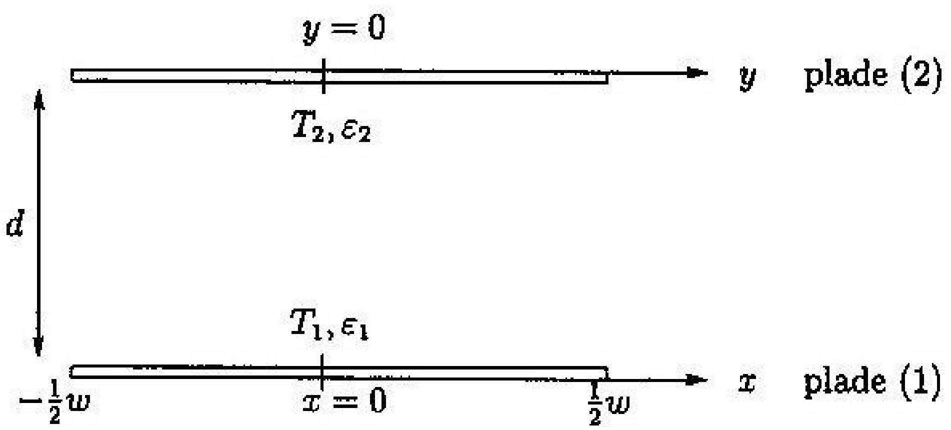
Mandatory 2

Spring 2019

Radiation balance between parallel plates

Two infinitely long paralllel plates of width w are placed with distance d.



The plates radiate and reflect diffusely and have uniform temperatures T_1 and T_2, and emmisivities ε_1 and ε_2, respectively. Let $u(x)$ denote the radiation per unit time per unit area from plate 1 at the distance x from the mid axis of the plate, and let $v(y)$ be the corresponding function for plate 2. Then u and v are solutions to the coupled integral equations:

$$u(x) = \varepsilon_1 \sigma T_1^4 + (1-\varepsilon_1) \int_{-\frac{1}{2}w}^{\frac{1}{2}w} F(x,y,d) v(y) dy \tag{1}$$

$$v(y) = \varepsilon_2 \sigma T_2^4 + (1-\varepsilon_2) \int_{-\frac{1}{2}w}^{\frac{1}{2}w} F(x,y,d) u(x) dx \tag{2}$$

where σ is the Stefan-Bolzmann constant, and F is given by

$$F(x,y,d) = \frac{1}{2} \frac{1}{(d^2 + (x-y)^2)^{\frac{3}{2}}} \tag{3}$$

The integrals

$$I_1(x) = \int_{-\frac{1}{2}w}^{\frac{1}{2}w} F(x,y,d)v(y)dy \quad (4)$$

$$I_2(y) = \int_{-\frac{1}{2}w}^{\frac{1}{2}w} F(x,y,d)u(x)dx \quad (5)$$

describe the incoming radiation per unit time per unit area from the other plate. The total energy (per unit time and unit length), which must be supplied to each plate in order to keep the temperature constant is

$$Q_1 = \int_{-\frac{1}{2}w}^{\frac{1}{2}w} (u(x) - I_1(x))dx \quad (6)$$

$$Q_2 = \int_{-\frac{1}{2}w}^{\frac{1}{2}w} (v(y) - I_2(y))dy \quad (7)$$

Data: $T_1 = 1000$, $T_2 = 500$, $\varepsilon_1 = 0.80$, $\varepsilon_2 = 0.60$, $\sigma = 1.712 \cdot 10^{-9}$, $d = 1.00$, $w = 1.00$.

Exercise a)

Use the trapezoidal method to write an approximation to (4) and (5). Hint: Introduce $(u_i)_{i=0}^N$ and $(v_i)_{i=0}^N$ describing the values of u and v in the points $(x_i)_{i=0}^N$ and $(y_i)_{i=0}^N$, respectively. Then write the discrete problem corresponding to (1) and (2) as a system of linear equations in the variables $(u_i)_{i=0}^N$ and $(v_i)_{i=0}^N$.

Exercise b)

Write a program that sets up and solves the system of linear equations from Exercise a) for a given value of N. Use the results to find Q_1 and Q_2.
For each value of N, state the found values of $u(x)$, $x = -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}$ and $v(y)$, $y = -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}$ as well as Q_1 and Q_2. Use $N = 4, 8, 16, 32, \ldots$.

Exercise c)

Determine Q_1 and Q_2 as precise as possible based on the results from Exercise b) together with an error estimate.

Practical information

You are supposed to work in groups of two (same groups as mandatory 1).
The program must be written in C++. The code as well as a report describing your solution and results of the exercises above must be handed in. Deadline for handing in (via SDU Assignment on Blackboard) is Tuesday April 23th at noon(12:00).