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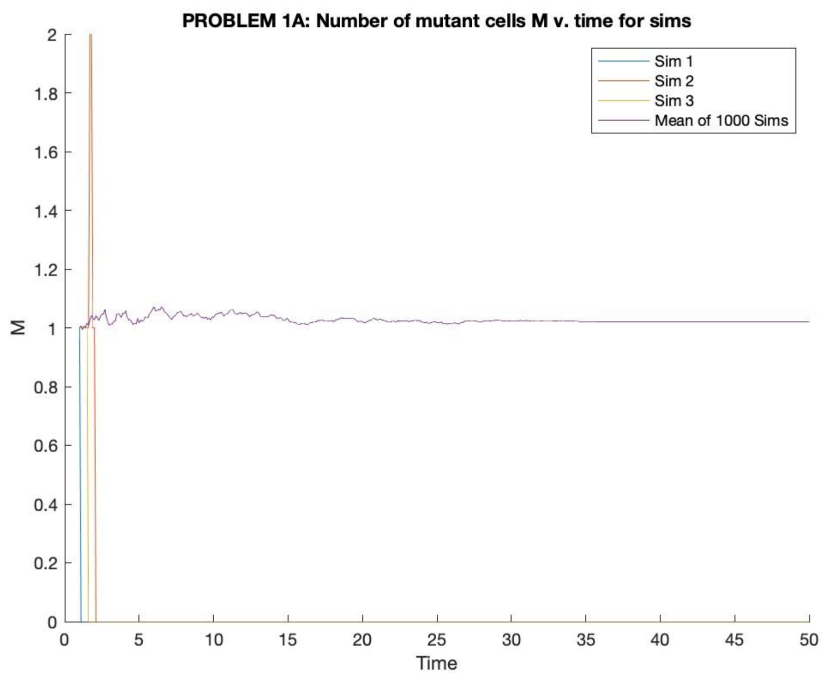
BIOE 446
13 October 2023

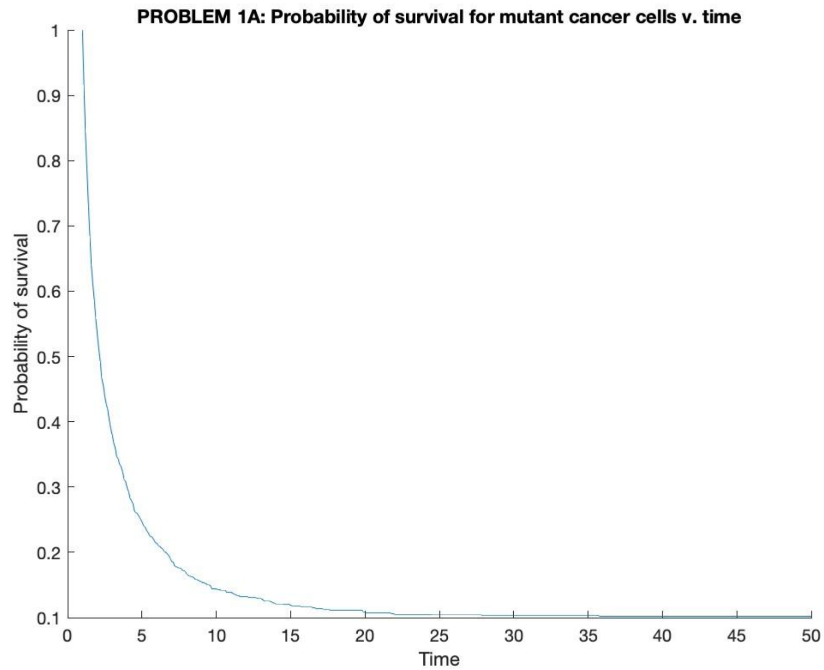
Lab 7: Evolutionary Population Dynamics

ANSWERS

1. **Moran model with no selection (DEMO)**
    a. Probability of cancer cells surviving at final time = 0.102

