Stuff with spectrums

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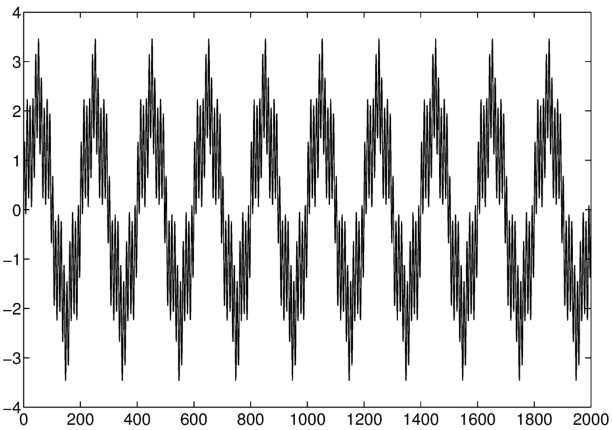
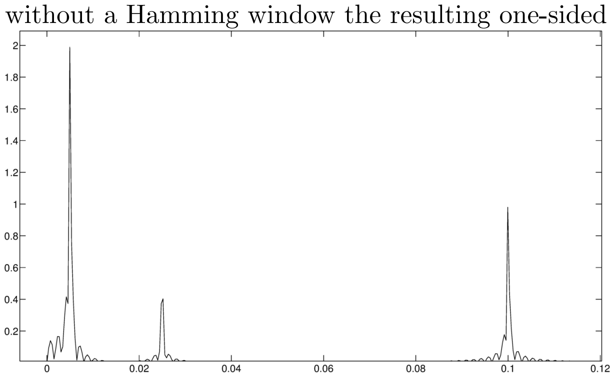
1 Single sided amplitude spectrum

A single sided amplitude spectrum is calculated in AmpSpec.m An example calculation on the signal:

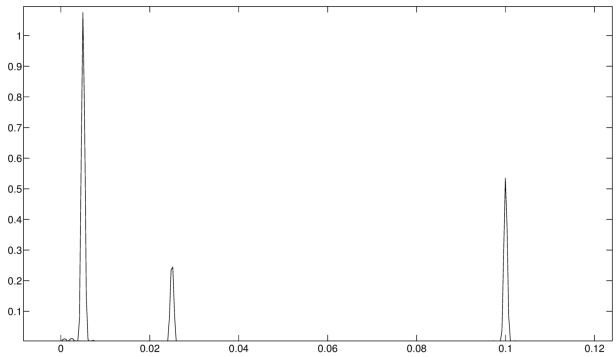
$$x = 0 : 0.01 : 2000$$
$$S = \sin 2\pi x/10 + 0.5 \sin 2\pi x/10 + 2 \sin 2\pi x/200$$



without a Hamming window the resulting one-sided amplitude spectrum is



Note that the amplitudes of the peaks approach the amplitudes of the sin waves at 1, 0.5 and 2. If we include a hamming window without doing any amplitude correction then we get.



Note that the side lobes have dissapeared but that the peak amplitudes have reduced. This is because the hamming window (which multiplies in the time domain) reduces the power of the signals by a constant amount. To correct, we multiply the spectrum by the amplitude correction factor being 1.855 for the hamming window. The result has peaks that approch the correct amplitude but without the side lobes.

2 Spectral Integral

The function periodiogram and bandpower can be used for this.

3 Discrete Fast Fourier Transform

The DFT on a signal of length N is defined as:

$$X_k = \sum_{j=0}^{N-1} x_j e^{-\frac{2\pi i j}{N} k} \tag{1}$$

Here the argument of the exponential runs from $j/N = 0$ to $j/N = N - 1/N$, and thus each value of k represents a sinusoid of period $T = \frac{2\pi}{\omega} = \frac{1}{k}$ on the domain $j/N = [0, 1)$. To get the inverse transform, we multiply by another sinusoid and sum over k:

$$\begin{aligned}
\sum_{k=0}^{N-1} X_k e^{\frac{2\pi i k}{N} m} &= \sum_{k=0}^{N-1} \sum_{j=0}^{N-1} x_j e^{-\frac{2\pi i j}{N} k} e^{\frac{2\pi i k}{N} m} && (2)\\
&= \sum_{j=0}^{N-1} x_j \left(\sum_{k=0}^{N-1} e^{-\frac{2\pi i j}{N}(k-m)} \right) && (3)\\
&= \sum_{j=0}^{N-1} x_j \left(N \delta_{km} \right) && (4)\\
\implies x_m &= \sum_{k=0}^{N-1} \frac{X_k}{N} e^{\frac{2\pi i k}{N} m} && (5)
\end{aligned}$$

Thus we see that the original signal x_j is now expressed as a sum of sinusoids with amplitude $|X_k|/N$ and phase $arg(X_k)$.