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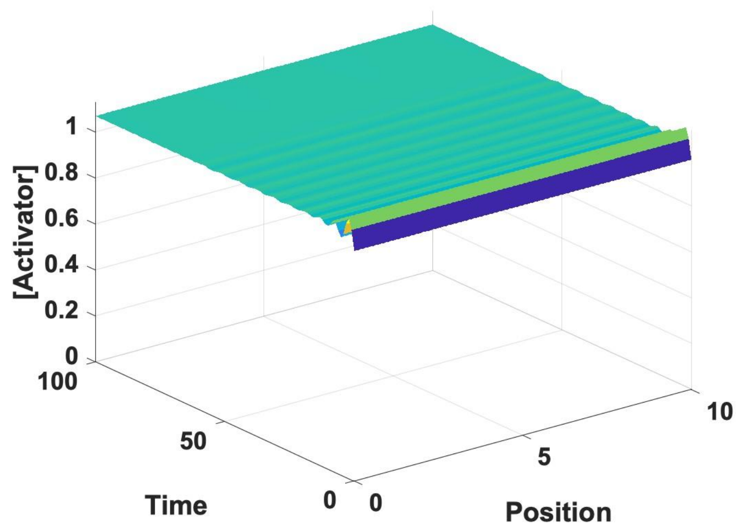
BIOE 446
17 November 2023

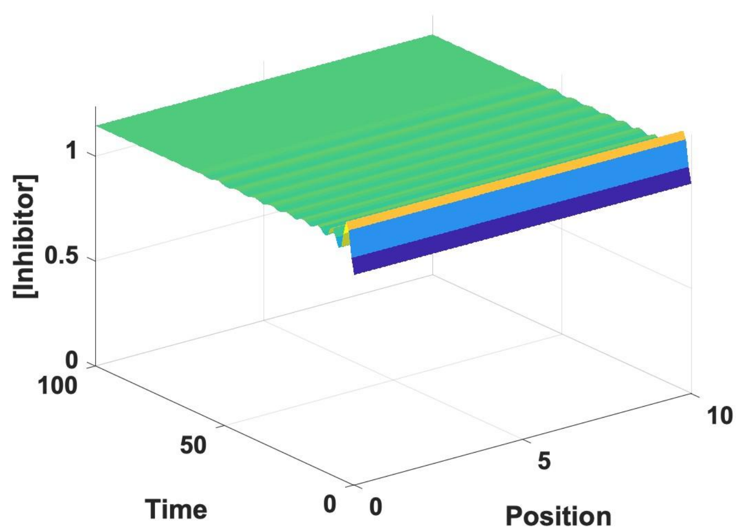
Lab 12: Reaction-Diffusion Systems and Pattern Formation