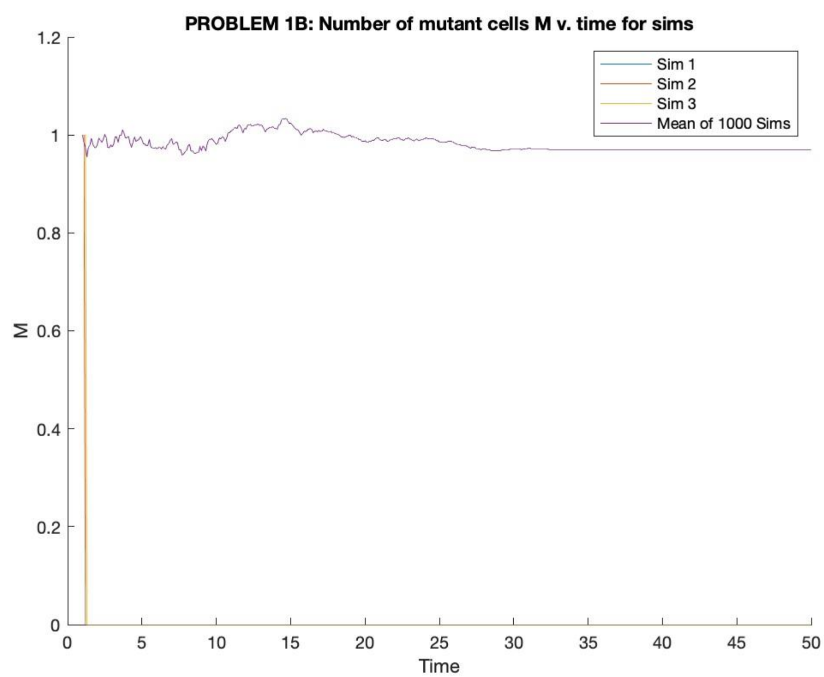
        Figures

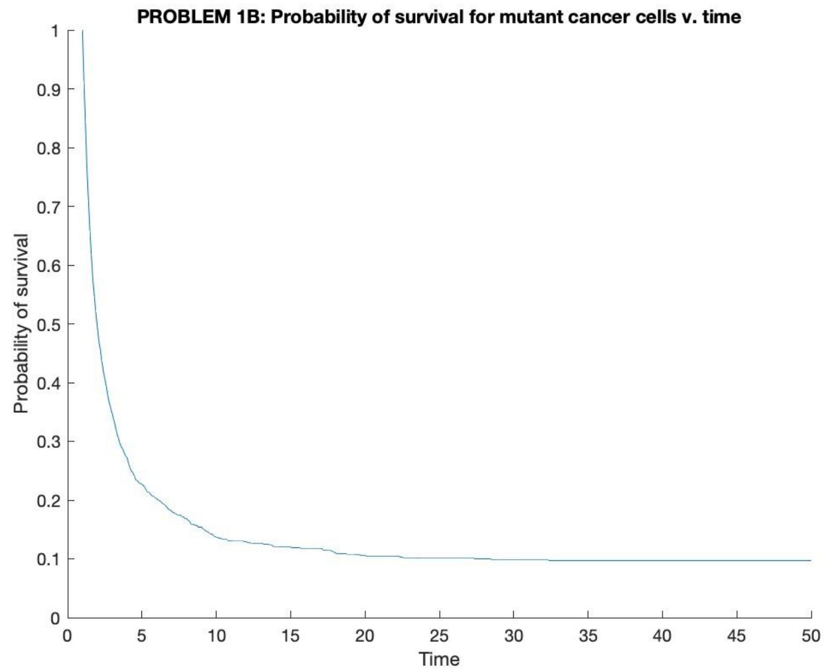




b. Probability of cancer cells surviving at final time = 0.097

Figures

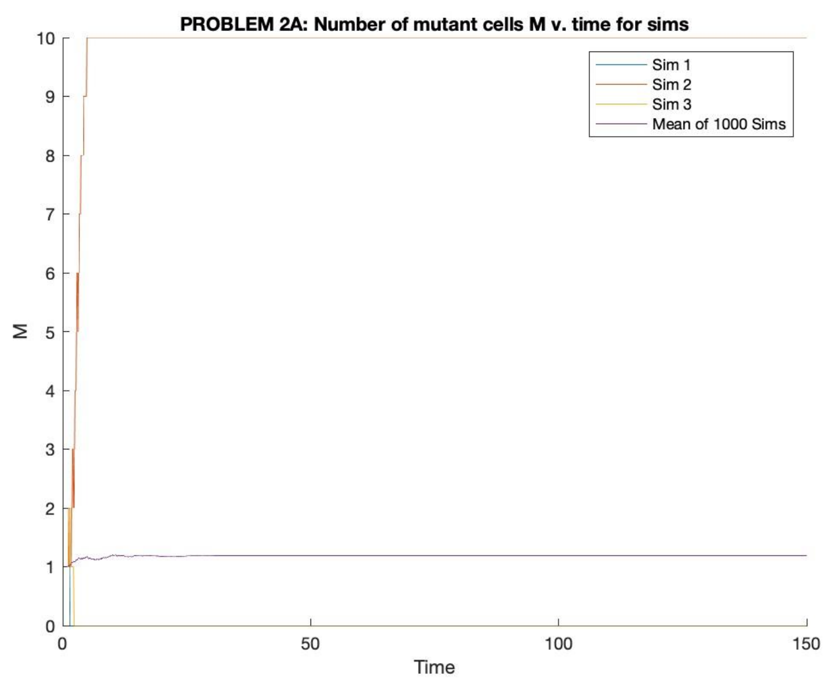


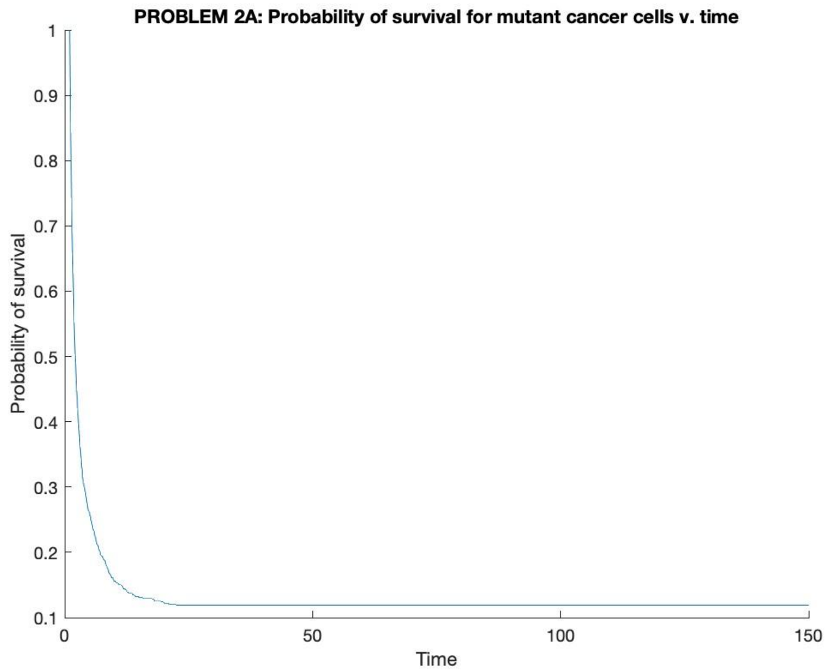


