

# Boosting Classifier Cascades

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## **Abstract**

The problem of optimal and automatic design of a detector cascade is considered. A novel mathematical model is introduced for a cascaded detector. This model is analytically tractable, leads to recursive computation, and accounts for both classification and complexity. A boosting algorithm, FCBoost, is proposed for fully automated cascade design. It exploits the new cascade model, minimizes a Lagrangian cost that accounts for both classification risk and complexity. It searches the space of cascade configurations to automatically determine the optimal number of stages and their predictors, and is compatible with bootstrapping of negative examples and cost sensitive learning. Experiments show that the resulting cascades have state-of-the-art performance in various computer vision problems.

## **1 Paper Body**

There are many applications where a classifier must be designed under computational constraints. One problem where such constraints are extreme is that of object detection in computer vision. To accomplish tasks such as face detection, the classifier must process thousands of examples per image, extracted from all possible image locations and scales, at a rate of several images per second. This problem has been the focus of substantial attention since the introduction of the detector cascade architecture by Viola and Jones (VJ) in [13]. This architecture was used to design the first real time face detector with state-of-the-art classification accuracy. The detector has, since, been deployed in many practical applications of broad interest, e.g. face detection on low-complexity platforms such as cameras or cell phones. The outstanding performance of the VJ detector is the result of 1) a cascade of simple to complex classifiers that reject most non-faces with a few machine operations, 2) learning with a combination of boosting and Haar features of extremely low complexity, and 3) use of bootstrapping to efficiently deal with the extremely large class of non-face examples. While the resulting detector is fast and accurate, the process of designing a cascade is not. In particular, VJ did not address the problem of how to automatically

determine the optimal cascade configuration, e.g. the numbers of cascade stages and weak learners per stage, or even how to design individual stages so as to guarantee optimality of the cascade as a whole. In result, extensive manual supervision is required to design cascades with good speed/accuracy trade off. This includes trial-and-error tuning of the false positive/detection rate of each stage, and of the cascade configuration. In practice, the design of a good cascade can take up several weeks. This has motivated a number of enhancements to the VJ training procedure, which can be organized into three main areas: 1) enhancement of the boosting algorithms used in cascade design, e.g. cost-sensitive variations of boosting [12, 4, 8], float Boost [5] or KLBoost [6], 2) post processing of a learned cascade, by adjusting stage thresholds, to improve performance [7], and 3) specialized cascade architectures which simplify the learning process, e.g. the embedded cascade (ChainBoost) of [15], where each stage contains all weak learners of previous stages. These enhancements do not address the fundamental limitations of the VJ design, namely how to guarantee overall cascade optimality. 1

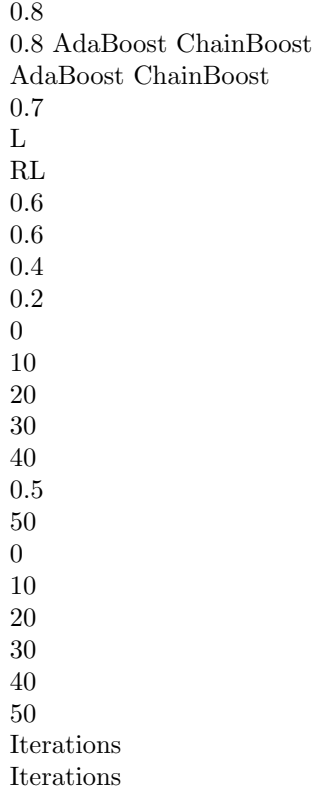


Figure 1: Plots of RL (left) and L (right) for detectors designed with AdaBoost and ChainBoost. More recently, various works have attempted to address this problem [9, 8, 1, 14, 10]. However, the proposed algorithms still rely on sequential learning of cascade stages, which is suboptimal, sometimes require manual supervision, do not search over cascade configurations, and frequently

lack a precise mathematical model for the cascade. In this work, we address these problems, through two main contributions. The first is a mathematical model for a detector cascade, which is analytically tractable, accounts for both classification and complexity, and is amenable to recursive computation. The second is a boosting algorithm, FCBoost, that exploits this model to solve the cascade learning problem. FCBoost solves a Lagrangian optimization problem, where the classification risk is minimized under complexity constraints. The risk is that of the entire cascade, which is learned holistically, rather than through sequential stage design, and FCBoost determines the optimal cascade configuration automatically. It is also compatible with bootstrapping and cost sensitive boosting extensions, enabling efficient sampling of negative examples and explicit control of the false positive/detection rate trade off. An extensive experimental evaluation, covering the problems of face, car, and pedestrian detection demonstrates its superiority over previous approaches.

