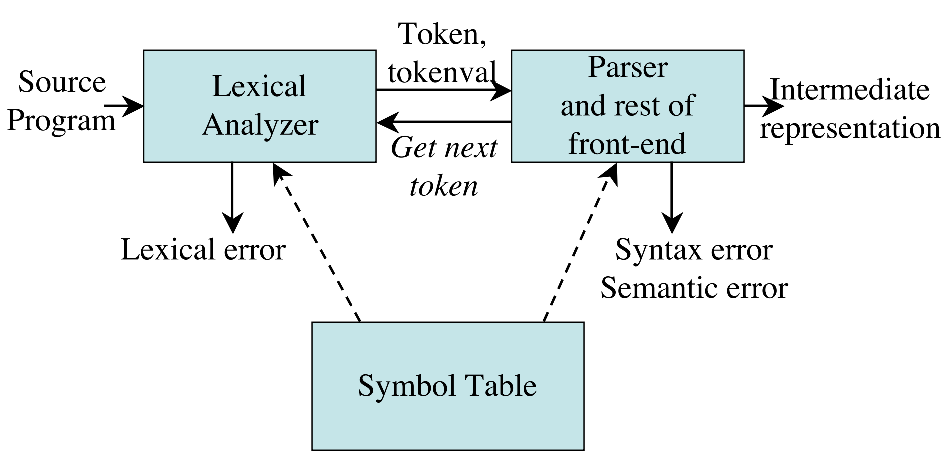
Syntax Analysis
Part I

Chapter 4

Position of a Parser in the Compiler Model



The Parser

- The task of the parser is to check syntax
- The syntax-directed translation stage in the compiler's front-end checks static semantics and produces an intermediate representation (IR) of the source program
  - Abstract syntax trees (ASTs)
  - Control-flow graphs (CFGs) with triples, three-address code, or register transfer lists
  - WHIRL (SGI Pro64 compiler) has 5 IR levels!

Error Handling

- A good compiler should assist in identifying and locating errors
  - *Lexical errors*: important, compiler can easily recover and continue
  - *Syntax errors*: most important for compiler, can almost always recover
  - *Static semantic errors*: important, can sometimes recover
  - *Dynamic semantic errors*: hard or impossible to detect at compile time, runtime checks are required
  - *Logical errors*: hard or impossible to detect

Viable-Prefix Property

- The *viable-prefix property* of LL/LR parsers allows early detection of syntax errors
  - Goal: detection of an error as soon as possible without consuming unnecessary input
  - How: detect an error as soon as the prefix of the input does not match a prefix of any string in the language

Error is detected here

```
...
for (;)
...
```

Prefix

Error is detected here

```
...
DO 10 I = 1;0
...
```

Prefix

Error Recovery Strategies

- *Panic mode*
  - Discard input until a token in a set of designated synchronizing tokens is found
- *Phrase-level recovery*
  - Perform local correction on the input to repair the error
- *Error productions*
  - Augment grammar with productions for erroneous constructs
- *Global correction*
  - Choose a minimal sequence of changes to obtain a global least-cost correction

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Grammars (Recap)

- Context-free grammar is a 4-tuple $G=(N,T,P,S)$ where
  - T is a finite set of tokens (*terminal* symbols)
  - N is a finite set of *nonterminals*
  - P is a finite set of *productions* of the form
    $\alpha \rightarrow \beta$
    where $\alpha \in (N \cup T)^* \, N \, (N \cup T)^*$
    and $\beta \in (N \cup T)^*$
  - S is a designated *start symbol* $S \in N$

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Notational Conventions Used

- Terminals
  $a,b,c,\ldots \in T$
  specific terminals: **0**, **1**, **id**, **+**
- Nonterminals
  $A,B,C,\ldots \in N$
  specific nonterminals: *expr*, *term*, *stmt*
- Grammar symbols
  $X,Y,Z \in (N \cup T)$
- Strings of terminals
  $u,v,w,x,y,z \in T^*$
- Strings of grammar symbols
  $\alpha,\beta,\gamma \in (N \cup T)^*$

Derivations (Recap)

