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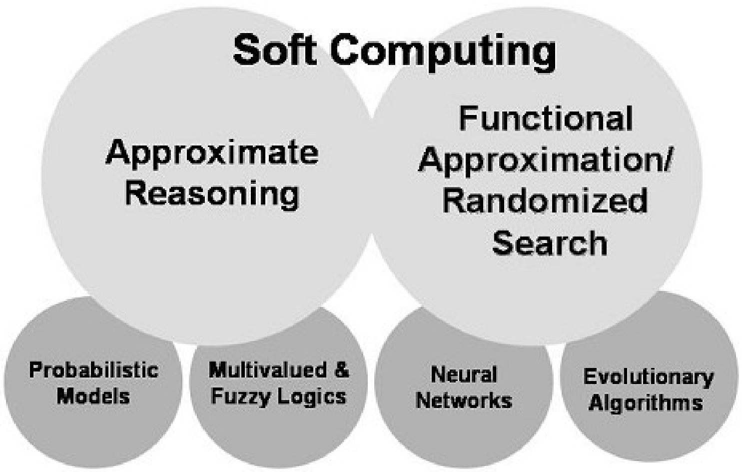

Soft Set Theory for Soft Computing

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1 History

Molodtsov initiated the concept of soft set as a new mathematical tool for dealing with uncertainties. Most of our traditional tools for formal modeling, reasoning and computing are crisp, deterministic and precise in character. However, there are many complicated problems in economics, engineering, environment, social science, medical science etc. that involve uncertainties. The Theory of Probability, Theory of Fuzzy Sets,Theory of Intuitionistic Fuzzy Sets,Theory of Vague Sets, Theory of Interval Mathematics and Theory of Rough Sets are considered as mathematical tools for dealing with uncertainties.

In 1999, Molodstov pointed out that these theories which are considered as mathematical tools for dealing with uncertainties, have certain limitations. He further pointed out that the reason for these limitations is, possibly, the inadequacy of the parameterization tool of the theory. The Soft Set Theory introduced by Molodstov is quite different from these theories in this context. The absence of any restrictions on the approximate description in Soft Set Theory makes this theory very convenient and easily applicable. Fuzzy set theory proposed by Professor L. A. Zadeh in 1965 is considered as a special case of the soft sets.

Current and soft mathematics can co-exist and can be used consistently to solve real world problems.

2 Soft Computing

Definition

Soft computing differs from conventional (hard) computing in that, unlike hard computing, it is tolerant of imprecision, uncertainty, partial truth, and approximation. In effect, the role model for soft computing is the human mind. The guiding principle of soft computing is: Exploit the tolerance for imprecision, uncertainty, partial truth, and approximation to achieve tractability, robustness and low solution cost. The basic ideas underlying soft computing in its current incarnation have links to many earlier influences, among them Zadeh's 1965 paper on fuzzy sets; the 1973 paper on the analysis of complex systems and decision processes; and the 1979 report (1981 paper) on possibility theory and soft data analysis.

Words of Wisdom

"Basically, soft computing is not a homogeneous body of concepts and techniques. Rather, it is a partnership of distinct methods that in one way or another conform to its guiding principle. At this juncture, the dominant aim of soft computing is to exploit the tolerance for imprecision and uncertainty to achieve tractability, robustness and low solutions cost. The principal constituents of soft computing are fuzzy logic, neurocomputing, and probabilistic reasoning, with the latter subsuming genetic algorithms, belief networks, chaotic systems, and parts of learning theory. In the partnership of fuzzy logic, neurocomputing, and probabilistic reasoning, fuzzy logic is mainly concerned with imprecision and approximate reasoning; neurocomputing with learning and curve-fitting; and probabilistic reasoning with uncertainty and belief propagation".

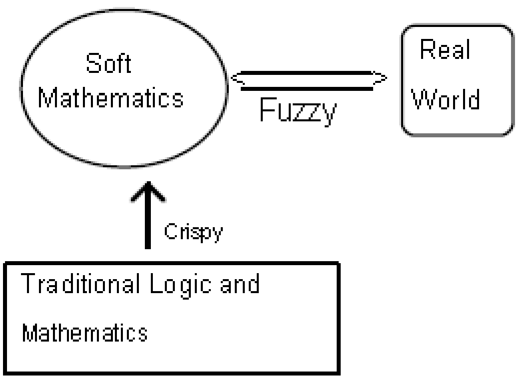
Components of Soft Computing

At this juncture, the principal constituents of Soft Computing (SC) are Fuzzy Logic (FL), Neural Computing (NC), Evolutionary Computation (EC) Machine Learning (ML) and Probabilistic Reasoning (PR), with the latter subsuming belief networks, chaos theory and parts of learning theory. What is important to note is that soft computing is not a melange. Rather, it is a partnership in which each of the partners contributes a distinct methodology for addressing problems in its domain. In this perspective, the principal constituent methodologies in SC are complementary rather than competitive. Furthermore, soft computing may be viewed as a foundation component for the emerging field of conceptual intelligence.