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#### Problem Definition

A binary classifier  $h(x)$  maps an example  $x$  into a class label  $y \in \{1, -1\}$  according to  $h(x) = \text{sign}[f(x)]$ , where  $f(x)$  is a continuous-valued predictor. Optimal classifiers minimize a risk  $R(f) = E_{X,Y} [L[y, f(x)]]$  where  $L[y, f(x)]$  is the loss function.  $(x_1, y_1), \dots, (x_n, y_n)$  is a set of training examples,  $y_i \in \{1, -1\}$  the class label of example  $x_i$ , and  $L[y, f(x)]$  a loss function. Commonly used losses are upper bounds on the zero-one loss, whose risk is the probability of classification error. Hence,  $R$  is a measure of classification accuracy. For applications with computational constraints, optimal classifier design must also take into consideration the classification complexity. This is achieved by defining a computational risk  $R_C(f) = E_{X,Y} [L_C[y, C(f(x))]]$  where  $C(f(x))$  is the complexity of evaluating  $f(x)$ , and  $L_C[y, C(f(x))]$  a loss function that encodes the cost of this operation. In most detection problems, targets are rare events and contribute little to the overall complexity. In this case, which we assume throughout this work,  $L_C[1, C(f(x))] = 0$  and  $L_C[-1, C(f(x))]$  is denoted  $L_C[C(f(x))]$ . The computational risk is thus  $R_C(f) = E_{X,Y} [L_C[C(f(x))]]$ .

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where  $S^+$

contains the negative examples of  $S^+$ . Usually, more accurate classifiers are more complex. For example in boosting, where the decision rule is a combination of weak rules, a finer approximation of the classification boundary (smaller error) requires more weak learners and computation. Optimal classifier design under complexity constraints is a problem of constrained optimization, which can be solved with Lagrangian methods. These minimize a Lagrangian  $J(\lambda) = R(f) + \lambda R_C(f)$  where  $\lambda$  is a Lagrange multiplier, which controls the trade-off between error rate and complexity. Figure 1 illustrates this trade-off, by plotting the evolution

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of  $RL$  and  $L$  as a function of the boosting iteration, for the AdaBoost algorithm [2]. While the risk always decreases with the addition of weak learners, this is not true for the Lagrangian. After a small number of iterations, the gain in accuracy does not justify the increase in classifier complexity. The design of classifiers under complexity constraints has been addressed through the introduction of detector cascades. A detector cascade  $H(x)$  implements a sequence of binary decisions  $h_i(x)$ ,  $i = 1 \dots m$ . An example  $x$  is declared a target ( $y = 1$ ) if and only if it is declared a target by all stages of  $H$ , i.e.  $h_i(x) = 1, \forall i$ . Otherwise, the example is rejected. For applications where the majority of examples can be rejected after a small number of cascade stages, the average classification time is very small. However, the problem of designing an optimal detector cascade is still poorly understood. A popular approach, known as ChainBoost or embedded cascade [15], is to 1) use standard boosting algorithms to design a detector, and 2) insert a rejection point after each weak learner. This is simple to implement, and creates a cascade with as many stages as weak learners. However, the introduction of the intermediate rejection points, a posteriori of detector design, sacrifices the risk-optimality of the detector. This is illustrated in Figure 1, where the evolution of  $RL$  and  $L$  are also plotted for ChainBoost. In this example,  $L$  is monotonically decreasing, i.e. the addition of weak learners no longer carries a large complexity penalty. This is due to the fact that most negative examples are rejected in the earliest cascade stages. On the other hand, the classification risk is more than double that of the original boosted detector. It is not known how close ChainBoost is to optimal, in the sense of (4).