2. Moran model with selection

a. Probability of cancer cells surviving at final time = 0.119

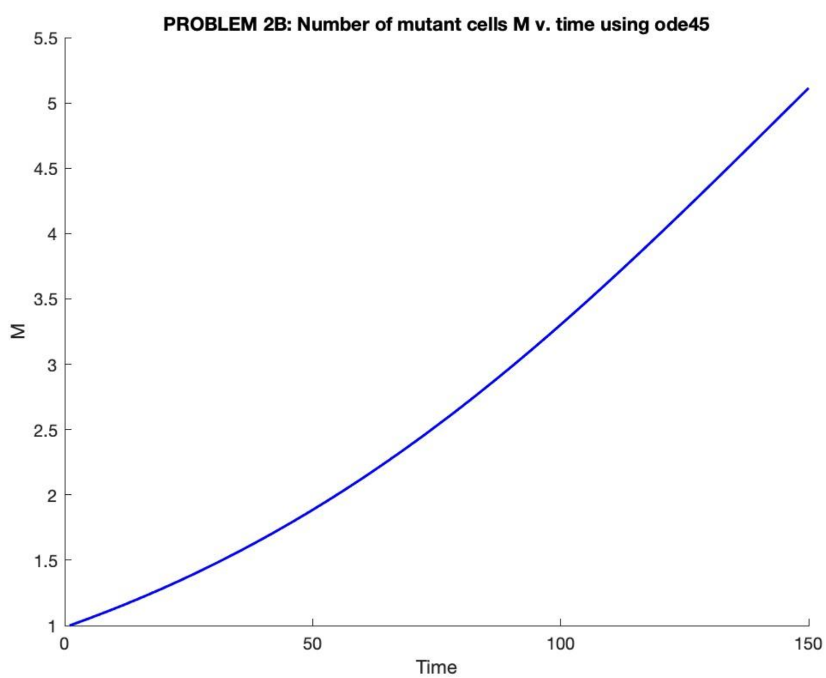
Figures





b. Mean number of mutants after 150 days = 2.7301
Final number of predicted mutants by ODE = 5.1136