- The *one-step derivation* is defined by
  $\alpha\, A\, \beta \Rightarrow \alpha\, \gamma\, \beta$
  where $A \rightarrow \gamma$ is a production in the grammar
- In addition, we define
  - $\Rightarrow$ is *leftmost* $\Rightarrow_{lm}$ if α does not contain a nonterminal
  - $\Rightarrow$ is *rightmost* $\Rightarrow_{rm}$ if β does not contain a nonterminal
  - Transitive closure $\Rightarrow^{*}$ (zero or more steps)
  - Positive closure $\Rightarrow^{+}$ (one or more steps)
- The *language generated by G* is defined by
  $L(G) = \{w \mid S \Rightarrow^{+} w\}$

Derivation (Example)

$E \rightarrow E \,\mathbf{+}\, E$
$E \rightarrow E \,\mathbf{*}\, E$
$E \rightarrow \mathbf{(}\, E\, \mathbf{)}$
$E \rightarrow \mathbf{-}\, E$
$E \rightarrow \mathbf{id}$

$E \Rightarrow \mathbf{-}\, E \Rightarrow \mathbf{-\ id}$
$E \Rightarrow_{rm} E \,\mathbf{+}\, E \Rightarrow_{rm} E \,\mathbf{+\ id} \Rightarrow_{rm} \mathbf{id + id}$
$E \Rightarrow^{*} E$
$E \Rightarrow^{+} \mathbf{id * id + id}$

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Chomsky Hierarchy: Language Classification

- A grammar G is said to be
  - *Regular* if it is *right linear* where each production is of the form
    $A \rightarrow w\,B$ or $A \rightarrow w$
    or *left linear* where each production is of the form
    $A \rightarrow B\,w$ or $A \rightarrow w$
  - *Context free* if each production is of the form
    $A \rightarrow \alpha$
    where $A \in N$ and $\alpha \in (N \cup T)^*$
  - *Context sensitive* if each production is of the form
    $\alpha\, A\, \beta \rightarrow \alpha\, \gamma\, \beta$
    where $A \in N$, $\alpha,\gamma,\beta \in (N \cup T)^*$, $|\gamma| > 0$
  - *Unrestricted*

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Chomsky Hierarchy

$$\mathcal{L}(\textit{regular}) \subseteq \mathcal{L}(\textit{context free}) \subseteq \mathcal{L}(\textit{context sensitive}) \subseteq \mathcal{L}(\textit{unrestricted})$$

Where $\mathcal{L}(T) = \{\, L(G) \mid G \text{ is of type } T \,\}$
That is, the set of all languages
generated by grammars G of type T

Examples:

Every *finite language* is regular

$L_1 = \{\, \mathbf{a}^n\mathbf{b}^n \mid n \geq 1 \,\}$ is context free

$L_2 = \{\, \mathbf{a}^n\mathbf{b}^n\mathbf{c}^n \mid n \geq 1 \,\}$ is context sensitive

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Parsing

- *Universal* (any C-F grammar)
  - Cocke-Younger-Kasimi
  - Earley
- *Top-down* (C-F grammar with restrictions)
  - Recursive descent (predictive parsing)
  - LL (Left-to-right, Leftmost derivation) methods
- *Bottom-up* (C-F grammar with restrictions)
  - Operator precedence parsing
  - LR (Left-to-right, Rightmost derivation) methods
    - SLR, canonical LR, LALR

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Top-Down Parsing

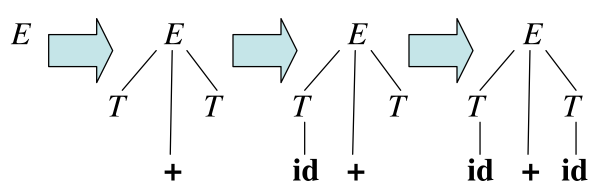
- LL methods (Left-to-right, Leftmost derivation) and recursive-descent parsing

Grammar:

$E \rightarrow T\,\mathbf{+}\,T$
$T \rightarrow \mathbf{(}\,E\,\mathbf{)}$
$T \rightarrow \mathbf{-}\,E$
$T \rightarrow \mathbf{id}$

Leftmost derivation:

$E \Rightarrow_{lm} T\,\mathbf{+}\,T$
$\Rightarrow_{lm} \mathbf{id}\,\mathbf{+}\,T$
$\Rightarrow_{lm} \mathbf{id}\,\mathbf{+}\,\mathbf{id}$



Left Recursion (Recap)

- Productions of the form

  $A \to A\ \alpha$

  $\quad\mid \beta$

  $\quad\mid \gamma$

  are left recursive
- When one of the productions in a grammar is left recursive then a predictive parser may loop forever

