Laboratory in Oceanography: Data and Methods

Additional Topics - Rotary Spectra

MAR550, Spring 2013

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Additional Topics – Rotary Spectra

Rotary Spectra decompose vector time series (e.g., current or wind data) into clockwise and counter-clockwise components.

Suppose we have *u* and *v* components of velocity:

$$u(t) = A\cos(\omega t) + B\sin(\omega t)$$
$$v(t) = C\cos(\omega t) + D\sin(\omega t)$$

These can be written in complex form as:

$$\begin{aligned} R &= u + iv \\ &= A\cos(\omega t) + B\sin(\omega t) + i\left[C\cos(\omega t) + D\sin(\omega t)\right] \\ &= (A + iC)\cos(\omega t) + (B + iD)\sin(\omega t) \end{aligned}$$

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Now write R as a sum of clockwise and counter-clockwise rotating components as follows:

$$
\begin{aligned}
R &= R^+ e^{i\omega t} + R^- e^{-i\omega t} \\
&= R^+\left(\cos(\omega t) + i\sin(\omega t)\right) + R^-\left(\cos(\omega t) - i\sin(\omega t)\right) \\
&= (R^+ + R^-)\cos(\omega t) + (R^+ - R^-) i\sin(\omega t)
\end{aligned}
$$

(Note: $e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$ rotates counter-clockwise in the complex plane, and $e^{-i\omega t} = \cos(\omega t) - i\sin(\omega t)$ rotates clockwise.)

Comparing this to the final expression on the previous slide, we had:

$$R = (A + iC)\cos(\omega t) + (B + iD)\sin(\omega t)$$

Equating the coefficients of the cosine and sine parts, we find:

$$
\begin{aligned}
R^+ &= \frac{1}{2}\left[A + D + i(C - B)\right] \\
R^- &= \frac{1}{2}\left[A - D + i(C + B)\right]
\end{aligned}
$$

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The magnitudes of the rotary components follow as:

$$\left|R^{+}\right| = \frac{1}{2}\left[\left(A+D\right)^{2}+\left(C-B\right)^{2}\right]^{1/2}$$

$$\left|R^{-}\right| = \frac{1}{2}\left[\left(A-D\right)^{2}+\left(C+B\right)^{2}\right]^{1/2}$$

Note: since the CW and CCW components are rotating at the same frequency but in opposite directions there will be times when they are additive (pointing in the same direction) and times when they are opposing (pointing in opposite direction) and tend to cancel each other out. These additive and opposing times define the major axis = (R^{+}+ R^{-}) and the minor axis = (R^{+}- R^{-}**)** of an ellipse**.**

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While the orientation and phase of the ellipses:

$$\text{orientation}: \quad \theta = \frac{1}{2}\left(\varepsilon^{+} + \varepsilon^{-}\right)$$

$$\text{phase}: \quad \phi = \frac{1}{2}\left(\varepsilon^{+} - \varepsilon^{-}\right)$$

where:

$$\varepsilon^{+} = \tan^{-1}\left(\frac{C-B}{A+D}\right)$$

$$\varepsilon^{-} = \tan^{-1}\left(\frac{C+B}{A-D}\right)$$

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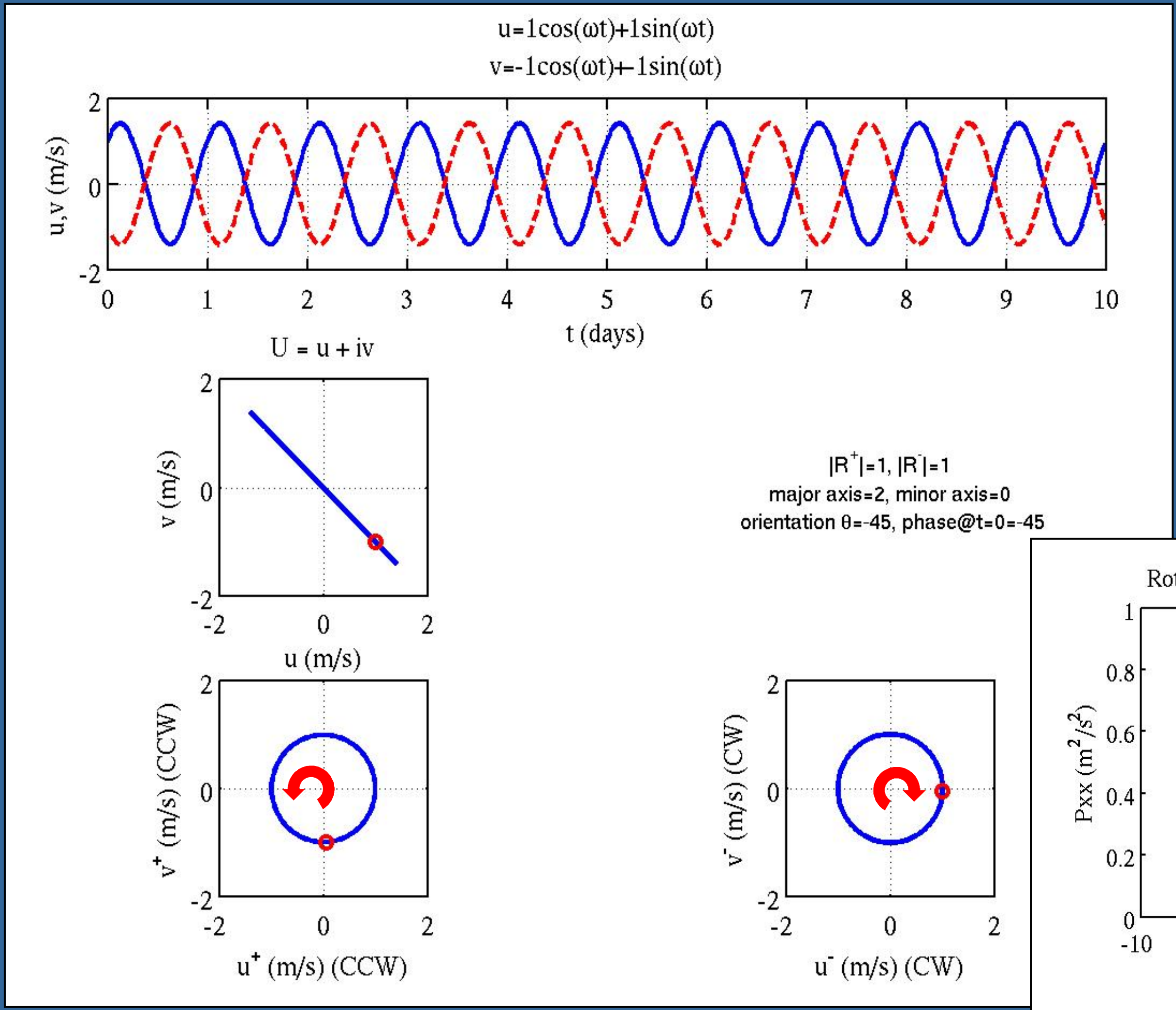
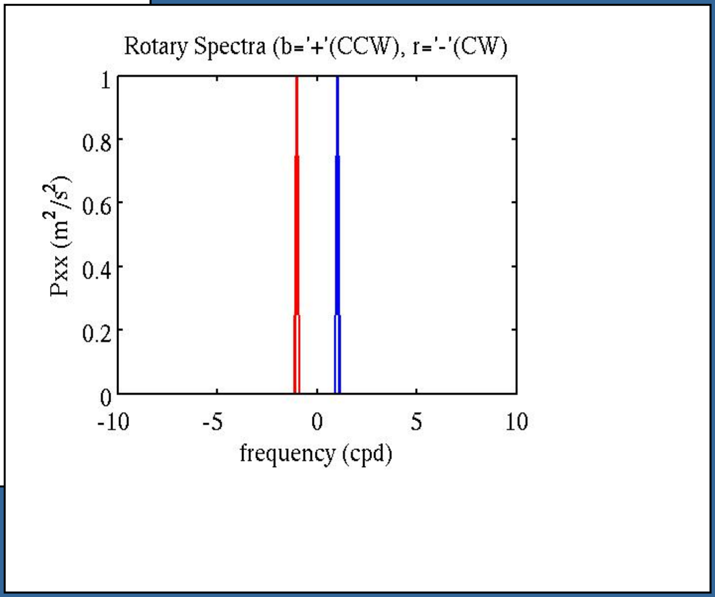
Examples:





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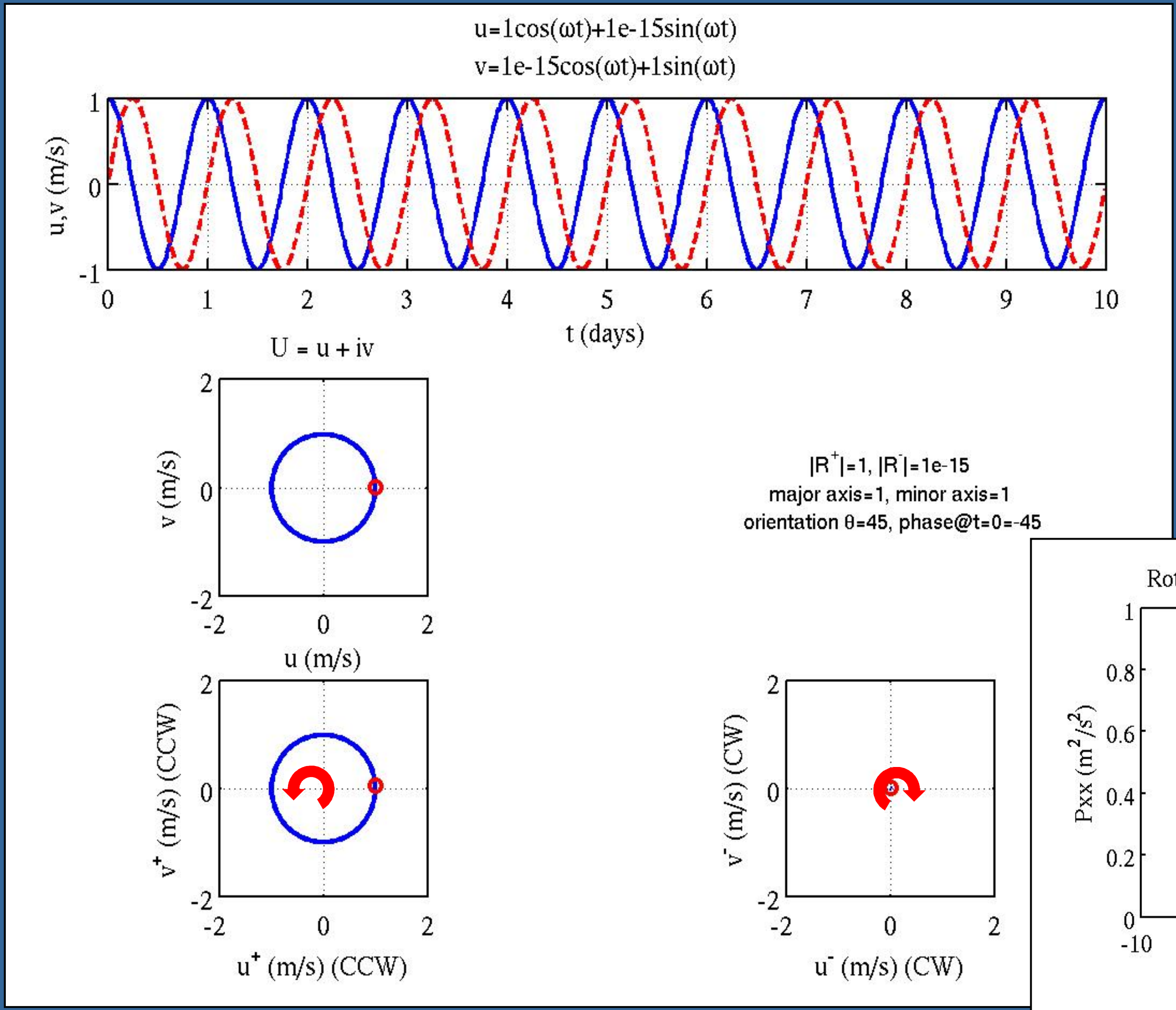
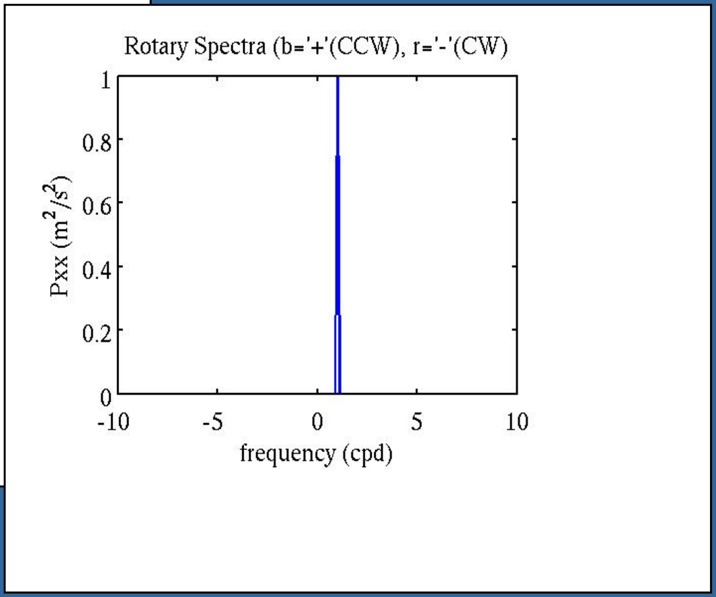
Examples:





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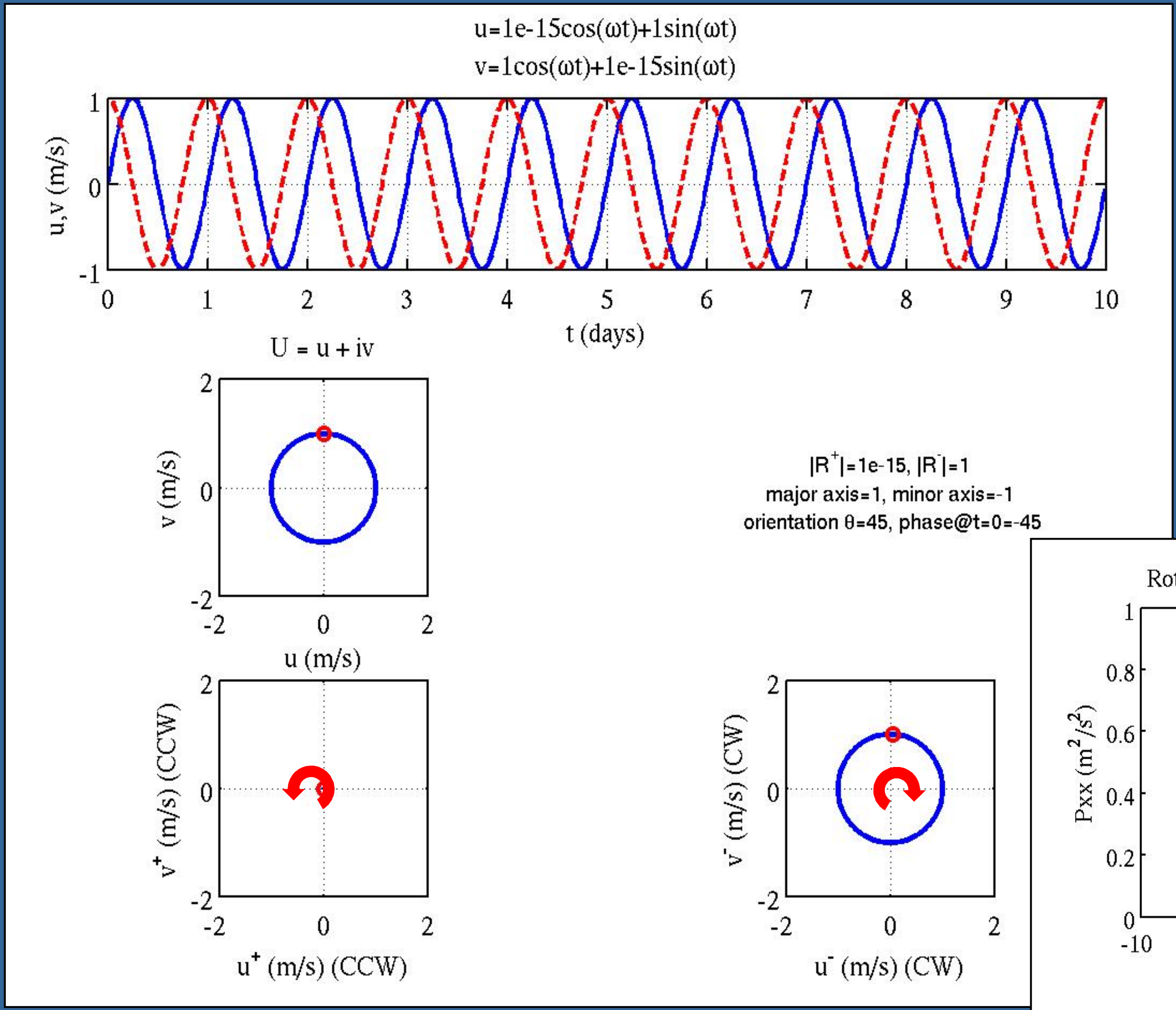
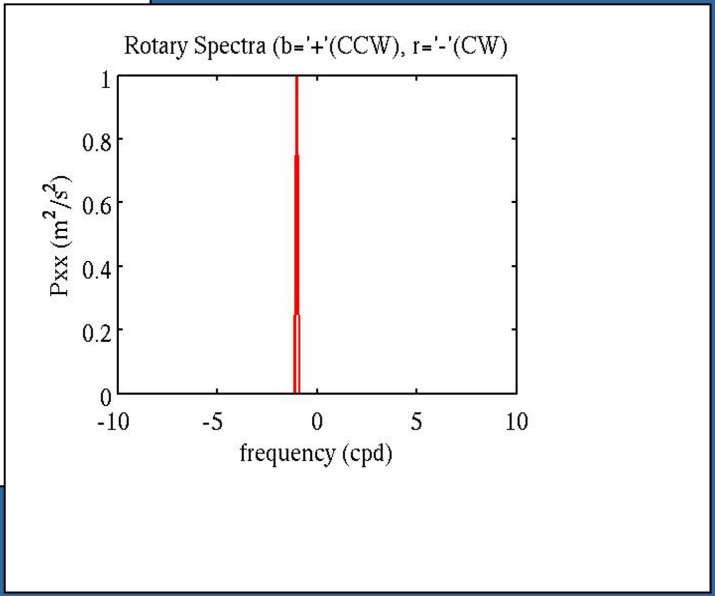
Examples:





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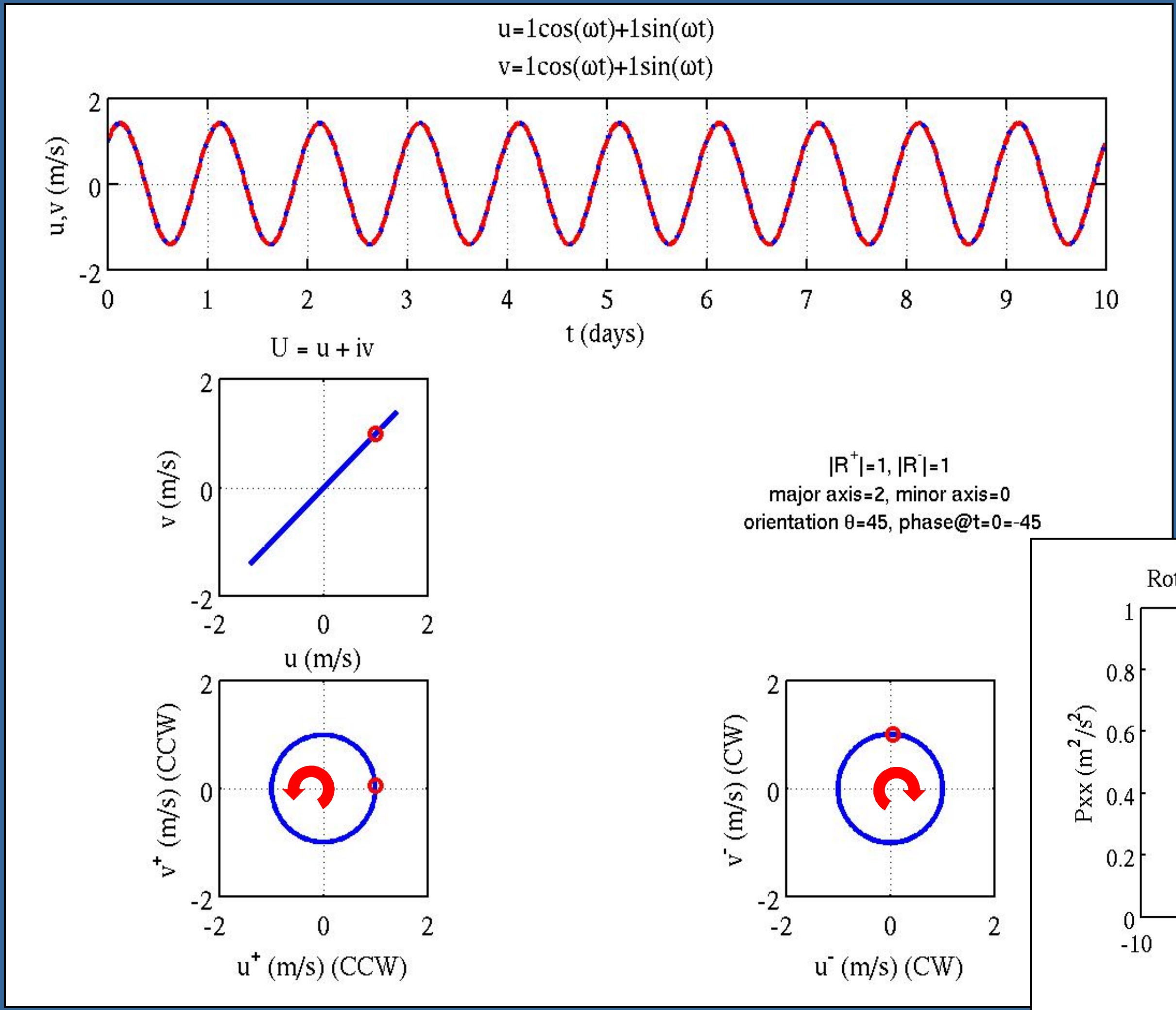
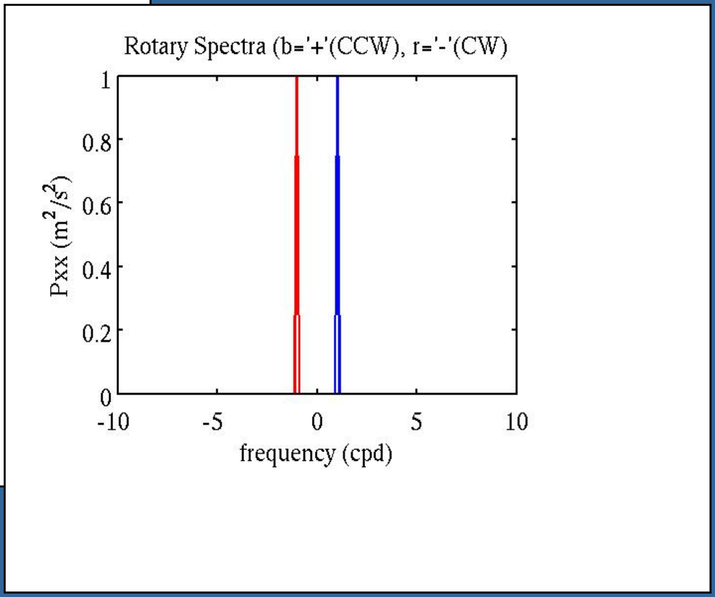
Examples:





Additional Topics – Rotary Spectra

Suppose now that we have two time series, ocean current, and wind ...

Autospectrum: The autospectrum for each time series is:

$$S_{cc} = \begin{cases} [A_c^+(f)]^2, & f \geq 0 \\ [A_c^-(f)]^2, & f \leq 0 \end{cases}$$

$$S_{ww} = \begin{cases} [A_w^+(f)]^2, & f \geq 0 \\ [A_w^-(f)]^2, & f \leq 0 \end{cases}$$

e.g., S_{cc} ($f \geq 0$) is the power spectral density of the counter-clockwise component of the current time-series. The area under this curve versus frequency will equal the variance of the cross-shore and along-shore current velocity components.

Additional Topics – Rotary Spectra

Inner cross-spectrum: The inner cross-spectrum of two time series compares the joint energy of the two time series for the rotary components rotating in the same direction (e.g. the clockwise component of one vector to the clockwise component of the other vector):

$$S_{cw}(f) = \left\langle W_c^*(f) W_w(f) \right\rangle$$

$$= \begin{cases} A_c^+(f) A_w^+(f) e^{[-i(\theta_c^+ - \theta_w^+)]}, & f \geq 0 \\ A_c^-(f) A_w^-(f) e^{[-i(\theta_c^- - \theta_w^-)]}, & f \leq 0 \end{cases}$$

with * denoting the complex conjugate and <•> representing an ensemble average.

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Inner coherence squared: The inner coherence-squared between the wind and current time series at frequency (*f*) is calculated according to:

$$C_{cw} = \begin{cases} \left\{ \left\langle A_c^+ A_w^+ \cos(\theta_c^+ - \theta_w^+) \right\rangle^2 + \left\langle A_c^+ A_w^+ \sin(\theta_c^+ - \theta_w^+) \right\rangle^2 \right\} / \left\langle {A_c^+}^2 \right\rangle \left\langle {A_w^+}^2 \right\rangle, & f \geq 0 \\ \left\{ \left\langle A_c^- A_w^- \cos(\theta_c^- - \theta_w^-) \right\rangle^2 + \left\langle A_c^- A_w^- \sin(\theta_w^- - \theta_w^-) \right\rangle^2 \right\} / \left\langle {A_c^-}^2 \right\rangle \left\langle {A_w^-}^2 \right\rangle, & f \leq 0 \end{cases}$$

The coherence ranges from 0 to 1, and represents the similarity (or variability) of the two time series to each other. A value near unity indicates a high degree of correlation, while a coherence near zero indicates a negligible correlation. Using a 95% confidence interval, a limiting value, or level to which coherence-squared values occur by chance is given by:

$$C_{cw}^{significant} = 1 - 0.5^{[2/(DOF-2)]}$$

where DOF represents the degrees of freedom contained in the time-series.

