```
A[a1_, a2_, a3_, a4_] = {a1, a2, a3, a4}; b[b1_, b2_, b3_, b4_] = {b1, b2, b3, b4};
```

Regular case

```
a1 = {2, 4, 1, 5}; a2 = {4, 1, 6, 2}; a3 = {2, -3, 2, 1}; a4 = {6, -2, -7, 3};

b1 = 1; b2 = 3; b3 = 4; b4 = -3;

Astart = A[a1, a2, a3, a4]; bstart = b[b1, b2, b3, b4];

{A[a1, a2, a3, a4] // MatrixForm, b[b1, b2, b3, b4] // MatrixForm}
```

$$\left\{ \begin{pmatrix} 2 & 4 & 1 & 5 \\ 4 & 1 & 6 & 2 \\ 2 & -3 & 2 & 1 \\ 6 & -2 & -7 & 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 4 \\ -3 \end{pmatrix} \right\}$$

```
p = a2[[1]] / a1[[1]];

a2 = a2 - p a1; b2 = b2 - p b1;

{A[a1, a2, a3, a4] // MatrixForm, b[b1, b2, b3, b4] // MatrixForm}
```

$$\left\{ \begin{pmatrix} 2 & 4 & 1 & 5 \\ 0 & -7 & 4 & -8 \\ 2 & -3 & 2 & 1 \\ 6 & -2 & -7 & 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 4 \\ -3 \end{pmatrix} \right\}$$

```
p = a3[[1]] / a1[[1]];

a3 = a3 - p a1; b3 = b3 - p b1;

p = a4[[1]] / a1[[1]];

a4 = a4 - p a1; b4 = b4 - p b1;

{A[a1, a2, a3, a4] // MatrixForm, b[b1, b2, b3, b4] // MatrixForm}
```

$$\left\{ \begin{pmatrix} 2 & 4 & 1 & 5 \\ 0 & -7 & 4 & -8 \\ 0 & -7 & 1 & -4 \\ 0 & -14 & -10 & -12 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 3 \\ -6 \end{pmatrix} \right\}$$

```
p = a3[[2]] / a2[[2]];

a3 = a3 - p a2; b3 = b3 - p b2;

p = a4[[2]] / a2[[2]];

a4 = a4 - p a2; b4 = b4 - p b2;

{A[a1, a2, a3, a4] // MatrixForm, b[b1, b2, b3, b4] // MatrixForm}
```

$$\left\{ \begin{pmatrix} 2 & 4 & 1 & 5 \\ 0 & -7 & 4 & -8 \\ 0 & 0 & -3 & 4 \\ 0 & 0 & -18 & 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \\ -8 \end{pmatrix} \right\}$$

```
p = a4[[3]] / a3[[3]];

a4 = a4 - p a3; b4 = b4 - p b3;
```

```
{A[a1, a2, a3, a4] // MatrixForm, b[b1, b2, b3, b4] // MatrixForm}
```

$$\left\{ \begin{pmatrix} 2 & 4 & 1 & 5 \\ 0 & -7 & 4 & -8 \\ 0 & 0 & -3 & 4 \\ 0 & 0 & 0 & -20 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \\ -20 \end{pmatrix} \right\}$$

```
x4 = b4 / a4[[4]]; x3 = (b3 - a3[[4]] x4) / a3[[3]];
x2 = (b2 - a2[[3]] x3 - a2[[4]] x4) / a2[[2]];
x1 = (b1 - a1[[2]] x2 - a1[[3]] x3 - a1[[4]] x4) / a1[[1]];
```

```
x = {x1, x2, x3, x4}
```

$$\left\{ -\frac{11}{21}, -\frac{19}{21}, \frac{2}{3}, 1 \right\}$$

```
Astart.x - bstart
```

```
{0, 0, 0, 0}
```

Pivoting case

```
a1 = {2, 4, 1, 5}; a2 = {4, 8, 6, 2}; a3 = {2, -3, 2, 1}; a4 = {6, -2, -7, 3};
```

```
b1 = 1; b2 = 4; b3 = 4; b4 = -3;
```

```
Astart = A[a1, a2, a3, a4]; bstart = b[b1, b2, b3, b4];
```

```
{A[a1, a2, a3, a4] // MatrixForm, b[b1, b2, b3, b4] // MatrixForm}
```

$$\left\{ \begin{pmatrix} 2 & 4 & 1 & 5 \\ 4 & 8 & 6 & 2 \\ 2 & -3 & 2 & 1 \\ 6 & -2 & -7 & 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 4 \\ -3 \end{pmatrix} \right\}$$

```
p = a2[[1]] / a1[[1]];
```

```
a2 = a2 - p a1; b2 = b2 - p b1;
```

```
{A[a1, a2, a3, a4] // MatrixForm, b[b1, b2, b3, b4] // MatrixForm}
```

$$\left\{ \begin{pmatrix} 2 & 4 & 1 & 5 \\ 0 & 0 & 4 & -8 \\ 2 & -3 & 2 & 1 \\ 6 & -2 & -7 & 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 4 \\ -3 \end{pmatrix} \right\}$$

```
p = a3[[1]] / a1[[1]];
```

```
a3 = a3 - p a1; b3 = b3 - p b1;
```

```
p = a4[[1]] / a1[[1]];
```

```
a4 = a4 - p a1; b4 = b4 - p b1;
```

```
{A[a1, a2, a3, a4] // MatrixForm, b[b1, b2, b3, b4] // MatrixForm}
```

