Signal Processing Challenge

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Problem Statement

- Assume a data matrix X with the following model:

$$X = A_s B_s^T + A_n B_n^T + Z \tag{1}$$

  where
  - X is a real $(m \times n)$ matrix;
  - A_n is an unknown $(m \times d)$ matrix, where the exact value of d is not known, but $d << m, n$;
  - B_n is an unknown $(n \times d)$ matrix;
  - A_s is an unknown $(m \times q)$ matrix, where q is known, and $q << m, n$. Also each column of A_s is in the column span of a *known* matrix S;
  - B_s is an unknown $(n \times q)$ matrix. But each row of B_s has at most one non-zero element.
  - It is assumed that $\text{span}(A_n) \not\subset \text{span}(A_s)$.

- Find a computation efficient method to estimate A_s and B_s.

Approach

- Use the technique of Singular Value Decomposition to estimate A_s and B_s.

- Matrix X can be expressed as:

$$X = U \Sigma V^T \tag{2}$$

  where
  - U is a real $(m \times m)$ matrix of eigenvectors of XX^T ;
  - Σ is a real $(m \times m)$ diagonal matrix of eigenvalues of XX^T;
  - V is a real $(n \times m)$ matrix of eigenvectors of $X^T X$ that correspond to non-zero eigenvalues.

- Setting $A_s = U\Sigma$, places A_s in the column span of matrix U, a *known* matrix. This implies $B_s = V$. When the eigenvectors of the highest q eigenvalues are selected, U is an $(m \times q)$, Σ is a $(q \times q)$ and V is a $(n \times q)$ matrix.

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Simulation Framework

- The performance was tested for the following simulation parameters:

  • Entries of A_s are derived from the class of random staircase functions, of integer step heights and width 32.
  In the Matlab notation: kron(randi(8, m/32, 1), ones(32, 1)), and in R notation: kronecker(runif(m/32, 1, 8), rep(1, 32))
  • Rows of B_s are zero except for possibly one random location where it is one.
  • Entries of B_n are i.i.d samples from a Gaussian process of a given variance.
  • Columns of A_n are random traces of a random walk process.
  • $m = 256, q = 4, d = 16, n = 1024$.

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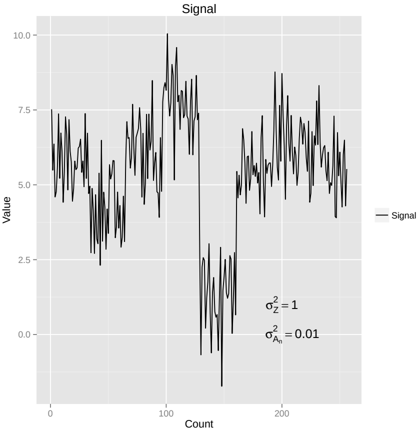
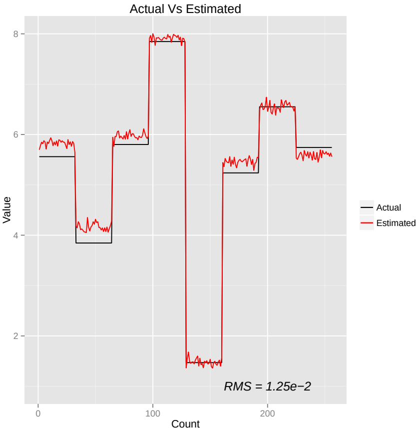
Results

- LHS: Plot of Signal (typical column of matrix X);
  RHS: Actual and Estimated values of $A_s B_s^T$ for the Signal.





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Results

contd.

- Similarity between the actual $(A_s B_s^T)$ and estimated matrix $(\hat{A}_s \hat{B}_s^T)$ is used as a performance measure. Similarity is evaluated using Root-Mean-Square,

$$rms = \frac{1}{nm}\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left((A_s B_s^T)_{j,i} - (\hat{A}_s \hat{B}_s^T)_{j,i}\right)^2} \qquad (3)$$

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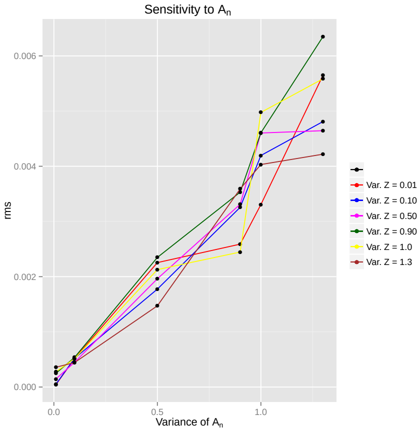
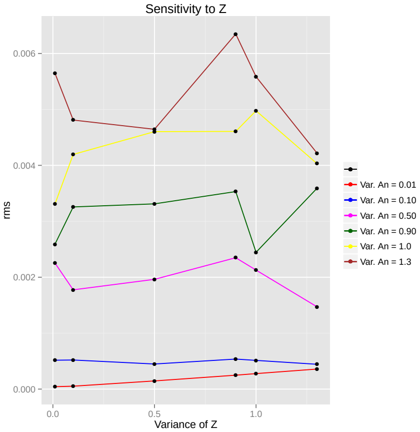
Results

contd.

- Sensitivity to variance in A_n and in Z,

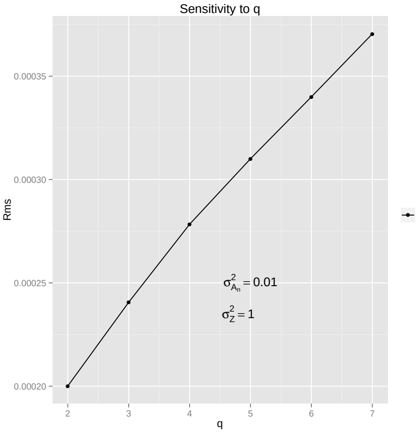


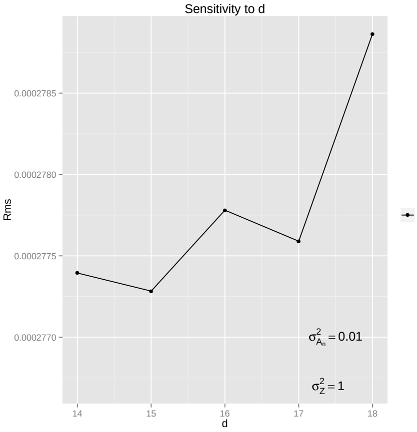


Results

contd.

- Sensitivity to variance in q and in d,





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