Report on Integration: Monte Carlo

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1 Introduction

This report details the calculation of the ratio of the volume of an n-sphere to its corresponding circumscribed n-cube using the Monte Carlo integration method. In the Monte Carlo integration method, points are randomly selected in the n-cube, and the proportion of points inside the n-sphere approximates the volume ratio between the two objects.

Empirically, the volume ratio can be found as

$$V(n) = \frac{2(2\pi)^{\frac{n-1}{2}}}{n!!2^n} \tag{1}$$

where $n!!$ represents the double factorial with a scale factor of $\sqrt{\frac{2}{\pi}}$ for even n.

1.1 Various Notes

In this project, all code was completed in Python and all plots were generated with Matplotlib. To verify the balanced nature of the psuedorandom number generator used, a histogram with 1000 equally sized bins over the interval $[-2, 2]$ was produced for 2000000 points, which is presented in Fig. 1.

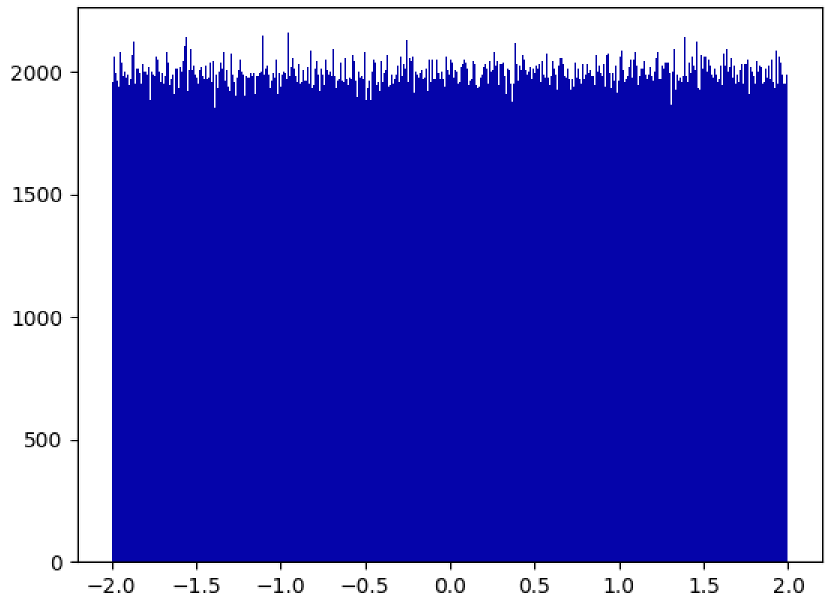

Figure 1: Figure 1 depicts the RNG histogram for 1000 equally sized bins over the interval $[-2, 2]$ with 2000000 points.

As seen in Fig. 1, the distribution is relatively even, implying that the RNG should be effective for a Monte Carlo simulation.

2 Monte Carlo Tests

For the traditional 2-dimensional case, the model converged well to the value of $\frac{\pi}{4} = 0.785398$ over 100000 epochs of size 40, reaching a value of 0.785764, with a log-scale error on the order of e^{-8}. Convergence graphs are presented below in Fig. 2.

Similar to the two-dimensional case, the Monte Carlo achieved similar convergence in higher dimensions. For example, in six dimensions, the model converged well to the value of $\frac{\pi^3}{384} = 0.08074$ over 400000 epochs of size 40, reaching a value of 0.08065, with a log-scale error on the order of e^{-9}. Convergence graphs are presented below in Fig. 3.