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#### Classifier cascades

In this work, we seek the design of cascades that are provably optimal under (4). We start by introducing a mathematical model for a detector cascade. 3.1

##### Cascade predictor

Let  $H(x) = \{h_1(x), \dots, h_m(x)\}$  be a cascade of  $m$  detectors  $h_i(x) = \text{sgn}[f_i(x)]$ . To develop some intuition, we start with a two-stage cascade,  $m = 2$ . The cascade implements the decision rule  $H(F)(x) = \text{sgn}[F(x)]$

(5)

$$\begin{aligned} \text{where } F(x) &= F(f_1, f_2)(x) = \\ &f_1(x) f_2(x) \\ &\text{if } f_1(x) \neq 0 \text{ if } f_1(x) = 0 \\ &= f_1 u(f_1) + u(f_1) f_2 \end{aligned}$$

(6) (7)

is denoted the cascade predictor,  $u(\cdot)$  is the step function and we omit the dependence on  $x$  for notational simplicity. This equation can be extended to a cascade of  $m$  stages, by replacing the predictor of the second stage, when  $m = 2$ , with the predictor of the remaining cascade, when  $m$  is larger. Letting  $F_j = F(f_j, \dots, f_m)$  be the cascade predictor for the cascade composed of stages  $j$  to  $m$   $F = F_1 = f_1 u(f_1) + u(f_1) F_2$ .

(8)

More generally, the following recursion holds  $F_k = f_k u(f_k) + u(f_k) F_{k+1}$

(9)

with initial condition  $F_m = f_m$ . In Appendix A, it is shown that combining (8) and (9) recursively leads to  $F$

$$= T_{1,m} + T_{2,m} f_m = T_{1,k} + T_{2,k} f_k u(?f_k) + T_{2,k} F_{k+1} u(f_k), \quad k \leq m. \quad (10) \quad (11)$$

with initial conditions  $T_{1,0} = 0$ ,  $T_{2,0} = 1$  and  $T_{1,k+1} = T_{1,k} + f_k u(?f_k) T_{2,k}$ ,

$$T_{2,k+1} = T_{2,k} u(f_k). \quad (12)$$

Since  $T_{1,k}$ ,  $T_{2,k}$ , and  $F_{k+1}$  do not depend on  $f_k$ , (10) and (11) make explicit the dependence of the cascade predictor,  $F$ , on the predictor of the  $k$ th stage. <sup>3</sup>

### 3.2

#### Differentiable approximation

Letting  $F(f_k + ?g) = F(f_1, \dots, f_k + ?g, \dots, f_m)$ , the design of boosting algorithms requires the evaluation of both  $F(f_k + ?g)$ , and the functional derivative of  $F$  with respect to each  $f_k$ , along any direction  $g$

$d_i ?F(f_k)$ ,  $g_i = F(f_k + ?g) - F(f_k) = ?F(f_k)$ . These are straightforward for the last stage since, from (10),  $F(f_m + ?g) = a_m + ?b_m g$ ,  $i = m$ ,  $g_i = b_m g$ , (13) where  $a_m = T_{1,m} + T_{2,m} f_m = F(f_m)$ ,  $b_m = T_{2,m}$ . (14) In general, however, the right-hand side of (11) is non-differentiable, due to the  $u(\cdot)$  functions. A differentiable approximation is possible by adopting the classic sigmoidal approximation  $u(x) \approx \tanh(?x) + 1$ , where  $?$  is a relaxation parameter. Using this approximation in (11),  $2 F = F(f_k) = T_{1,k} + T_{2,k} f_k (1 + ? u(f_k)) + T_{2,k} F_{k+1} u(f_k)$  (15)  $1 + ? T_{1,k} + T_{2,k} f_k + T_{2,k} [F_{k+1} - f_k] [\tanh(?f_k) + 1]$ . (16) <sup>2</sup> It follows that  $i = ?F(f_k)$ ,  $g_i = b_k g$  (17)

$1 T_{2,k} [1 + ? \tanh(?f_k)] + ?[F_{k+1} - f_k] [1 + ? \tanh^2(?f_k)]$ . (18)  $b_k = 2 F(f_k + ?g)$  can also be simplified by resorting to a first order Taylor series expansion around  $f_k$   $F(f_k + ?g) \approx a_k + ?b_k g$  (19)

$$1 a_k = F(f_k) = T_{1,k} + T_{2,k} f_k + [F_{k+1} - f_k] [\tanh(?f_k) + 1]. \quad (20) \quad 2 \quad 3.3$$

