Equations of Motion: Booster

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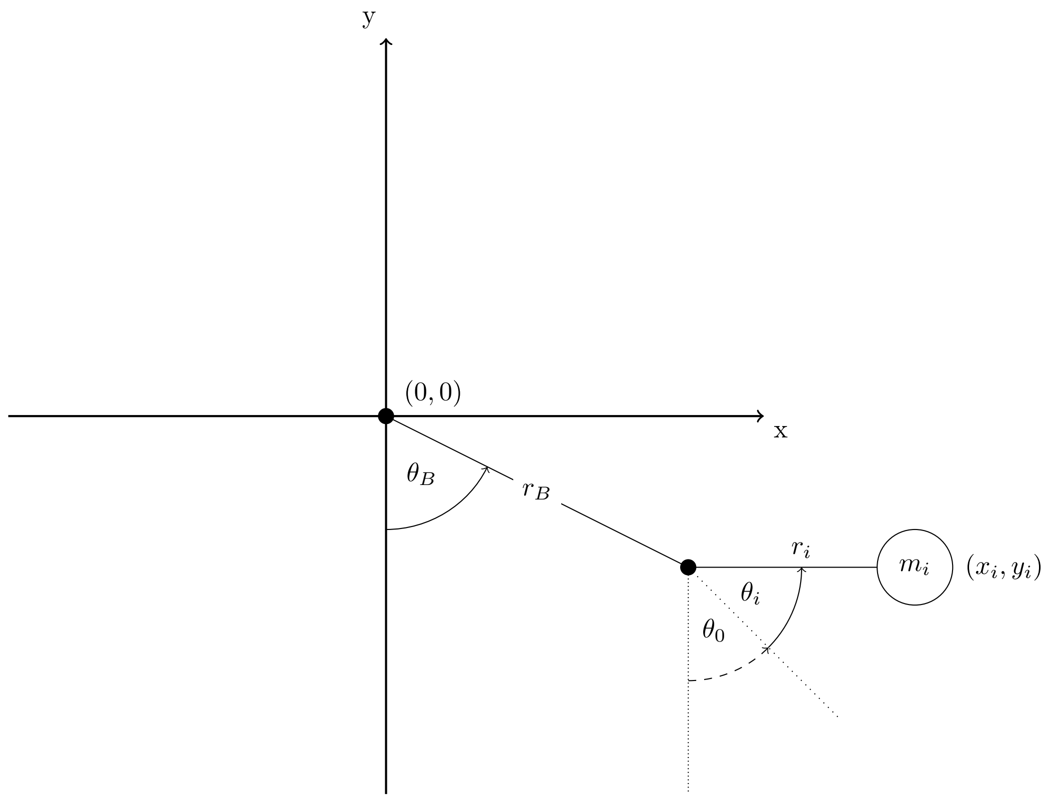
1 Diagram



This represents one half of a typical booster. It assumes that the mass difference between the booster arm and the gondolas is big enough that the effect of the gondolas on the booster arm is negligible.
Furthermore, since we don't know the mass distribution of the gondola we model n masses being connected to the booster arm. Their positions are defined relative to θ_0 so their movement can be tracked using one equation.

2 Assumptions

- Point masses
- Massless, rigid rods
- $\dot{\theta_B}$ and $\ddot{\theta_B}$ are not affected by the motion of the masses. In practice, $\dot{\theta_B}$ will be constant and $\ddot{\theta_B}$ will be 0, but we'll not assume that so they stay present in the final equations
- Gravity is present

3 Trigonometric Identities

$$\cos a \cos b + -\sin a \sin b = \cos a - +b \tag{3.1}$$

$$\sin a \cos b + -\cos a \sin b = \sin a + -b \tag{3.2}$$

$$\sin^2 a + \cos^2 a = 1 \tag{3.3}$$

4 Kinematic Constrains

$$x_i = r_B \sin \theta_B + r_i \sin(\theta_0 + \theta_i) \tag{4.1}$$

$$y_i = -r_B \cos \theta_B - r_i \cos(\theta_0 + \theta_i) \tag{4.2}$$

5 Velocities

$$\dot{x_i} = r_B \cos \theta_B \dot{\theta_B} + r_i \cos(\theta_0 + \theta_i)\dot{\theta_0} \tag{5.1}$$

$$\dot{y_i} = r_B \sin \theta_B \dot{\theta_B} + r_i \sin(\theta_0 + \theta_i)\dot{\theta_0} \tag{5.2}$$