Figure

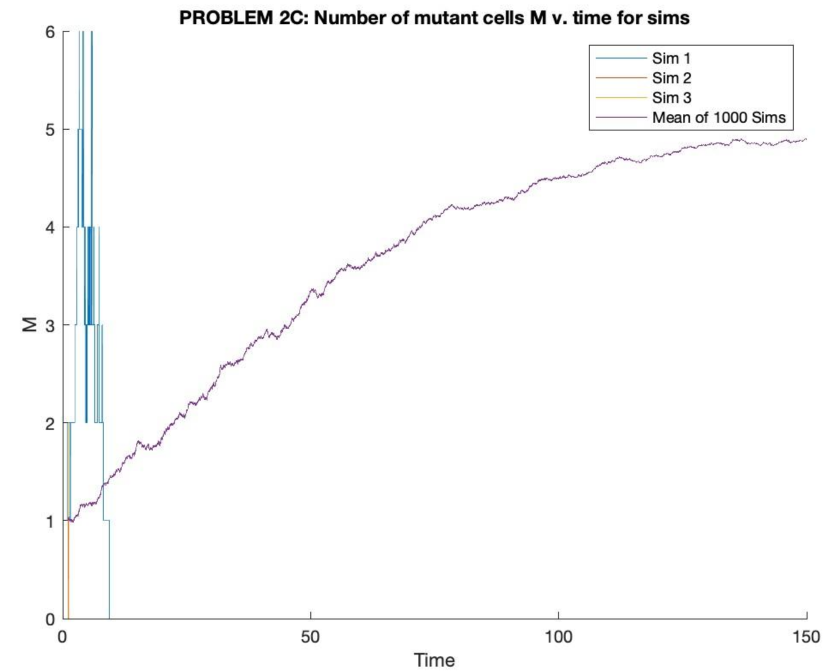


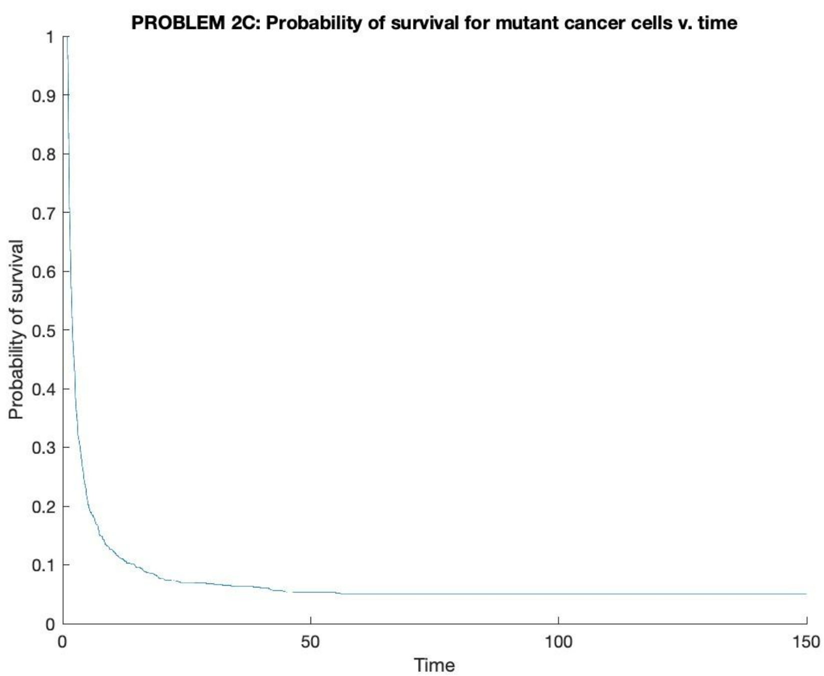
Compared to the stochastic simulation in part A, solving this ODE gives a higher average number of mutants (2.7301) compared to the simulation

(~1.3). The ODE solution's number of mutants also increases gradually with time, whereas the stochastic simulation equilibrates around the average number of mutants.

c. Probability of cancer cells surviving at final time = 0.051

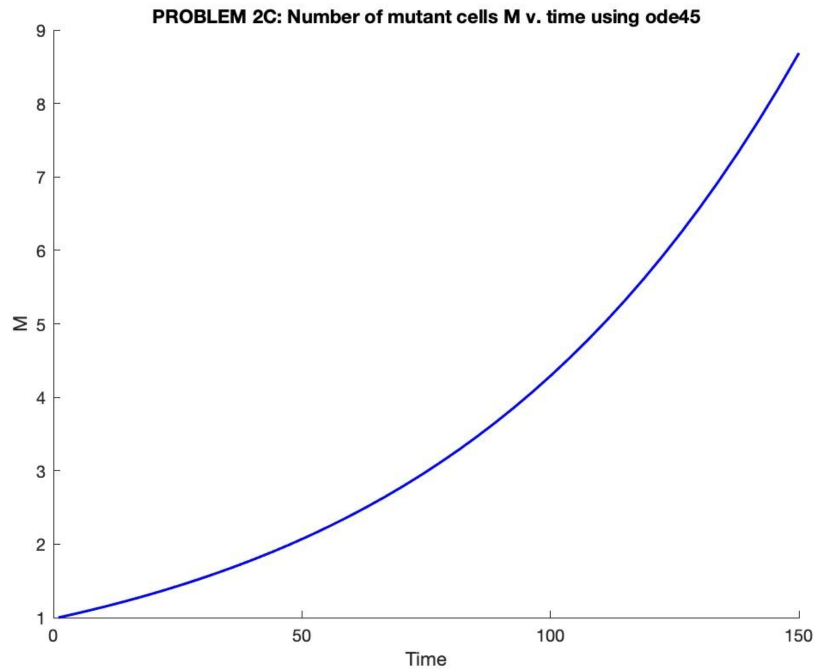
Figures





Mean number of mutants after 150 days = 3.5734
Final number of predicted mutants by ODE = 8.6872