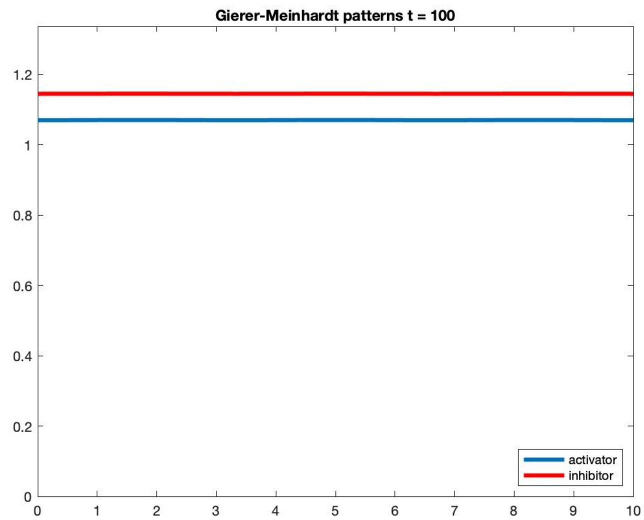
ANSWERS

1. **1D pattern formation in the Gierer-Meinhardt model**
    a. Figures

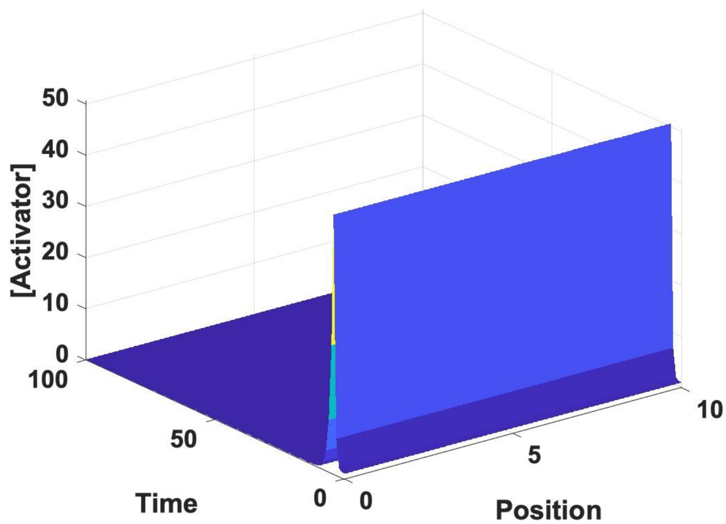
Run 1

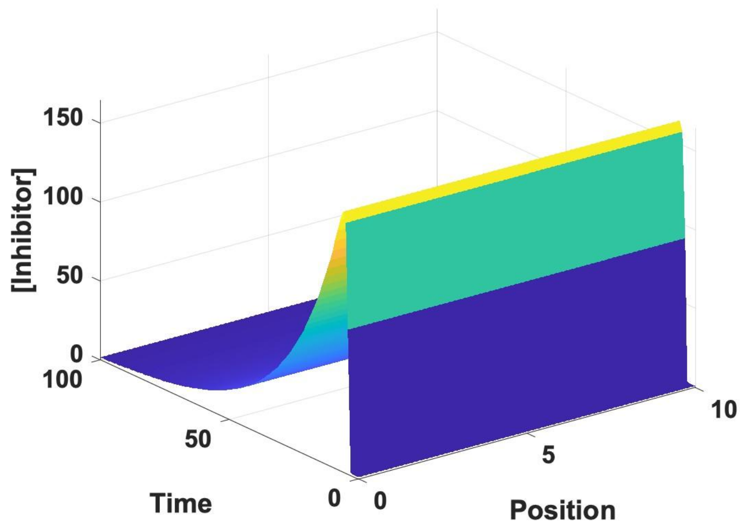


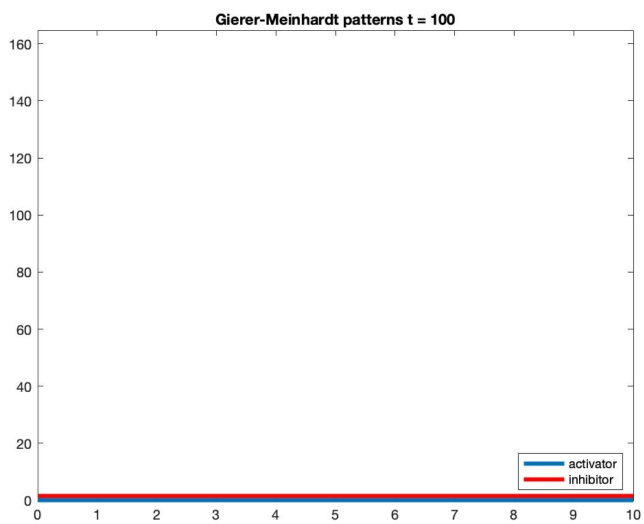



Gierer-Meinhardt patterns t = 100
activator
inhibitor
0
0.2
0.4
0.6
0.8
1
1.2
0 1 2 3 4 5 6 7 8 9 10

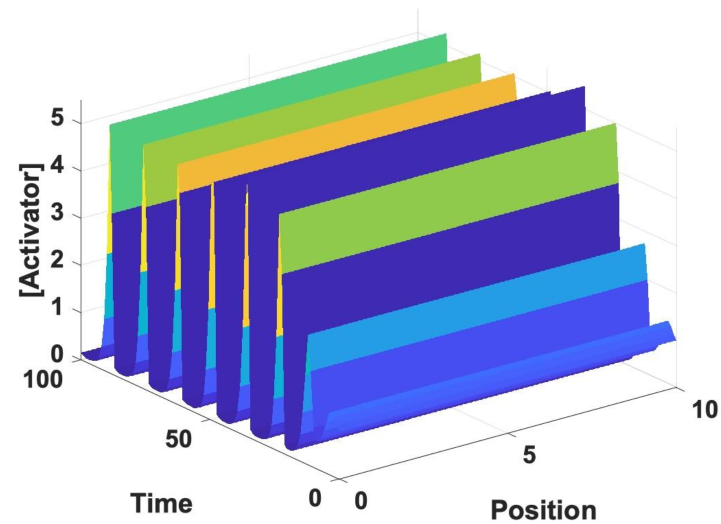
Run 2


[Activator]
0
10
20
30
40
50
Time
100
50
0
Position
0
5
10

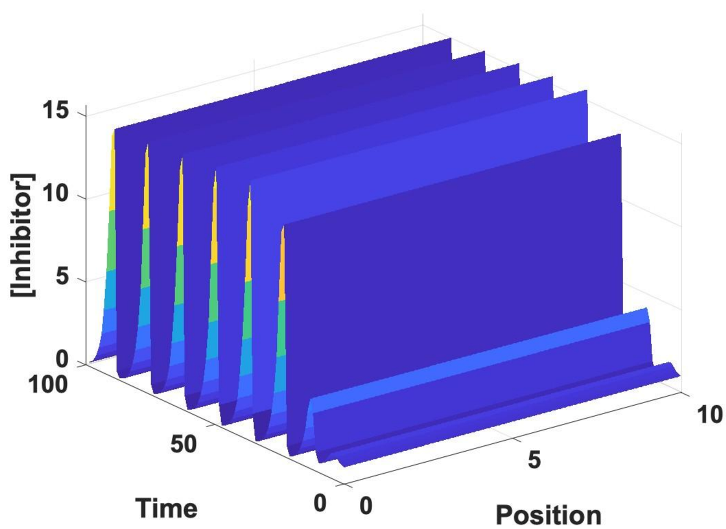

150
100
50
0
[Inhibitor]
100
50
0
Time
0
5
10
Position


Gierer-Meinhardt patterns t = 100
160
140
120
100
80
60
40
20
0
0
1
2
3
4
5
6
7
8
9
10
activator
inhibitor

