Predator-Prey Dynamics

For this lab, we will be analyzing different models of predator prey dynamics to determine the effects of different assumptions on long-term behavior. Complete the following problems and address any included questions. For your submission, include any requested figures, answers to all questions, and your MATLAB code.

Problem 1: Simulation of Lotka-Volterra Model

The simplest possible model of predator-prey interactions is the Lotka-Volterra model, in which the abundance of predator, P, and prey, H, are given by the ordinary differential equations:

1. $\frac{dH}{dt} = aH - bHP$
2. $\frac{dP}{dt} = cHP - mP$

Where a is the prey reproduction time, b is the prey death due to predation, c is the predator reproduction based on consumption of prey, and m is predator death. Note that here predator survival is dependent on the abundance of prey. For the following problems, use $a = 0.5, b = 3, c = 3, m = 0.1$. For all parameters and species time is in units of $days^{-1}$ and species abundances are in units of $cells/mL$.

a) Write a function in MATLAB to be passed into an ODE solver that given time t, a vector of abundances $y = [H, P]$, and a vector of parameters $p = [a, b, c, d]$ outputs the rates of change $dydt = [\frac{dH}{dt}, \frac{dP}{dt}]$ based on the Lotka-Volterra model. Calculate the rates of change given $t = 0$, $y = [0.4. 0.4]$, and the parameters specified above.
b) Using the function that you wrote in part a) and MATLAB's ode solver ode15s (or ode45), simulate the model given initial conditions $[H_0, P_0] = [0.4, 0.4], [0.3, 0.3], [0.2, 0.2]$ over a time period of 500 days. Plot H and P v. t for your first initial conditions of $[H_0, P_0] = [0.4, 0.4]$ on a semiology scale and also P v. H for all your initial conditions on a single plot on a log-log scale. On your second plot, include the nullclines for the differential equations ($\frac{dH}{dt} = 0, \frac{dP}{dt} = 0$) using MATLAB's fimplicit function. You should see that your system never intersects the steady state (intersection of the nullclines). What does this mean?
c) How do the initial conditions affect the long term behavior of predator and prey abundances? Is this realistic? Explain. Hint: Realistically there should be other limiting factors to species abundances than the presence of predators or prey, i.e., a carrying capacity.

Problem 2: Introducing a carrying capacity on prey growth.

We will now more realistically model growth of prey abundance with logistic growth, i.e. add a carrying capacity to this term. This system is now modeled by the ordinary differential equations:

1. $\frac{dH}{dt} = aH\left(1 - \frac{H}{K}\right) - bHP$
2. $\frac{dP}{dt} = cHP - mP$

Where K is the carrying capacity of the prey population. For this problem, use $a = 0.5, b = 3, c = 3, m = 0.1, K = 0.4$.

a) Write a function in MATLAB to be passed into an ODE solver that given time t, a vector of abundances $y = [H, P]$, and a vector of parameters $p = [a, b, c, d, K]$ outputs the rates of change

$dydt = [\frac{dH}{dt}, \frac{dP}{dt}]$ based on the modified model. Calculate the rates of change given $t = 0$, $y = [0.4, 0.4]$, and the parameters specified above.

b) Using the function that you wrote in part a) and MATLAB's ode solver ode15s (or ode45), simulate the model given initial conditions $[H_0, P_0] =$ over a time period of 500 days. Plot H and P v. t for your first initial conditions of $[H_0, P_0] = [0.4, 0.4], [0.3\ 0.3], [0.2\ 0.2]$ on a semiology scale and also P v. H for all your initial conditions on a single plot on a log-log scale. On your second plot, include the nullclines for the differential equations ($\frac{dH}{dt} = 0, \frac{dP}{dt} = 0$) using MATLAB's fimplicit function. How do these simulations compare to those in Problem 1? Do they seem more realistic? In what ways?

<u>Problem 3: Simulating a Macarthur-Rozenzweig model.</u>

Now we will make one more modification to our model to make it more realistic. Here, predator consumption of prey is modified to include an efficiency ϵ and a timescale τ. Our differential equations now take the form:

1. $$\frac{dH}{dt} = aH\left(1 - \frac{H}{K}\right) - \frac{bH}{1+b\tau H}P$$
2. $$\frac{dP}{dt} = \frac{\epsilon bH}{1+b\tau H}P - mP$$

For this problem, use $a = 0.5, b = 3, \epsilon = 1, \tau = 3, m = 0.1, K = 0.4$

a) Write a function in MATLAB to be passed into an ODE solver that given time t, a vector of abundances $y = [H, P]$, and a vector of parameters $p = [a, b, \epsilon, \tau, d, K]$ outputs the rates of change $dydt = [\frac{dH}{dt}, \frac{dP}{dt}]$ based on the modified model. Calculate the rates of change given $t = 0$, $y = [0.4, 0.4]$, and the parameters specified above.

b) Using the function that you wrote in part a) and MATLAB's ode solver ode15s (or ode45), simulate the model given initial conditions $[H_0, P_0] = [0.4, 0.4], [0.3, 0.3], [0.2, 0.2]$ over a time period of 500 days Plot H and P v. t for your first initial conditions of $[H_0, P_0] = [0.4\ 0.4]$ on a semiology scale and also P v. H for all your initial conditions on a single plot on a log-log scale.. On your second plot, include the nullclines for the differential equations ($\frac{dH}{dt} = 0, \frac{dP}{dt} = 0$) using MATLAB's fimplicit function. Compare these simulations to those from Problems 1 and 2. Do these seem more realistic?