

Task for lecture 13

Consider the following differential equation

$$\left. \begin{aligned} y''(x) &= -\cos(y(x)) \cdot \sin(y'(x)) \quad \text{for } 0 < x < 10 \\ y(0) &= 0 \quad , \quad y(10) = 3 \end{aligned} \right\} \quad (1)$$

Solve the equation, numerically, following the procedure outlined by HGP today. That is

- Set Initial Guess, $y_i^{(0)}$ for $i = 0, \dots, N$
- Define \mathbf{F} , \mathbf{F}_y , $\mathbf{F}_{y'}$
- Define $\mathbf{J}_{i,j}(y_0, \dots, y_N)$
- Define $\mathbf{f}_i(y_0, \dots, y_N)$
- Solve the system of equations $\mathbf{J}(\mathbf{y}^k)\Delta\mathbf{y} = -\mathbf{f}(\mathbf{y}^{(k)})$
- Update your \mathbf{y} , using $\mathbf{y}^{(k+1)} = \mathbf{y}^{(k)} + \Delta\mathbf{y}$
- Run iterations until a satisfying result appears

If time allows

- Perform Richardson to verify error estimate and order
- Plot the found solution $y(x)$ for $0 \leq x \leq 10$
- Compare with the solution from last lecture