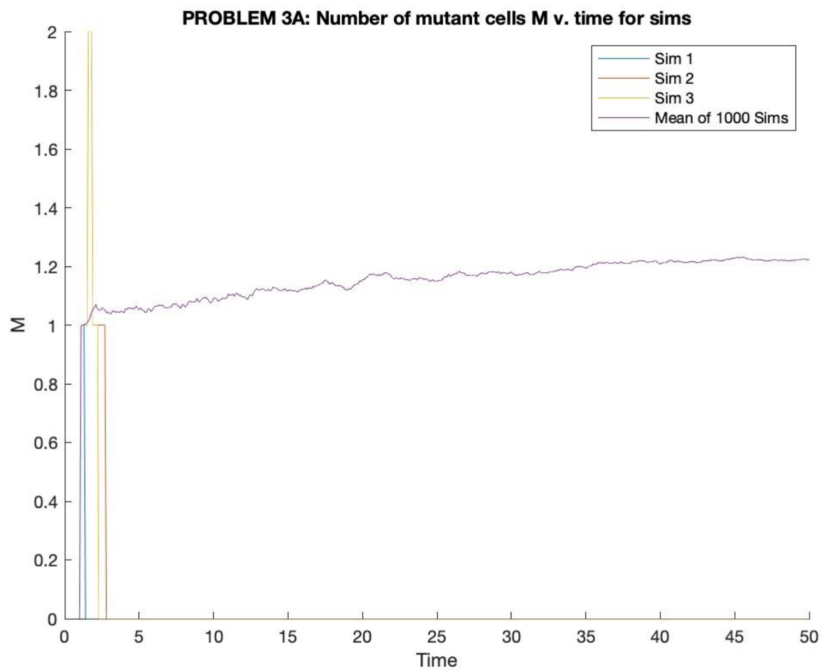
Figure

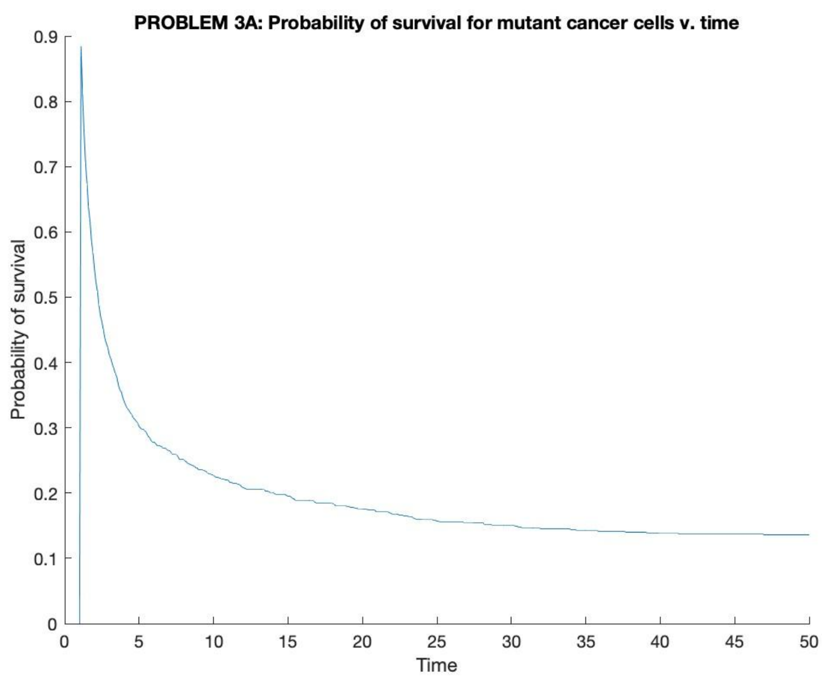


With a greater number of cells (N=100), the results are significantly altered. The probability of cancer cells surviving at the final time is about half (5.1% with N=100 compared to 11.9% with N=10). However, due to the greater number of starting cells, the mean number of mutant cells from both the stochastic simulation and ODE solution is higher with N=100 than for N=10. This is likely due to the fact that in both cases (N=10 and N=100), there is only a single starting mutant cell, but in the N=100 environment there is a larger proportion of wildtype cells than mutant cells, which makes it harder for the mutant to proliferate.

3. **Spatial Moran model**
    a. Probability of cancer cells surviving at final time = 0.136