#### Cascade complexity

In Appendix B, a similar analysis is performed for the computational complexity. Denoting by  $C(f_k)$  the complexity of evaluating  $f_k$ , it is shown that  $C(F) = P_{1,k} + P_{2,k} C(f_k) + P_{2,k} u(f_k) C(F_{k+1})$ . (21) with initial conditions  $C(F_{m+1}) = 0$ ,  $P_{1,1} = 0$ ,  $P_{2,1} = 1$  and  $P_{1,k+1} = P_{1,k} + C(f_k)$   $P_{2,k} P_{2,k+1} = P_{2,k} u(f_k)$ . (22) This makes explicit the dependence of the cascade complexity on the complexity of the  $k$ th stage. <sup>P</sup> In practice,  $f_k = l \cdot c_l \cdot g_l$  for  $g_l \in U$ , where  $U$  is a set of functions of approximately identical complexity. For example, the set of projections into Haar features, in which  $C(f_k)$  is proportional to the number of features  $g_l$ . In general,  $f_k$  has three components. The first is a predictor that is also used in a previous cascade stage, e.g.  $f_k(x) = f_{k-1}(x) + c_g(x)$  for an embedded cascade. In this case,  $f_{k-1}(x)$  has already been evaluated in stage  $k - 1$  and is available with no computational cost. The second is the set  $O(f_k)$  of features that have been used in some stage  $j \leq k$ . These features are also available and require minimal computation (multiplication by the weight  $c_l$  and addition to the running sum). The third is the set  $N(f_k)$  of features that have not been used in any stage  $j \leq k$ . The overall computation is  $C(f_k) = |N(f_k)|$

$(f_k) \rightarrow + \gamma O(f_k) \rightarrow$ , (23) where  $\gamma \geq 1$  is the ratio of computation required to evaluate a used vs. new feature. For Haar 1 wavelets,  $\gamma \approx 20$ . It follows that updating the predictor of the  $k$ th stage increases its complexity to

$C(f_k) + \gamma$  if  $\gamma \leq O(f_k)$   $C(f_k + \gamma g) = (24) C(f_k) + 1$  if  $\gamma \geq N(f_k)$ , and the complexity of the cascade to  $C(F(f_k + \gamma g)) = P_{1,k} + P_{2,k} C(f_k + \gamma g) + P_{2,k} u(f_k + \gamma g) C(F_{k+1})$  (25)  $= \gamma_k + \gamma_k C(f_k + \gamma g) + \gamma_k u(f_k + \gamma g)$  with

$$\begin{aligned} \gamma_k &= P_{1,k} \\ \gamma_k &= P_{2,k} 4 \\ \gamma_k &= P_{2,k} C(F_{k+1}). \end{aligned} \quad (26) \quad (27)$$

### 3.4

#### Neutral predictors

The models of (10), (11) and (21) will be used for the design of optimal cascades. Another observation that we will exploit is that  $H[F(f_1, \dots, f_m), f_m] = H[F(f_1, \dots, f_m)]$ . This implies that repeating the last stage of a cascade does not change its decision rule. For this reason  $n(x) = f_m(x)$  is referred to as the neutral predictor of a cascade of  $m$  stages.

### 4

#### Boosting classifier cascades

In this section, we introduce a boosting algorithm for cascade design. 4.1

#### Boosting

Boosting algorithms combine weak learners to produce a complex decision boundary. Boosting iterations are gradient descent steps towards the predictor  $f(x)$  of minimum risk for the loss  $L[y, f(x)] = e^{-yf(x)}$  [3]. Given a set  $U$  of weak learners, the functional derivative of RL along the direction of weak learner  $g$  is

