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Introduction to Recommender Systems
Assignment 1

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1. Problem 1

   We have a linear model that gives us score for every pair of user and item:

$$r(\text{user}, \text{item}) = \theta x_{(\text{user}, \text{item})} + \epsilon \tag{1}$$

   If we suppose that item's and user's features can be collected independently:

$$r(\text{user}, \text{item}) = \theta_u x_u + \theta_i x_i + \epsilon \tag{2}$$

   Where θ_u and θ_i are user's and item's parameters and x_u, x_i are user's and item's features respectively.
   Personalization task becomes:

$$\begin{aligned} \text{toprec}(u, n) &= \underset{i}{\overset{n}{\operatorname{argmax}}}\, r(u, i) = \underset{i}{\overset{n}{\operatorname{argmax}}}(\theta_u x_u + \theta_i x_i + \epsilon) = \underset{i}{\overset{n}{\operatorname{argmax}}}(\theta_u x_u + \theta_i x_i) = & (3) \\ &= \underset{i}{\overset{n}{\operatorname{argmax}}}(\theta_i x_i) & (4) \end{aligned}$$

   First we drop the noise ϵ since it does not depend on user or item, and then we drop the term corresponding to the user's features because it does not affect the argmax taken over all the items. As we can see our $\text{toprec}(u, n)$ does not actually depend on user u.

2. Problem 2

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{U} \underbrace{\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}}_{V} \tag{5}$$

   V corresponds to items in some latent space. Recommendations are defined by an orthogonal projection of a user's preferences p onto the latent features space of items:

$$r = V^T V p \tag{6}$$