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On the use of shunted piezo actuators for mitigation of distribution errors in resonator arrays

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Abstract:

Earlier work has shown that an array of very small attached resonators can be designed to alter the dynamic response of a primary structure. The altered response can be designed to make the primary structure appear heavily damped or to have a particular spectral shape such as a band-pass response. However, small errors in the distribution of mass and stiffness distribution of the attachments can have a significant effect, degrading the intended performance. This presentation discusses a concept of correcting small property distribution errors using shunted piezoelectric strip actuators bonded to the attachments.

Keywords: array dynamics, fuzzy structures

Mitigation of Distribution Errors in Subordinate Oscillator Arrays

1 Introduction

Many investigators have discussed how a system's resonant response can be altered by attaching a designed array of substantially smaller subordinate resonators[1, 2, 3, 4]. The concept is based on selectively drawing mechanical energy away from a primary structure in a prescribed portion of a resonant band or at a prescribed rate that achieves a design objective for the overall system response. Early work was related to increasing the damping in a system by rapidly channeling mechanical energy into the subordinate set[5, 6, 7]. This work will focus on a similar application: creating a mechanical band-rejection filter. We show that system disorder (such as small fabrication errors) has a profound effect on predicted performance of such systems. We propose a compensation scheme to mitigate the effect of the disorder. This scheme uses capacitively shunted piezoelectric strip actuators to make small changes in the stiffness of the attachments.

2 Prototype System for Study

The system model under consideration in Fig. 1 consists of a primary resonator with mass m_p, stiffness k_p, and proportional damping constant c_p, to which an array of N substantially smaller resonators are attached. The distributions of the attachment masses m_n, stiffnesses k_n, and proportional damping constants c_n have been chosen to alter dynamic behavior of the primary resonator such that response is a flat bandpass. The responses of the primary structure's impulse response are shown in Fig. 2 where the desired bandwidths Δ are 1.25%, 2.5%, 5%, and 10% of the natural frequency of the primary system, $\omega_p = \sqrt{k_p/m_p}$. Mass and stiffness of each subordinate element is normalized to the primary resonator's properties such that $\alpha_n = m_n/m_p$ and $\gamma_n = k_n/k_p$ respectively. The non-dimensional form of the system response (Eq. 1) is determined using distributions of the normalized mass and stiffness, α and γ:

$$\frac{\hat{X}_p(\omega)\,k_p}{\hat{F}_p(\omega)} = \left(1 - \Omega^2 + \frac{i\Omega}{Q_p} + i\Omega^2 \sum_{n=1}^{N} \alpha_n \hat{r}_n\right)^{-1} \tag{1}$$

where

$$\hat{r}_n(\Omega) = \frac{1 + i\frac{\Omega}{Q_n}\sqrt{\frac{\alpha_n}{\gamma_n}}}{\frac{\Omega}{Q_n}\sqrt{\frac{\alpha_n}{\gamma_n}} + i\left(\frac{\alpha_n}{\gamma_n}\Omega^2 - 1\right)} \tag{2}$$

and

$$\text{element quality factor}, Q_n = \frac{\sqrt{m_n k_n}}{c_n} = \frac{1}{2\zeta_n} \tag{3}$$

Our earlier work[1] describes the process of designing the property distributions and has shown that the total attachment mass required to produce such a bandpass response is given by $\sum m_n/m_p \approx \Delta^2/3.5$. For the discussion that follows, errors will be introduced to the mass, stiffness and damping distributions of the subordinate set to represent disorder due fabrication variation.

3 Simulation of the Effect of Fabrication Disorder

Numerical simulation of the performance of such systems that include the effect of fabrication tolerances indicate that small errors in geometric dimensions have a profound effect on the degree to which the oscillator array alters the system response. Evaluation of Eq. 1 is used to illustrate the effects of introducing prescribed errors in the property distributions of the attached resonators (mass m_n, stiffness k_n, and resonator quality factor $Q_n = \sqrt{m_n k_n}/c_n$). The prescribed errors are specified relative to the *as-designed* mass, stiffness, and quality factor values.

Figure 3 demonstrates the effect of increasing disorder on system response. This example is the impulse response of a primary structure with a 25 element array designed to result in a 20% bandpass response. The discorded property distributions are specified by adding normally distributed random values of specified standard deviations to the original 'as-designed' distributions.