General Left Recursion Elimination

Arrange the nonterminals in some order $A_1, A_2, \ldots, A_n$
for $i = 1, \ldots, n$ **do**
  for $j = 1, \ldots, i\text{-}1$ **do**
    replace each
      $A_i \to A_j\ \gamma$
    with
      $A_i \to \delta_1\ \gamma \mid \delta_2\ \gamma \mid \ldots \mid \delta_k\ \gamma$
    where
      $A_j \to \delta_1 \mid \delta_2 \mid \ldots \mid \delta_k$
  enddo
  eliminate the immediate left recursion in A_i
enddo

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Immediate Left-Recursion Elimination

Rewrite every left-recursive production

$$
\begin{aligned}
A &\rightarrow A\ \alpha \\
&\mid \beta \\
&\mid \gamma \\
&\mid A\ \delta
\end{aligned}
$$

into a right-recursive production:

$$
\begin{aligned}
A &\rightarrow \beta\ A_R \\
&\mid \gamma\ A_R \\
A_R &\rightarrow \alpha\ A_R \\
&\mid \delta\ A_R \\
&\mid \varepsilon
\end{aligned}
$$

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Example Left Rec. Elimination

$$
\left.\begin{aligned}
A &\rightarrow B\ C \mid \mathbf{a} \\
B &\rightarrow C\ A \mid A\ \mathbf{b} \\
C &\rightarrow A\ B \mid C\ C \mid \mathbf{a}
\end{aligned}\right\}\ \text{Choose arrangement: } A, B, C
$$

$i = 1$:	nothing to do
$i = 2, j = 1$:	$B \rightarrow C\ A \mid \underline{A}\ \mathbf{b}$
	$\Rightarrow \quad B \rightarrow C\ A \mid \underline{B\ C}\ \mathbf{b} \mid \underline{\mathbf{a}}\ \mathbf{b}$
	$\Rightarrow_{(\text{imm})}\ B \rightarrow C\ A\ B_R \mid \mathbf{a}\ \mathbf{b}\ B_R$
	$B_R \rightarrow C\ \mathbf{b}\ B_R \mid \varepsilon$
$i = 3, j = 1$:	$C \rightarrow \underline{A}\ B \mid C\ C \mid \mathbf{a}$
	$\Rightarrow \quad C \rightarrow \underline{B\ C}\ B \mid \underline{\mathbf{a}}\ B \mid C\ C \mid \mathbf{a}$
$i = 3, j = 2$:	$C \rightarrow \underline{B}\ C\ B \mid \mathbf{a}\ B \mid C\ C \mid \mathbf{a}$
	$\Rightarrow \quad C \rightarrow \underline{C\ A\ B_R}\ C\ B \mid \underline{\mathbf{a}\ \mathbf{b}\ B_R}\ C\ B \mid \mathbf{a}\ B \mid C\ C \mid \mathbf{a}$
	$\Rightarrow_{(\text{imm})}\ C \rightarrow \mathbf{a}\ \mathbf{b}\ B_R\ C\ B\ C_R \mid \mathbf{a}\ B\ C_R \mid \mathbf{a}\ C_R$
	$C_R \rightarrow A\ B_R\ C\ B\ C_R \mid C\ C_R \mid \varepsilon$

Left Factoring

- When a nonterminal has two or more productions whose right-hand sides start with the same grammar symbols, the grammar is not LL(1) and cannot be used for predictive parsing
- Replace productions
  $A \to \alpha\ \beta_1 \mid \alpha\ \beta_2 \mid \ldots \mid \alpha\ \beta_n \mid \gamma$
  with
  $A \to \alpha\ A_R \mid \gamma$
  $A_R \to \beta_1 \mid \beta_2 \mid \ldots \mid \beta_n$

Predictive Parsing

- Eliminate left recursion from grammar
- Left factor the grammar
- Compute FIRST and FOLLOW
- Two variants:
  - Recursive (recursive calls)
  - Non-recursive (table-driven)

FIRST (Revisited)

- FIRST(α) = the set of terminals that begin all strings derived from α

FIRST(a) = {a} if $a \in T$
FIRST(ε) = {ε}
FIRST(A) = $\cup_{A\to\alpha}$ FIRST(α) for $A\to\alpha \in P$
FIRST($X_1X_2\ldots X_k$) =
  if for all j = 1, …, i-1 : $\varepsilon \in$ FIRST(X_j) **then**
    add non-ε in FIRST(X_i) to FIRST($X_1X_2\ldots X_k$)
  if for all j = 1, …, k : $\varepsilon \in$ FIRST(X_j) **then**
    add ε to FIRST($X_1X_2\ldots X_k$)

