Math 2BL Project Proposal

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1 Project Name

Adjusted Learning Matrix Model

2 Project Description

The project will create a graphical user interface (GUI) designed for an instructor (tutor or teacher) who is looking to create an adjustable learning plan for an individual student. This program is focused on providing equitable, accessible, individualized learning plans for students with learning differences, although this program is made in the Universal Design for Learning (UDL) style to benefit all students. The program will use linear algebra and matrices, with an instructor input matrix, a transformation matrix, and an adapted instruction output matrix to provide guidance to the instructor.

2.1 Overview of Matrices

$$y = Ax$$

Input Matrix:

$$x = \begin{bmatrix} \frac{h-5}{2} \\ \frac{c-5}{2} \\ \frac{p-5}{2} \\ \frac{m-5}{2} \end{bmatrix}$$

where:

- h = independence/help provided (0 = no independence, 10 = full independence)
- c = observed student confidence (0 = no confidence, 10 = full confidence)
- p = persistence (0 = gave up immediately, 10 = never gave up)
- m = accuracy & mistake severity/type (0 = full conceptual error, 10 = no errors)

Transformation Matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \text{*need to determine matrix values}$$

Output Matrix:

$$y = \begin{bmatrix} d \\ s \\ t \end{bmatrix}$$

where:

- d = adjusted difficulty level, $[-1, 1]$ ($-$ = decrease, 0 = same, $+$ = increase)
- s = adjusted support level, $[-1, 1]$ ($-$ = increase, 0 = same, $+$ = decrease)
- t = adjusted time/pacing, $[-1, 1]$ ($-$ = slow down, 0 = same, $+$ = speed up)

2.2 Instructor Input Matrix

The instructor will be given prompts to enter in values based on easily measurable observations on how the student did on that assignment. For example, on the user (instructor) end, this could look like:

- Enter student's independence/amount of help provided (0=no independence, 10=full independence):
- Enter observed student confidence (0=no confidence, 10=full confidence):
- Enter student's persistence (0=gave up immediately, 10=never gave up):
- Enter student's accuracy & mistake severity/type* (0=full conceptual error, 10=no errors):

On the programming end, this corresponds to a matrix x:

$$x = \begin{bmatrix} \frac{h-5}{2} \\ \frac{c-5}{2} \\ \frac{p-5}{2} \\ \frac{m-5}{2} \end{bmatrix}$$

where the variables h, c, p, and m range from 0 to 10 (inclusive) and correspond to the values entered in by the user (instructor), where h = independence/help, c = confidence, p = persistence, and m = accuracy & mistake severity/type. These values are then normalized so that the values in the matrix x range from -1 to 1 (inclusive).

Note: Need to refine the "mistake severity/type" values to account for things like arithmetic errors (minor), forgetting units (minor), incorrect units (moderate), minor conceptual error (moderate), full conceptual error (severe).

2.3 Adapted Instruction Output Matrix

Once the instructor has entered in the values for matrix x, the program will output recommendations on how the difficulty level, support level, and pace of material should be adjusted. For example, on the user (instructor) end, this could look like:

- The difficulty level should: increase/decrease slightly/moderately/significantly OR stay the same
- The support provided should: increase/decrease slightly/moderately/significantly OR stay the same
- The pacing of the material should: speed up/slow down slightly/moderately/significantly OR stay the same

On the programming end, this corresponds to a matrix y:

$$y = \begin{bmatrix} d \\ s \\ t \end{bmatrix}$$

where d, s, t range from -1 to 1, and guide the recommendations given to the user (instructor).

2.3.1 Pseudocode for Output Matrix

```
Print "The difficulty level should "
If d = 0, print "stay the same."
Else, if d < 0, print "decrease "
    If d <= -0.7, print "significantly."
    Else, if d < -0.3, print "moderately."
    Else, print "slightly."
Else, if d > 0, print "increase "
    If d >= 0.7, print "significantly."
    Else, if d > 0.3, print "moderately."
    Else, print "slightly."
Else, error

Print "The support provided should "
If s = 0, print "stay the same."
Else, if s < 0, print "increase "
    If s <= -0.7, print "significantly."
    Else, if s < -0.3, print "moderately."
    Else, print "slightly."
Else, if s > 0, print "decrease "
    If s >= 0.7, print "significantly."
    Else, if s > 0.3, print "moderately."
    Else, print "slightly."
Else, error

Print "The pacing should "
If t = 0, print "stay the same."
Else, if t < 0, print "slow down "
    If t <= -0.7, print "significantly."
    Else, if t < -0.3, print "moderately."
    Else, print "slightly."
Else, if t > 0, print "speed up "
    If t >= 0.7, print "significantly."
    Else, if t > 0.3, print "moderately."
    Else, print "slightly."
Else, error
```

2.4 Transformation Matrix

This matrix is only visible on the programmer's end. This matrix will include values that represent how each input from the instructor affects the output for how the instruction should be adapted.