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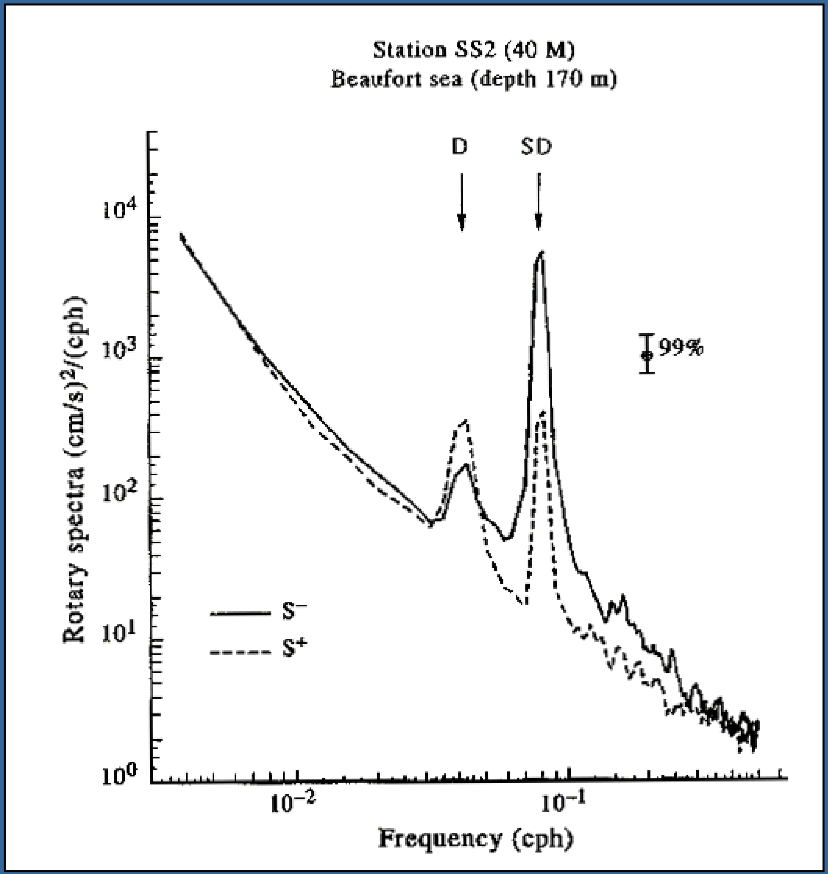
Inner phase: The inner phase for the cross spectrum and coherence measures the phase lead of the rotary component of the one time-series with respect to the other time-series. It can be calculated according to the following equation:

$$\tan(\phi_{cw}) = \begin{cases} \left\{ \left\langle A_c^+ A_w^+ \sin(\theta_c^+ - \theta_w^+) \right\rangle^2 / \left\langle A_c^+ A_w^+ \cos(\theta_c^+ - \theta_w^+) \right\rangle^2 \right\}, & f \geq 0 \\ \left\{ \left\langle - A_c^- A_w^- \sin(\theta_c^- - \theta_w^-) \right\rangle^2 / \left\langle A_c^- A_w^- \cos(\theta_c^- - \theta_w^-) \right\rangle^2 \right\}, & f \leq 0 \end{cases}$$

Additional Topics – Rotary Spectra

Example:



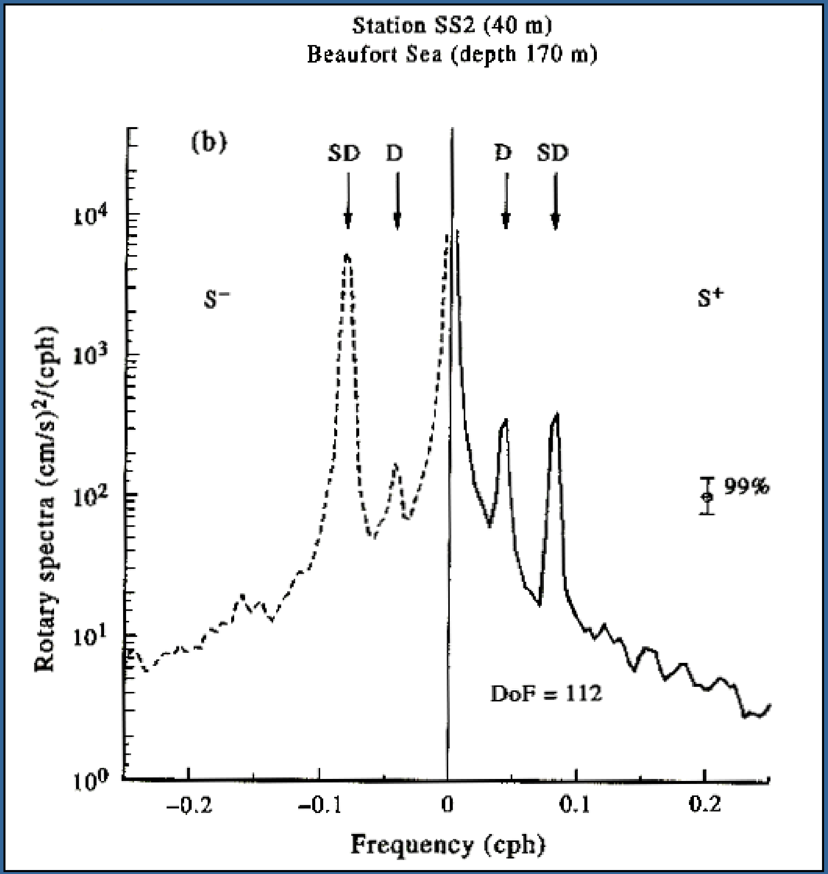


Figure 5.6.13. Rotary current spectra for hourly currents measured at 40-m depth in the Beaufort Sea, Arctic Ocean (water depth = 170 m). Peaks are at the diurnal (D) and semidiurnal (SD) tidal frequencies. Frequency resolution is 0.0005 cph and there are 112 degrees of freedom per spectral band. Vertical bar gives the 99% level of confidence. (a) One-sided rotary spectra, $S^-(f)$ and $S^+(f)$, versus logf for positive frequency, f; (b) two-sided rotary spectra, $S(f_k^+)=S^+$ and $S(f_k^-)=S^-$ versus logf for positive and negative frequencies, $f_k^\pm$. (Courtesy E. Carmack, A. Rabinovich, and E. Kolikov.)