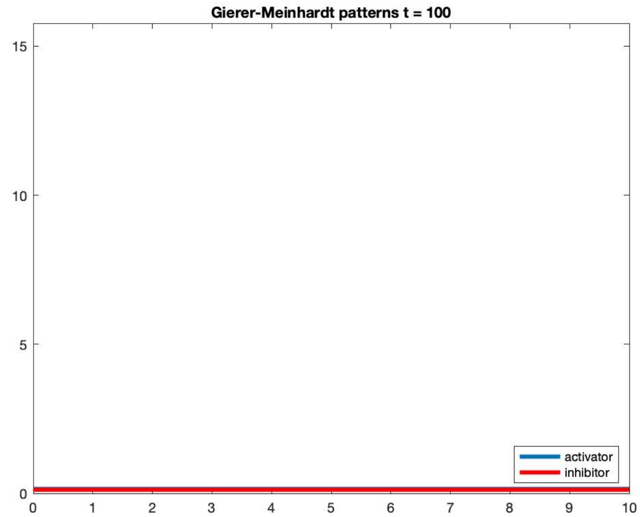
Run 3



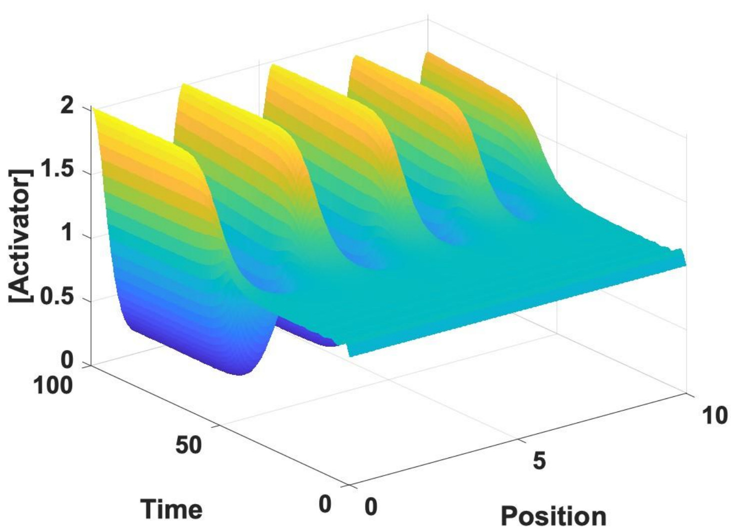
[Activator]
5
4
3
2
1
0
100
50
0
Time
0
5
10
Position

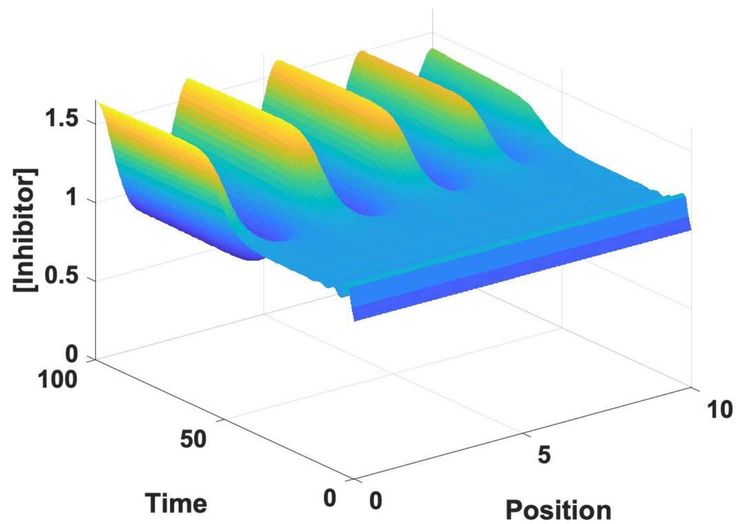


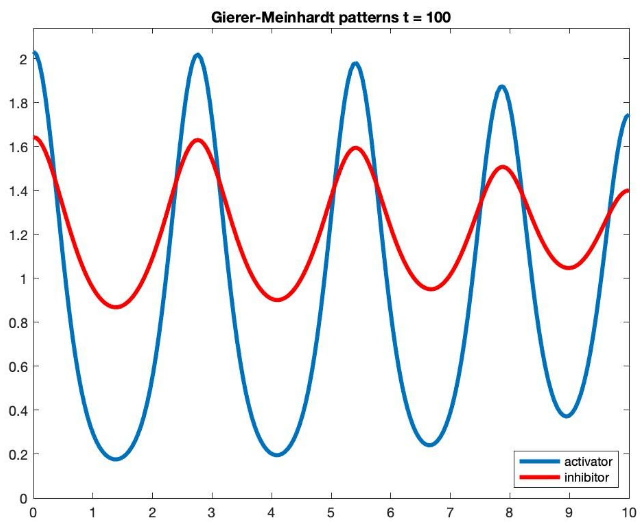
[Inhibitor]
15
10
5
0
100
50
0
Time
0
5
10
Position


Gierer-Meinhardt patterns t = 100
15
10
5
0
0
1
2
3
4
5
6
7
8
9
10
activator
inhibitor

