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The ratio of small to large separations of acoustic oscillations as a diagnostic of the interior of solar-like stars

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Abstract. By considering stellar models with the same interior structure but different outer layers we demonstrate that the ratio of the small to large separations of acoustic oscillations in solar-like stars is essentially independent of the structure of the outer layers, and is determined solely by the interior structure. Defining the scaled Eulerian pressure perturbation $\psi_\ell(\omega, t) = rp'/(\rho c)^{1/2}$ we define the internal phase shift $\delta_\ell(\omega, t)$ through the relation $\omega\psi/(\mathrm{d}\psi/\mathrm{d}t) = \tan(\omega t - \pi\ell/2 + \delta_\ell)$. The δ_ℓ are almost independent of acoustic radius $t = \int \mathrm{d}r/c$ outside the stellar core and can be determined as a continuous functions of ω from *partial wave* solutions for the interior – that is solutions of the oscillation equations for any ω that satisfy the Laplace boundary condition at a sufficiently large acoustic radius t_f outside the stellar core. If the ω are eigenfrequencies then they satisfy the *Eigenfrequency Equation* $\omega T = (n + \ell/2)\pi + \alpha(\omega) - \delta_\ell(\omega)$ where $\alpha(\omega)$ is the ℓ independent surface phase shift (Roxburgh & Vorontsov 2000). Using this result we show that the ratio of small to large separations is determined to high accuracy solely by the internal phase shifts $\delta_\ell(\omega)$ and hence by the interior structure alone. The error in this result is estimated and shown to be smaller than that associated with the errors in the determination of the frequencies ($\approx$0.1–0.3 μHz) from the upcoming space missions MOST, COROT and *Eddington*.

Key words. stars: oscillations

1. Introduction

The upcoming space missions MOST (Mathews 1998), COROT (Baglin et al. 1998) and *Eddington* (Roxburgh et al. 2000; Favata et al. 2000) will observe the photometric time series of flux integrated over the stellar disc, enabling us to determine the oscillation frequencies $\nu_{n\ell m}$ of modes of low degree ℓ. The surface geometry of the modes is described by a spherical harmonic $Y_{\ell m}$, and n is the radial order of the mode. For spherical stars the frequencies are m-degenerate $\nu_{n\ell m} = \nu_{n\ell}$, this degeneracy being lifted by rotation. Low degree modes penetrate deep inside the star, each frequency being a different functional of the variation of density, pressure and adiabatic exponent with radius. In principle, knowledge of a set of frequencies should provide knowledge on the internal structure that can be used to test and develop our understanding of stellar evolution. This is the goal of asteroseismology.

For acoustic p-modes in solar like stars combinations of frequencies often proposed for diagnostic purposes are the large separations Δ_ℓ, and small separations d_{02} and d_{13}, defined by

$$\Delta_\ell(n) = \nu_{n,\ell} - \nu_{n-1,\ell} \tag{1}$$

$$d_{02}(n) = (\nu_{n,0} - \nu_{n-1,2}) \tag{2}$$

$$d_{13}(n) = (\nu_{n,1} - \nu_{n-1,3}). \tag{3}$$

Additionally we will consider the separations d_{01}, initially defined as the 3 point separation $(\nu_{n,1} - 2\nu_{n,0} + \nu_{n+1,1})/2$ (Roxburgh 1993), but here defined as the smoother 5 point separations

$$d_{01}(n) = \frac{1}{8}(\nu_{n-1,0} - 4\nu_{n-1,1} + 6\nu_{n,0} - 4\nu_{n,1} + \nu_{n+1,0}) \tag{4}$$

$$d_{10}(n) = -\frac{1}{8}(\nu_{n-1,1} - 4\nu_{n,0} + 6\nu_{n,1} - 4\nu_{n+1,0} + \nu_{n+1,1}). \tag{5}$$

The large separations for the standard solar reference modelS of Christensen-Dalsgaard et al. (1996) are shown in Fig. 1, whereas Fig. 2 shows the scaled small separations for the same model.

