# Assignment 1: Probability Theory

L645/B659, Sandra Kuebler

DUE: at beginning of class on Thursday, September 11

1. **Conditional Probability**

   (a) If two fair dice are rolled, what is the conditional probability that the first one lands on 6 given that the sum of the dice is $i$? Compute for all values of $i$ between 2 and 12.

   (b) A bin contains 25 lightbulbs, 5 of which are in good condition and will function for at least 30 days, 10 of which are partially defective and will fail in their second day of use, and 10 of which are totally defective and will not light up. Given that a randomly chosen bulb initially lights what is the probability that it will still be working after one week?

   **10 pts.**

2. **Bayes Theorem**

   English and American spelling are 'rigour' and 'rigor' respectively. A man staying at a Parisian hotel writes this word, and a letter taken at random from his spelling is found to be a vowel. If 40 percent of the English-speaking men at the hotel are English and 60 percent are Americans, what is the probability that the writer is an Englishman?

   **5 pts.**

3. **Expected Value and Variance**

   Take the following text and

   (a) calculate the probabilities of the letters (approximated by their relative frequencies). Ignore punctuation signs and numbers. Treat upper case and lower case letters as equal.

   (b) come up with a random variable for the letters and write it down. How many digits do you need?

   (c) calculate the Expected Value and the Variance of this code.

   > Bach was the most famous composer in the world, and so was Handel. Handel was half German, half Italian and half English. He was very large. Bach died from 1750 to the present. Beethoven wrote music even though he was deaf. He was so deaf he wrote loud music. He took long walks in the forest even when everyone was calling for him. Beethoven expired in 1827 and later died for this.

   If you are enrolled in B659, write a program to calculate this. Submit your code and an output.

   **10pts.**

4. Additional Exercise: **Entropy**

   Use the probabilities from the last experiment as distribution $p$. Then calculate the distribution $q$ over the following text:

   > The sun never set on the British Empire because the British Empire is in the East and the sun sets in the West. Queen Victoria was the longest queen. She sat on a thorn for 63 years. He reclining years and finally the end of her life were exemplatory of a great personality. Her death was the final event which ended her reign.

   Replace all the zeroes by 0.0001. What is the KL divergence between the two distributions? If you are enrolled in B659, write a program to calculate this. Submit your code and an output.

   **additional 10pts.**