Components of soft computing include:

- Neural networks (NN)
- Fuzzy logics (FL)
- Evolutionary computation (EC), including:
  - Evolutionary algorithms
    - Genetic algorithms
    - Differential evolution
  - Metaheuristic and Swarm Intelligence
    - Ant colony optimization
    - Bees algorithms
    - Bat algorithm
    - Cuckoo search
    - Harmony search
    - Firefly algorithm
    - Artificial immune systems
    - Particle swarm optimization
- Ideas about probability including Bayesian networks
- Chaos theory
- Perceptron

Generally speaking, soft computing techniques resemble biological processes more closely than traditional techniques, which are largely based on formal logical systems, such as sentential logic and predicate logic, or rely heavily on computer-aided numerical analysis (as in finite element analysis). Soft computing techniques are intended to complement each other. Unlike



hard computing schemes, which strive for exactness and full truth, soft computing techniques exploit the given tolerance of imprecision, partial truth, and uncertainty for a particular problem. Another common contrast comes from the observation that inductive reasoning plays a larger role in soft computing than in hard computing.

3 Soft Set Theory

Structured Subsets

According to the authors of [1], ideal membership functions for real world fuzzy sets are made from elastic material, because fuzzy sets should be able to tolerate certain amount of perturbation or stretching. Mathematically, each perturbation is a new function, that consists of a set of functions, each of which can be continuously stretched into another function. Such a set of functions is a highly structured subset of membership function space.

Soft Sets

A fuzzy set, which is also called, in different parts of the world, a soft set, is an *abstract structured set* of membership function space. Such a structured subset may be an equivalence class, a neighborhood, or fuzzified structures in the membership function space. Soft sets are intended to capture and to defuse the conflicts among existing fuzzy theories. So a fuzzy set could be defined by

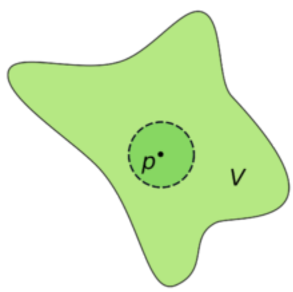
1. A collection of membership functions
2. that should be able to transform among themselves

Neighbourhood Systems

It is an abstraction of “near” or “negligible distances” in geometry. A neighborhood system is an association that assigns each datum a list of data that may or may not contain the datum. It is natural and implementable, and used for approximate retrieval in databases and approximate reasoning in knowledge bases

The central notion here is neighborhood systems. It is an abstraction of “near” or “negligible distances” in geometry. A neighborhood system is an association that assigns each datum a list of data that may or may not contain the datum. It is natural and implementable. So current and soft mathematics can co-exist and can be used consistently to solve real world problems.

Neighborhood systems express the semantics of nearby spaces. Let p be an object (or datum) in the universe or space X.



- A neighborhood, denoted by N(p), or simply N, of p is a non-empty subset of X, which may or may not contain the object p.
- Any subset that contains a neighborhood is a neighborhood.
- A neighborhood system of object p, denoted by NS(p), is a non-empty maximal family of neighborhoods of p.
- A neighborhood system of X, denoted by NS(X) is the collection of NS (p) for all objects p in X.

- If a neighborhood system NS(X) that satisfies certain axioms, then X is a topological space. In general, X is a Frechet (V) space.
- From this view, rough set theory is a special case of the neighborhood system theory.

Soft sets, defined using Neighbourhood Systems

A real world fuzzy set is defined abstractly by a neighborhood systems of membership function space. Neighborhood systems translate the real world problem into a mathematical problem. So our ultimate goal is to axiomatize fuzzy sets through such neighborhood systems.

Membership Function Space

Let U be the universe of discourse, and let FX : U $\to$ M be a map, where M is, in general, a membership space.

FX is called a membership function; FX(x) is called the grade or degree of membership of x $\in$ U.

If M is the set of two elements $\{0, 1\}$, then FX is the characteristic function of a classical crispy set. If M is a unit interval [0, 1], then FX is the membership function of a classical fuzzy set.