A plot of the average values of d_{02} vs. Δ as a diagnostic tool was introduced by Christensen-Dalsgaard (1988) (see also Mazumdar & Roxburgh 2003), and other combinations of frequencies were considered by Audard & Provost (1994) and Audard et al. (1994).
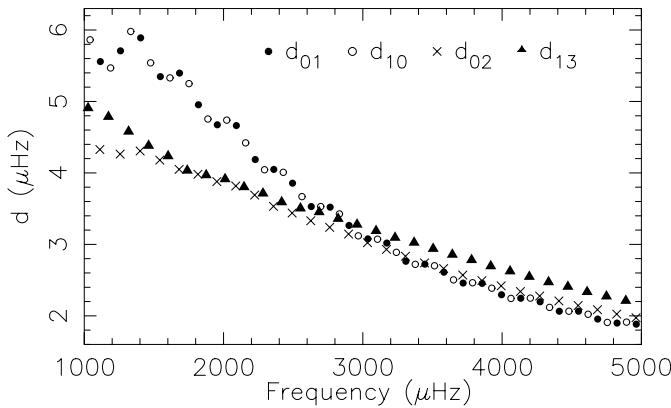
In the asymptotic limit of high frequency Tassoul (1980) found that

$$d_{\ell,\ell+2} \approx \frac{2\ell + 3}{4\pi^2\nu_{n\ell}T}\left(\frac{c(R)}{R} - \int_0^R \frac{\mathrm{d}c}{r}\right); \quad T = \int_0^R \frac{\mathrm{d}r}{c} \tag{6}$$

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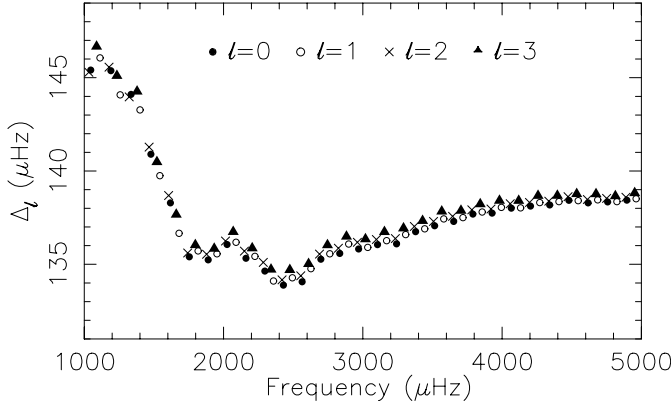
Fig. 1. Large separations Δ_ℓ for ModelS and $\ell = 0, 1, 2, 3$.

Fig. 2. Scaled small separations $d_{02}/3$, $d_{13}/5$, d_{01}, d_{10}: ModelS.

Fig. 3. Large separations Δ_ℓ, $\ell = 0, 1, 2, 3$, for all 4 models.

Fig. 4. Small separations d_{02} for all 4 models.

Fig. 5. Small separations d_{13} for all 4 models.

being the acoustic radius of the star and c the sound speed. The dependence on the derivative of c, which changes sign in the solar core, suggests that the small separations give a diagnostic of the deep interior of the star. In fact the Tassoul asymptotic result gives a poor fit both to the small separations of stellar models and to the observed values for the Sun. A much better fit was obtained by Roxburgh & Vorontsov (1994) using a distorted wave Born approximation.
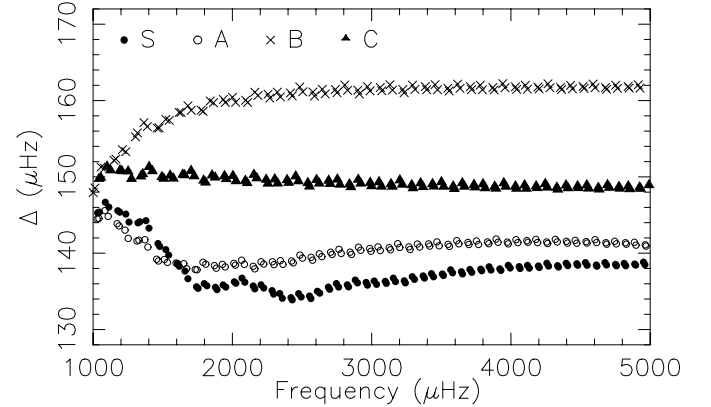
2. Contribution of the outer layers of a star

To examine the effect of the outer layers of a star on the separations we construct a set of 4 models with exactly the same interior structure but with different outer envelopes for $r \geq r_f$. That is $P(r), \rho(r), M_r(r)$ are unchanged for $r \leq r_f$. One model is modelS itself, the other three are:

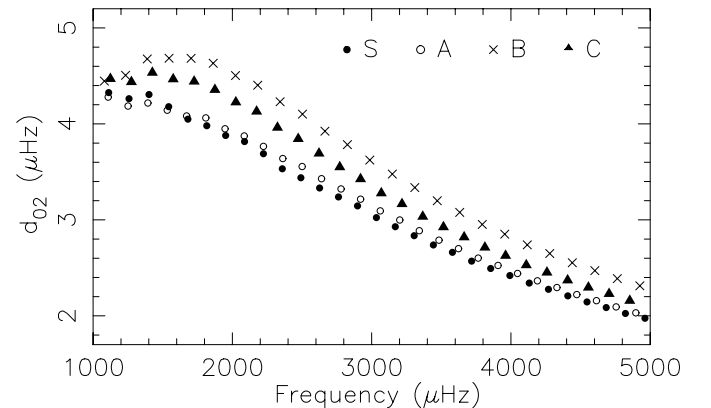**Model A:** P, ρ, M_r unchanged for all r but $\Gamma_1 = 5/3$ for all r. Since $\Gamma_1 \approx 5/3$ for $r < 0.95\,R_\odot$ this is almost the same as just changing the value of Γ_1 for $r > 0.95\,R_\odot$.