FOLLOW

- FOLLOW(A) = the set of terminals that can immediately follow nonterminal A

FOLLOW(A) =
  for all ($B \to \alpha\ A\ \beta$) $\in P$ **do**
    add FIRST(β)\{ε} to FOLLOW(A)
  for all ($B \to \alpha\ A\ \beta$) $\in P$ and $\varepsilon \in$ FIRST(β) **do**
    add FOLLOW(B) to FOLLOW(A)
  for all ($B \to \alpha\ A$) $\in P$ **do**
    add FOLLOW(B) to FOLLOW(A)
  if A is the start symbol S **then**
    add **$** to FOLLOW($A$)

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LL(1) Grammar

- A grammar G is LL(1) if for each collections of productions
  $A \rightarrow \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$
  for nonterminal A the following holds:

  1. $\text{FIRST}(\alpha_i) \cap \text{FIRST}(\alpha_j) = \varnothing$ for all $i \neq j$
  2. if $\alpha_i \Rightarrow^* \varepsilon$ then
     2.a. $\alpha_j \not\Rightarrow^* \varepsilon$ for all $i \neq j$
     2.b. $\text{FIRST}(\alpha_j) \cap \text{FOLLOW}(A) = \varnothing$ for all $i \neq j$

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Non-LL(1) Examples

Grammar	Not LL(1) because
$S \rightarrow S\,\mathbf{a} \mid \mathbf{a}$	Left recursive
$S \rightarrow \mathbf{a}\,S \mid \mathbf{a}$	$\text{FIRST}(\mathbf{a}\,S) \cap \text{FIRST}(\mathbf{a}) \neq \varnothing$
$S \rightarrow \mathbf{a}\,R \mid \varepsilon$ $R \rightarrow S \mid \varepsilon$	For R: $S \rightarrow^* \varepsilon$ and $\varepsilon \rightarrow^* \varepsilon$
$S \rightarrow \mathbf{a}\,R\,\mathbf{a}$ $R \rightarrow S \mid \varepsilon$	For R: $\text{FIRST}(S) \cap \text{FOLLOW}(R) \neq \varnothing$

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Recursive Descent Parsing

- Grammar must be LL(1)
- Every nonterminal has one (recursive) procedure responsible for parsing the nonterminal's syntactic category of input tokens
- When a nonterminal has multiple productions, each production is implemented in a branch of a selection statement based on input look-ahead information

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Using FIRST and FOLLOW to Write a Recursive Descent Parser

expr → *term rest*
rest → **+** *term rest*
| **-** *term rest*
| ε
term → **id**

procedure *rest*();
begin
  if *lookahead* in FIRST(**+** *term rest*) **then**
    match('**+**'); *term*(); *rest*()
  else if *lookahead* in FIRST(**-** *term rest*) **then**
    match('**-**'); *term*(); *rest*()
  else if *lookahead* in FOLLOW(*rest*) **then**
    return
  else error()
end;

FIRST(**+** *term rest*) = { **+** }
FIRST(**-** *term rest*) = { **-** }
FOLLOW(*rest*) = { **$** }

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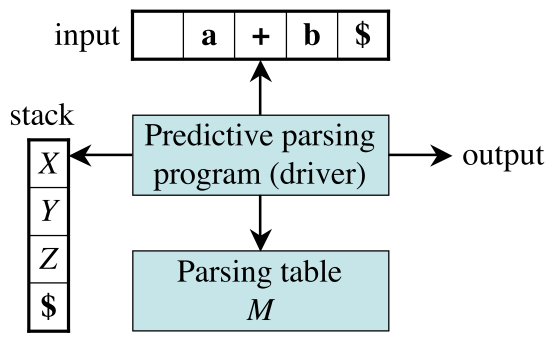
Non-Recursive Predictive Parsing

- Given an LL(1) grammar G=(N,T,P,S) construct a table $M[A,a]$ for $A \in N$, $a \in T$ and use a driver program with a stack