Let F be a collection of fuzzy sets on U. Each fuzzy set is defined by a neighborhood (may be a singleton) in the membership function space; let Col be a neighborhood system. MF(U) is the total union of all membership functions representing F, namely the union of members in Col : MF(U) = $\bigcup$ Col
Col is a neighborhood system on MF(U).

Types of Soft Sets

Depending on the nature of Col, we have various types of fuzzy sets (sofsets).

1. **W-Sofset** - Weighted Soft Set (Mathematical Soft Set or Quantitative Soft Set) occur when Col consists of singletons, i.e., the membership function space of a given fuzzy set consists only of a single set. Every membership function is treated as a characteristic function of a soft set. This is the naive definition of fuzzy sets.

2. **F-Sofset** - Finite-multi Soft Set occurs when Col consists of finite sets.

3. **P-Sofset** - Partitioned Soft Set, occurs when Col forms a crispy partition. The space of membership functions is partitioned into equivalence classes.

4. **B-Sofset** - Basic Neighborhood Soft Set occurs when Col forms a basic neighborhood system (i.e., the neighborhood system is defined by a binary relation). Basic neighborhoods are geometric view of binary relation. Two membership functions, FX and GX, (for the same soft set X) measure theoretically near, if for any given $\varepsilon > 0$, $\|FX(X) - GX(X)\| < \varepsilon$

5. **C-Sofset** - Covering Soft Set occurs when Col forms a covering.

6. **N-Sofset** - Neighborhood Soft Set (Real World Soft Set or Qualitative Soft Set) occurs when if forms a neighborhood system.

Note that a W-Sofset is a special case of an F-Sofset, which is a special case of a P-Sofset, which is a special case of a B-Sofset, which is a special case of an N-Sofset.

One Fuzzy Set - Several Membership Functions

There are two conflicting notions:

1. A Fuzzy set is defined by one and only one membership function, as in W-Sofsets. Since only one membership function is permitted for each fuzzy set, membership functions have to be selected so carefully that they are closed under extended operations

2. A Fuzzy set could have several membership functions. The method of selection of membership functions is more relaxed.

One Membership Function - Several Fuzzy Sets

As of December 1, 2012 AD, there are two conflicting notions:

1. A membership function can represent at most one fuzzy set. So the space MF(U) of membership functions is partitioned into equivalence classes. Each equivalence class represents one and only one fuzzy set. Such fuzzy sets are P-sofsets. Mathematically, this view is the most elegant one. Unfortunately, it may not be realistic. One will realize, however after a moment of reflection, that MF(U) may be a continuous family of real valued functions. Very unlikely there is a natural partition on such MF(U)s. We are forced to accept more general and realistic view. Of course, there are situations in which MF(U) has very natural partitions.

2. a real world fuzzy set should be defined by an elastic membership function. Mathematically, one can express such an elastic membership function by a set of membership functions that are very near to each other. In other words, they belong to the same neighborhood. In general, is a neighborhood system, not a crispy partition. Overlapping does occur in neighborhoods. So a membership function could lie in two neighborhoods. In other words, a membership function may represent two fuzzy sets. Such fuzzy sets are N-sofsets.

The developments of various generalized set theories form a beginning of a "soft mathematics and may provide a foundation for soft computing. This paper is one of our attempt to provide a solid set theory for soft computing.

4 Application of Soft sets in Soft computing

Molodtsov presented some applications of the soft set theory in several directions viz. study of smoothness of functions , game theory, operations research, Riemann-integration, Perron integration, probability, theory of measurement, etc. In this section, we present an application of soft set theory in a decision making problem with the help of rough approach. The problem we consider is as below.

Let U = $\{h_l, h_2, h_i, h_i, h_g, h_s\}$, be a set of six houses, E = {expensive; beautiful; wooden; cheap; in the green surroundings: modern; in good repair: in bad repair} be a set of parameters.

Consider the soft set (F, E) which describes the *attractiveness of the houses*, given by (F, E) = {expensive houses = 4, beautiful houses = $\{h_l, h_z, h_g, h_4, h_s, h_e\}$, wooden houses = $\{h_l, h_z, h_G\}$, modern houses = { hl , hp, hs}, in bad repair houses = { hz, hd, hs}, cheap houses = { hl , hz, h3,h4, h5, hi}, in good repair houses = { hl , h3, hG}, in the green surroundings houses = { /tl, /x2, h3,lb, ho}.