Model B: For $r \geq r_f = 0.9\,R_\odot$ the structure of the envelope is determined by a linear variation of polytropic index $n = n_0 + n_1(r - r_f)$ with n continuous at $r = r_f$. The model has a radius $1\,R_\odot$ and mass of $1\,M_\odot$ and Γ_1 is the same as in modelS.
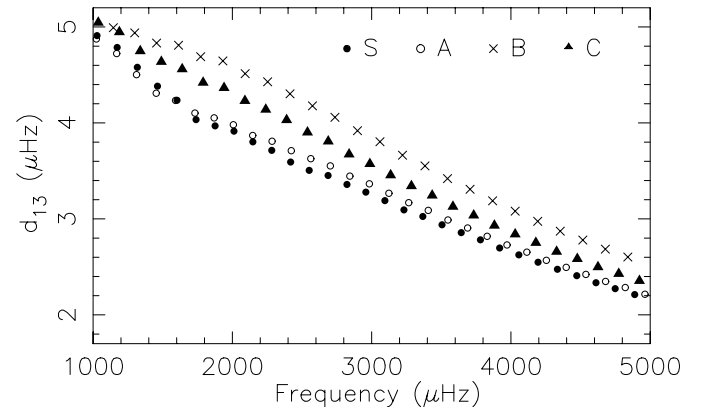
Model C: For $r \geq r_f = 0.72$ the envelope is adiabatic with $\Gamma_1 = 5/3$. This model has radius of $0.995\,R_\odot$.

Figure 3 shows the large separations Δ_ℓ for all 4 models and for $\ell = 0, 1, 2, 3$. The small separations d_{02}, d_{13}, d_{01} and d_{10} are shown in Figs. 4–6. It is clear that the structure of the outer layers plays a significant role in determing both large and small separations.

3. The ratio of small to large separations

We define the ratios r_{ij} of small to large separations as

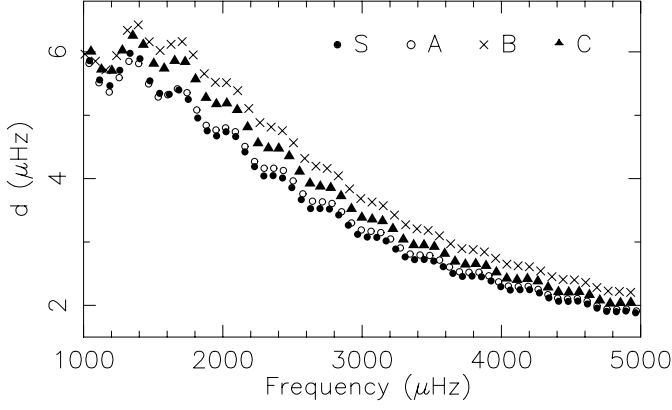$$r_{02}(n) = \frac{d_{02}(n)}{\Delta_1(n)}, \quad r_{13}(n) = \frac{d_{13}(n)}{\Delta_0(n+1)} \tag{7}$$

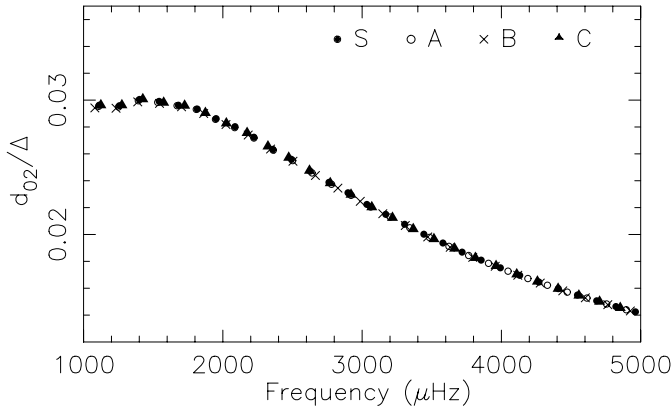**Fig. 6.** Small separations d_{01}, d_{10} for all 4 models.

