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Dispatching policies for last-mile distribution with stochastic supply and demand

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ABSTRACT

Relief distribution has received considerable attention in the disaster operations management literature. However, the majority of this literature assumes that supply is always available. In reality, a significant portion of the materials that flow through the humanitarian relief chain are donations, which represent an uncertain supply source in terms of both quantity and timing. This paper investigates a two-stage relief chain consisting of a single staging area (SA) where donations arrive over time in uncertain quantities, which are periodically distributed to random numbers of disaster survivors located at a point of distribution (POD). A single vehicle travels back and forth between the SA and POD transporting relief supplies during a finite horizon. The goal of this study is to identify dispatching policies for the vehicle with the sole purpose of minimizing unsatisfied demand at the POD. To this end, we examine the effectiveness of two common-sense heuristic policies relative to the optimal dispatching policy, the latter of which is determined via stochastic dynamic programming. Our findings indicate that although continuously dispatching the vehicle between the SA and POD is not an optimal policy, it is either optimal or close to optimal in most situations.

1. Introduction

Following any large-scale disaster, large quantities of food, water, clothing, and medical supplies are needed to avoid extreme suffering and loss of life. Especially at first, other considerations such as cost are insignificant compared to the consequences of having a need go unmet; all parts of the humanitarian supply chain must work efficiently from the initial collection of supplies to the final delivery to beneficiaries. While there are other important aspects of the disaster relief process, relief agencies estimate that logistics makes up about 80% of the total relief effort (Trunick, 2005).

Attention to disaster relief logistics greatly increased after the 2004 tsunami in the Indian Ocean. News networks broadcast the tragic story around the world, which in turn sparked an influx of financial and in-kind donations from sympathetic viewers. Truly, the outpouring of concern and support was one of the brightest moments in human history; unfortunately, the surge of in-kind donations posed very significant logistical problems. The Sri Lankan airport was overwhelmed by the amount of aid donated from around the world, and the existing warehouses were unable to hold the goods that made it in; ultimately the supply chain collapsed due to a combination of excess volume and/or damage (Thomas and Kopczak, 2005). Because of the urgency of the situation, the relief chain was designed and implemented before decision makers had sufficient information about the extent of the disaster. This uncertainty prevented them from using traditional

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for-profit techniques, and instead they had to improvise ad hoc solutions as best they could. This was far from the first disaster in the world, but the visibility and severity of this event led logisticians around the globe to realize the need for specialized strategies for disaster relief logistics. Since then, many studies have contributed to our understanding of the unique characteristics of humanitarian supply chains, which are comprised of the following four functional stages (e.g. Vanajakumari et al., 2016):

1. *Major Distribution Centers* – Permanent distribution centers strategically located throughout the world, holding water and food before they are sent to Pre-Staging Areas. For example, the Federal Emergency Management Agency (FEMA) has 8 storage centers in the continental U.S. and 3 offshore locations: Guam, Hawaii, and Puerto Rico.
2. *Pre-Staging Areas* – Temporary locations to hold inventory in anticipation of a disaster. The facilities help aid agencies quickly move needed supplies to main Staging Areas. An example of this is moving supplies near Florida before hurricane season.
3. *Staging Areas (SAs)* – While major distribution centers and pre-staging areas are maintained independent of whether or not an active disaster is taking place, SAs are established once disaster events occur. Staging areas are temporary constructs chosen from a pre-determined set of locations such as schools, shopping mall parking lots, or faith-based organizations (e.g. churches). They are intended to reduce the lead time and cost of distributing aid to disaster survivors by bringing supplies closer to affected areas.
4. *Points of Distribution (PODs)* – Locations where aid is distributed to affected people. Like SAs, POD locations are selected after the disaster but identified before. Similar to SAs, promising locations for PODs are schools, parking lots, and churches, ideally situated close to affected areas, but can sometimes be as far as 600 miles away.

This study addresses the final stage of the humanitarian relief chain, last-mile distribution, which refers to the delivery of relief supplies from staging areas to local distribution centers (Knott, 1987). Specifically, this paper considers the impact of unsolicited donations on last-mile distribution in a humanitarian relief supply chain. The aforementioned logistical difficulties associated with the Sri Lankan airport give some sense of how unsolicited donations can affect relief distribution. Generally speaking, the most significant drawbacks associated with unsolicited donations can be attributed to *material convergence*, which refers to the influx of supplies and equipment in response to perceived needs during the aftermath of major disasters (Holguín-Veras et al., 2012). Based on interviews with practitioners, Holguín-Veras et al. (2012) found that the challenges of material convergence have more to do with the massive volume of donations than donation uncertainty; the presence of donated items in overwhelming quantities causes congestion at entry points into affected areas and at end sites where donated items reach their destination. Incoming donations have to be inspected and sorted before being distributed to beneficiaries, but because the majority of donated items are not needed, they eventually end up being destroyed. These and other issues related to material convergence unnecessarily add complexity to the disaster relief supply chain, which in turn causes delays in delivering relief supplies to the people who urgently need them. Although this paper addresses unsolicited donations in last-mile distribution, material convergence is not considered directly; in particular, this paper takes unsolicited donations into account by modeling supply at the Staging Area as an exogenous stochastic process. Thus our focus is managing donation uncertainty as opposed to handling the large amounts of donations specific to material convergence. On the other hand, the random donation arrivals considered in this study can be thought of as those that prior to being transported to the SA, have emerged out of material convergence after clearing inspection and sorting processes.