The four examples shown in Fig. 2 indicate increasing ripple across the band with increasing disorder levels. This unwanted deviation becomes apparent at error levels of approximately one part in one thousand.

Any of the fabrication processes under consideration for producing oscillator arrays, such as laser cutting, water jet, or traditional machining have fabrication tolerances no better than 0.002 inches (0.05 mm). This is problematic, particularly as size scales shrink for higher frequencies or low mass systems. For example, an oscillator array designed to filter a 10kg, 1000Hz system would require individual elements with mean length of less than an inch,a mean width of less than a quarter inch, and thickness of approximately 0.025 inches. A device of this size will exhibit undesirable response characteristics regardless of the manufacturing method used.

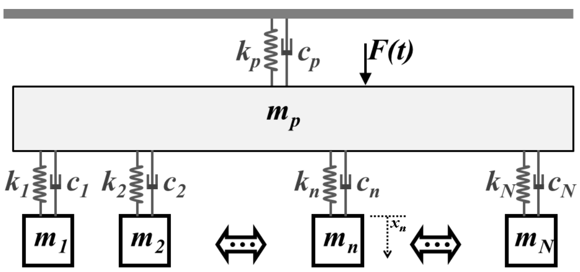

Figure 1: $N+1$ degree of freedom model of a mechanical system with N subordinate elements attached to the primary resonator. Each element has a distinct mass m_n, stiffness k_n, and damping c_n. The ranges of subordinate element properties are specified by prescribed distributions.

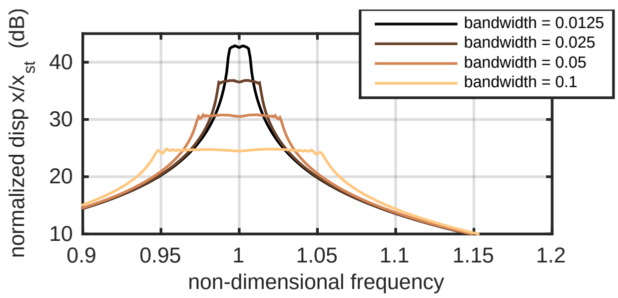

Figure 2: The array of attachments can be designed to make the response of the primary element behave as a bandpass

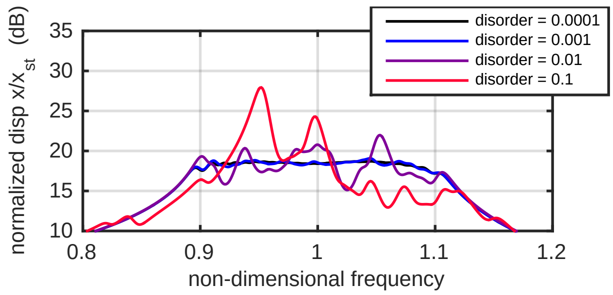

Figure 3: Performance degradation as errors in the property distributions increase from one part in 10^4 (where bandpass performance is unaffected) to 10% (where bandpass response is no longer apparent).

One parameter of importance to the performance of such systems is the modal overlap, defined as the width of a resonance peak of a single element of the array over the separation of adjacent peaks. Mathematically this is given by

$$\eta = \frac{1}{Q_n \Delta} \sqrt{\frac{\alpha_n}{\gamma_n}} \tag{4}$$

While it has been predicted that a modal overlap, $\eta < 2$ will cause significant performance degradation, Fig. 4 details the relationship with disorder across many values of modal overlap, η. This is generated by taking the in-band difference between a zero-disorder response and the response of a statistically randomized error allocation in the frequency distribution. Of interest is the fact that the system shows a linear relationship between disorder and RMS difference at low levels of disorder when $\eta \geq 2$. In this regime, the system will perform at or very close to the 'as-designed' response. In the second regime, the RMS difference is insensitive to increasing disorder and the system begins to exhibit the visible response degradation shown in Fig. 3. This transition also corresponds to the same one part in one-thousand threshold discussed earlier. Beyond this region there is another region that exhibits the linear one-to-one relationship between disorder and RMS difference of the transfer functions. In this region the SOA is considered out of its performance range and inoperable. It is also worth noting, that as η decreases below two, the 'as-designed' response begins in the second regime and can be outside of its performance threshold and would be ineffective even at very low levels of disorder.