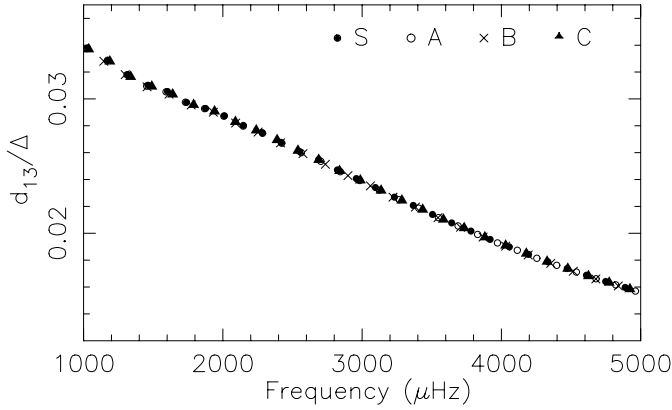**Fig. 7.** Ratio $r_{02} = d_{02}/\Delta_1$ for all 4 models.

Fig. 8. Ratio $r_{13} = d_{13}/\Delta_0$ for all 4 models.

$$r_{01}(n) = \frac{d_{01}(n)}{\Delta_1(n)} \quad r_{10}(n) = \frac{d_{10}(n)}{\Delta_0(n+1)}. \tag{8}$$

Figures 7–9 show these ratios for all 4 models. As can be seen from these figures the ratios r_{ij} are essentially the same for all 4 models. Since the modified models are identical to the unmodified modelS in the inner layers, but differ in the outer layers, this demonstrates empirically that the ratios r_{ij} of small to large separations are independent of the structure of the outer layers of a star, and therefore provide a diagnostic of the stellar interior alone.
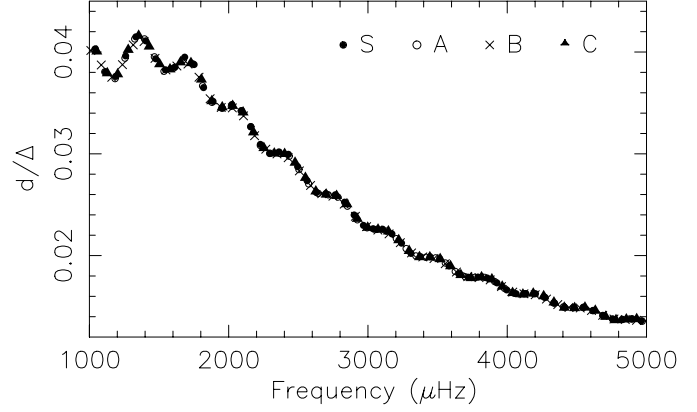

Fig. 9. Ratio $r_{01} = d_{01}/\Delta_1, r_{10} = d_{10}/\Delta_0.$ for all 4 models.

4. Phase shifts and the Eigenfrequency Equation

To understand this result we introduce the concepts of phase shifts and partial waves, and derive the *Eigen-frequency Equation* of Roxburgh & Vorontsov (2000).

The equations governing the oscillations of a spherical star can be expressed as (see e.g. Unno et al. 1979)

$$\frac{d\xi}{dr} + \frac{2}{r}\xi - \frac{g}{c^2}\xi + \left(1 - \frac{\ell(\ell+1)c^2}{\omega^2 r^2}\right)\frac{p'}{\rho c^2} = \frac{\ell(\ell+1)}{\omega^2 r^2}\phi' \tag{9}$$

$$\frac{dp'}{dr} + \frac{g}{c^2}p' + \left(N^2 - \omega^2\right)\rho\xi + \rho\frac{d\phi'}{dr} = 0 \tag{10}$$

$$\frac{d^2\phi'}{dr^2} + \frac{2}{r}\frac{d\phi'}{dr} - \frac{\ell(\ell+1)}{r^2}\phi' = 4\pi G\rho\left(\frac{p'}{\rho c^2} + \frac{N^2}{g}\xi\right) \tag{11}$$

where the perturbations in radius δr, pressure δP and gravitational potential $\delta\Phi$ are decomposed in the form

$$\delta r = \xi(r)Y_{\ell m}e^{i\omega t}, \delta P = p'(r)Y_{\ell m}e^{i\omega t}, \delta\Phi' = \phi(r)Y_{\ell m}e^{i\omega t} \tag{12}$$

with $Y_{\ell m}$ spherical harmonics and ω the angular frequency ($\omega = 2\pi\nu$). The sound speed c, Brunt-Väisälä frequency N, and acceleration due to gravity g, are defined as

$$c^2 = \Gamma_1\frac{P}{\rho}, \quad N^2 = \frac{g^2}{c^2}\left(1 - \Gamma_1\frac{d\log\rho}{d\log P}\right) \quad g = \frac{GM_r}{r^2}. \tag{13}$$