$$\left\{ \begin{pmatrix} 2 & 4 & 1 & 5 \\ 0 & 0 & 4 & -8 \\ 0 & -7 & 1 & -4 \\ 0 & -14 & -10 & -12 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ -6 \end{pmatrix} \right\}$$

```
abuf = a2; a2 = a3; a3 = abuf; bbuf = b2; b2 = b3; b3 = bbuf;
```

```
{A[a1, a2, a3, a4] // MatrixForm, b[b1, b2, b3, b4] // MatrixForm}
```

$$\left\{ \begin{pmatrix} 2 & 4 & 1 & 5 \\ 0 & -7 & 1 & -4 \\ 0 & 0 & 4 & -8 \\ 0 & -14 & -10 & -12 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 2 \\ -6 \end{pmatrix} \right\}$$

```
p = a3[[2]] / a2[[2]];
```

```
a3 = a3 - p a2; b3 = b3 - p b2;
```

```
p = a4[[2]] / a2[[2]];
```

```
a4 = a4 - p a2; b4 = b4 - p b2;
```

```
{A[a1, a2, a3, a4] // MatrixForm, b[b1, b2, b3, b4] // MatrixForm}
```

$$\left\{ \begin{pmatrix} 2 & 4 & 1 & 5 \\ 0 & -7 & 1 & -4 \\ 0 & 0 & 4 & -8 \\ 0 & 0 & -12 & -4 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 2 \\ -12 \end{pmatrix} \right\}$$

```
p = a4[[3]] / a3[[3]];
```

```
a4 = a4 - p a3; b4 = b4 - p b3;
```

```
{A[a1, a2, a3, a4] // MatrixForm, b[b1, b2, b3, b4] // MatrixForm}
```

$$\left\{ \begin{pmatrix} 2 & 4 & 1 & 5 \\ 0 & -7 & 1 & -4 \\ 0 & 0 & 4 & -8 \\ 0 & 0 & 0 & -28 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 2 \\ -6 \end{pmatrix} \right\}$$

```
x4 = b4 / a4[[4]]; x3 = (b3 - a3[[4]] x4) / a3[[3]];
x2 = (b2 - a2[[3]] x3 - a2[[4]] x4) / a2[[2]];
x1 = (b1 - a1[[2]] x2 - a1[[3]] x3 - a1[[4]] x4) / a1[[1]];
```

```
x = {x1, x2, x3, x4}
```

$$\left\{ \frac{33}{98}, -\frac{41}{98}, \frac{13}{14}, \frac{3}{14} \right\}$$

```
Astart.x - bstart
```

```
{0, 0, 0, 0}
```

Singular case

```
a1 = {2, 4, 1, 5}; a2 = {4, 8, 6, 2}; a3 = {2, 4, 2, 1}; a4 = {6, 12, -7, 3};
```

```
b1 = 1; b2 = 4; b3 = 4; b4 = -3;
```

```
Astart = A[a1, a2, a3, a4]; bstart = b[b1, b2, b3, b4];
```

```
{A[a1, a2, a3, a4] // MatrixForm, b[b1, b2, b3, b4] // MatrixForm}
```

$$\left\{ \begin{pmatrix} 2 & 4 & 1 & 5 \\ 4 & 8 & 6 & 2 \\ 2 & 4 & 2 & 1 \\ 6 & 12 & -7 & 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 4 \\ -3 \end{pmatrix} \right\}$$

```
p = a2[[1]] / a1[[1]];
```

```
a2 = a2 - p a1; b2 = b2 - p b1;
```

```
{A[a1, a2, a3, a4] // MatrixForm, b[b1, b2, b3, b4] // MatrixForm}
```

$$
\left\{
\begin{pmatrix}
2 & 4 & 1 & 5 \\
0 & 0 & 4 & -8 \\
2 & 4 & 2 & 1 \\
6 & 12 & -7 & 3
\end{pmatrix},
\begin{pmatrix}
1 \\
2 \\
4 \\
-3
\end{pmatrix}
\right\}
$$

```
p = a3[[1]] / a1[[1]];
```

```
a3 = a3 - p a1; b3 = b3 - p b1;
```

```
p = a4[[1]] / a1[[1]];
```

```
a4 = a4 - p a1; b4 = b4 - p b1;
```

```
{A[a1, a2, a3, a4] // MatrixForm, b[b1, b2, b3, b4] // MatrixForm}
```

$$
\left\{
\begin{pmatrix}
2 & 4 & 1 & 5 \\
0 & 0 & 4 & -8 \\
0 & 0 & 1 & -4 \\
0 & 0 & -10 & -12
\end{pmatrix},
\begin{pmatrix}
1 \\
2 \\
3 \\
-6
\end{pmatrix}
\right\}
$$

```
LinearSolve[Astart, bstart]
```

LinearSolve::nosol : Linear equation encountered that has no solution. ≫

LinearSolve[{{2, 4, 1, 5}, {4, 8, 6, 2}, {2, 4, 2, 1}, {6, 12, -7, 3}}, {1, 4, 4, -3}]