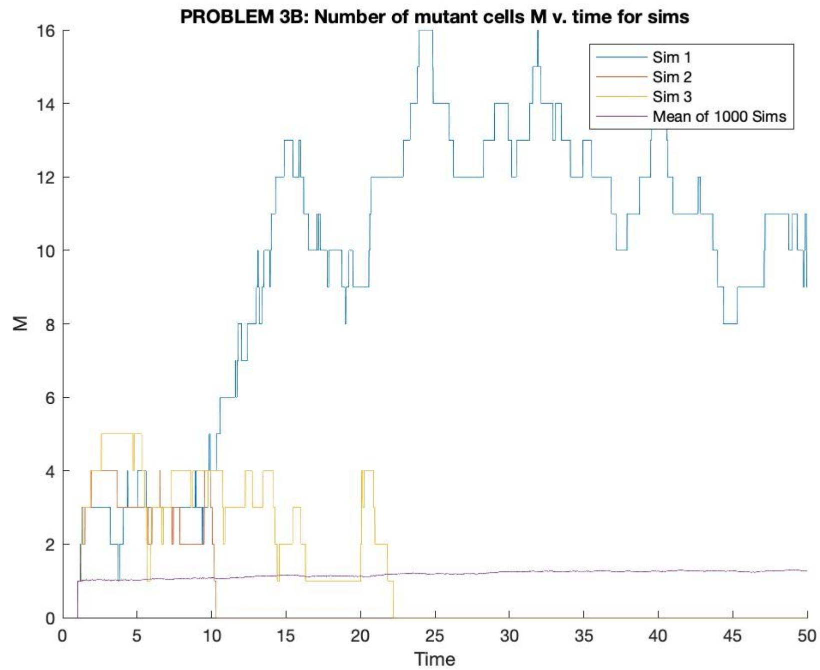
    Figures

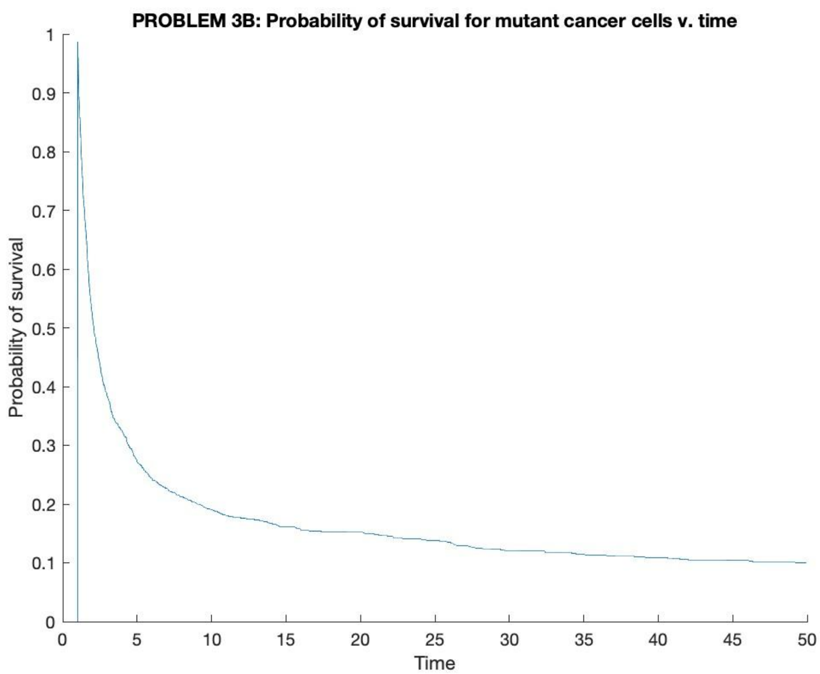




b. Probability of cancer cells surviving at final time = 0.100

Figures





With a larger number of cells N=100, the probability of cancer cells surviving at the final time (10.0%) is slightly lower than with fewer total cells (N=10) (13.6%), likely due to similar reasoning as in problem 2. Looking at the figures, the spatial Moran model stochastic simulations seem to have a higher variance with a larger number of cells and compared to the transition matrix model in problem 2.

c. *The mutant survival probability in the spatial Moran model does not significantly change depending on the number of cells (i.e. between the N=10 and N=100 cases) since the spatial model design does is not influenced as much by the size of the lattice (since each cell only has 1 or 2 neighbors). In the homogenous situation in problem 2, the system and mutant survival probability is much more dependent on the number of cells; with a higher number of cells (i.e. for N=100), the survival probability is much lower (5.1%) compared to the case with a tenth of the number of cells (i.e. for N=10, where the probability of survival is 11.9%).*