These equations are governed by boundary conditions of regularity at the centre, and that at the surface the potential ϕ' matches onto the corresponding ℓ dependent solution of Laplace's equation, and the wave is reflected high in the atmospheric layers. This reflective wave condition is often approximated to the vanishing of the Lagrangian pressure perturbation $p' - \rho g\xi = 0$ at $r = R$ (see e.g. Unno et al. 1979; Vorontsov & Zharkov 1989).
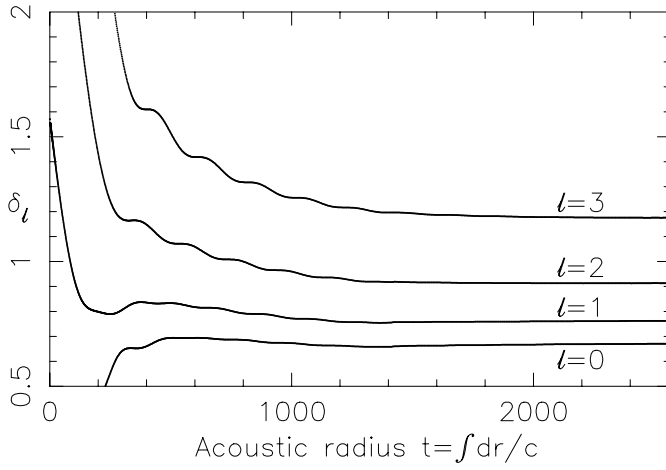
We now define the scaled pressure perturbation ψ, and acoustic radius t, as

$$\psi = \frac{p'r}{(\rho c)^{1/2}}, \quad t = \int_0^r \frac{dr}{c}. \tag{14}$$

The solution $\psi_\ell(t)$ for a particular eigenmode is shown in Fig. 10. $\psi_\ell(t)$ behaves like a Spherical Bessel function $J_\ell(\omega t)$ in the interior which in turn behaves like $\sin(\omega t - \pi\ell/2)$ for
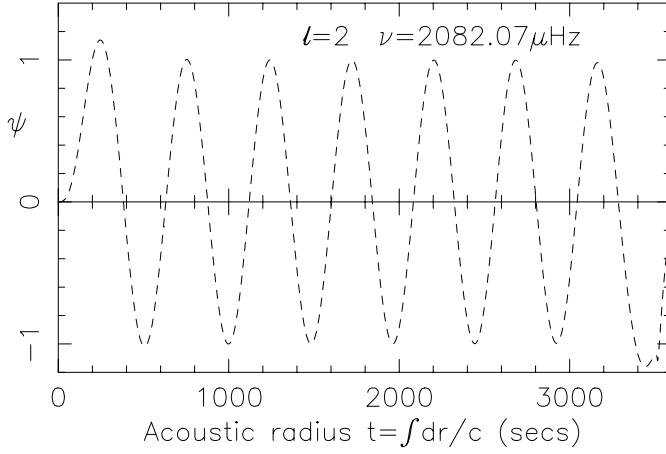
Fig. 10. Eigensolution $\psi_\ell(t)$; $\ell = 2$, $\nu = 2082.07$ μHz.

Fig. 11. Inner phase shifts δ_ℓ versus acoustic radius at $\nu = 2500$ μHz.

Fig. 12. Inner phase shifts $\delta_\ell(\nu)$ at $r/R_\odot = 0.9$.

Fig. 13. Partial wave solution superposed on eigensolution.

$\omega t > 1$. The J_ℓ are the exact solutions for a uniform non gravitating sphere (Rayleigh 1896) and the p-mode eigenfunctions for a real star can be represented as distorted Bessel functions (Roxburgh & Vorontsov 1994).
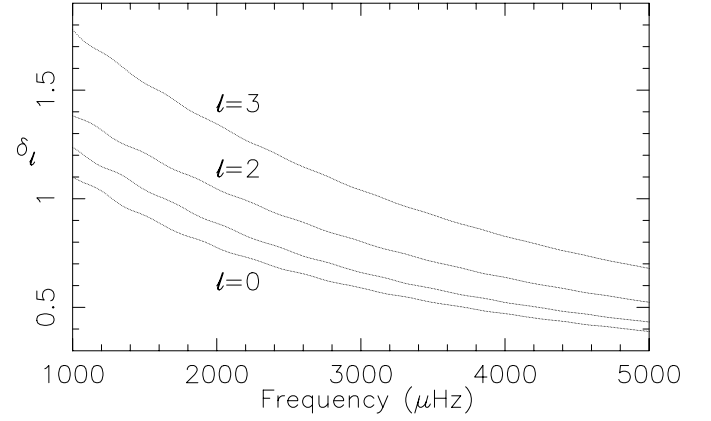
We now represent the ψ in terms of internal phase shifts $\delta_\ell(\omega, t)$ defined by