Run 4


2
1.5
1
0.5
0
[Activator]
100
50
0
Time
0
5
10
Position



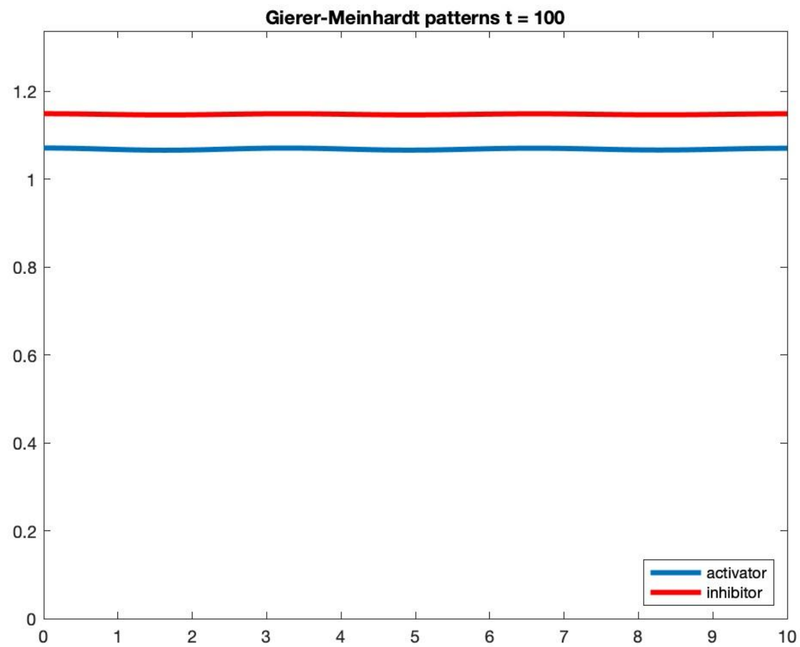


The first two runs produce a moving plot where the activator and inhibitor levels shift with time and decay/stabilize to a near-zero amount of activator and inhibitor. In both of these simulations, the amounts of activator and inhibitor are spatially homogenous/constant. In the third run, the amounts of activator and repressor oscillate with time, but still remain spatially homogenous/constant. The surface plots show the dynamics of these simulations both spatially and temporally; the oscillations in the first three simulations only occur temporally (i.e., the levels of activator and inhibitor remain the same/homogenous across all positions, but the levels of both activator and inhibitor change with time).

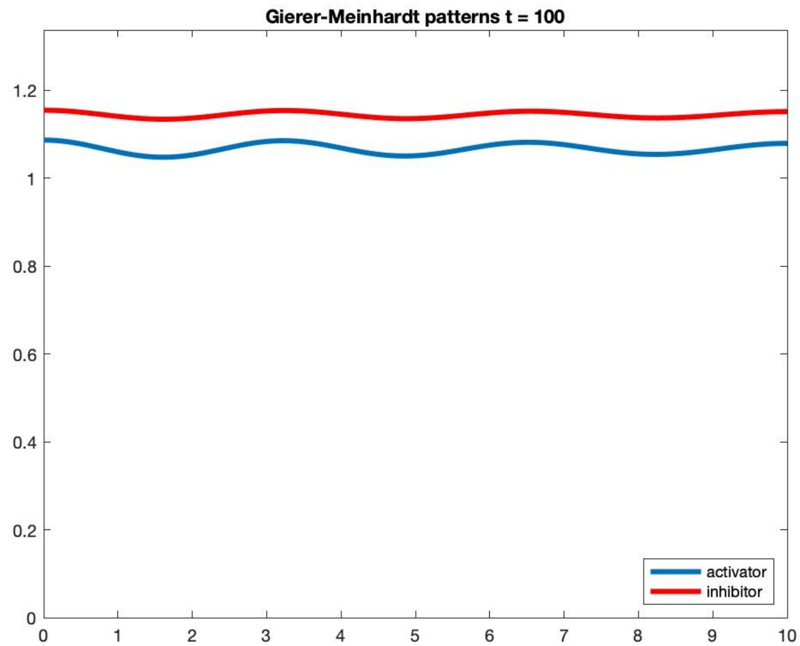
In the fourth simulation (parameter set), however, the amounts of activator and inhibitor are initially constant/homogenous across all positions (and oscillate slightly with time), but as time progresses the *spatial distribution becomes non-homogenous and produces a sinusoidal pattern seen in the final figure. Unlike the first 3 parameter sets, the solution to the PDEs using this final parameter set is spatially sinusoidal.*

