

Solution 1a:

Let E – Event that sum of the 2 numbers of dice landings is 'i'

E1 – Event that dice lands on '6'.

$P(E1|E)$ is the required probability.

Case -1:

We know by conditional probability that, $p(E|E1) = \frac{p(E \cap E1)}{p(E)}$

If 'i' lies in $[2,6]$, then $P(E \cap E1) = 0$

$\Rightarrow P(E|E1) = 0$

Case – 2:

Here, 'i' belongs to $[7,12]$.

$$P(E \cap E1) = (1/6) * (1/6) = 1/36$$

Note, that if dice - 1 lands on 6, then dice - 2 must land on 'i-6', for the sum to be 'i'. Therefore, the second dice only has one choice.

Notation:

$[(x,y)]$ is a tuple means, dice – 1 landed on x, and dice – 2 landed on y]

Note that $P(E)$, depends on the value of 'i'.

a) i=7:

All possible scenarios are (1,6), (6,1), (2,5), (5, 2), (3,4), (4,3).

$\Rightarrow p(E) = 6/36$

$\Rightarrow p(E1|E) = 1/6$

b) i=8:

All possible scenarios are (2,6), (6,2), (3,5), (5,3), (4,4).

$\Rightarrow p(E) = 5/36$.

$\Rightarrow p(E1|E) = 1/5$.

c) i=9:

All possible scenarios are (3,6), (6,3), (5,4), (4,5)

$\Rightarrow p(E) = 4/36$.

$\Rightarrow p(E1|E) = 1/4$.

d) i=10:

All possible scenarios are (4,6), (6,4), (5,5).

$\Rightarrow p(E) = 3/36$.

$\Rightarrow p(E1|E) = 1/3$.

e) i=11:

All possible scenarios are (6,5), (5,6).

$\Rightarrow p(E) = 2/36$.

$\Rightarrow p(E1|E) = 1/2$.

f) i=12:

The only case possible is (6,6).

$\Rightarrow p(E) = 1/36$.

$\Rightarrow p(E1|E) = 1$.

Solution 1b:

Let E1 – randomly chosen bulb initially lights.

Let E2 – bulb chosen will still be working after 1 week.

By conditional probability formula, we know that,

$$p(E2|E1) = p(E1 \cap E2) / p(E1)$$

$$\begin{aligned} p(E1) &= (\text{No. of bulbs in good condition} + \text{Number of partially defective bulbs}) / \text{total number of bulbs.} \\ &= (5+10)/25. \\ &= 3/5. \end{aligned}$$

$$\begin{aligned} p(E1 \cap E2) &= \text{Number of good bulbs} / \text{Total number of bulbs.} \\ &= 5/25 = 1/5. \end{aligned}$$

=> $p(E2|E1) = 1/3$, which is the desired result.

Solution 2:

Let E – letter taken at random is a vowel.

Let A – man is an englishman.

Let B – man is an american.

$$p(A) = 0.4$$

$$p(B) = 0.6$$

$$\begin{aligned} p(E) &= p(E \cap A) + p(E \cap B) \\ &= p(A) * p(E|A) + p(B) * p(E|B) \\ &= 0.4 * 0.5 + 0.6 * 0.4 \\ &= 0.44 \end{aligned}$$

$$\begin{aligned} p(E \cap A) &= 0.4 * 0.5 \\ &= 0.2 \end{aligned}$$

$$\begin{aligned} \Rightarrow p(A|E) &= 0.2/0.44 \\ &= (5/11). \end{aligned}$$