6 Accelerations

$$\ddot{x_i} = -r_B \sin \theta_B \dot{\theta_B}^2 + r_B \cos \theta_B \ddot{\theta_B} - r_i \sin(\theta_0 + \theta_i)\dot{\theta_0}^2 + r_i \cos(\theta_0 + \theta_i)\ddot{\theta_0} \tag{6.1}$$

$$\ddot{y_i} = r_B \cos \theta_B \dot{\theta_B}^2 + r_B \sin \theta_B \ddot{\theta_B} + r_i \cos(\theta_0 + \theta_i)\dot{\theta_0}^2 + r_i \sin(\theta_0 + \theta_i)\ddot{\theta_0} + g \tag{6.2}$$

7 Potential Energy

$$V = g \sum_{i=1}^{n} (m_i y_i) \tag{7.1}$$

Substitute 4.2 into 7.1:

$$V = g \sum_{i=1}^{n} (-m_i r_B \cos \theta_B - m_i r_i \cos(\theta_0 + \theta_i)) \tag{7.2}$$

8 Kinetic Energy

Base formula:

$$T = \frac{1}{2} \sum_{i=1}^{n} (m_i v_i^2) \tag{8.1}$$

Substitute v_1, v_2:

$$T = \frac{1}{2} \sum_{i=1}^{n} (m_i \dot{x_i}^2 + m_i \dot{y_i}^2) \tag{8.2}$$

Substitute 5.1, 5.2 into 8.2:

$$T = \frac{1}{2}\sum_{i=1}^{n}(m_i(r_B\cos\theta_B\dot{\theta_B} + r_i\cos(\theta_0+\theta_i)\dot{\theta_0})^2 + m_i(r_B\sin\theta_B\dot{\theta_B} + r_i\sin(\theta_0+\theta_i)\dot{\theta_0})^2) \tag{8.3}$$

expand the squares:

$$\begin{aligned} T = \frac{1}{2}\sum_{i=1}^{n}(& m_i r_B^2\cos^2\theta_B\dot{\theta_B}^2 + m_i r_i^2\cos^2(\theta_0+\theta_i)\dot{\theta_0}^2 + 2m_i r_B\cos\theta_B\dot{\theta_B}r_i\cos(\theta_0+\theta_i)\dot{\theta_0} \\ &+ m_i r_B^2\sin^2\theta_B\dot{\theta_B}^2 + m_i r_i^2\sin^2(\theta_0+\theta_i)\dot{\theta_0}^2 + 2m_i r_B\sin\theta_B\dot{\theta_B}r_i\sin(\theta_0+\theta_i)\dot{\theta_0}) \end{aligned} \tag{8.4}$$

Simplify using 3.1 and 3.3:

$$T = \frac{1}{2}\sum_{i=1}^{n}(m_i r_B^2\dot{\theta_B}^2 + m_i r_i^2\dot{\theta_0}^2 + 2m_i r_B\dot{\theta_B}r_i\dot{\theta_0}\cos(\theta_B-\theta_0-\theta_i)) \tag{8.5}$$

9 Lagrangian

$$L = T - V \tag{9.1}$$

Substitute 8.5, 7.2 into 9.1:

$$\begin{aligned} L = &\frac{1}{2}\sum_{i=1}^{n}(m_i r_B^2\dot{\theta_B}^2 + m_i r_i^2\dot{\theta_0}^2 + 2m_i r_B\dot{\theta_B}r_i\dot{\theta_0}\cos(\theta_B-\theta_0-\theta_i)) \\ &- g\sum_{i=1}^{n}(-m_i r_B\cos\theta_B - m_i r_i\cos(\theta_0+\theta_i)) \end{aligned} \tag{9.2}$$