Additional Topics – Rotary Spectra

The following are similarly defined for the rotary components rotating in the opposite direction (e.g. the clockwise component of one vector to the anticlockwise component of the other vector)

- *Outer cross-spectrum*
- *Outer coherence squared*
- *Outer phase*

Additional Topics – Rotary Spectra

Key Points:

- Rotary spectra decompose complex time series into CW and CCW rotating components.
- Complex data could be wind, currents, T & S, etc.
- Can be used to analyze wind, waves or currents and/or to isolate inertial motions, tidal motions, and certain classes of waves.
- Rotary spectra are invariant under coordinate rotation.

References:

- Mooers, C. N. K., 1973. A technique for the cross spectrum analysis of pairs of complex-valued time series, with emphasis on properties of polarized components and rotational invariants. *DSR*, 1973, Vol. 20, 1129-1141

Laboratory in Oceanography: Data and Methods

Additional Topics
Other Cool Stuff in Matlab

MAR550, Spring 2013

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Additional Topics – Other Cool Stuff

Functions

- A function in Matlab is an m-file that allows passing of variables as input and output.
- Advantage/disadvantage is that the workspace within a function is independent from the workspace calling the function (use ‘global’ variable declarations, or ‘evalin’ command to circumvent this)
- Good for tasks that have to be done frequently, or for complicated tasks to help make code more readable.

Additional Topics – Other Cool Stuff

Function Syntax

- Enclose the input argument list in parentheses
- Separate the inputs with commas
- Enclose string arguments with single quotation marks
- Optionally assign any output from the function to one or more output arguments

Examples:

- out = *functionname*(variable, 'string', expression, ...);
- [out1, out2, ..., outN] = *functionname*(in1, in2, ..., inN);

Additional Topics – Other Cool Stuff

Clearing Functions from Memory

- You can use ‘clear’ in any of the following ways to remove functions from the MATLAB workspace.

Syntax	Description
clear <function name>	Remove specified function from workspace.
clear functions	Remove all compiled M-functions.
clear all	Remove all variables and functions.

- Any functions called must be within the scope of (i.e., visible to) the calling function or your MATLAB session.
- The function precedence order determines the precedence of one function over another based on the type of function and its location in MATLAB’s path
- Find which function MATLAB calls using the ‘which’ command, e.g.,

```
>> which mean
C:\Program Files\MATLAB\R2007a\toolbox\matlab\datafun\mean.m
```

Additional Topics – Other Cool Stuff

Example

- Suppose we wish to fit a Gaussian curve to data ...
- Our model is:

$$y = Ae^{\frac{-(x-x_o)^2}{2\sigma^2}}$$

where we wish to determine three parameters: A, x_o, and σ

- To use a *linear* least squares, we would first need to take logarithm of both sides:

$$\ln(y) = \ln\left(Ae^{\frac{-(x-x_o)^2}{2\sigma^2}} \right)$$
$$= \ln(A) - \frac{(x-x_o)^2}{2\sigma^2}$$

- Instead use ‘lsqnonlin’, or similar ‘by hand’ approach to find best fit for three parameters simultaneously

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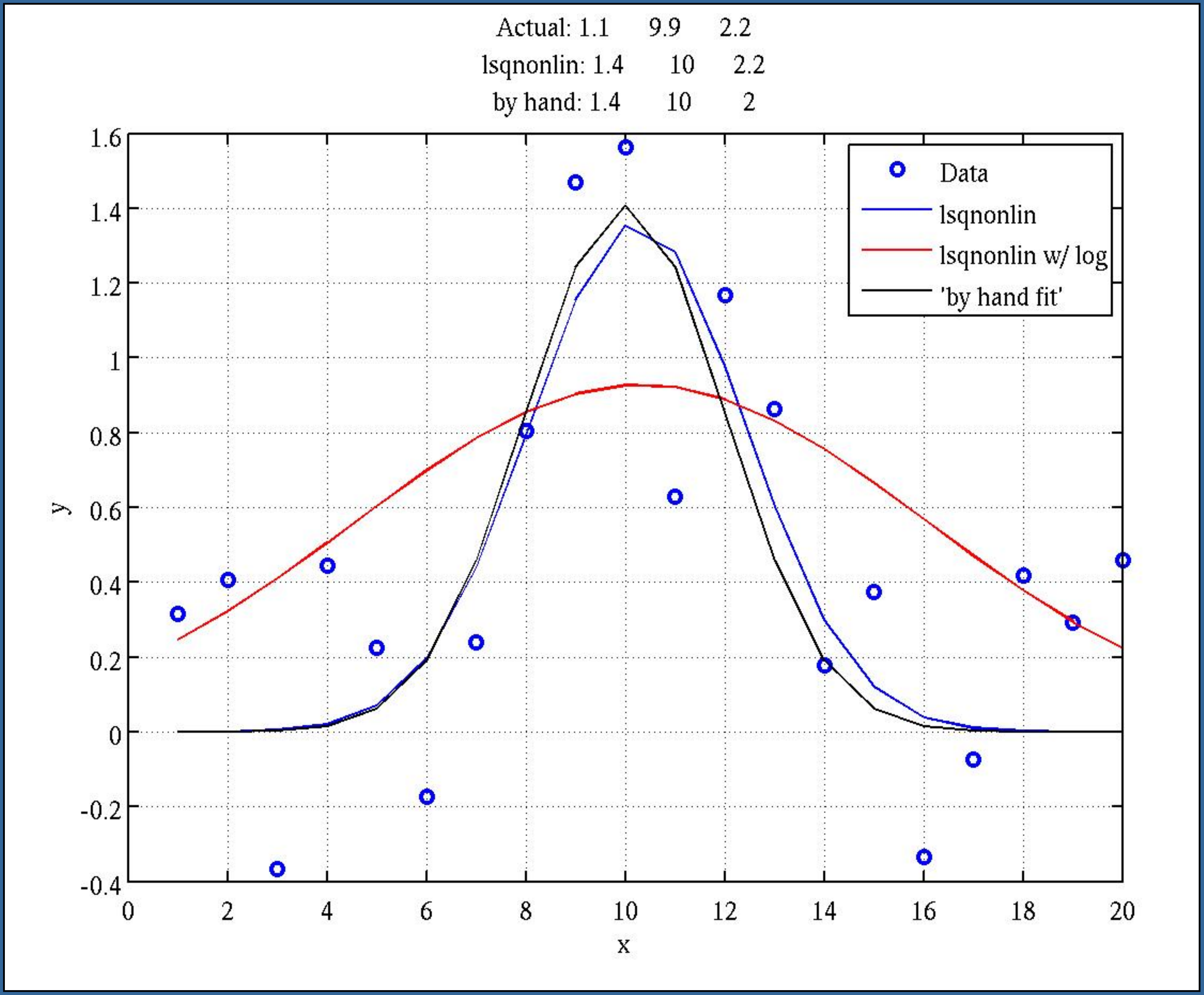