CODE

```
close all
clc

%% Problem 1: Moran model with no selection (DEMO)
disp('PROBLEM 1')
clear

%% Problem 1, Part A
% initializing parameters
N = 10;
t0 = 1; % day
dt = t0/N;
tf = 50;
tspan = 1:dt:tf;
Nsim = 1000;
n_mutants = zeros(length(tspan), Nsim);

% simulate model
for i = 1:Nsim
    cells = zeros(N,1);
    cells(randi(N)) = 1;
    n_mutants(1, i) = 1;

    for j = 2:length(tspan)
        r = randi(N, 2, 1);
        cells(r(1)) = cells(r(2));
        n_mutants(j, i) = sum(cells);

        if n_mutants(j, i) == 0
            break
        elseif n_mutants(j, i) == N
            n_mutants(j+1:length(tspan), i) = N;
        end
    end
end

m_mutants = mean(n_mutants, 2);
prob_survival = sum(n_mutants(:,:)' > 0)/Nsim;
prob_survival_tf = prob_survival(end);

figure(1)
hold on
plot(tspan, n_mutants(:, 1:3))
plot(tspan, m_mutants)
xlabel('Time')
ylabel('M')
legend('Sim 1','Sim 2', 'Sim 3', 'Mean of 1000 Sims')
title('PROBLEM 1A: Number of mutant cells M v. time for sims')

figure(2)
```

```
hold on
plot(tspan, prob_survival)
xlabel('Time')
ylabel('Probability of survival')
title('PROBLEM 1A: Probability of survival for mutant cancer cells v. time')

fprintf("Probability of mutant survival at final time: %.3f\n", prob_survival_tf)

%% Problem 1, Part B
N = 10;
t0 = 1;
dt = t0/N;
tf = 50;
tspan = 1:dt:tf;
Nsim = 1000;
n_mutants = zeros(length(tspan), Nsim);

for i = 1:Nsim
    n_mutants(1, i) = 1;
    for j = 2:length(tspan)
        prob_increase = n_mutants(j-1, i)/N * (1-n_mutants(j-1, i)/N);
        prob_decrease = n_mutants(j-1, i)/N * (1-n_mutants(j-1, i)/N);

        r = rand;
        if r <= prob_increase
            n_mutants(j, i) = n_mutants(j-1, i)+1;
        elseif r<= prob_increase + prob_decrease
            n_mutants(j, i) = n_mutants(j-1, i)-1;
        else
            n_mutants(j, i) = n_mutants(j-1, i);
        end

        if n_mutants(j, i) ==0
            break
        elseif n_mutants(j, i) == N
            n_mutants(j+1:length(tspan), i) = N;
            break
        end
    end
end

m_mutants = mean(n_mutants, 2);
prob_survival = sum(n_mutants(:,:)' > 0)/Nsim;
prob_survival_tf = prob_survival(end);

figure(3)
hold on
plot(tspan, n_mutants(:, 1:3))
plot(tspan, m_mutants)
xlabel('Time')
ylabel('M')
```

```matlab
legend('Sim 1','Sim 2', 'Sim 3', 'Mean of 1000 Sims')
title('PROBLEM 1B: Number of mutant cells M v. time for sims')

figure(4)
hold on
xlabel('Time')
ylabel('Probability of survival')
plot(tspan, prob_survival)
title('PROBLEM 1B: Probability of survival for mutant cancer cells v. time')

fprintf("Probability of mutant survival at final time: %.3f\n", prob_survival_tf)


%% Problem 2: Moran model with selection
disp('PROBLEM 2')
clear

%% Problem 2, Part A
N = 10;
t0 = 1;
dt = t0/N;
tf = 150;
tspan = 1:dt:tf;
Nsim = 1000;
n_mutants = zeros(length(tspan), Nsim);

s = 0.05;

for i = 1:Nsim
    n_mutants(1, i) = 1;
    for j = 2:length(tspan)
        prob_increase = (1+s) * n_mutants(j-1, i)/N * (1-n_mutants(j-1, i)/N);
        prob_decrease = n_mutants(j-1, i)/N * (1-n_mutants(j-1, i)/N);

        r = rand;
        if r <= prob_increase
            n_mutants(j, i) = n_mutants(j-1, i)+1;
        elseif r<= prob_increase + prob_decrease
            n_mutants(j, i) = n_mutants(j-1, i)-1;
        else
            n_mutants(j, i) = n_mutants(j-1, i);
        end

        if n_mutants(j, i) ==0
            break
        elseif n_mutants(j, i) == N
            n_mutants(j+1:length(tspan), i) = N;
            break
        end
    end
end
```

```matlab
m_mutants = mean(n_mutants, 2);
prob_survival = sum(n_mutants(:,:)' > 0)/Nsim;
prob_survival_tf = prob_survival(end);

figure(5)
hold on
plot(tspan, n_mutants(:, 1:3))
plot(tspan, m_mutants)
xlabel('Time')
ylabel('M')
legend('Sim 1', 'Sim 2', 'Sim 3', 'Mean of 1000 Sims')
title('PROBLEM 2A: Number of mutant cells M v. time for sims')

figure(6)
hold on
xlabel('Time')
ylabel('Probability of survival')
plot(tspan, prob_survival)
title('PROBLEM 2A: Probability of survival for mutant cancer cells v. time')

fprintf("Probability of mutant survival at final time: %.3f\n", prob_survival_tf)

%% Problem 2, Part B
N = 10;
t0 = 1;
g = log10(2)/t0;
s = 0.05;
M0 = 1;
tf = 150;

p = [g, s, N];
[t, M] = ode45(@(t, M) ode2B(t, M, p), [t0 tf], M0);

M_mean = mean(M)
M_150 = M(end)

figure(7)
hold on
plot(t, M, '-b', LineWidth=1.5)
xlabel('Time')
ylabel('M')
title('PROBLEM 2B: Number of mutant cells M v. time using ode45')


%% Problem 2, Part C
N = 100;
t0 = 1;
dt = t0/N;
tf = 150;
tspan = 1:dt:tf;
```

```
Nsim = 1000;
n_mutants = zeros(length(tspan), Nsim);

s = 0.05;

for i = 1:Nsim
    n_mutants(1, i) = 1;
    for j = 2:length(tspan)
        prob_increase = (1+s) * n_mutants(j-1, i)/N * (1-n_mutants(j-1, i)/N);
        prob_decrease = n_mutants(j-1, i)/N * (1-n_mutants(j-1, i)/N);

        r = rand;