Rearrange:

$$\begin{aligned} L = \sum_{i=1}^{n}(&\frac{1}{2}m_i r_B^2\dot{\theta_B}^2 + \frac{1}{2}m_i r_i^2\dot{\theta_0}^2 + m_i r_B\dot{\theta_B}r_i\dot{\theta_0}\cos(\theta_B-\theta_0-\theta_i) \\ &+ g m_i r_B\cos\theta_B + g m_i r_i\cos(\theta_0+\theta_i)) \end{aligned} \tag{9.3}$$

10 Lagrange's Equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0 \tag{10.1}$$

10.1 θ_0

$$\frac{\partial L}{\partial \dot{\theta_0}} = \sum_{i=1}^{n}(m_i r_i^2 \dot{\theta_0} + m_i r_B \dot{\theta_B} r_i \cos(\theta_B - \theta_0 - \theta_i)) \tag{10.2}$$

$$\begin{aligned}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta_0}}\right) = \sum_{i=1}^{n}(m_i r_i^2 \ddot{\theta_0} + m_i r_B \ddot{\theta_B} r_i \cos(\theta_B - \theta_0 - \theta_i) \\ -m_i r_B \dot{\theta_B} r_i \sin(\theta_B - \theta_0 - \theta_i)(\dot{\theta_B} - \dot{\theta_0}))\end{aligned} \tag{10.3}$$

$$\frac{\partial L}{\partial \theta_0} = \sum_{i=1}^{n}(m_i r_B \dot{\theta_B} r_i \dot{\theta_0} \sin(\theta_B - \theta_0 - \theta_i) - g m_i r_i \sin(\theta_0 + \theta_i)) \tag{10.4}$$

Substitute 10.3, 10.4 into 10.1:

$$\begin{aligned}&\sum_{i=1}^{n}(m_i r_i^2 \ddot{\theta_0} + m_i r_B \ddot{\theta_B} r_i \cos(\theta_B - \theta_0 - \theta_i) - m_i r_B \dot{\theta_B} r_i \sin(\theta_B - \theta_0 - \theta_i)(\dot{\theta_B} - \dot{\theta_0})) \\ -&\sum_{i=1}^{n}(m_i r_B \dot{\theta_B} r_i \dot{\theta_0} \sin(\theta_B - \theta_0 - \theta_i) - g m_i r_i \sin(\theta_0 + \theta_i)) \\ =&0\end{aligned} \tag{10.5}$$

Rearrange:

$$\begin{aligned}&\sum_{i=1}^{n}(m_i r_i^2 \ddot{\theta_0} + m_i r_B \ddot{\theta_B} r_i \cos(\theta_B - \theta_0 - \theta_i) - m_i r_B \dot{\theta_B} r_i \sin(\theta_B - \theta_0 - \theta_i)(\dot{\theta_B} - \dot{\theta_0})) \\ &\quad -m_i r_B \dot{\theta_B} r_i \dot{\theta_0} \sin(\theta_B - \theta_0 - \theta_i) + g m_i r_i \sin(\theta_0 + \theta_i)) \\ &= 0\end{aligned} \tag{10.6}$$

Simplify:

$$\begin{aligned}&\sum_{i=1}^{n}(m_i r_i^2 \ddot{\theta_0} + m_i r_B \ddot{\theta_B} r_i \cos(\theta_B - \theta_0 - \theta_i) \\ &\quad -m_i r_B \dot{\theta_B}^2 r_i \sin(\theta_B - \theta_0 - \theta_i) + g m_i r_i \sin(\theta_0 + \theta_i)) \\ &\quad =0\end{aligned} \tag{10.7}$$

Separate sums for separate forces:

$$\begin{aligned}&\ddot{\theta_0} \sum_{i=1}^{n}(m_i r_i^2) + r_B \ddot{\theta_B} \sum_{i=1}^{n}(m_i r_i \cos(\theta_B - \theta_0 - \theta_i)) \\ -&r_B \dot{\theta_B}^2 \sum_{i=1}^{n}(m_i r_i \sin(\theta_B - \theta_0 - \theta_i)) + g \sum_{i=1}^{n}(m_i r_i \sin(\theta_0 + \theta_i))) = 0\end{aligned} \tag{10.8}$$