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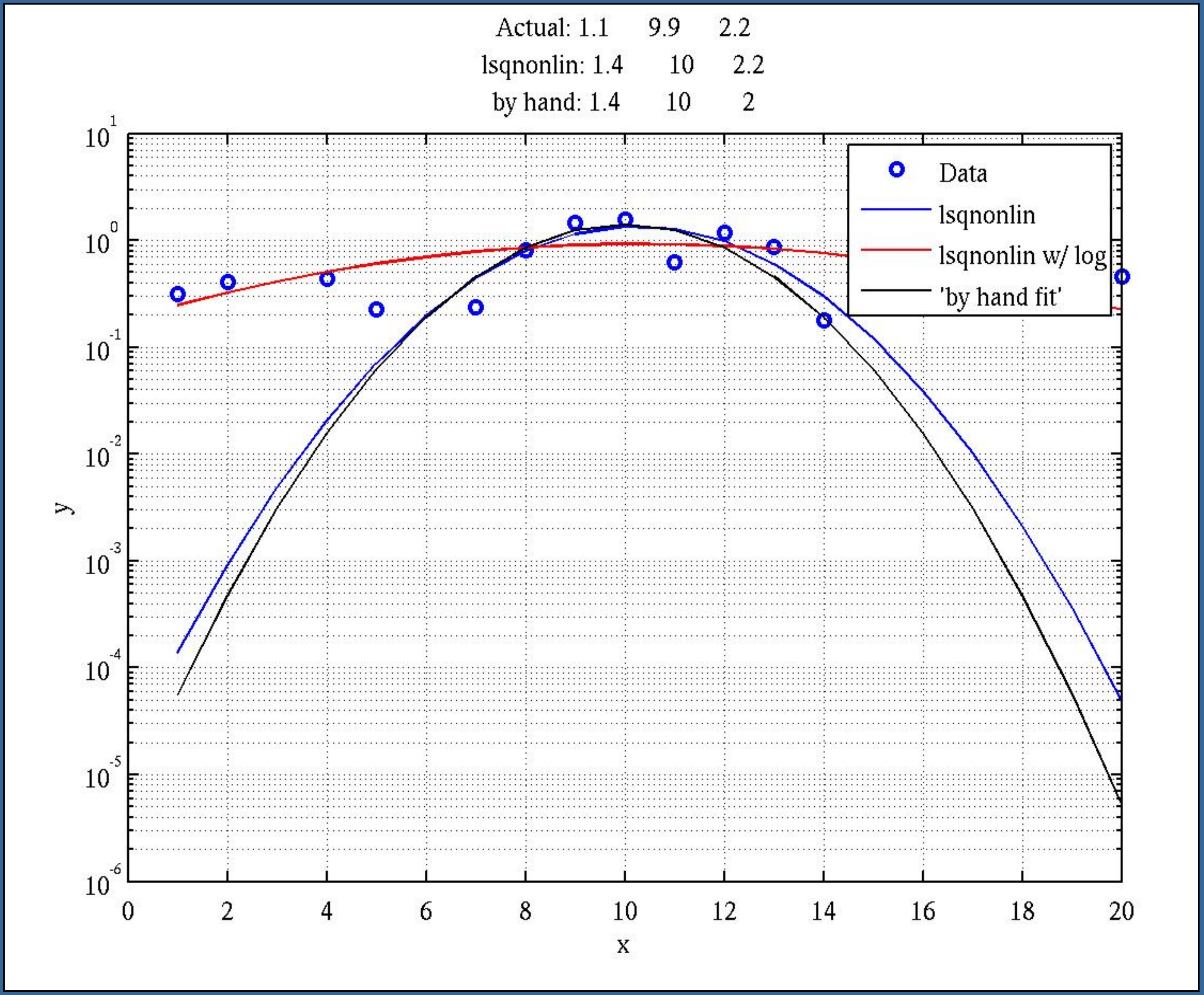

Additional Topics – Other Cool Stuff

Notable Points Re: Functions:

- Use nargin to allow variable number of inputs
- Sometimes useful to pass a function reference to another function, e.g., lsqnonlin

Other Miscellaneous Stuff …

- guide - GUI development environment
- beep.m - beep
- sound.m - play sounds