Calibration of a planar 2-axis robot using SVD

Consider a planar 2-axis robot with the following relations between the joint angles θ_1, θ_2 and the "tool-position" (x, y):

$$x = x_0 + a\cos\theta_1 + b\cos(\theta_1 + \theta_2)$$

$$y = y_0 + a\sin\theta_1 + b\sin(\theta_1 + \theta_2)$$

where (x_0, y_0) is the position of the "robot-base" and a, b denote the lengths of the two robot parts.

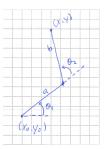


Figure 1: Parameter relations

Now we wish to experimentally determine x_0, y_0, a, b by measuring coordinated values $(\theta_1^{(i)}, \theta_2^{(i)}, x^{(i)}, y^{(i)})$ i = 1, ..., N.

Note that x, y are in cm and that θ_1, θ_2 are in radian.

Hereby, we may establish 2N linear equations in the four unknowns x_0, y_0, a, b :

$$x^{(1)} = x_0 + a\cos\theta_1^{(1)} + b\cos(\theta_1^{(1)} + \theta_2^{(1)})$$

$$y^{(1)} = y_0 + a\sin\theta_1^{(1)} + b\sin(\theta_1^{(1)} + \theta_2^{(1)})$$

$$x^{(2)} = x_0 + a\cos\theta_1^{(2)} + b\cos(\theta_1^{(2)} + \theta_2^{(2)})$$

$$y^{(2)} = y_0 + a\sin\theta_1^{(2)} + b\sin(\theta_1^{(2)} + \theta_2^{(2)})$$

$$\dots = \dots$$

$$x^{(N)} = x_0 + a\cos\theta_1^{(N)} + b\cos(\theta_1^{(N)} + \theta_2^{(N)})$$

$$y^{(N)} = y_0 + a\sin\theta_1^{(N)} + b\sin(\theta_1^{(N)} + \theta_2^{(N)})$$

- 1. Find expressions for the elements in the matrix A and the right-hand side, say z, for the associated systems of linear equations for the parameters $q = (x_0, y_0, a, b)$.
- 2. Read the files with values, insert them in A and compute U, W og V using the method from NR. State (with arguments based on the SVD matrices) if there are issues with close to linear dependencies between the parameters x_0, y_0, a, b .
- 3. Estimate the parameters $q = (x_0, y_0, a, b)$ and state your results. State also the residual error ||Aq z||.
- 4. The values of the $x^{(i)}, y^{(i)}$'s are measured with a camera, and there is a measurement uncertainty that is estimated to 1mm on each coordinate. Estimate the resulting errors on the found parameters using NR, Eq. (15.4.19).
- 5. Compare the results from both datasets.