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Constructing a Predictive Parsing Table

for each production $A \rightarrow \alpha$ **do**
    for each $a \in$ FIRST(α) **do**
        add $A \rightarrow \alpha$ to $M[A,a]$
    enddo
    if $\varepsilon \in$ FIRST(α) **then**
        for each $b \in$ FOLLOW(A) **do**
            add $A \rightarrow \alpha$ to $M[A,b]$
        enddo
    endif
enddo
Mark each undefined entry in M error

Example Table

$E \rightarrow T\, E_R$
$E_R \rightarrow \mathbf{+}\, T\, E_R \mid \varepsilon$
$T \rightarrow F\, T_R$
$T_R \rightarrow \mathbf{*}\, F\, T_R \mid \varepsilon$
$F \rightarrow \mathbf{(}\, E\, \mathbf{)} \mid \mathbf{id}$

$A \rightarrow \alpha$	FIRST(α)	FOLLOW(A)
$E \rightarrow T\, E_R$	**(id**	**$)**
$E_R \rightarrow \mathbf{+}\, T\, E_R$	**+**	**$)**
$E_R \rightarrow \varepsilon$	ε	
$T \rightarrow F\, T_R$	**(id**	**+ $)**
$T_R \rightarrow \mathbf{*}\, F\, T_R$	*	**+ $)**
$T_R \rightarrow \varepsilon$	ε	
$F \rightarrow \mathbf{(}\, E\, \mathbf{)}$	**(**	*** + $)**
$F \rightarrow \mathbf{id}$	**id**	

	id	**+**	*	**(**	**)**	**$**
E	$E \rightarrow T\, E_R$			$E \rightarrow T\, E_R$		
E_R		$E_R \rightarrow \mathbf{+}\, T\, E_R$			$E_R \rightarrow \varepsilon$	$E_R \rightarrow \varepsilon$
T	$T \rightarrow F\, T_R$			$T \rightarrow F\, T_R$		
T_R		$T_R \rightarrow \varepsilon$	$T_R \rightarrow \mathbf{*}\, F\, T_R$		$T_R \rightarrow \varepsilon$	$T_R \rightarrow \varepsilon$
F	$F \rightarrow \mathbf{id}$			$F \rightarrow \mathbf{(}\, E\, \mathbf{)}$		

LL(1) Grammars are Unambiguous

Ambiguous grammar
$S \rightarrow \mathbf{i}\, E\, \mathbf{t}\, S\, S_R \mid \mathbf{a}$
$S_R \rightarrow \mathbf{e}\, S \mid \varepsilon$
$E \rightarrow \mathbf{b}$

$A \rightarrow \alpha$	FIRST(α)	FOLLOW(A)
$S \rightarrow \mathbf{i}\, E\, \mathbf{t}\, S\, S_R$	**i**	**e $**
$S \rightarrow \mathbf{a}$	**a**	
$S_R \rightarrow \mathbf{e}\, S$	**e**	**e $**
$S_R \rightarrow \varepsilon$	ε	
$E \rightarrow \mathbf{b}$	**b**	**t**

Error: duplicate table entry

	a	**b**	**e**	**i**	**t**	**$**
S	$S \rightarrow \mathbf{a}$			$S \rightarrow \mathbf{i}\, E\, \mathbf{t}\, S\, S_R$		
S_R			$S_R \rightarrow \varepsilon$ $S_R \rightarrow \mathbf{e}\, S$			$S_R \rightarrow \varepsilon$
E		$E \rightarrow \mathbf{b}$				

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Predictive Parsing Program (Driver)

push(**$**)
push(S)
a := *lookahead*
repeat
    X := pop()
    if X is a terminal or X = **$** **then**
        match(X) // move to next token, a := *lookahead*
    else if $M[X,a] = X \rightarrow Y_1Y_2\ldots Y_k$ **then**
        push($Y_k, Y_{k\text{-}1}, \ldots, Y_2, Y_1$) // such that Y_1 is on top
        produce output and/or invoke actions
    else error()
    endif
until X = **$**

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Example Table-Driven Parsing

