Pendulum Report

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1 Introduction

This report details the approximation of the motion of a pendulum using both Euler's and the Euler-Cromer method, approximating solutions to the differential equation

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} + \frac{g}{L}\sin(\theta) = 0 \tag{1}$$

As the empirical solution, the small-angle solution for the pendulum was used:

$$\begin{aligned} \theta(t) &= A\cos(\Omega t + \phi) \\ \omega(t) &= -A\Omega\sin(\Omega t + \phi) \end{aligned} \tag{2}$$

For Euler's method, the angular velocity and position were calculated using

$$\begin{aligned} \omega(t+\Delta t) &= \omega(t) - \frac{g}{L}\sin(\theta(t))\Delta t \\ \theta(t+\Delta t) &= \theta(t) + \omega(t)\Delta t \end{aligned} \tag{3}$$

For the Euler-Cromer Method, the same equation for $\omega(t)$ was used, but instead

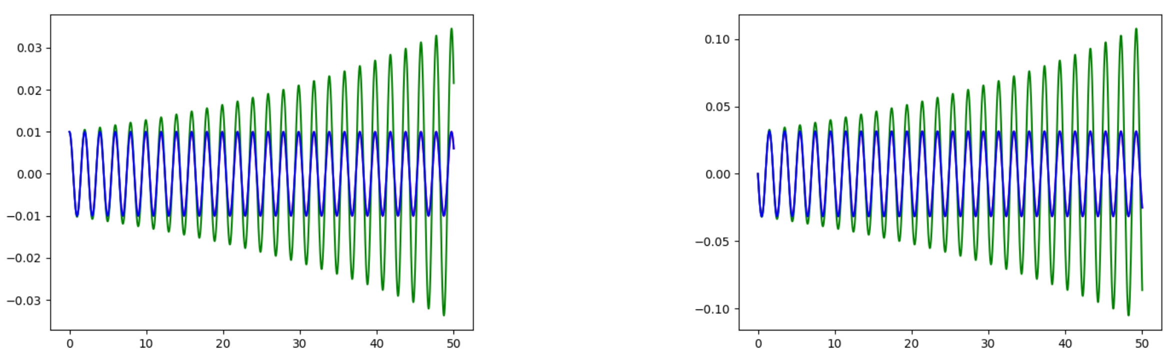
$$\theta(t+\Delta t) = \theta(t) + \omega(t+\Delta t)\Delta t \tag{4}$$

1.1 Various Notes

In this project, all code was completed in Python and all plots were generated with Matplotlib.

2 Small Angle Tests

For small angle measurements of $A = 0.01$, $\phi = 0$, and $\Omega = 3.16$ Hz, both $\omega(t)$ and $\theta(t)$ were plotted for each of the three equations with $\Delta t = 0.005$, yielding as shown in Fig. 1: As seen in both



(a) $\theta(t)$ graphed over 10000 iterations.

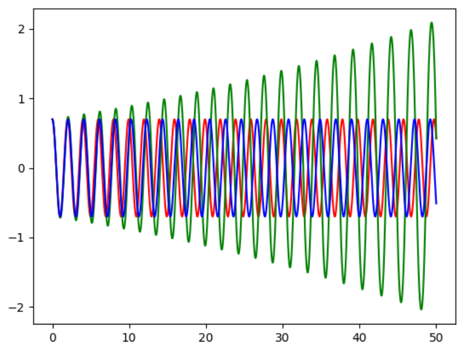
(b) $\omega(t)$ graphed over 10000 iterations.

Figure 1: The plots of $\omega(t)$ and $\theta(t)$ for small angles.

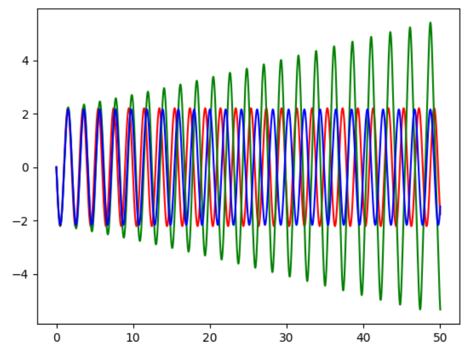
plots, while Euler-Cromer accurately models the behavior of the small-angle solution, Euler grows out of control.

3 Large Angle Tests

For small angle measurements of $A = 0.7$, $\phi = 0$, and $\Omega = 3.16$ Hz, both $\omega(t)$ and $\theta(t)$ were plotted for each of the three equations with $\Delta t = 0.005$, yielding as shown in Fig. 2: As seen in



(a) $\theta(t)$ graphed over 10000 iterations.



(b) $\omega(t)$ graphed over 10000 iterations.

Figure 2: The plots of $\omega(t)$ and $\theta(t)$ for large angles.

both plots, while Euler-Cromer accurately models the behavior of the solution, Euler grows out of control. In addition, Euler-Cromer accurately models the fact that the true large-angle period should be longer than the prediction of the extrapolated small-angle empirical solution, which is shown in the plot.

4 Conclusion

Thus, overall, the Euler-Cromer method accurately models the behavior of a pendulum in both the small and large angle regimes, unlike Euler, which grows out of control.