b. Figures

D = 0.09



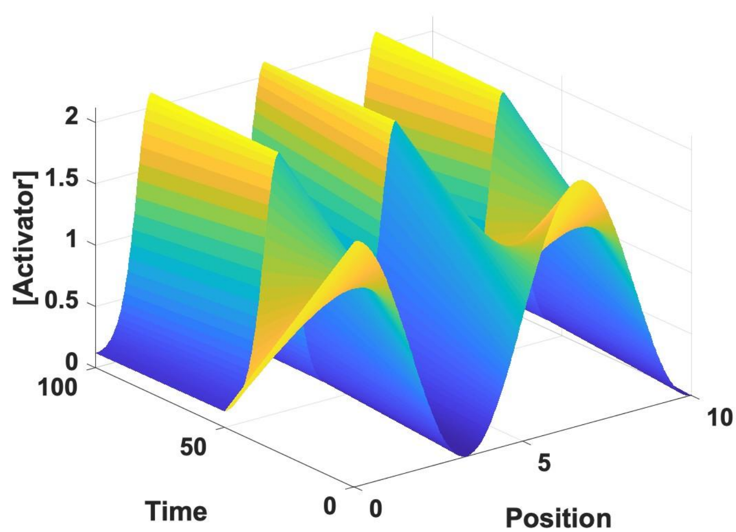
D = 0.085

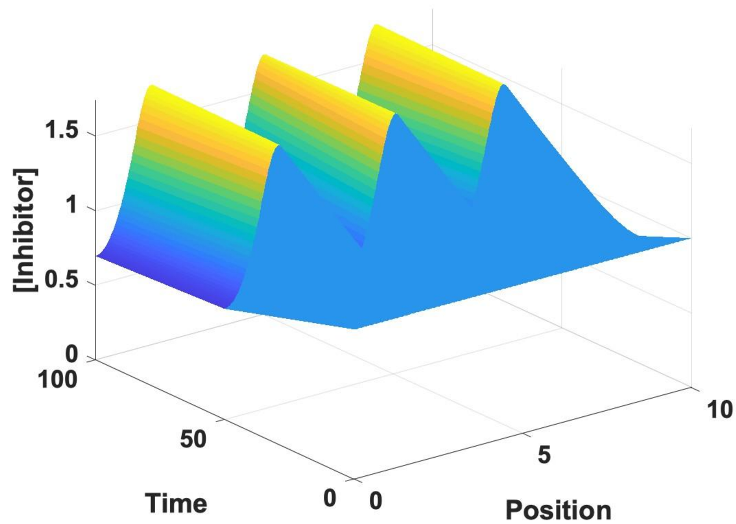


A diffusivity value of approximately D = 0.09 results in a non-stable (non-homogenous) steady state solution in the spatial dimension. For smaller diffusivity values, the spatial distribution becomes more and more sinusoidally-distributed. Slow diffusion may cause this to happen since the amounts of activator and inhibitor cannot spread as quickly/effectively relative to the dynamics of the reactions, resulting in areas of higher and lower concentrations of both.