$\frac{1}{N} \sum_{i=1}^N \frac{\partial}{\partial y_i} (f(x_i) + \gamma g(x_i)) = \frac{1}{N} \sum_{i=1}^N \gamma \text{RL}(f), g_i = -\frac{\partial}{\partial y_i} L = e^{-y_i} w_i g(x_i)$ , (28) —St — i d? —St — i ?=0 where  $w_i = e^{-y_i} f(x_i)$  is the weight of  $x_i$ . Hence, the best update is

$$g^*(x) = \arg \max_{g \in U} \gamma \text{RL}(f), g_i \cdot g^* U \quad (29)$$

Letting  $I(x)$  be the indicator function, the optimal step size along the selected direction,  $g^*(x)$ , is  $P \sum_{i=1}^N w_i I(y_i = g^*(x_i)) = \frac{1}{N} \sum_{i=1}^N y_i (f(x_i) + c g^*(x_i))$   $c = \arg \min_e = \log \frac{P_i}{P}$ . (30)  $\gamma \text{RL}(F) = \sum_{i=1}^N w_i I(y_i \neq g(x_i))$

The predictor is updated into  $f(x) = f(x) + c g^*(x)$  and the procedure iterated. 4.2

#### Cascade risk minimization

To derive a boosting algorithm for the design of detector cascades, we adopt the loss  $L[y, F(f_1, \dots, f_m)(x)] = e^{-yF(f_1, \dots, f_m)(x)}$ , and minimize the cascade risk  $\frac{1}{N} \sum_{i=1}^N F(f_1, \dots, f_m)(x_i) = e^{-\text{RL}(F)} = \mathbb{E}_{X,Y} \{e^{-yF(f_1, \dots, f_m)}\}$  —St — i Using (13) and (19),

$$\frac{1}{N} \sum_{i=1}^N \frac{\partial}{\partial y_i} [a_k(x_i) + b_k(x_i) g(x_i)] = \frac{1}{N} \sum_{i=1}^N \gamma \text{RL}(F(f_k)), g_i = -\frac{\partial}{\partial y_i} L = e^{-y_i} w_i b_k(x_i) g(x_i) \quad (31) \quad \text{—St — i d? —St — i ?=0}$$

where  $w_i = e^{-y_i} a(x_i)$ ,  $b_k = b_k(x_i)$  and  $a_k, b_k$  are given by (14), (18), and (20). The optimal descent direction and step size for the  $k$ th stage are then  $g_k^*$



(38) (39)

$k$

with  $w_k = e^{y_i a(x_i)}$  and  $a_k$  and  $b_k$  given by (14), (18), and (20). Given a set of weak learners  $U$ , the optimal descent direction and step size for the  $k$ th stage are then  $g_k = \arg \max_{g \in U} -L(F(f_k) + g)$ ,  $\eta_k$

(40)

$g \in U$

$c_k$

$= \arg \min_{c \in \mathbb{R}} R$

(

$\frac{1}{n} \sum_{i=1}^n X_k^T y_i b_k c_k(x_i) - \frac{1}{n} \sum_{i=1}^n X_k^T s_k c_k(x_i) y_i \eta_k e^{w_i} + \eta_k - \eta_k$

)

.

(41)

A pair  $(g_{k,1}, c_{k,1})$  is found among the set  $O(f_k)$  and another among the set  $U \cap O(f_k)$ . The one that most reduces (4) is selected as the best update for the  $k$ th stage and the stage with the largest impact is updated. This gradient descent procedure is denoted Fast Cascade Boosting (FCBoost).

5

Extensions

FCBoost supports a number of extensions that we briefly discuss in this section. 5.1

Cost Sensitive Boosting

As is the case for AdaBoost, it is possible to use cost sensitive risks in FCBoost. For example, the risk of CS-AdaBoost:  $RL(f) = \mathbb{E}_{X,Y} \{y c e^{y f(x)}\}$  [12] or Asym-AdaBoost:  $RL(f) = c \mathbb{E}_{X,Y} \{e^{y f(x)}\}$  [8], where  $y c = CI(y = 1) + (1 - C)I(y = 0)$  and  $C$  is a cost factor. 6

0.36

Train Set pos neg 9,000 9,000 1,000 10,000 1,000 10,000

Test Set pos neg 832 832 100 2,000 200 2,000

0.32

RL

Data Set Face Car Pedestrian

0.28

0.24

0

10

20

30

RC

Figure 2: Left: data set characteristics. Right: Trade-off between the error (RL) and complexity (RC) components of the risk as  $\eta$  changes in (4). Table 1: Performance of various classifiers on the face, car, and pedestrian test sets. Method AdaBoost ChainBoost FCBoost ( $\eta = 0.02$ )