        if r <= prob_increase
            n_mutants(j, i) = n_mutants(j-1, i)+1;
        elseif r<= prob_increase + prob_decrease
            n_mutants(j, i) = n_mutants(j-1, i)-1;
        else
            n_mutants(j, i) = n_mutants(j-1, i);
        end

        if n_mutants(j, i) ==0
            break
        elseif n_mutants(j, i) == N
            n_mutants(j+1:length(tspan), i) = N;
            break
        end
    end
end

m_mutants = mean(n_mutants, 2);
prob_survival = sum(n_mutants(:,:)' > 0)/Nsim;
prob_survival_tf = prob_survival(end);

figure(8)
hold on
plot(tspan, n_mutants(:, 1:3))
plot(tspan, m_mutants)
xlabel('Time')
ylabel('M')
legend('Sim 1', 'Sim 2', 'Sim 3', 'Mean of 1000 Sims')
title('PROBLEM 2C: Number of mutant cells M v. time for sims')

figure(9)
hold on
xlabel('Time')
ylabel('Probability of survival')
plot(tspan, prob_survival)
title('PROBLEM 2C: Probability of survival for mutant cancer cells v. time')

fprintf("Probability of mutant survival at final time: %.3f\n", prob_survival_tf)
```

```
N = 100;
t0 = 1;
g = log10(2)/t0;
s = 0.05;
M0 = 1;
tf = 150;

p = [g, s, N];
[t, M] = ode45(@(t, M) ode2B(t, M, p), [t0 tf], M0);

M_mean = mean(M)
M_150 = M(end)

figure(10)
hold on
plot(t, M, '-b', LineWidth=1.5)
xlabel('Time')
ylabel('M')
title('PROBLEM 2C: Number of mutant cells M v. time using ode45')


%% Problem 3: Spatial Moran model
disp('PROBLEM 3')
clear

%% Problem 3, Part A
N = 10;
t0 = 1;
dt = t0/N;
tf = 50;
tspan = 1:dt:tf;
Nsim = 1000;

s = 0.05;

n_mutants = zeros(length(tspan), Nsim);

for i = 1:Nsim
    lattice = zeros(1, N);
    lattice(1, randi(N)) = 1;
    n_mutants(1, Nsim) = 1;

    for j = 2:length(tspan)

        idx = randi(N);

        if idx ~= 1 && idx ~= N
            neighbors = [idx-1, idx+1];
            if lattice(neighbors(1)) == 1
                prob = (1+s)/(2+s);
            else
```

```
            prob =  1/(2+s);
        end

        if rand < prob
            lattice(idx) = lattice(neighbors(1));
        else
            lattice(idx) = lattice(neighbors(2));
        end

    elseif idx == 1
        lattice(idx) = lattice(idx+1);
    elseif idx == N
        lattice(idx) = lattice(idx-1);
    end

    n_mutants(j, i) = sum(lattice);

    if n_mutants(j, i) == 0
        break
    elseif n_mutants(j, i) == N
        n_mutants(j+1:length(tspan), i) = N;
        break
    end
  end
end

m_mutants = mean(n_mutants, 2);
prob_survival = sum(n_mutants(:,:)' > 0)/Nsim;
prob_survival_tf = prob_survival(end);

figure(11)
hold on
plot(tspan, n_mutants(:, 1:3))
plot(tspan, m_mutants)
xlabel('Time')
ylabel('M')
legend('Sim 1', 'Sim 2', 'Sim 3', 'Mean of 1000 Sims')
title('PROBLEM 3A: Number of mutant cells M v. time for sims')

figure(12)
hold on
xlabel('Time')
ylabel('Probability of survival')
plot(tspan, prob_survival)
title('PROBLEM 3A: Probability of survival for mutant cancer cells v. time')

fprintf("Probability of mutant survival at final time: %.3f\n", prob_survival_tf)

%% Problem 3, Part B
N = 100;
t0 = 1;
```

```matlab
dt = t0/N;
tf = 50;
tspan = 1:dt:tf;
Nsim = 1000;

s = 0.05;

n_mutants = zeros(length(tspan), Nsim);

for i = 1:Nsim
    lattice = zeros(1, N);
    lattice(1, randi(N)) = 1;
    n_mutants(1, Nsim) = 1;

    for j = 2:length(tspan)

        idx = randi(N);

        if idx ~= 1 && idx ~= N
            neighbors = [idx-1, idx+1];
            if lattice(neighbors(1)) == 1
                prob = (1+s)/(2+s);
            else
                prob =  1/(2+s);
            end

            if rand < prob
                lattice(idx) = lattice(neighbors(1));
            else
                lattice(idx) = lattice(neighbors(2));
            end

        elseif idx == 1
            lattice(idx) = lattice(idx+1);
        elseif idx == N
            lattice(idx) = lattice(idx-1);
        end

        n_mutants(j, i) = sum(lattice);

        if n_mutants(j, i) == 0
            break
        elseif n_mutants(j, i) == N
            n_mutants(j+1:length(tspan), i) = N;
            break
        end
    end
end

m_mutants = mean(n_mutants, 2);
prob_survival = sum(n_mutants(:,:)' > 0)/Nsim;
```

```matlab
prob_survival_tf = prob_survival(end);

figure(13)
hold on
plot(tspan, n_mutants(:, 1:3))
plot(tspan, m_mutants)
xlabel('Time')
ylabel('M')
legend('Sim 1', 'Sim 2', 'Sim 3', 'Mean of 1000 Sims')
title('PROBLEM 3B: Number of mutant cells M v. time for sims')

figure(14)
hold on
xlabel('Time')
ylabel('Probability of survival')
plot(tspan, prob_survival)
title('PROBLEM 3B: Probability of survival for mutant cancer cells v. time')

fprintf("Probability of mutant survival at final time: %.3f\n", prob_survival_tf)

%% Problem 3, Part C
% No MATLAB code required.


%% Functions

% Problem 2
function dMdt = ode2B(t, M, p)
    g = p(1);
    s = p(2);
    N = p(3);

    g_av = ((M/N)*(1+s)*g) + (((N-M)/N)*g);
    dMdt = (g*M*(1+s)) - (g_av*M);
end
```