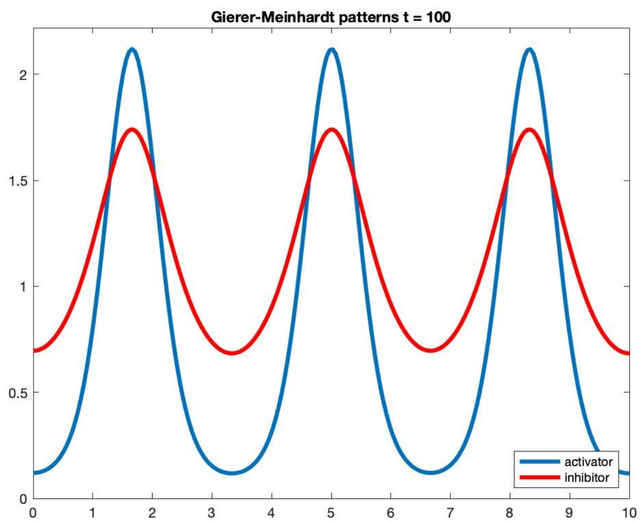
c. Figures

n = 3



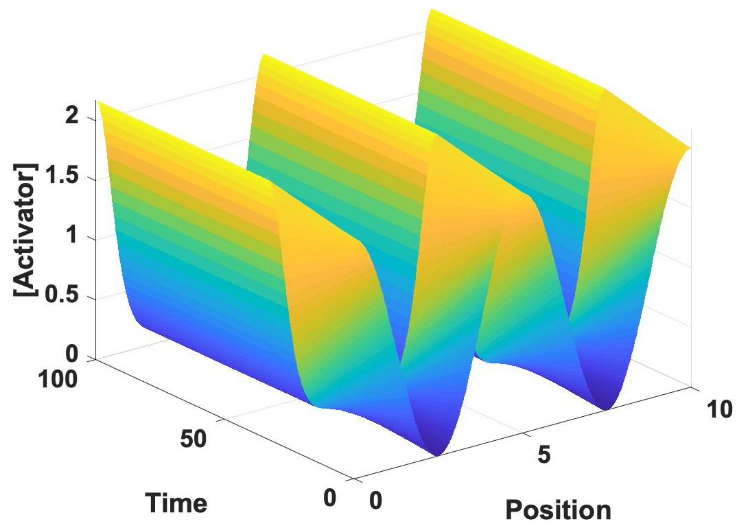


[Inhibitor]
1.5
1
0.5
0
100
50
Time
0
0
5
10
Position

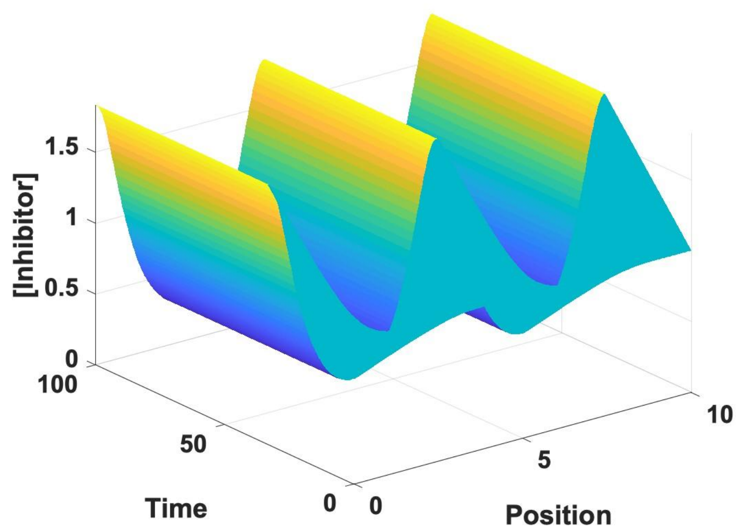


Gierer-Meinhardt patterns t = 100
activator
inhibitor

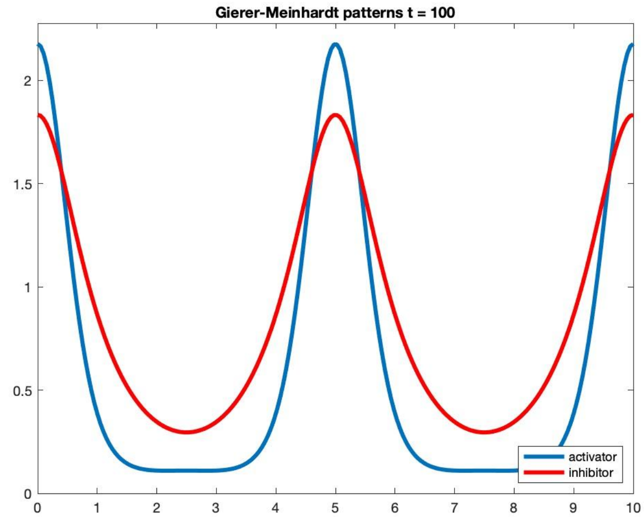
n = 4



[Activator]
2
1.5
1
0.5
0
100
50
0
Time
0
5
10
Position



[Inhibitor]
1.5
1
0.5
0
100
50
0
Time
0
5
10
Position

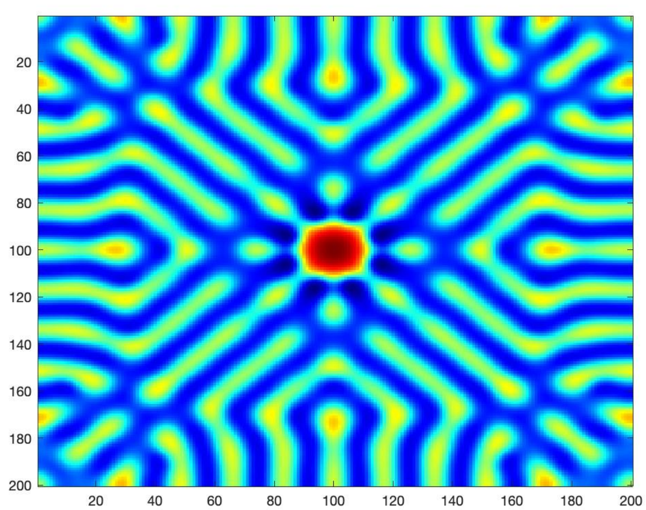


As n increases, the period of the spatial distribution sinusoid (technically the "Gierer-Meinhardt pattern") of the system increases (the peaks and valleys in the distribution of the activator and inhibitor become more spread out). In the given position range from 0 to 10, for n = 3, there are roughly 3 full periods in the distribution at the final time point; for n = 4, there are only 2 periods at the final time point.

2. 2D pattern formation in the Gray-Scott model

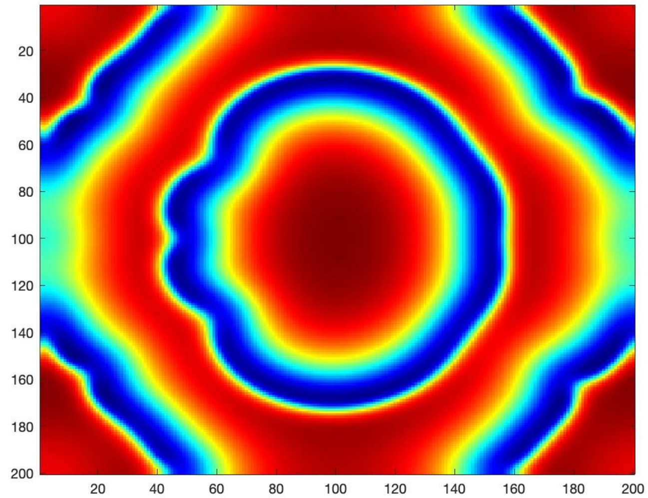
a. Figures

Run 1



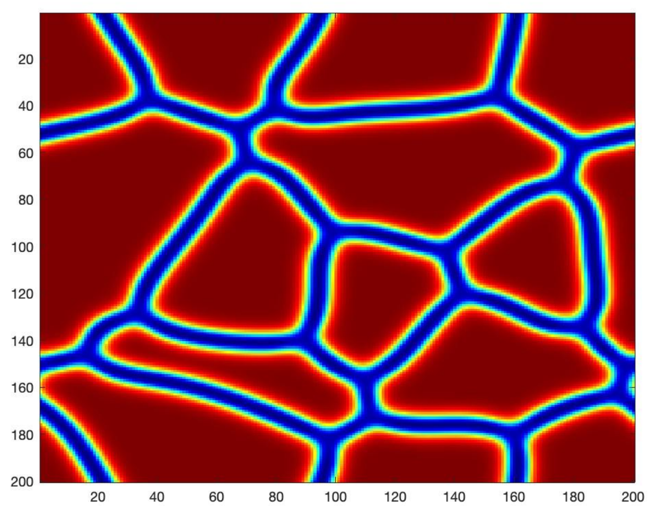
For this simulation, the pattern continually changed with vertical and horizontal symmetry about the center point. The pattern started as a *square that expanded and "bounced" off the edges of the space, producing an increasingly complex pattern.*

Run 2



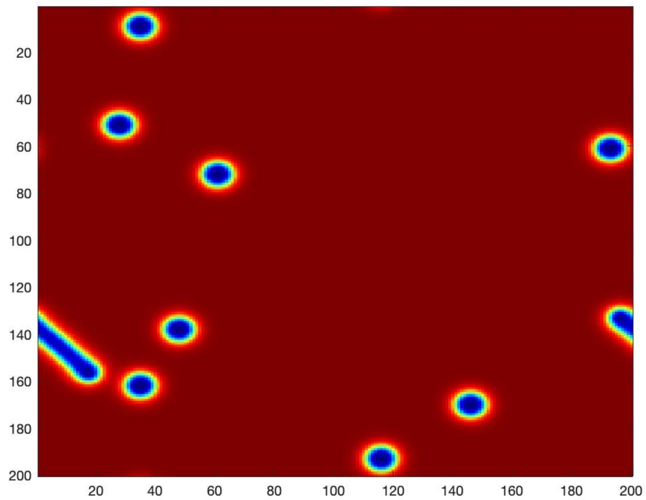
In this run, a distorted circle/oval wave expanded/curled from the center of the system. The simulation appeared to have symmetry across the horizontal axis, and the initial "wave" began to smoothen to appear more circular towards the end of the simulation.

