Extended Abstracts of the 2018 IISE Annual Conference
K. Barker, D. Berry, C. Rainwater, eds.

A decomposition scheme in production planning based on linear programming, which couples the concept of clearing function and the dynamic economic environment

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Keywords
Production Planning, Clearing Function, Linear Programming.

1. Problem Description

The present work proposes a decomposition scheme in production planning based on linear programming model which couples the concept of decomposition and clearing function in a dynamic economic environment to plan production along a planning horizon, where planning parameters such as demand, production capacity, production resources, and other may change over the planning horizon. In general, this requires the planner, to freeze production plan for some periods or rebuild the production plan of each period, with obvious implications on production planning costs. This happens in general, because problems of production planning require constant modifications due to the environment in which they are inserted and even the responsible planner.

Due to the need of updated information as input on the model, several reworks are required within the planning horizon and this is, in every productive process, a waste of resources. In long planning horizons where there is a great variation in the input parameters, the planning periods to be executed are moving further away from the reality found, which requires the need to either re-plan the schedule, or redo the planning according to the new reality. Therefore, at the beginning of any period from the first, a set of decisions on a production is taken based on current information under a supervision of a decision support system represented by the mathematical model that acts according to the latest available information. This implies that the number of planned periods is much larger than the number of periods to be performed, because for each execution period, the remainder of the planning horizon is re-planned. At the end, the number of planned periods is of order square of n while the number of periods to be executed is of order n, meaning an exponential growing with the number of periods.

The scheme we are proposing in this paper addresses this drawback and provides a lower cost solution to this production planning problem. The proposed scheme is implemented by an algorithm which is analyzed in detail and after numerically illustrated. The decisions variables of the model are the production and inventory levels at each period. The scheme is quite flexible for planning parameters such as demand, production capacity, production resources, and others, and the proposed decomposition assumes that the planning horizon is fixed, and it is shortening over time, which is customary in cases of concessions of public services to individuals.

Based on a scheme initially proposed by [1] and extended by [2–4] , which describe cumulatively the resources whose leftovers can be transferred from one period to the next, we consider the following production planning problem: Let X_{ij} be the production level of product i in period j, I_{ij} the inventory level of product i in period j and c_{ij} the unit cost income one unit of product i in period j. Let b_{ki} be the number of components k used to produce one unit of product i, and h_i the standard time required to produce one unit of product i. R_j is the amount of labor resource (in units of standard time) available during period j, and that any unused labor resource from period j cannot be carried out to period $j+1$. Let S_{kj} be the supply of components k available for consumption in period j, and let D_i be the maximum demand for product i until the end of the planning horizon, and d_{ij} be the minimum demand for product i in period j, suppose furthermore that γ_{ij} is the available production capacity for each product i in period j, prescribed by external clearing function. Hence, the model of production planning coupled with clearing function that can be proposed to present our approach is formulated as:

$$
\begin{aligned}
& Maximize \sum_{j=1}^{J}\sum_{i=1}^{I}\{c_{ij}X_{ij}-(c_{ij}I_{ij}+h_{ij}I_{ij})\} \\
s.t. \quad & \sum_{j=1}^{t}\sum_{i=1}^{N} b_{ki}X_{ij} \le \sum_{j=1}^{t} S_{kj} && \forall k,t=1,2,\ldots,T \\
& I_{ij} = I_{ij-1}+X_{ij}-D_{ij} && \forall i,\forall j \\
& X_{ij}-\gamma_{ij} \le 0 && \forall i,\forall j \\
& \sum_{j=1}^{T} X_{ij} \le D_i && \forall i,t=1,2,\ldots,T \\
& X_{ij} \ge d_{ij} && \forall i,\forall j \\
& \sum_{i=1}^{I} h_i X_{ij} \le R_j && \forall i,\forall j \\
& X_{ij}\ge 0, I_{ij}\ge 0, I_{i0}=0 \\
& i=1,2,\ldots,N, \quad j=1,2,\ldots,T
\end{aligned}
\tag{1}
$$

To discuss decomposition for model (1) we begin by defining the following vectors representation, for each period of the planning horizon:

1. $x^{(j)} = (x_{1j}, x_{2j}, \ldots, x_{Nj})^T, j = 1,2,\ldots,T;$
2. $B = (b_{ki}), k = 1,2,\ldots,K, i = 1,2,\ldots,N;$
3. $S^{(j)} = (S_{1j}, S_{2j}, \ldots, S_{kj})^T, j = 1,2,\ldots,T;$
4. $R^{(j)} = R_j, j = 1,2,\ldots,T;$
5. $D^{(j)} = (D_1, D_2, \ldots, D_N)^T;$
6. $c^{(j)} = (c_{1j}, c_{2j}, \ldots, c_{Nj}), j = 1,2,\ldots,T;$
7. $h = (h_1, h_2, \ldots, h_N);$
8. $\gamma^{(j)} = (\gamma_{1j}, \gamma_{2j}, \ldots, \gamma_{Nj}), j = 1,2,\ldots,T;$
9. $\theta^{(j)} = (\theta_{1j}, \theta_{2j}, \ldots, \theta_{Nj}), j = 1,2,\ldots,T;$
10. I, Identity matrix (Accordingly sized).