Stack	Input	Production applied
$\$E$	**id+id*id$**	
$\$E_RT$	**id+id*id$**	$E \rightarrow T\ E_R$
$\$E_RT_RF$	**id+id*id$**	$T \rightarrow F\ T_R$
$\$E_RT_R$**id**	**id+id*id$**	$F \rightarrow$ **id**
$\$E_RT_R$	**+id*id$**	
$\$E_R$	**+id*id$**	$T_R \rightarrow \varepsilon$
$\$E_RT$**+**	**+id*id$**	$E_R \rightarrow$ **+** $T\ E_R$
$\$E_RT$	**id*id$**	
$\$E_RT_RF$	**id*id$**	$T \rightarrow F\ T_R$
$\$E_RT_R$**id**	**id*id$**	$F \rightarrow$ **id**
$\$E_RT_R$	***id$**	
$\$E_RT_RF$*****	***id$**	$T_R \rightarrow$ ***** $F\ T_R$
$\$E_RT_RF$	**id$**	
$\$E_RT_R$**id**	**id$**	$F \rightarrow$ **id**
$\$E_RT_R$	**$**	
$\$E_R$	**$**	$T_R \rightarrow \varepsilon$
$	**$**	$E_R \rightarrow \varepsilon$

Panic Mode Recovery

Add synchronizing actions to undefined entries based on FOLLOW

FOLLOW(E) = { **$** **)** }
FOLLOW(E_R) = { **$** **)** }
FOLLOW(T) = { **+** **$** **)** }
FOLLOW(T_R) = { **+** **$** **)** }
FOLLOW(F) = { ***** **+** **$** **)** }

	id	**+**	*****	**(**	**)**	**$**
E	$E \rightarrow T\ E_R$			$E \rightarrow T\ E_R$	***synch***	***synch***
E_R		$E_R \rightarrow \mathbf{+}\ T\ E_R$			$E_R \rightarrow \varepsilon$	$E_R \rightarrow \varepsilon$
T	$T \rightarrow F\ T_R$	***synch***		$T \rightarrow F\ T_R$	***synch***	***synch***
T_R		$T_R \rightarrow \varepsilon$	$T_R \rightarrow \mathbf{*}\ F\ T_R$		$T_R \rightarrow \varepsilon$	$T_R \rightarrow \varepsilon$
F	$F \rightarrow$ **id**	***synch***	***synch***	$F \rightarrow$ **(** E **)**	***synch***	***synch***

synch: pop A and skip input till synch token
or skip until FIRST(A) found

Phrase-Level Recovery

Change input stream by inserting missing *****
For example: **id id** is changed into **id * id**

	id	**+**	*****	**(**	**)**	**$**
E	$E \rightarrow T\ E_R$			$E \rightarrow T\ E_R$	***synch***	***synch***
E_R		$E_R \rightarrow \mathbf{+}\ T\ E_R$			$E_R \rightarrow \varepsilon$	$E_R \rightarrow \varepsilon$
T	$T \rightarrow F\ T_R$	***synch***		$T \rightarrow F\ T_R$	***synch***	***synch***
T_R	***insert ****	$T_R \rightarrow \varepsilon$	$T_R \rightarrow \mathbf{*}\ F\ T_R$		$T_R \rightarrow \varepsilon$	$T_R \rightarrow \varepsilon$
F	$F \rightarrow$ **id**	***synch***	***synch***	$F \rightarrow$ **(** E **)**	***synch***	***synch***

insert *: insert missing ***** and redo the production

Error Productions

$E \rightarrow T\, E_R$
$E_R \rightarrow \mathbf{+}\, T\, E_R \mid \varepsilon$
$T \rightarrow F\, T_R$
$T_R \rightarrow \mathbf{*}\, F\, T_R \mid \varepsilon$
$F \rightarrow (\, E\,) \mid \mathbf{id}$

Add error production:

$T_R \rightarrow F\, T_R$

to ignore missing *****, e.g.: **id id**

	id	**+**	*****	**(**	**)**	**$**
E	$E \rightarrow T\, E_R$			$E \rightarrow T\, E_R$	***synch***	***synch***
E_R		$E_R \rightarrow \mathbf{+}\, T\, E_R$			$E_R \rightarrow \varepsilon$	$E_R \rightarrow \varepsilon$
T	$T \rightarrow F\, T_R$	***synch***		$T \rightarrow F\, T_R$	***synch***	***synch***
T_R	$T_R \rightarrow F\, T_R$	$T_R \rightarrow \varepsilon$	$T_R \rightarrow \mathbf{*}\, F\, T_R$		$T_R \rightarrow \varepsilon$	$T_R \rightarrow \varepsilon$
F	$F \rightarrow \mathbf{id}$	***synch***	***synch***	$F \rightarrow (\, E\,)$	***synch***	***synch***