$$\chi_\ell(\omega, t) = \frac{\omega\psi}{\mathrm{d}\psi/\mathrm{d}t} = \tan[\omega t - \ell\pi/2 + \delta_\ell(\omega, t)] \qquad (15)$$
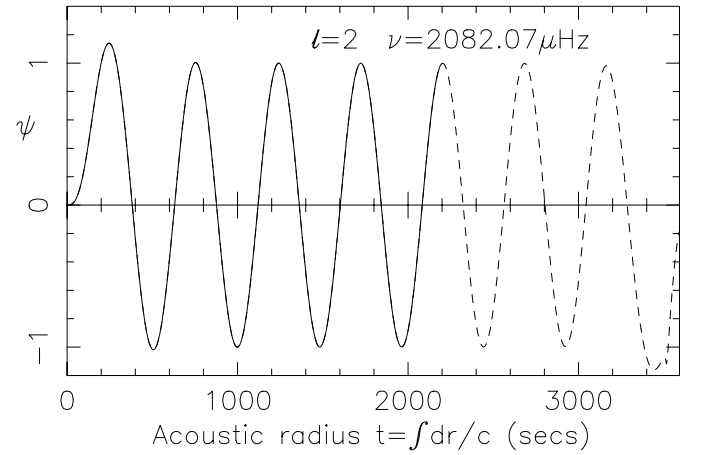
where $\omega = 2\pi\nu$ is the angular frequency. The introduction of the term $\ell\pi/2$ is for convenience and captures the fact that the eigenfunctions behave like Spherical Bessel functions $J_\ell(\omega t)$ in the interior.

In Fig. 11 we show the variation of the $\delta_\ell(\omega, t)$ with acoustic radius t for modelS; the δ_ℓ become almost independent of t outside the stellar core. Figure 12 shows the variation of $\delta_\ell(\omega)$ at $r/R_\odot = 0.9$.

The δ_ℓ can be defined as continuous functions of ω using *partial wave* solutions of the equations governing the oscillations, that is solutions for any ω out to some $t = t_f$ that satisfy the central conditions of regularity, and where the perturbation in gravitational potential ϕ' matches on to the appropriate ℓ dependent solution of Laplace's equation at t_f. In the outer layers of a star the density ρ is very small and the partial wave

solutions reproduce the actual eigensolutions to very high accuracy. This is seen in Fig. 13 where we have superposed the partial wave solution out to $r = 0.9$ $R_\odot$ on the full eigensolution.

We similarly define surface phase shifts α_ℓ by

$$\chi_\ell(\omega, t) = \frac{\omega\psi}{\mathrm{d}\psi/\mathrm{d}t} = -\tan[\omega t_e - \alpha_\ell(\omega, t_e)] \qquad (16)$$

where $t_e = T - t$ is the acoustic depth. In Fig. 14 we show the surface phase shifts α_ℓ as a function of acoustic depth t_e, and in Fig. 15 the variation with frequency at a depth of $r/R_\odot = 0.9$; the α_ℓ collapse to a single function of frequency which is almost independent of acoustic depth t_e below the near surface layers. This behaviour in the outer layers is well known (e.g. Gough 1986; Vorontsov & Zarkhov 1989; Christensen-Dalsgaard & Perez Hernandez 1992) and is due to the very weak dependence of the eigensolution on ℓ when $\ell(\ell + 1)c^2/(\omega r)^2 \ll 1$ for small c and large r, and the perturbation in gravitational potential is also small.

Equations (15) and (16) are two representations of the same function $\chi_\ell(\omega, t)$; equating these at some intermediate acoustic radius t, and recalling that $\tan(\theta + n\pi) = \tan(\theta)$, we obtain the *Eigenfrequency Equation*

