Duration Hedging with Interest Rate Swaps

This tutorial walks through a simple ALM example: an insurer holds a small portfolio of bonds backing two insurance liabilities, and uses an interest rate swap to close a duration gap when rates change overnight.

```python
import sys
from pathlib import Path

# make the alm package importable when running from docs/tutorial/
sys.path.insert(0, str(Path.cwd().parents[1] / "src"))

import polars as pl
from great_tables import GT

from alm.asset import Bond
from alm.liability import SPIA, Term
from alm.core import InterestRateSwap
```

Step 1 — Build the portfolio

Assets

The insurer holds two bonds:

Bond	Face	Coupon	Maturity	Frequency
A	$5M	3.5 %	5 yr	Semi-ann
B	$3M	4.5 %	10 yr	Semi-ann

```python
bond_a = Bond(face_value=5_000_000, coupon_rate=0.035, maturity=5, frequency=2)
bond_b = Bond(face_value=3_000_000, coupon_rate=0.045, maturity=10, frequency=2)
```

Liabilities

The insurer has written two policies:

- A **SPIA** paying $300 000 / year for up to 30 years (monthly, life only).
- A **20-year Term Life** policy with a $1 000 000 face value and $5 000 annual premium.

We use a flat 1 % annual mortality rate for simplicity.

```
qx = [0.01] * 30

spia = SPIA(
    premium=4_000_000,
    annual_payout=300_000,
    qx=qx,
    frequency=12,
    certain_period=0,
)

term = Term(
    face_value=1_000_000,
    annual_premium=5_000,
    term=20,
    qx=qx,
    frequency=12,
)
```

Step 2 — Day 1: measure the duration gap

Suppose the market discount rate on Day 1 is **4.00 %**.

```
rate_day1 = 0.04

# Asset side
pv_a1 = bond_a.present_value(rate_day1)
pv_b1 = bond_b.present_value(rate_day1)
dur_a1 = bond_a.duration(rate_day1)
dur_b1 = bond_b.duration(rate_day1)
pv_assets1 = pv_a1 + pv_b1

# Liability side
pv_spia1 = spia.present_value(rate_day1)
```

```python
pv_term1 = term.present_value(rate_day1)
dur_spia1 = spia.duration(rate_day1)
dur_term1 = term.duration(rate_day1)
pv_liab1 = pv_spia1 + pv_term1

# Weighted-average durations
dur_assets1 = (pv_a1 * dur_a1 + pv_b1 * dur_b1) / pv_assets1
dur_liab1 = (pv_spia1 * dur_spia1 + pv_term1 * dur_term1) / pv_liab1
gap1 = dur_assets1 - dur_liab1

day1_df = pl.DataFrame(
    {
        "Section": [
            "Assets",
            "Assets",
            "Assets",
            "Liabilities",
            "Liabilities",
            "Liabilities",
            "Summary",
        ],
        "Item": [
            "Bond A",
            "Bond B",
            "Total",
            "SPIA",
            "Term Life",
            "Total",
            "Duration Gap (A - L)",
        ],
        "PV": [pv_a1, pv_b1, pv_assets1, pv_spia1, pv_term1, pv_liab1, None],
        "Duration": [
            dur_a1,
            dur_b1,
            dur_assets1,
            dur_spia1,
            dur_term1,
            dur_liab1,
            gap1,
        ],
    }
)
```

```
(
    GT(day1_df, rowname_col="Item", groupname_col="Section")
    .tab_header(title="Day 1 Portfolio", subtitle=f"Market rate: {rate_day1:.2%}")
    .fmt_currency(columns="PV", decimals=0)
    .fmt_number(columns="Duration", decimals=2)
    .sub_missing(missing_text="")
)
```

Day 1 Portfolio
Market rate: 4.00%

	PV	Duration
Assets		
Bond A	$4,887,718	4.63
Bond B	$3,122,636	8.21
Total	$8,010,353	6.02
Liabilities		
SPIA	$4,652,414	11.43
Term Life	$63,720	8.40
Total	$4,716,134	11.39
Summary		
Duration Gap (A − L)		−5.36

The asset portfolio has shorter duration than the liabilities — a negative duration gap. If rates fall, liability PV rises faster than asset PV, eroding surplus.

Step 3 — Enter a receive-fixed swap to close the gap

A **receive-fixed** swap adds duration: it is economically equivalent to buying a fixed-rate bond and funding it at the floating rate.

We size the swap notional using **DV01 matching** — the standard approach for hedging interest-rate risk. We need the swap's DV01 to fill the gap between asset DV01 and liability DV01:

$$\text{Notional} = \frac{\text{DV01}_L - \text{DV01}_A}{\text{DV01}_{\text{swap per \$1 notional}}}$$

```python
# Compute asset and liability DV01s (dollar value of 1 bp)
def dv01(pv_func, rate):
    return (pv_func(rate - 0.0001) - pv_func(rate + 0.0001)) / 2

dv01_a = dv01(bond_a.present_value, rate_day1) + dv01(bond_b.present_value, rate_day1)
dv01_l = dv01(spia.present_value, rate_day1) + dv01(term.present_value, rate_day1)
dv01_gap = dv01_l - dv01_a

# Set up a unit-notional 10-year receive-fixed swap at 4 %
swap_tenor = 10
unit_swap = InterestRateSwap(
    notional=1, fixed_rate=0.04, tenor=swap_tenor,
    frequency=2, pay_fixed=False,
)

# Flat floating forecast equal to the current rate
flat_floating = [rate_day1] * unit_swap.n_periods
dv01_per_dollar = unit_swap.dv01(flat_floating, rate_day1)

swap_notional = dv01_gap / dv01_per_dollar
swap_notional = round(swap_notional / 100_000) * 100_000  # round to $100k

# Create the sized swap
swap = InterestRateSwap(
    notional=swap_notional, fixed_rate=0.04, tenor=swap_tenor,
    frequency=2, pay_fixed=False,
)

dv01_df = pl.DataFrame({
    "Section": ["Assets", "Liabilities", "Gap", "Swap", "Swap"],
    "Metric": ["DV01", "DV01", "DV01 Gap", "DV01 per $1", "Required Notional"],
    "Value": [dv01_a, dv01_l, dv01_gap, dv01_per_dollar, swap_notional],
})

(
    GT(dv01_df, rowname_col="Metric", groupname_col="Section")
    .tab_header(title="DV01 Matching")
    .fmt_number(columns="Value", rows=[0, 1, 2], decimals=2)
    .fmt_number(columns="Value", rows=[3], decimals=10)
    .fmt_currency(columns="Value", rows=[4], decimals=0)
)
```

DV01 Matching	Value
Assets	
DV01	4,728.16
Liabilities	
DV01	5,351.69
Gap	
DV01 Gap	623.53
Swap	
DV01 per $1	0.0008175718
Required Notional	$800,000

Hedged portfolio on Day 1

```python
pv_swap1 = swap.present_value(flat_floating, rate_day1)
dv01_swap1 = swap.dv01(flat_floating, rate_day1)

# Portfolio duration including the swap's duration contribution
# Swap duration contribution = swap DV01 * 10000 / PV_assets (approx)
dur_hedged1 = dur_assets1 + (swap.dv01(flat_floating, rate_day1) * 10_000) / pv_assets1

hedged_df = pl.DataFrame({
    "Section": ["Swap", "Swap", "Duration", "Duration", "Duration"],
    "Metric": ["PV", "DV01", "Hedged Assets", "Liabilities", "Gap"],
    "Value": [pv_swap1, dv01_swap1, dur_hedged1, dur_liab1, dur_hedged1 - dur_liab1],
})

(
    GT(hedged_df, rowname_col="Metric", groupname_col="Section")
    .tab_header(title="Day 1 - Hedged Portfolio")
    .fmt_currency(columns="Value", rows=[0], decimals=0)
    .fmt_number(columns="Value", rows=[1, 2, 3, 4], decimals=2)
)
```

Day 1 — Hedged Portfolio	Value
Swap	
PV	$0
DV01	654.06

Day 1 — Hedged Portfolio Value	
Duration	
Hedged Assets	6.84
Liabilities	11.39
Gap	−4.55

Step 4 — Day 2: rates move

Overnight the market rate drops by 50 bps to **3.50 %**.

```
rate_day2 = 0.035
flat_floating_day2 = [rate_day2] * swap.n_periods

# Re-value everything at the new rate
pv_a2 = bond_a.present_value(rate_day2)
pv_b2 = bond_b.present_value(rate_day2)
pv_assets2 = pv_a2 + pv_b2

pv_spia2 = spia.present_value(rate_day2)
pv_term2 = term.present_value(rate_day2)
pv_liab2 = pv_spia2 + pv_term2

pv_swap2 = swap.present_value(flat_floating_day2, rate_day2)

surplus_unhedged_d1 = pv_assets1 - pv_liab1
surplus_unhedged_d2 = pv_assets2 - pv_liab2
surplus_hedged_d1 = pv_assets1 + pv_swap1 - pv_liab1
surplus_hedged_d2 = pv_assets2 + pv_swap2 - pv_liab2

day2_df = pl.DataFrame({
    "Metric": ["Day 1", "Day 2", "Change"],
    "Assets": [pv_assets1, pv_assets2, pv_assets2 - pv_assets1],
    "Liabilities": [pv_liab1, pv_liab2, pv_liab2 - pv_liab1],
    "Hedge": [pv_swap1, pv_swap2, pv_swap2 - pv_swap1],
    "Unhedged": [surplus_unhedged_d1, surplus_unhedged_d2, surplus_unhedged_d2 - surplus_unhe
    "Hedged": [surplus_hedged_d1, surplus_hedged_d2, surplus_hedged_d2 - surplus_hedged_d1],
})

(
    GT(day2_df, rowname_col="Metric")
```

```
    .tab_header(title="Day 2 - Rate Impact", subtitle=f"Market rate: {rate_day2:.2%} (down 50
    .tab_spanner(label="Surplus", columns=["Unhedged", "Hedged"])
    .fmt_currency(columns=["Assets", "Liabilities", "Hedge", "Unhedged", "Hedged"], decimals=
)
```

Day 2 — Rate Impact
Market rate: 3.50% (down 50 bps)

	Assets
Day 1	$8,010,353
Day 2	$8,251,293
Change	$240,940

Key takeaways

- **Without the swap**, the 50 bp rate drop causes liabilities to rise faster than assets, shrinking surplus.
- **With the receive-fixed swap**, the swap gains value as rates fall, offsetting the liability increase and stabilising surplus.
- The swap notional was sized so that the portfolio's duration gap is approximately zero — this is classical **duration matching**.