Run 3



In this simulation, spots at random locations in the system expanded with radial symmetry/concentrically (shown in red) until they intersected neighboring spots/circles and filled the entire system. A gap was left between the spots at the end of the simulation, shown in blue.

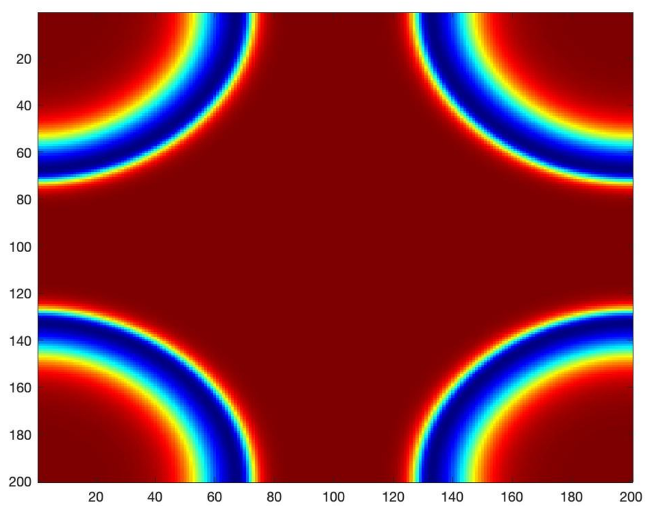
Run 4



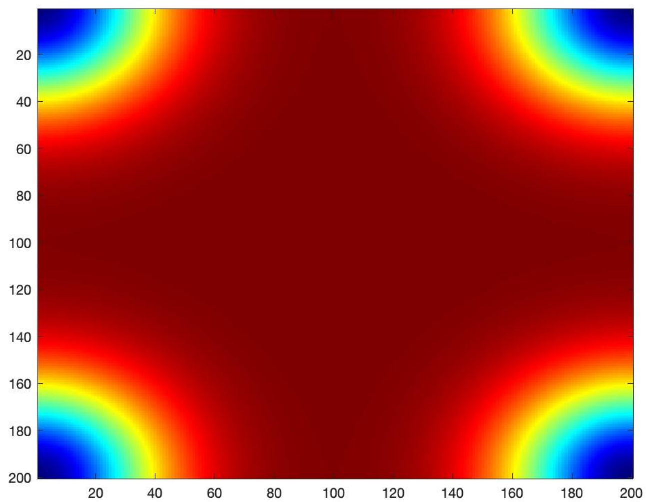
This run started similarly to run 3 with spots on the screen; spots that were significantly close to other spots combined and began to extend along the line that connected their initial locations. Overall, the dynamics of this system were more subtle than the first 3 simulations.

b. Figures

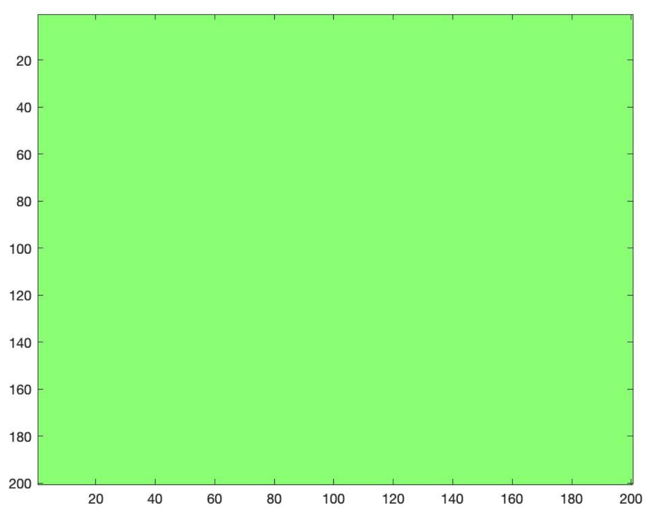
D_v = 0.7 (final solution)



D_v = 0.8 (before homogenous solution)



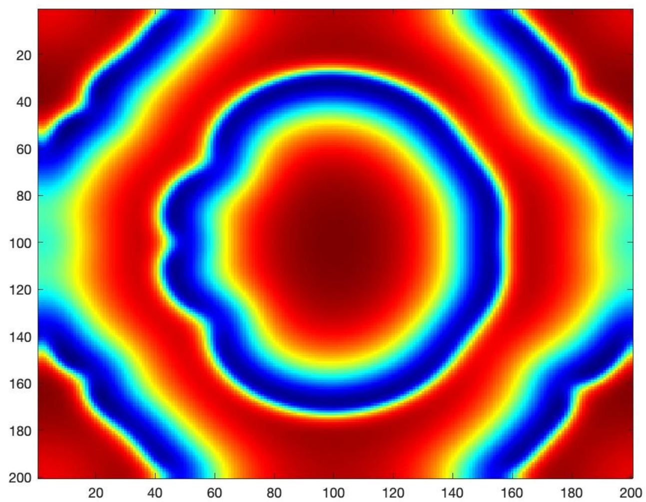
D_v = 0.8 (at homogenous solution)