$$\omega_{n,\ell}T = (n + \ell/2)\pi + \alpha(\omega_{n,\ell}) - \delta_\ell(\omega_{n,\ell}), \qquad (17)$$
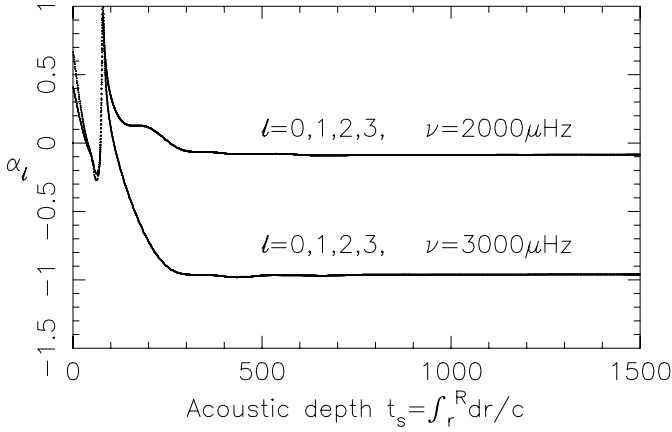
Fig. 14. Outer phase shifts α_ℓ as a function of acoustic depth for frequencies $\nu = 2000, 3000\ \mu$Hz and $\ell = 0, 1, 2, 3$.
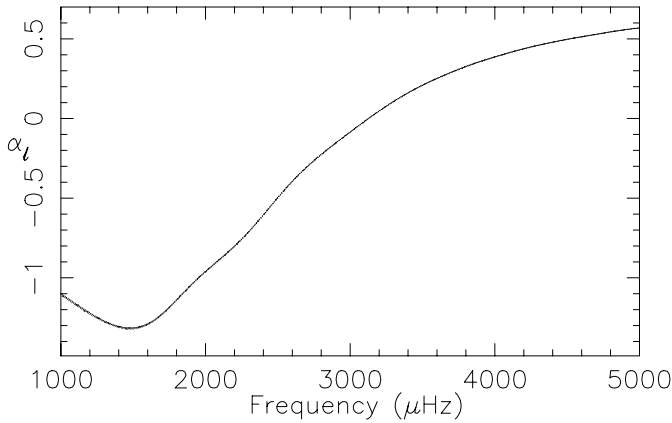
Fig. 15. Outer phase shifts α_ℓ as a function of frequency at $r/R_\odot = 0.9$ for $\ell = 0, 1, 2, 3$.

where T is the acoustic radius, $\delta_\ell(\omega)$ the internal phase shifts, $\alpha(\omega)$ the surface phase shift, and n an integer.

5. Separations in terms of phase shifts

We now derive expressions for the large and small separations in terms of the phase shifts $\alpha(\omega), \delta_\ell(\omega)$, using the *Eigenfrequency Equation* (17).

To lowest order we neglect the variation of α and δ_ℓ with ω between neighbouring frequencies and obtain the approximations

$$\Delta_\ell = \frac{1}{2\pi}[\omega_{n,\ell} - \omega_{n-1,\ell}] = \frac{1}{2T} \tag{18}$$

$$\frac{d_{02}}{\Delta_1} = \frac{1}{\pi}(\delta_2 - \delta_0), \quad \frac{d_{13}}{\Delta_0} = \frac{1}{\pi}(\delta_3 - \delta_1) \tag{19}$$

$$\frac{d_{01}}{\Delta_1} = \frac{1}{\pi}(\delta_1 - \delta_0), \quad \frac{d_{10}}{\Delta_0} = \frac{1}{\pi}(\delta_1 - \delta_0). \tag{20}$$

The ratios depend only on the internal phase shifts δ_ℓ and hence only on the interior structure of the star.

For the calculation to higher order we take, as an example, the ratio r_{02}. We first define a mean angular frequency ω and angular large separation Δ such that

$$\omega_{n,1} = \omega + \frac{1}{2}\Delta, \quad \omega_{n-1,1} = \omega - \frac{1}{2}\Delta \tag{21}$$

and expand α and δ_ℓ around ω to obtain

$$\left(T - \frac{\partial\alpha}{\partial\omega} + \frac{\partial\delta_1}{\partial\omega}\right)\Delta = \pi + \frac{\partial^3}{\partial\omega^3}(\alpha - \delta_1)\frac{\Delta^3}{24} + ... \tag{22}$$

We now define the small separations in angular frequency e, d, as

$$\omega_{n,0} - \omega = e, \quad \omega_{n,0} - \omega_{n-1,2} = d. \tag{23}$$

On expanding α and δ_ℓ about ω we obtain

$$Td = [\delta_2 - \delta_0] + \frac{\partial\alpha}{\partial\omega}d + \frac{\partial\delta_2}{\partial\omega}(e - d) - \frac{\partial\delta_0}{\partial\omega}e + O(d^2) \tag{24}$$

which, on neglecting terms of order d^2, gives

$$\left(T - \frac{\partial\alpha}{\partial\omega} + \frac{\partial\delta_1}{\partial\omega}\right)d = (\delta_2 - \delta_0) + \left(\frac{\partial\delta_1}{\partial\omega} - \frac{\partial\delta_0}{\partial\omega}\right)d \tag{25}$$

where $\delta_2 - \delta_0$ is evaluated at $\omega_{n-1,2} = \omega + e - d$ and all other terms are evaluated at ω.