11 State space

solve for $\ddot{\theta_0}$:

$$\ddot{\theta_0} = \frac{\begin{array}{r} -r_B\ddot{\theta_B}\sum_{i=1}^{n}(m_i r_i\cos(\theta_B-\theta_0-\theta_i)) \\ +\,r_B\dot{\theta_B}^2\sum_{i=1}^{n}(m_i r_i\sin(\theta_B-\theta_0-\theta_i)) \\ -\,g\sum_{i=1}^{n}(m_i r_i\sin(\theta_0+\theta_i)) \end{array}}{\sum_{i=1}^{n}(m_i r_i^2)} \tag{11.1}$$

rewrite as state space:

$$\dot{\vec{y}} = \frac{d}{dt}\left\{\begin{array}{c}\theta_0 \\ \dot{\theta_0}\end{array}\right\} = \left\{\begin{array}{c}\dot{\theta_0} \\ \ddot{\theta_0}\end{array}\right\} \tag{11.2}$$

12 Sensor data

Like all the masses, the Sensor is located at (θ_s, r_s) which is defined relative to θ_0 in the gondola frame. That however, only specifies the location of the sensor and not it's orientation. The sensor is expected to have the same orientation as θ_0.

12.1 Accelerometer

12.1.1 x

Rotate $\ddot{x_s}$ into inertial frame:

$$\ddot{x}_{sinertial} = \ddot{x_s}\cos(-\theta_0) - \ddot{y_s}\sin(-\theta_0) \tag{12.1}$$

Substitute:

$$\begin{aligned} \ddot{x}_{sinertial} = & \\ & -r_B\sin\theta_B\cos(-\theta_0)\dot{\theta_B}^2 + r_B\cos\theta_B\cos(-\theta_0)\ddot{\theta_B} \\ & -r_s\sin(\theta_0+\theta_s)\cos(-\theta_0)\dot{\theta_0}^2 + r_s\cos(\theta_0+\theta_s)\cos(-\theta_0)\ddot{\theta_0} \\ & -r_B\cos\theta_B\sin(-\theta_0)\dot{\theta_B}^2 - r_B\sin\theta_B\sin(-\theta_0)\ddot{\theta_B} \\ & -r_s\cos(\theta_0+\theta_s)\sin(-\theta_0)\dot{\theta_0}^2 - r_s\sin(\theta_0+\theta_s)\sin(-\theta_0)\ddot{\theta_0} \\ & -g\sin(-\theta_0) \end{aligned} \tag{12.2}$$

Simplify using 3.1 and 3.2:

$$\begin{aligned} \ddot{x}_{sinertial} = & \\ & -r_B\dot{\theta_B}^2\sin(\theta_B-\theta_0) + r_B\ddot{\theta_B}\cos(\theta_B-\theta_0) \\ & -r_s\dot{\theta_0}^2\sin(\theta_0+\theta_s-\theta_0) + r_s\ddot{\theta_0}\cos(\theta_0+\theta_s-\theta_0) \\ & -g\sin(-\theta_0) \end{aligned} \tag{12.3}$$

Remove 0/1-terms:

$$\begin{aligned}\ddot{x_s}_{inertial} = & \\ & - r_B\dot{\theta_B}^2 \sin(\theta_B - \theta_0) + r_B\ddot{\theta_B}\cos(\theta_B - \theta_0) \\ & - r_s\dot{\theta_0}^2 \sin\theta_s + r_s\ddot{\theta_0}\cos\theta_s \\ & + g\sin\theta_0 \end{aligned} \tag{12.4}$$

12.1.2 y

Rotate $\ddot{y_s}$ into inertial frame:

$$\ddot{y_s}_{inertial} = \ddot{x_s}\sin(-\theta_0) + \ddot{y_s}\cos(-\theta_0) \tag{12.5}$$

