Calibration of a planar 2-axis robot using SVD

Numerical Methods
6. Semester

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Task data

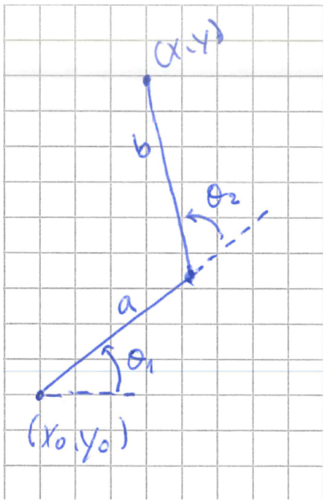

Figure 1: *Parameter relations.*

$x = x_0 + a * cos(\Theta_1) + b * cos(\Theta_1 + \Theta_2)$
$y = y_0 + a * sin(\Theta_1) + b * sin(\Theta_1 + \Theta_2)$

Task 1

The following matrix is the A-matrix deduced from the x, and y-equations.

$$A = \begin{bmatrix} 1 & 0 & cos(\Theta_1^1) & cos(\Theta_1^1 + \Theta_2^1) \\ 0 & 1 & sin(\Theta_1^1) & sin(\Theta_1^1 + \Theta_2^1) \\ 1 & 0 & cos(\Theta_1^2) & cos(\Theta_1^2 + \Theta_2^2) \\ 0 & 1 & sin(\Theta_1^2) & sin(\Theta_1^2 + \Theta_2^2) \\ . & . & . & . \\ . & . & . & . \\ 1 & 0 & cos(\Theta_1^N) & cos(\Theta_1^N + \Theta_2^N) \\ 0 & 1 & sin(\Theta_1^N) & sin(\Theta_1^N + \Theta_2^N) \end{bmatrix}$$

The following is the q-vector containing the parameters which we want to estimate with Singular Value Decomposition [SVD].
$q = (x_0, y_0, a, b)$
The following is the z-vector, also known as the right-hand side. This contain the measured results which we will use to estimate the parameters in q.
$z = (x^1, y^1, x^2, y^2 .....x^N, y^N)$

Task 2

Dataset 1

The following is the W-matrix extracted with SVD. It is shown as a vector since the matrix only contains values along its diagonal.
$W = (23.3788, 22.6835, 21.8248, 21.5074)$
From the W-vector it can be seen that W1 > W2 > W3 > W4 > 0. This property indicates that there are not any linear dependencies in the A-matrix, ie. matrix A is linear independent.
The U, W and V matrix can be seen by executing the included 'mandatory.cpp' file.

Dataset 2

$W = (31.6427, 22.7755, 21.9092, 0.0456)$
Looking at the W-matrix from dataset 2 the A-matrix is close to linear dependend. This is seen though the fourth element of W which is close to 0.

Task 3

Dataset 1

The following is the estimated parameters, ei. the q-vector extrated with svd.solve.
$q = (4.09826, 6.09964, 50.0933, 40.0896)$

Dataset 2

The following is the estimated parameters, ei. the q-vector extrated with svd.solve.
$q = (2.40792, 5.11981, 52.0294, 40.0893)$

Task 4

Dataset 1

A measurement error of 1mm equals 0.1 cm which is the value both the A-matrix and the z-vector is corrected with. The new q-vector equals the following:
$q\_corr = (40.9826, 60.9964, 500.933, 400.896)$
From Numerical Recipes, equation 15.4.19 is used to estimate the resulting error found on the parameters from the q-vector. The equation is stated below.

$$\sigma^2(aj) = \sum_{i=0}^{M-1} \frac{1}{w_i^2}[V_{(i)}]_j^2 = \sum_{i=0}^{M-1} (\frac{V_{ji}}{w_i})^2$$

The resulting errors are as follows:
$err\_est = (0.00448463, 0.00447814, 0.00448083, 0.00448329)$

Dataset 2

The new q-vector equals the following:
$q\_corr = (24.0792, 51.1981, 520.294, 400.893)$

The resulting errors from equation 15.4.19 are as follows:
$err\_est = (1.33811, 0.781668, 1.54969, 0.00448065)$

Task 5

From Task 2 the W-matrix of dataset 1 shows that the A-matrix is Linear independent. From dataset 2 the W-matrix shows that the A-matrix is close to Linear dependent.

From Task 3 the mean and variance of the residual errors is calculated and stated below:

	Dataset 1	Dataset 2
$Mean$	0.247414	0.252372
$Variance$	0.0192309	0.0206311

The residual errors are a representation of how well the datapoints fit a linear line.
From the mean and variance it can be seen that the first dataset is a better fit than the second, as the values are smaller, though the difference is quite slim.

From Task 4 the error estimate shows a larger error in dataset 2 than dataset 1. This difference indicates a larger uncertaincy in dataset 2 measured values.