5.2



RL	0.20	0.45	0.30
Face RC	50	2.65	4.93
L	1.20	0.50	0.40
RL	0.22	0.65	0.44
Car RC	50	2.40	5.38
L	1.22	0.70	0.55
RL	0.35	.052	0.46
Pedestrian RC L	50	1.35	3.34
	0.59	4.23	0.54

Bootstrapping

Bootstrapping is a procedure to augment the training set, by using false positives of the current classifier as the training set for the following [11]. This improves performance, but is feasible only when the bootstrapping procedure does not affect previously rejected examples. Otherwise, the classifier will forget the previous negatives while learning from the new ones. Since FCBoost learns all cascade stages simultaneously, and any stage can change after bootstrapping, this condition is violated. To overcome the problem, rather than replacing all negative examples with false positives, only a random subset is replaced. The negatives that remain in the training set prevent the classifier from forgetting about the previous iterations. This method is used to update the training set whenever the false positive rate of the cascade being learned reaches 50%.

6

#### Evaluation

Several experiments were performed to evaluate the performance of FCBoost, using face, car, and pedestrian recognition data sets, from computer vision. In all cases, Haar wavelet features were used as weak learners. Figure 2 summarizes the data sets. Effect of  $\gamma$ : We started by measuring the impact of  $\gamma$ , see (4), on the accuracy and complexity of FCBoost cascades. Figure 2 plots the accuracy component of the risk, RL, as a function of the complexity component, RC, on the face data set, for cascades trained with different  $\gamma$ . The leftmost point corresponds to  $\gamma = 0.05$ , and the rightmost to  $\gamma = 0$ . As expected, as  $\gamma$  decreases the cascade has lower error and higher complexity. In the remaining experiments we used  $\gamma = 0.02$ . Cascade comparison: Figure 3 (a) repeats the plots of the Lagrangian of the risk shown in Figure 1, for classifiers trained with 50 boosting iterations, on the face data. In addition to AdaBoost and ChainBoost, it presents the curves of FCBoost with ( $\gamma = 0.02$ ) and without ( $\gamma = 0$ ) complexity constraints. Note that, in the latter case, performance is in between those of AdaBoost and ChainBoost. This reflects the fact that FCBoost ( $\gamma = 0$ ) does produce a cascade, but this cascade has worse accuracy/complexity trade-off than that of ChainBoost. On the other hand, the inclusion of complexity constraints, FCBoost ( $\gamma = 0.02$ ), produces a cascade with the best trade-off. These results are confirmed by Table 1, which compares classifiers trained on all data sets. In all cases, AdaBoost detectors have the lowest error, but at a tremendous computational cost. On the other hand, ChainBoost cascades are always the fastest, at the cost of the highest classification error. Finally, FCBoost ( $\gamma = 0.02$ ) achieves the best accuracy/complexity trade-off: its cascade has the lowest risk Lagrangian L. It is close to ten times

faster than the AdaBoost detector, and has half of the increase in classification error (with respect to AdaBoost) of the ChainBoost cascade. Based on these results, FCBBoost ( $\eta = 0.02$ ) was used in the last experiment. 7