We claim that the following general procedure decomposes problem (1) and assure the rolling-planning scheme as defined here: Assume that the planning-horizon T is divided into M blocks of P periods each, that is, $T = M \times P$. Then, define a block matrix $\mathbb{A}$ from the constraints of (1) and the corresponding right-hand side vector $\mathbb{F}$ as,

$$
\mathbb{A} = \begin{bmatrix}
B & & & & \\
I & & & & \\
h & & & & \\
\mathbf{1} & & & & \\
B & B & & & \\
I & I & & & \\
0 & h & & & \\
0 & \mathbf{1} & & & \\
\vdots & & & & \\
B & B & B & \ldots & B \\
I & I & I & \ldots & I \\
0 & 0 & 0 & & h \\
0 & 0 & 0 & & \mathbf{1}
\end{bmatrix},
\mathbb{F} = \begin{bmatrix}
S^{(1)} & & & & & & \\
D^{(1)} & & & & & & \\
R^{(1)} & & & & & & \\
\gamma^{(1)} & & & & & & \\
S^{(1)} & + & S^{(2)} & & & & \\
D^{(2)} & & & & & & \\
0 & + & R^{(2)} & & & & \\
0 & + & \gamma^{(2)} & & & & \\
\vdots & & & & & & \\
S^{(1)} & + & S^{(2)} & + & \ldots & + & S^{(M)} \\
D^{(M)} & & & & & & \\
0 & + & 0 & + & \ldots & + & R^{(M)} \\
0 & + & 0 & + & \ldots & + & \gamma^{(M)}
\end{bmatrix}
\tag{2}
$$

where each block-matrix, *B, I, h*, is respectively a matrix $B_{K\times N}$, a matrix $I_{N\times N}$, a vector $h_{\mathbf{1}\times N}$, and a vector $\mathbf{1}$ is a sum vector. Hence the matrix $\mathbb{A}$ is a $\mathbb{A}_{[(K+N+\mathbf{1}+\mathbf{1})\times T]\times[(T\times N)]}$ matrix, which is then decomposed into M blocks of size P, and accordingly, the vector $\mathbb{F}$. Each block of matrix $\mathbb{A}$ will be numerated as $\mathbb{A}^{(j)}$, and the right hand side vector $\mathbb{F}$, as $\mathbb{F}^{(j)}$. Since $\mathbb{A}^{(j)}$ is fixed, as long as the sizes of blocks are kept constant, it will simply be referred to as $\mathbb{G}$ matrix.

The decomposition scheme based on the present model assure that all planning horizon have the sub-problems as sub-horizons of the whole planning horizon and use the current available information at each period of the planning horizon. Defining the matrix notation for each period of the planning horizon, we consider $\mathbb{A}$ as the matrix of the constraints and $\mathbb{F}$ as the right-hand side matrix of the constraints.

Let assume $D^{(j)}$ is the maximum demand for the remainder of the planning horizon, i.e., the demand from period j until to the end of the planning horizon. Then $\mathbb{L}_{\mathbb{NC}}^{(j)} = D^{(j)} - \sum_{t=j+1}^{T} \gamma^{(t)}$, $j = 1,...,M-1$, $\mathbb{L}^{(M)} = 0$, is the vector data update, where $\mathbb{L}_{\mathbb{NC}}^{(j)}$ stands for the least production for period j, and another vector data update $\mathbb{L}_{\mathbb{MO}}^{(j)} \leftarrow h^T D^{(j)} - h^T \sum_{t=j+1}^{T} \theta^{(t)}$, where $\mathbb{L}_{\mathbb{MO}}^{(j)}$ stands for the least labor resource for period j, to assure feasibility for the problem. Thus, we define the following algorithm.

2. Algorithm

- Initial step

  Solve the sub-problem,

$$\begin{aligned} &Maximize\ \ C^{(1)}X^{(1)} \\ s.t. \qquad &\mathbb{G}X^{(1)} \leq \mathbb{F}^{(1)} \\ &X^{(1)} \geq \mathbb{L}_{\mathbb{NC}}^{(1)} \\ &X^{(1)} \geq \mathbb{L}_{\mathbb{MO}}^{(1)} \end{aligned} \tag{3}$$

- Iterative step

  For $j = 2, \ldots, M$, update the vectors $\mathbb{F}^{(j)}$, $\mathbb{L}_{\mathbb{NC}}^{(j)}$ and $\mathbb{L}_{\mathbb{MO}}^{(j)}$ as,

$$\begin{gathered} \mathbb{F}^{(j)} \leftarrow \mathbb{F}^{(j)} + [\mathbb{F}^{(j-1)} - \mathbb{G}\hat{X}^{(j-1)}], \\ \mathbb{L}_{\mathbb{NC}}^{(j)} \leftarrow D^{(j)} - \textstyle\sum_{t=j+1}^{T} \gamma^{(j)}, \\ \mathbb{L}_{\mathbb{MO}}^{(j)} \leftarrow h^T D^{(j)} - h^T \textstyle\sum_{t=j}^{T} \theta^{(t)}, \\ D^{(j)} \leftarrow [D^{(j)} - \hat{X}^{(j-1)}], (D^{(1)} = D), D^{(j)} = D^{(1)} - \textstyle\sum_{i=1}^{j-1} \hat{X}^{(i)} \end{gathered}$$

  And solve the sub-problems,

$$\begin{aligned} &Maximize\ \ C^{(j)}X^{(j)} \\ s.t. \qquad &\mathbb{G}X^{(j)} \leq \mathbb{F}^{(j)} \\ &X^{(j)} \geq \mathbb{L}_{\mathbb{NC}}^{(j)} \\ &X^{(j)} \geq \mathbb{L}_{\mathbb{MO}}^{(j)} \end{aligned} \tag{4}$$

  $X^{(j)}$ stands for an optimal solution for $j^{(th)}$ sub problem $j = 1, \ldots, M$.

This decomposition algorithm receives the same set of information as the original problem and since both have the same resolution rules, this implying that the solution obtained upon the decomposition is an optimal solution. In addition, since the algorithm presented covers a fixed planning horizon, which reduces the production planning on every period of its execution. Therefore, it can also conclude the greater is the variability in planning parameters on a dynamic environment, the greater is the resource savings, when compared to a more stable environment.

References

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