Suppose that, Mr. X is interested to buy a house on the basis of his choice parameters beautiful, wooden, cheap!, in the green surroundings, in good repair, etc., which constitute the subset P = beautiful; wooden; cheap; in the green surroundings; in good repair of the set E.

That means, out of available houses in U, he is to select that house which qualifies with all (or with maximum number of) parameters of the soft set P.

The problem is to select the house which is most suitable with the choice parameters of Mr. X. The house which is most suitable for Mr. X, need not be most suitable for Mr. Y or Mr. Z as the selection is dependent upon the set of choice parameters of each buyer. To solve the problem, we do some theoretical characterizations of the soft set theory of Molodtsov, which we present below.

Tabular Representation of soft set

Consider the soft set (F, P) above on the basis of the set P of choice parameters of Mr. X. We can represent this soft set in a tabular form as shown below. This style of representation will be useful for storing a soft set in a computer memory. If h ∈ F(E) then h_i F(∈)= 1, otherwise h_{ij} = 0, where h_{ij} are the entries in Figure 1. Thus, a soft set now can be viewed as a knowledge

U	e_1	e_2	e_3	e_4	e_5
h_1	1	1	1	1	1
h_2	1	1	1	1	0
h_3	1	0	1	1	1
h_4	1	0	1	1	0
h_5	1	0	1	0	0
h_6	1	1	1	1	1

Figure 1: Table 1

representation system where the set of attributes is to be replaced by a set of parameters.

Reduct-Table for a Soft Set

Consider the soft set (F, E). Clearly, for any $P \subset E$, (F, P) is a soft subset of (F, E). We will now define a reduct-soft-set of the soft set (F, P).

Consider the tabular representation of the soft set (F, P). If Q is a reduct of P, then the soft set (F, Q) is called the reduct-soft-set of the soft set (F, P).

Intuitively, a reduct-soft-set (F, Q) of the soft set (F, P) is the essential part, which suffices to describe all basic approximate descriptions of the soft set (F, P).

The core soft set of (F, P) is the soft set (I;: C)! where C is the CORE (P).

Choice Value for an Object h_i

The choice value of an object $h_i \in U$ is c_i, given by

$$c_i = \sum_j h_{ij}$$

where h_{ij} are the entries in the table of the reduct-soft-set.

Algorithm for Selection for House

The following algorithm may be followed by Mr. X to select the house he wishes to buy

1. input the soft set (F, E),

2. input the set P of choice parameters of Mr. X which is a subset of E,
3. find all reduct-soft-sets of (F,P),
4. choose one reduct-soft-set say (F, Q) of (F, P),
5. find k, for which ck = maxci.

Then h_k is the optimal choice object. If k has more than one value, then any one of them could be chosen by Mr. X by using his option.

Now we use the algorithm to solve our original problem.

Clearly, from the table we see that $\{e_1, e_2, e_4, e_5\}, \{e_2, e_3, e_4, e5\}$ are the two reducts of P = $\{e_1, e_2, e_3, e_4, e_5\}$. Choose any one say, Q = $\{e_1, e_2, e_4, e_5\}$.

Incorporating the choice values, the reduct-soft-set can be represented in Table 2 below.

U	e_1	e_2	e_4	e_5	choice value
h_1	1	1	1	1	$c_1 = 4$
h_2	1	1	1	0	$c_2 = 3$
h_3	1	0	1	1	$c_3 = 3$
h_4	1	0	1	0	$c_4 = 2$
h_5	1	0	0	0	$c_5 = 1$
h_6	1	1	1	1	$c_6 = 4$

Figure 2: Table 2

Here max $c_i = c_i$ or c_6. Decision: Mr. X can buy either the house h_1 or the house h_2.

5 Conclusion

In the literature of fuzzy sets, there is no unified view on the relationship between fuzzy sets and membership functions. Some authors treat membership functions as characteristic functions. In other words, one fuzzy set has one and only one membership function, in our terminology, a W-sofset.

References

[1] T. Y. Lin *A set theory for soft computing* - http://xanadu.cs.sjsu.edu/~tylin/publications/paperList/86_18quality9.pdf

[2] Imada; Akira, *Introduction to Soft Computing.* http://neuro.bstu.by/ai/soft-computing.pdf

[3] Maji;P.K, Roy; A.R. *An Application of Soft Sets in Decision Making Problem.* Department of Mathematics, Indian Institute of Technology, Kharagpur, India.

[4] Soft Computing http://en.wikipedia.org/wiki/Soft-computing

[5] Soft Sets http://en.wikipedia.org/wiki/Soft_set