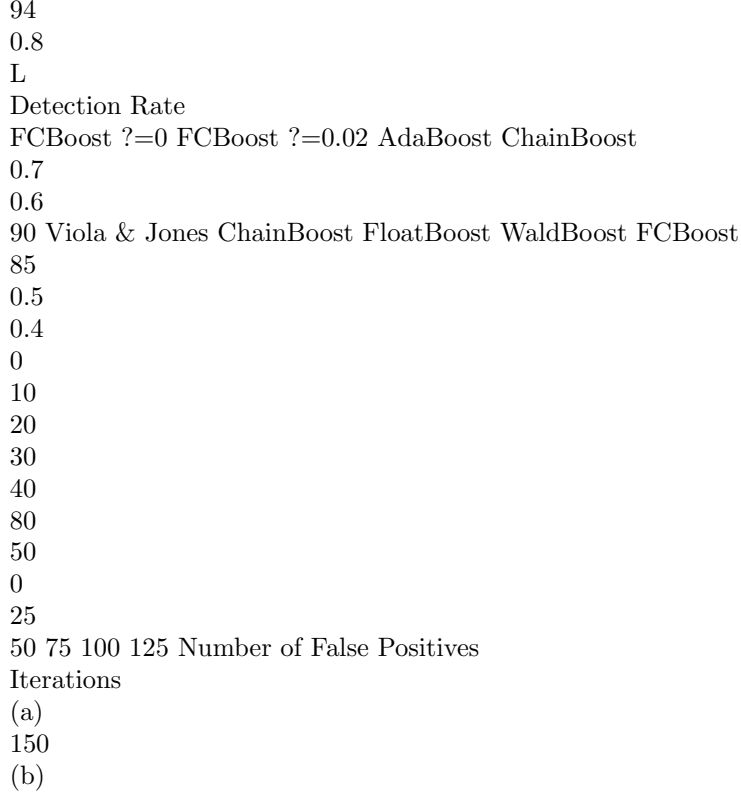


Figure 3: a) Lagrangian of the risk for classifiers trained with various boosting algorithms. b) ROC of various detector cascades on the MIT-CMU data set.

Table 2: Comparison of the speed of different detectors. Method Evals	
VJ [13]	8
FloatBoost [5]	18.9
ChainBoost [15]	18.1
WaldBoost [9]	10.84
[8]	15.45
FCBBoost	7.2

Face detection: We finish with a face detector designed with FCBBoost ( $\eta = 0.02$ ), bootstrapping, and 130K Haar features. To make the detector cost-sensitive, we used CS-AdaBoost with  $C = 0.99$ . Figure 3 b) compares the resulting ROC to those of VJ [13], ChainBoost [15], FloatBoost [5] and WaldBoost [9]. Table 2 presents a similar comparison for the detector speed (average number of features evaluated per patch). Note the superior performance of the FCBBoost cascade in terms of both accuracy and speed. To the best of our knowledge, this is the fastest face detector reported to date.

A Recursive form of cascade predictor Applying (9) recursively to (8)  $F = f_1 u(f_1) + u(f_1)F_2 = f_1 u(f_1) + u(f_1) [f_2 u(f_2) + u(f_2)F_3] = f_1 u(f_1) + f_2 u(f_1)u(f_2) + u(f_1)u(f_2) [f_3 u(f_3) + u(f_3)F_4] =$

$$\sum_{k=1}^K \sum_{i=1}^I u(f_i) \prod_{j=1}^j u(f_j) + F_k \quad (42) \quad (43) \quad (44) \quad (45)$$

$= T_{1,k} + T_{2,k} F_k$  (46)  $P_{k+1} = Q$  where  $T_{1,k} = \sum_{i=1}^I u(f_i) \prod_{j=1}^j u(f_j)$  and  $T_{2,k} = \prod_{j=1}^j u(f_j)$  satisfy the recursions of (12). Combining (46) and (9) then leads to (11). (10) follows from (46) and the initial condition  $F_m = f_m$ .

B Recursive form of cascade complexity Let  $C(f_k)$  be the complexity of evaluating  $f_k$ . Then  $C(F) = C(f_1) + u(f_1)C(F_2) = C(f_1) + u(f_1)[C(f_2) + u(f_2)C(F_3)] =$

$$\sum_{k=1}^K \sum_{i=1}^I C(f_i) \prod_{j=1}^j u(f_j) + C(F_k) \quad (47) \quad (48) \quad (49)$$

$$= P_{1,k} + P_{2,k} C(F_k) \quad (50)$$

with  $P_{1,k+1} = P_{1,k} + C(f_k)$   $P_{2,k+1} = P_{2,k} u(f_k)$  (51) and initial conditions  $P_{1,1} = 0$ ,  $P_{2,1} = 1$ . The relationship of (47) is a special case of  $C(F_k) = C(f_k) + u(f_k)C(F_{k+1})$  (52) with initial conditions  $C(F_m) = C(f_m)$  and  $C(F_{m+1}) = 0$ . Combining (52) with (50) leads to (21). 8

## 2 References

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