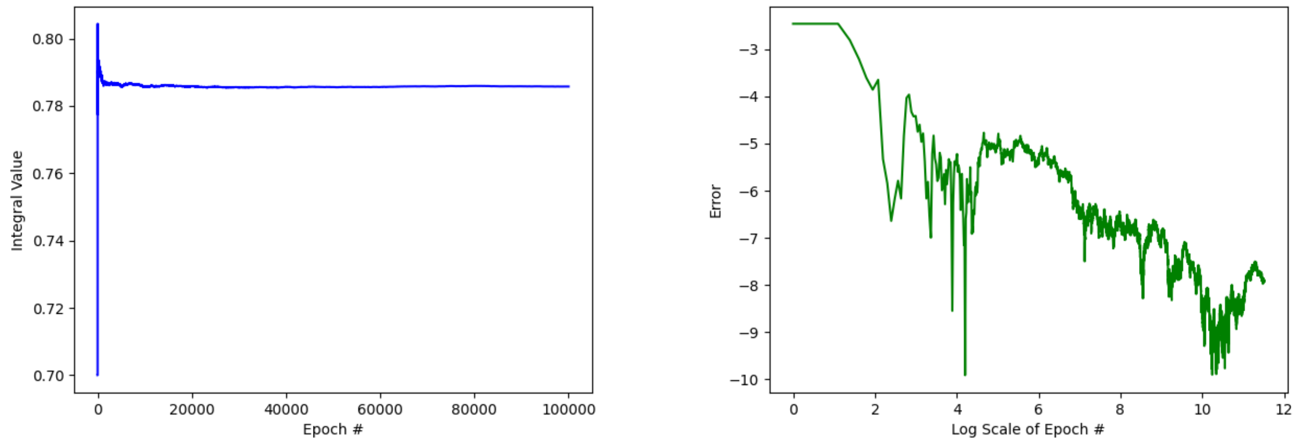

Figure 2: Figure 2 depicts the integral value and log-scale error as they converge as the number of epochs increases.

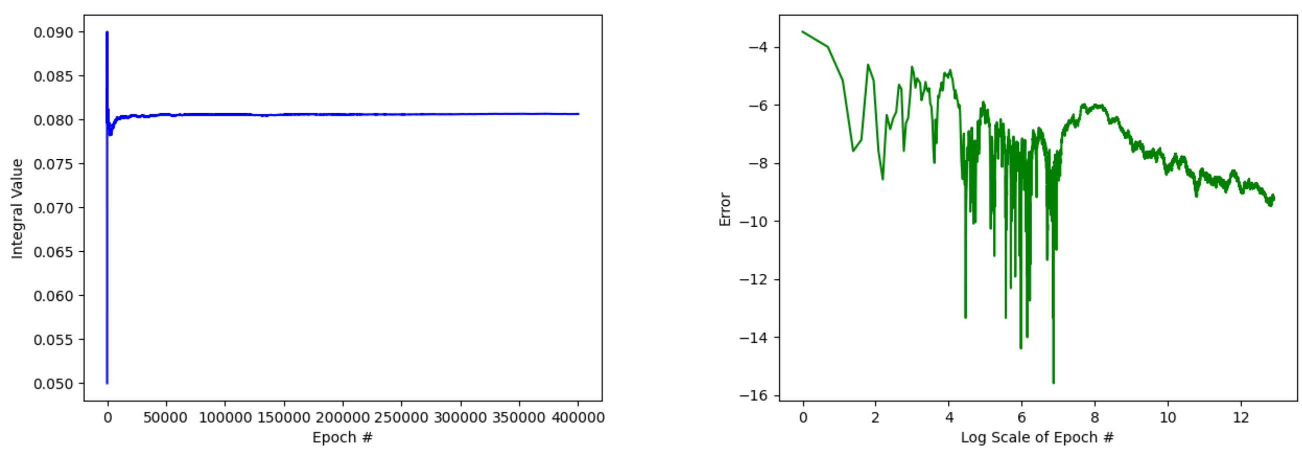

Figure 3: Figure 3 depicts the integral value and log-scale error for the six-dimensional Monte Carlo as they converge as the number of epochs increases.

3 Conclusion

Thus, overall, the Monte Carlo methods presented provided a way to approximate n-dimensional integrals with convergence based on a unbiased random number generator. Specifically, in this lab, the Monte Carlo approximations allowed for the calculation of the volume ratio of an n-sphere to its circumscribing n-cube, demonstrating significant convergence over a large number of epochs.