Substitute:

$$\begin{aligned}\ddot{y_s}_{inertial} = & \\ & - r_B\sin\theta_B\sin(-\theta_0)\dot{\theta_B}^2 + r_B\cos\theta_B\sin(-\theta_0)\ddot{\theta_B} \\ & - r_s\sin(\theta_0 + \theta_s)\sin(-\theta_0)\dot{\theta_0}^2 + r_s\cos(\theta_0 + \theta_s)\sin(-\theta_0)\ddot{\theta_0} \\ & + r_B\cos\theta_B\cos(-\theta_0)\dot{\theta_B}^2 + r_B\sin\theta_B\cos(-\theta_0)\ddot{\theta_B} \\ & + r_s\cos(\theta_0 + \theta_s)\cos(-\theta_0)\dot{\theta_0}^2 + r_s\sin(\theta_0 + \theta_s)\cos(-\theta_0)\ddot{\theta_0} \\ & + g\cos(-\theta_0) \end{aligned} \tag{12.6}$$

Simplify using 3.1 and 3.2:

$$\begin{aligned}\ddot{y_s}_{inertial} = & \\ & + r_B\dot{\theta_B}^2 \cos(\theta_B - \theta_0) + r_B\ddot{\theta_B}\sin(\theta_B - \theta_0) \\ & + r_s\dot{\theta_0}^2 \cos(\theta_0 + \theta_s - \theta_0) + r_s\ddot{\theta_0}\sin(\theta_0 + \theta_s - \theta_0) \\ & + g\cos(-\theta_0) \end{aligned} \tag{12.7}$$

Remove 0/1-terms:

$$\begin{aligned}\ddot{y_s}_{inertial} = & \\ & + r_B\dot{\theta_B}^2 \cos(\theta_B - \theta_0) + r_B\ddot{\theta_B}\sin(\theta_B - \theta_0) \\ & + r_s\dot{\theta_0}^2 \cos\theta_s + r_s\ddot{\theta_0}\sin\theta_s \\ & + g\cos\theta_0 \end{aligned} \tag{12.8}$$

12.1.3 Vector in ENU coordinates

$$Accelerometer_{ENU} = \begin{bmatrix} 0 \\ \ddot{x_s}_{inertial} \\ \ddot{y_s}_{inertial} \end{bmatrix} \tag{12.9}$$

12.2 Gyroscope

Since θ_0 is aligned with the sensor:

$$Gyroscope_{ENU} = \begin{bmatrix} \dot{\theta}_0 \\ 0 \\ 0 \end{bmatrix} \tag{12.10}$$

13 Simulate multi-mass booster using single-mass equations

Doing so allows us to determine θ_c and r_c for the center of mass using careful analysis of IMU recordings without ever knowing the exact mass distribution of the gondola. The math in this section serves as a proof that this is in fact possible and to find formulas for θ_c and r_c to then compare the two simulations to see if they really are identical.

The interesting finding here is that while - as expected - θ_c points toward the center of mass as calculated using cartesian coordinates, r_c is actually different from what the center-of-mass calculation tells us.

13.1 Convert from cartesian to polar coordinates

$$r = \sqrt{x^2 + y^2} \tag{13.1}$$

$$\theta = \arctan\left(\frac{x}{-y}\right) \tag{13.2}$$

13.2 Convert from polar to cartesian coordinates

$$x = r\sin\theta \tag{13.3}$$

$$y = -r\cos\theta \tag{13.4}$$

13.3 θ_c

total mass:

$$m_c = \sum_{i=1}^{n} m_i \tag{13.5}$$

cartesian center of mass:

$$R_x = \frac{1}{m_c}\sum_{i=1}^{n}(m_i r_i \sin\theta_i) \tag{13.6}$$

$$R_y = \frac{1}{m_c}\sum_{i=1}^{n}(-m_i r_i \cos\theta_i) \tag{13.7}$$

convert to polar angle(**doing this for the radius would yield a wrong result**):

$$\theta_c = \arctan\left(\frac{R_x}{-R_y}\right) \tag{13.8}$$

$$\theta_c = \arctan\left(\frac{\sum_{i=1}^{n}(m_i r_i \sin\theta_i)}{\sum_{i=1}^{n}(m_i r_i \cos\theta_i)}\right) \tag{13.9}$$

