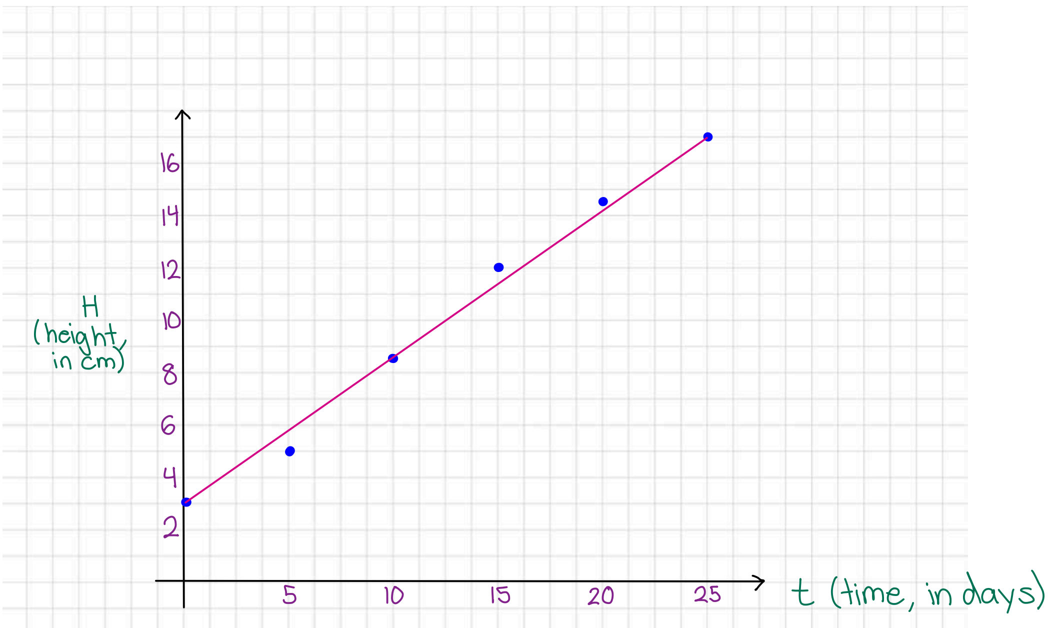
Days	Height (cm)
0	3
5	5
10	9
15	12
20	15
25	17
t	H

Part A

Add a scale and label axes. Scale is in purple
axes labels are in green

Plot the points. points are in blue

Draw a line of best fit (using a different color). line of best fit is in pink



Part B

Find the equation of the line of best fit.

$m = \frac{17-3}{25-0} = \frac{14}{25}$

$\frac{14}{25} = \frac{14 \cdot 4}{25 \cdot 4} = \frac{56}{100} = 0.56$

slope $= 0.56 \frac{\text{cm}}{\text{day}}$

$3 = 0.56(0) + b$

$b = 3$

y-intercept = 3 days

$H = 0.56t + 3$

Part C

Interpret the y-intercept:

At 0 days, the plant is 3 cm tall.

Part D

Calculate the residual for day 10:

$H = 0.56t + 3$

$H = 0.56(10) + 3 = 5.6 + 3 = 8.6$

actual $= 9$

residual $= 9 - 8.6 = 0.4$

Part E

Predict the height after 30 days:

$H = 0.56t + 3$

$H = 0.56(30) + 3 = 5.6(3) + 3 = 16.8 + 3 = 19.8$

19.8 cm

Part F

If the plant can only grow to a maximum of 25 cm, when will it reach this height according to your model?

$H = 0.56t + 3$

$25 = 0.56t + 3$

$-3 \qquad -3$

$22 = 0.56t$

$22 = \frac{14}{25}t$

$\cdot\frac{25}{14} \quad \cdot\frac{25}{14}$

$t = 22 \cdot \frac{25}{14} = \frac{\cancel{2} \cdot 11 \cdot 25}{\cancel{2} \cdot 7} = \frac{11 \cdot 25}{7} = \frac{275}{7} = 39\frac{2}{7}$

$39\frac{2}{7}$ days

$$\begin{array}{r} 39 \\ 7\overline{)275} \\ -21 \\ \hline 65 \\ -63 \\ \hline 2 \end{array}$$