4 Cantilever System with Shunted Piezoelectric Actuators

One implementation of such a system to which this model can be applied is shown in Fig. 5, where a cantilevered plate acts as the primary structure and an array of slender cantilevers as the array elements. To compensate for errors in the 'as-built' array element properties, we propose a technique in which each of the cantilevers has an piezoelectric patch actuator bonded to it that can be used to alter its effective properties. Further, we will show that it is sufficient to alter just one of the three property distribution. For example, the effective stiffness k_n of each array element can be adjusted by shunting the piezoelectric patch with a capacitor

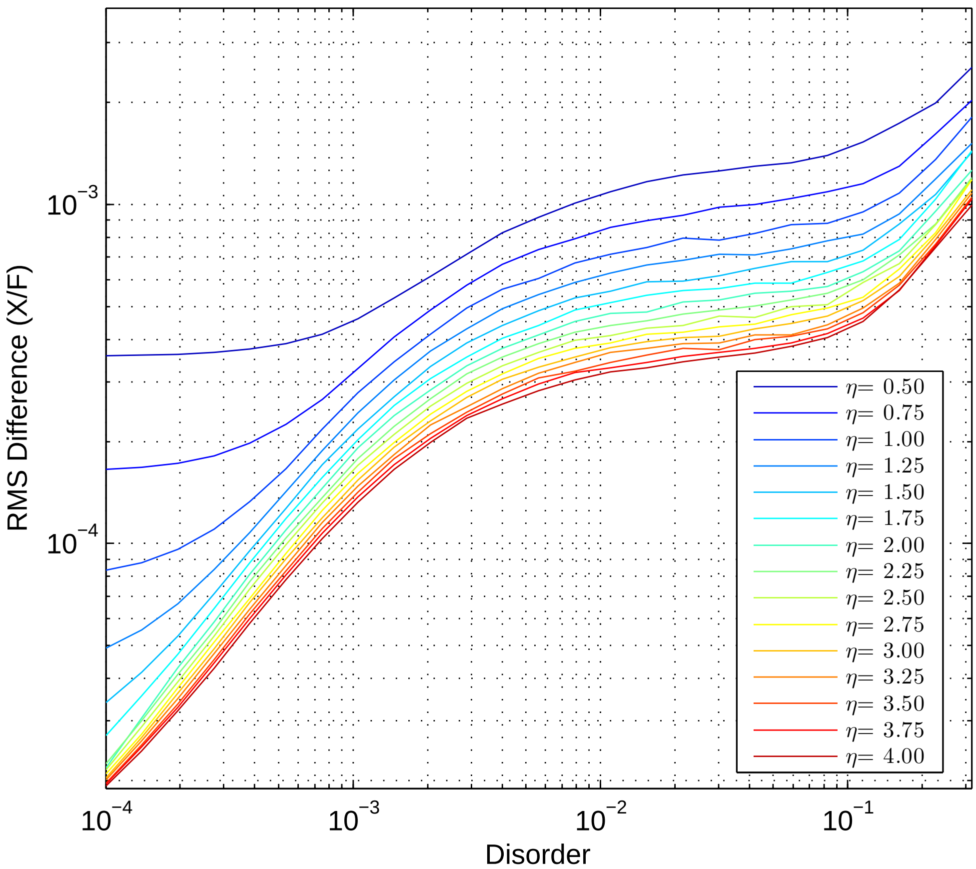

Figure 4: Each value of η generates a unique curve. As η increases, the system becomes less sensitive to the effects of disorder. The cost of this decrease sensitivity is that the system will be required to be manufactured with more oscillators for the same bandwidth.

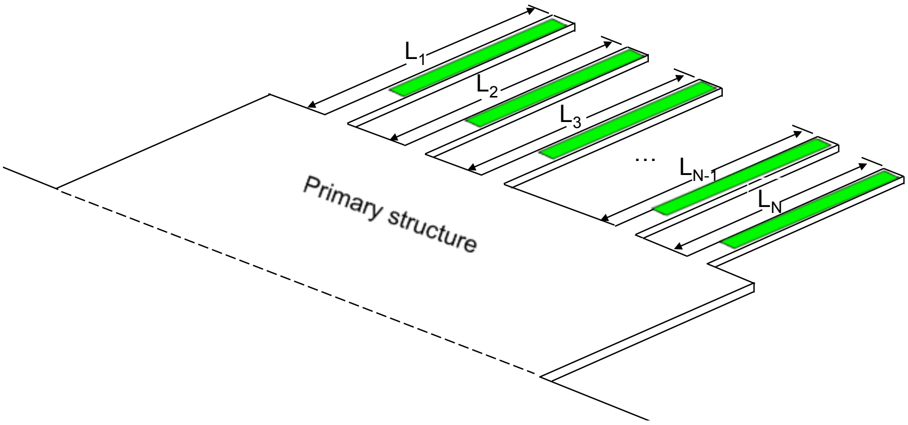

Figure 5: Schematic of a cantilevered plate to which an array of cantilevered beam resonators are attached. The green areas represent piezoelectric patch actuators, which are individually shunted with capacitors to adjust the effective stiffness of each cantilever.