Simplify 11.1 for the special case of a single mass:

$$\ddot{\theta}_{0c} = \frac{-r_B\ddot{\theta}_B \cos\left(\theta_B - \theta_0 - \theta_c\right) + r_B\dot{\theta}_B^{\,2} \sin\left(\theta_B - \theta_0 - \theta_c\right) - g\sin\left(\theta_0 + \theta_c\right)}{r_c} \tag{13.10}$$

If we put the second and the third force in a reference frame relative to θ_0, we can see that they both follow the same scheme with F being some force:

$$\frac{F}{r_c}\sin\theta_c \tag{13.11}$$

Do the same for 11.1:

$$\frac{F}{\sum_{i=1}^{n}(m_i r_i^2)}\sum_{i=1}^{n}(m_i r_i \sin\theta_i) \tag{13.12}$$

We want them to be equal, so let's do that:

$$\frac{F}{r_c}\sin\theta_c = \frac{F}{\sum_{i=1}^{n}(m_i r_i^2)}\sum_{i=1}^{n}(m_i r_i \sin\theta_i) \tag{13.13}$$

Solve for θ_c:

$$\sin\theta_c = \frac{r_c\sum_{i=1}^{n}(m_i r_i \sin\theta_i)}{\sum_{i=1}^{n}(m_i r_i^2)} \tag{13.14}$$

$$\theta_c = \arcsin\left(\frac{r_c\sum_{i=1}^{n}(m_i r_i \sin\theta_i)}{\sum_{i=1}^{n}(m_i r_i^2)}\right) \tag{13.15}$$

We now have two formulas for θ_c, one defined by the center of mass, and one by setting the forces for single- and multi-mass boosters equal. Now we can set those two formulas equal as well:

$$\arcsin\left(\frac{r_c\sum_{i=1}^{n}(m_i r_i \sin\theta_i)}{\sum_{i=1}^{n}(m_i r_i^2)}\right) = \arctan\left(\frac{\sum_{i=1}^{n}(m_i r_i \sin\theta_i)}{\sum_{i=1}^{n}(m_i r_i \cos\theta_i)}\right) \tag{13.16}$$

Now solve for r_c. Start by applying sin:

$$\frac{r_c\sum_{i=1}^{n}(m_i r_i \sin\theta_i)}{\sum_{i=1}^{n}(m_i r_i^2)} = \sin\left(\arctan\left(\frac{\sum_{i=1}^{n}(m_i r_i \sin\theta_i)}{\sum_{i=1}^{n}(m_i r_i \cos\theta_i)}\right)\right) \tag{13.17}$$

apply $\sin(\arctan x) = \frac{x}{\sqrt{x^2+1}}$:

$$\frac{r_c \sum_{i=1}^{n}(m_i r_i \sin\theta_i)}{\sum_{i=1}^{n}(m_i r_i^2)} = \frac{\sum_{i=1}^{n}(m_i r_i \sin\theta_i)}{\sum_{i=1}^{n}(m_i r_i \cos\theta_i)\sqrt{\frac{\left(\sum_{i=1}^{n}(m_i r_i \sin\theta_i)\right)^2}{\left(\sum_{i=1}^{n}(m_i r_i \cos\theta_i)\right)^2}+1}} \tag{13.18}$$

Pull in $\sum_{i=1}^{n}(m_i r_i \cos\theta_i)$ by squaring it:

$$\frac{r_c \sum_{i=1}^{n}(m_i r_i \sin\theta_i)}{\sum_{i=1}^{n}(m_i r_i^2)} = \frac{\sum_{i=1}^{n}(m_i r_i \sin\theta_i)}{\sqrt{\frac{\left(\sum_{i=1}^{n}(m_i r_i \sin\theta_i)\right)^2\left(\sum_{i=1}^{n}(m_i r_i \cos\theta_i)\right)^2}{\left(\sum_{i=1}^{n}(m_i r_i \cos\theta_i)\right)^2}+\left(\sum_{i=1}^{n}(m_i r_i \cos\theta_i)\right)^2}} \tag{13.19}$$