Dividing Eq. (25) by Eq. (22) gives

$$\frac{d_{02}}{\Delta_1} = \frac{d}{\Delta} = \frac{1}{\pi}(\delta_2 - \delta_0)(1 + \epsilon) \tag{26}$$

where, on using the zero order result $\pi d = \Delta(\delta_2 - \delta_0)$ (cf. Eq. (19)), the error term can be expressed as

$$\epsilon = \frac{1}{\pi}\left(\frac{\partial(\delta_1 - \delta_0)}{\partial\omega}\Delta - \frac{\partial^3}{\partial\omega^3}(\alpha - \delta_1)\frac{\Delta^3}{24}\right) \tag{27}$$

or, on converting to ν,

$$\epsilon = \frac{1}{\pi}\left(\frac{\partial(\delta_1 - \delta_0)}{\partial\nu}\Delta_1 - \frac{\partial^3}{\partial\nu^3}(\alpha - \delta_1)\frac{\Delta_1^3}{24}\right). \tag{28}$$

Referring to Figs. 12 and 15 with $\Delta_1 \approx 140\ \mu$Hz we can estimate the value of $\epsilon \approx 5 \times 10^{-3}$ at low frequencies, decreasing at higher frequencies over the frequency range 1000–5000 μHz. To this order of accuracy the ratio of small to large separation is given by

$$\frac{d_{02}(n)}{\Delta_1(n)} = \frac{1}{\pi}[\delta_2(\nu) - \delta_0(\nu)] \tag{29}$$

the same as obtained by the lower order analysis (Eq. (19)), and is determined solely by the interior structure of the star, independent of the structure of the outer layers.

A parallel analysis can be carried through for the other ratios of small to large separations with the same conclusions; to high accuracy the ratios depend only on the inner phase shifts and hence on the interior structure of the star.

The upcoming space missions MOST, COROT and *Eddington* have as one of their goals the determination of frequencies of solar like oscillations to a precision of 0.1–0.3 μHz. Since the small separations d_{02} are of order 10 μHz the determinations of the ratio d_{02}/Δ_1 have an error of order 1.5% which is somewhat greater than the error ϵ in the above analysis.

6. Conclusions

The oscillation frequencies, large, and small separations depend on the structure of both the inner and outer layers of a star, so model fitting and testing techniques to probe the interior structure of stars are dependent on our having a good understanding of the structure of the outer layers. But these are just the layers where our ignorance is greatest; non adiabatic convection is important but not understood, the oscillations are non adiabatic in the surface layers, and the structure of real stellar atmospheres is poorly understood. For example the oscillation frequencies predicted by the reference solar modelS differ from the observed values by up to 10 μHz at the higher end of the observed frequency range.

By constructing stellar models with the same interior structure $P(r), \rho(r), M_r(r)$ but with different outer layers we found that although the small and large separations differ between the models, the ratios of small to large separations are very closely equal for all the models. We then showed that the eigenmodes of oscillation can be expressed in terms of inner phase shifts $\delta_\ell(\nu)$ and an outer ℓ independent phase shift $\alpha(\nu)$ which satisfy an *Eigenfrequency Equation*

$$2\pi\nu_{n,\ell}T = (n + \ell/2)\pi + \alpha(\nu_{n,\ell}) - \delta_\ell(\nu_{n,\ell}). \tag{30}$$

Using this equation we are able to show that to an accuracy $\approx 5 \times 10^{-3}$ the ratios of small to large separations depend only on the inner phase shifts $\delta_\ell(\nu)$ which are determined solely by the interior structure of the star, and are uninfluenced by the unknown structure of the outer layers. Hence these ratios give valuable diagnostics that can be used to test models of the interior structure of stars using the high precision data sets of p-mode oscillation frequencies from the upcoming space missions MOST, COROT and *Eddington*.

The above analysis can be improved by least squares fitting an observed frequency set to Eq. (30) to extract the inner phase shift differences $\delta_\ell(\nu) - \delta_0(\nu)$ and the combination $2\pi\nu T - \alpha(\nu) + \delta_0(\nu)$. These functions can then be used as the basis of an inversion technique to map the interior structure of a star even though we cannot map the outer layers (Roxburgh & Vorontsov 2002a), and to reveal signatures of the location of regions of rapid change in acoustic variables such as the *HeII* ionisation zone and the base of a convective envelope (Roxburgh & Vorontsov 2001; Roxburgh 2002).

The property that the phases of the inner partial waves $\chi_\ell(\omega, t)$, defined in Eq. (15), collapse onto a function independent of ℓ in the outer layers, is the basis of the *Differential Response* Inversion Technique which has yielded high resolution inversion of helioseismic data (Vorontsov 1998, 2001), and has been successfully applied to simulated small frequency sets for solar-like stars (Roxburgh & Vorontsov 2002b,c). These techniques are currently being further refined and will be reported in subsequent publications.

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