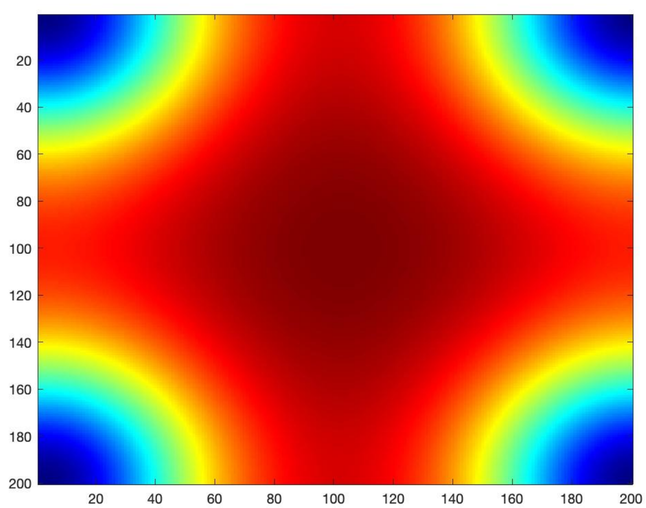
A diffusivity value of D_v = 0.8 produces a homogenous long-term state using the parameters from run 1. As the system approached the homogenous state, the system dynamics slowed, shown in the figures above. Right before reaching the homogenous solution, the pattern slowly expanded from the 4 corners of the system. Increasing the diffusivity may prevent a pattern from being formed since the dynamics of movement may occur much more quickly relative to the dynamics of reactions; this produces a more "mixed" system where detailed patterns cannot form as species diffuse too quickly.

c. Figures

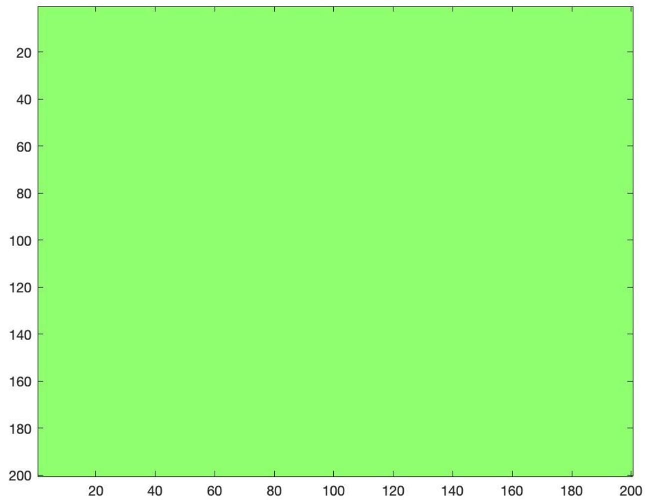
D_v = 0.5 (final solution)



D_v = 0.6 (before homogenous solution)



D_v = 0.6 (at homogenous solution)



The dynamics of this system (using the initial conditions from run 2) as it approached the homogenous state were very similar to the system dynamics in part B. A lower diffusivity value of about D_v = 0.6 was required to reach the homogenous solution, however, which could be attributed to the lower f or k system parameters compared to run 1. The reasoning for why increasing the diffusivity prevents pattern formation in this example is the same as in part B as well.

CODE

```
close all
clc

%% Problem 1: 1D pattern formation in the Gierer-Meinhardt model
disp('PROBLEM 1')
clear

%% Problem 1, Part A
close all
clc
clear

tmax = 100;
n = 100;
L = 10;

% P1 = [0.1 1.25 0.07];
% GM_solve(P1, tmax, n, L, [1 2 3]);

% P2 = [0.05 0.05 0.07];
% GM_solve(P2, tmax, n, L, [1 2 3]);

% P3 = [0.4 0.5 0.07];
% GM_solve(P3, tmax, n, L, [1 2 3]);

P4 = [0.05 1.4 0.07];
GM_solve(P4, tmax, n, L, [1 2 3]);

%% Problem 1, Part B
close all
clc
clear

tmax = 100;
n = 100;
L = 10;

P = [0.09 1.25 0.07];
GM_solve(P, tmax, n, L, [1 2 3]);

%% Problem 1, Part C
close all
clc
clear

tmax = 100;
n = 4;
L = 10;

P = [0.05, 1.4, 0.07];
```

```matlab
IC_func_sym = @(x, P) IC_func(x, P, n, L);
GM_solve_IC(P, tmax, n, L, [1 2 3], IC_func_sym);


%% Problem 2: 2D pattern formation in the Gray-Scott model
disp('PROBLEM 2')
clear

%% Problem 2, Part A
close all

% rxn_dfsn_gs(0.022, 0.051, 1, 0.5, InitState='square');

% rxn_dfsn_gs(0.01, 0.041, 1, 0.5, InitState='wavefront');

% rxn_dfsn_gs(0.09, 0.059, 1, 0.5, InitState='uSpots');

rxn_dfsn_gs(0.09, 0.059, 1, 0.5, InitState='vSpots');

%% Problem 2, Part B
close all

% rxn_dfsn_gs(0.022, 0.051, 1, 0.7, InitState='square');

rxn_dfsn_gs(0.022, 0.051, 1, 0.8, InitState='square');

%% Problem 2, Part C
close all

% rxn_dfsn_gs(0.01, 0.041, 1, 0.5, InitState='wavefront');

rxn_dfsn_gs(0.01, 0.041, 1, 0.6, InitState='wavefront');


%% Functions

% Problem 1
function y0 = IC_func(x, P, n, L)
    h = ones(size(x));
    a = 1+cos(n*pi*x/L);

    y0 = [a; h];
end
```