Simplify:

$$\frac{r_c \sum_{i=1}^{n}(m_i r_i \sin\theta_i)}{\sum_{i=1}^{n}(m_i r_i^2)} = \frac{\sum_{i=1}^{n}(m_i r_i \sin\theta_i)}{\sqrt{\left(\sum_{i=1}^{n}(m_i r_i \sin\theta_i)\right)^2+\left(\sum_{i=1}^{n}(m_i r_i \cos\theta_i)\right)^2}} \tag{13.20}$$

move the fraction from the left to the right:

$$r_c = \frac{\sum_{i=1}^{n}(m_i r_i^2)}{\sqrt{\left(\sum_{i=1}^{n}(m_i r_i \sin\theta_i)\right)^2+\left(\sum_{i=1}^{n}(m_i r_i \cos\theta_i)\right)^2}} \tag{13.21}$$

If we do the same calculation with the first force in 13.10 which multiplies by cos instead of sin we get the exact same result. This means that r_c and θ_c can be used to simulate any multi-mass booster.

14 Estimating parameters from IMU data

All equations in this chapter assume an IMU that is perfectly positioned and oriented with respect to r_s and θ_s.

14.1 θ_c

Given accelerometer data from a situation without any movement, 12.9 can substituted and be simplified:

$$\ddot{x}_{sinertial} = g\sin\theta_0 \tag{14.1}$$

$$\ddot{y}_{sinertial} = g\cos\theta_0 \tag{14.2}$$

Solving these for θ_0 we get two equations:

$$\theta_0 = \arcsin\left(\frac{\ddot{x}_{sinertial}}{g}\right) \tag{14.3}$$

$$\theta_0 = \arccos\left(\frac{\ddot{y}_{sinertial}}{g}\right) \tag{14.4}$$

Since our data will be noisy we can estimate the real value calculating the average for all recorded samples:

$$\theta_0 = \frac{\sum_{i=1}^{n} \left(\arcsin\left(\frac{\ddot{x}_{sinertial_i}}{g}\right) + \arccos\left(\frac{\ddot{y}_{sinertial_i}}{g}\right) \right)}{2n} \tag{14.5}$$

The center of mass is located at 0 degrees, so:

$$\theta_c = -\theta_0 \tag{14.6}$$

14.2 r_c

14.2.1 θ_0 at any given point

Since there's no closed form solution to 13.10, a root-finding algorithm has to be used to find θ_0 for any given acceleration. Since manual analysis has shown that there's two solutions for most accelerations I ended up brute-forcing that by iterating over all possible values for θ_0 in appropiately small steps and return the two closest solutions that have a certain distance from each other.
This can be done for the accelerations of both the x-axis and the y-axis. The correct result is the one that appears twice in the combined list of results. Since the results may have slight differences, they have to be identified by looking for the two values with the smallest difference. The approximation can be improved by using the average of these two values as the final result for θ_0.

14.2.2 r_c

Solve 13.10 for r_c:

$$r_c = \frac{-r_B\ddot{\theta_B}\cos(\theta_B - \theta_0 - \theta_c) + r_B\dot{\theta_B}^2\sin(\theta_B - \theta_0 - \theta_c) - g\sin(\theta_0 + \theta_c)}{\ddot{\theta}_{0c}} \tag{14.7}$$

Due to noise this has to be done for every sample and then averaged appropiately. Samples with small values for $\ddot{\theta}_{0c}$ should be skipped because of the division by that value. Negative or otherwise impossible results for r_c should also be skipped. The approximation is most accurate when $\dot{\theta_B}$ and $\ddot{\theta_B}$ are 0.

14.3 Fine-tuning r_c and finding friction

By running a simulation with the acquired values for a situation where $\ddot{\theta_B} = 0$ and overlaying it on top of the IMU data, r_c and friction can be fine tuned to be as close as possible. It's unlikely to simulate a whole ride correctly and that's not really necessary because that's what kalman filters are for.
Friction scales with $(\dot{\theta_0} - \dot{\theta_B})$ which is the velocity between the booster and the gondola.