and tuning the shunt capacitance. Davis and Lesieurtre[8] give the effective stiffness of the shunted cantilever as

$$k_{\text{effective}} = k_{\text{short}} \left(1 + \frac{\kappa_{e/m}^2}{1 - \kappa_{e/m}^2 + \zeta\left(i\omega\right)} \right) \tag{5}$$

where $\kappa_{e/m}$ is the electro-mechanical coupling, k_{short} is the stiffness of the element with the actuator short-circuited, and $\zeta\left(i\omega\right) = \hat{Z}_{\text{piezo}} / \hat{Z}_{\text{shunt}}$ is the ratio of the impedances of the piezoelectric actuator and the shunt capacitor.

Simulation results shown in Fig. 6 demonstrate that it is sufficient to adjust only the stiffness of the array elements to return the system response to the designed response. The required shunt capacitances are determined in an iterative process that uses the impulse response and minimizes the difference between the 'as-built' and 'as-designed' responses. This is the same performance metric calculated in Fig. 4.

The left side of Fig. 6 shows and example of a 1% stiffness and damping error and 5% mass error applied to each distribution of m, k, and Q, shown in green, and plotted as the sum of these errors in the top-right clearly demonstrating pure system performance. Stiffness corrections, shown in red, are then applied to only the stiffness distribution. The red curve in the FRF plot shows a corrected response and that it is only necessary to modify a single distribution to a new 'as-corrected' to offset the effect of error in all of the distributions.

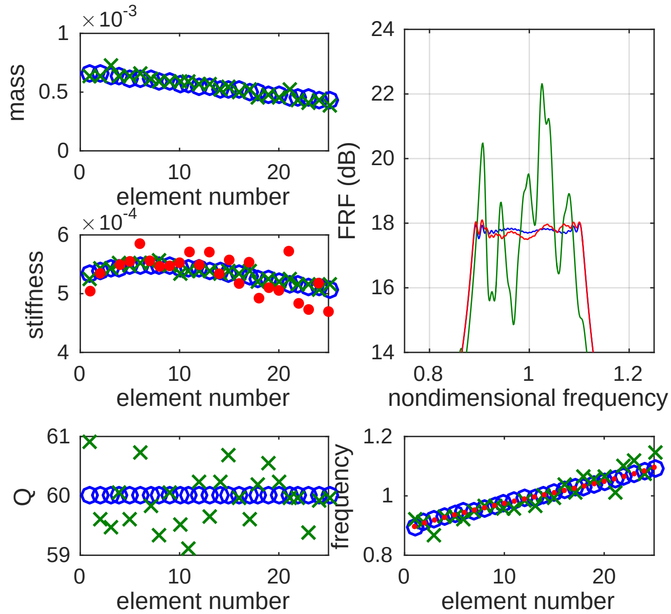

Figure 6: *As-designed* (o) and *as-built* (x) property distribution for the mass, stiffness, and damping for a 25 element array (*left*). A correction for the stiffness distribution (•) is calculated to achieve the desired frequency distribution (*lower right*). Frequency spectra (*top right*) show *as-designed* (blue), *as-built* (green), and *corrected* (red) responses.

5 Conclusion

Although the SOA is extremely sensitive to disorder, it has a predictable region that can be designed for to retain the desirable flat band-stop response. It is also shown that a adjusting the properties of a single distribution (stiffness) can offset the sum of all other errors and yield a response that behaves much more closely to an ideal system.

Secondly, knowledge of the approximate level of disorder in the system from Fig. 4 based on manufacturing tolerances allows selection of the proper η and therefore appropriate number of oscillators required to stay in the first regime of the graph shown in Fig. 5. If unable to achieve desired degree of precision, piezoelectric actuators use a capacitive control system to minimize the difference in impulse response between desired and 'as-built'.

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