On the programming end, this corresponds to a matrix A:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \text{**(need to determine what these values are)}$$

where values in the matrix range from -1 to 1.

For example, if a lot of help was given by the instructor meaning the student had no independence (h is close to 0), then this would correspond to decreasing the difficulty (negative d), increasing the support provided (negative s), and/or slowing down the pace (negative t).

A more specific example is if the instructor observes that the student's confidence is low (c close to 0) even though the student is getting the correct answer ($m = 10$), then this would correspond to increasing the support provided (negative s), but keeping the same difficulty ($d = 0$) and pace ($t = 0$), in order to build up the student's confidence.

Note: Need to determine the values in the matrix. Also, need to think about if output matrix y represents adjusted values (change) or an absolute value.

2.4.1 Matrix Equations

$$\begin{bmatrix} d \\ s \\ t \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \begin{bmatrix} \frac{h-5}{2} \\ \frac{c-5}{2} \\ \frac{p-5}{2} \\ \frac{m-5}{2} \end{bmatrix} = \frac{1}{5} \begin{bmatrix} a_{11}(h-5) + a_{12}(c-5) + a_{13}(p-5) + a_{14}(m-5) \\ a_{21}(h-5) + a_{22}(c-5) + a_{23}(p-5) + a_{24}(m-5) \\ a_{31}(h-5) + a_{32}(c-5) + a_{33}(p-5) + a_{34}(m-5) \end{bmatrix}$$

Expanding:

$$\begin{cases} d = a_{11}(h-5) + a_{12}(c-5) + a_{13}(p-5) + a_{14}(m-5) \\ s = a_{21}(h-5) + a_{22}(c-5) + a_{23}(p-5) + a_{24}(m-5) \\ t = a_{31}(h-5) + a_{32}(c-5) + a_{33}(p-5) + a_{34}(m-5) \end{cases}$$

2.4.2 Test to Determine Starting Values for Matrix A

Given:

- Full independence: $h = 10$
- Full confidence: $c = 10$
- Never gave up: $p = 10$
- No errors: $m = 10$

Then:

- Difficulty increases significantly: $d = 1$
- Support provided decreases significantly: $s = 1$
- Pacing speeds up significantly: $t = 1$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} + a_{12} + a_{13} + a_{14} \\ a_{21} + a_{22} + a_{23} + a_{24} \\ a_{31} + a_{32} + a_{33} + a_{34} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

This yields:

$$\begin{cases} a_{11} + a_{12} + a_{13} + a_{14} = 1 \\ a_{21} + a_{22} + a_{23} + a_{24} = 1 \\ a_{31} + a_{32} + a_{33} + a_{34} = 1 \end{cases}$$

Since the range of possible values for h, c, p, and m is $[0, 10]$, the range of possible values for each entry in matrix x is $[-1, 1]$.

Since the range of possible values for d, s, and t is $[-1, 1]$, the range of possible values for each entry in matrix y is $[-1, 1]$.

Therefore, if each entry in matrix A has the same weight, the approximate values in matrix A are:

$$A = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix}$$

2.4.3 Test to Refine Values for Matrix A

To refine these values, I will now consider how each output is affected by each input. All have a positive correlation, and I will rate the correlation as strong, moderate, or weak. I will then adjust the corresponding matrix values, keeping the sum of each matrix row as 1.

For difficulty (d):

- independence (h) – strong positive correlation (increase a_{11} to 0.35)
- confidence (c) – moderate positive correlation (decrease a_{12} to 0.15)
- perseverance (p) – moderate positive correlation (decrease a_{13} to 0.15)
- accuracy (m) – strong positive correlation (increase a_{14} to 0.35)

For support (s):

- independence (h) – strong positive correlation (increase a_{21} to 0.30)
- confidence (c) – strong positive correlation (increase a_{22} to 0.30)
- perseverance (p) – strong positive correlation (increase a_{23} to 0.30)
- accuracy (m) – moderate positive correlation (decrease a_{24} to 0.10)

For pacing (t):

- independence (h) – moderate positive correlation (increase a_{31} to 0.30)
- confidence (c) – weak positive correlation (decrease a_{32} to 0.15)
- perseverance (p) – weak positive correlation (decrease a_{33} to 0.15)
- accuracy (m) – strong positive correlation (increase a_{34} to 0.40)

Updated adjusted matrix A:

$$A = \begin{bmatrix} 0.35 & 0.15 & 0.15 & 0.35 \\ 0.30 & 0.30 & 0.30 & 0.10 \\ 0.30 & 0.15 & 0.15 & 0.40 \end{bmatrix}$$