The specific relief chain examined in this study consists of a single staging area and a single point of distribution as shown in Fig. 1. The point of distribution is located close enough to the affected area that it is readily accessible to disaster survivors (i.e., beneficiaries), but far enough away so that their safety is not compromised. Donations and beneficiaries arrive periodically to the staging area and point of distribution respectively, both in uncertain quantities. A single vehicle of limited capacity is available to transport donations from the staging area to the point of distribution with the goal of minimizing unsatisfied demand.

The system in Fig. 1 represents a stylized version of a more complex relief logistics network observed in practice. In the real-world system, there are multiple SAs, PODs, and vehicles that are dispatched among them. The motivation for limiting the scope of this study to the simplified system shown in Fig. 1 is the possibility of being able to characterize the structure of optimal policies analytically, computationally, or both. The ability to express the form of an optimal policy, albeit in a stylized environment, can be very useful. For one, if we discover that optimal policies within the context of the stylized system are complicated and unstructured, then we know it would be highly unlikely for complex models that more closely resemble the real-world system to have optimal policies that are simple or structured. Another advantage is that if the stylized model leads to the identification of simple or structured optimal policies, then the resulting insights could be used to design effective solution procedures for more complex models in future studies.

It is also worth mentioning that the two-stage relief chain portrayed in Fig. 1 has an analogous counterpart in a for-profit supply chain context, namely shipment consolidation. Also known as inventory consolidation or temporal consolidation, shipment consolidation refers to holding small loads that arrive randomly at different times and transporting them in a single larger load (Lai et al., 2016). From this perspective, the humanitarian aid vehicle dispatching problem introduced in this paper is equivalent to a shipment consolidation problem with stochastic supply and demand. However similar to most humanitarian logistics papers that have their own commercial sector counterparts, our study can be distinguished from the shipment consolidation literature by its objective function. Specifically, the objective considered in this study is mini-

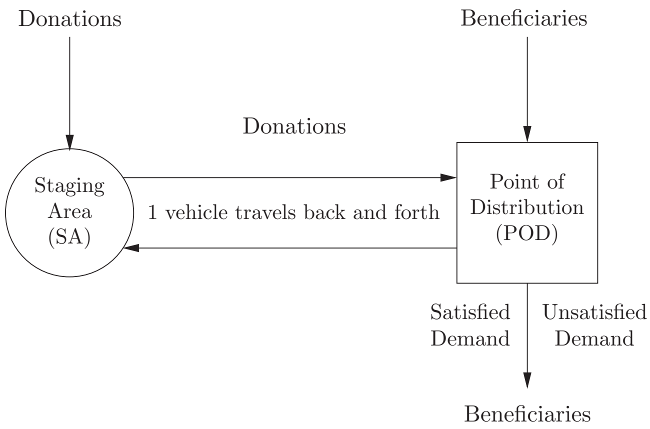

Fig. 1. Supply chain network addressed in this study.

mization of unsatisfied demand whereas the objective in most inventory consolidation papers is to minimize cost. Further details concerning the relationship between this study and the shipment consolidation literature are given in Section 2.3.

The goal of this paper is to determine vehicle dispatching policies that minimize unsatisfied demand at the point of distribution within the context of the two-stage humanitarian supply chain depicted in Fig. 1. Two simple and practical policies would be to (i) dispatch the vehicle on a continuous basis or (ii) dispatch the vehicle only when it is full. Both policies are identical when the vehicle is at the point of demand; regardless of policy, the vehicle returns to the staging area immediately after unloading its cargo. At the staging area, on the other hand, the policies diverge: under the continuous dispatching (CD) policy, all available inventory is loaded onto the vehicle (capacity permitting) and immediately dispatched to the point of demand even if it is not full. Under full truckload dispatching (FTD), the vehicle waits at the staging area until enough inventory accumulates to fill the vehicle to capacity, at which time it leaves for the point of demand.

The primary objective of this study is to assess the performance of the above-mentioned pragmatic policies relative to the optimal policy and to each other. To this end, we pose the following research questions (note that Questions 1 and 2 are concerned with decisions made at specific points in time, while Questions 3 and 4 deal with the overall performance of policies during a finite horizon):

Question 1. *If the current position of the vehicle is at the point of demand, is dispatching the vehicle to the staging area always an optimal decision?*

Since supply arrivals only occur at the staging area, there seems to be no incentive for the vehicle to remain at the point of demand after making a delivery. Doing so would only delay the vehicle's eventual return to the point of demand with additional supplies, which is most likely counterproductive given the objective of minimizing unsatisfied demand. This leads us to a bigger question: is there ever any merit in delaying dispatch? Questions 1 and 2 address this issue depending if the vehicle is located at the point of demand or the staging area respectively.

Question 2. *If the vehicle is at the staging area, are there any situations in which deferring dispatch to a future period is an optimal decision?*

In general, delaying dispatch of the vehicle for any reason seems unwise given the objective of minimizing unsatisfied demand. Perhaps postponing dispatch from the staging area to the point of distribution is an appropriate decision if the accumulation of donations at the staging area is low, which would result in only a few items being shipped. For example, delaying dispatch is clearly a good decision if the inventory level at the staging area is zero.

Question 3. *If the average rate of demand at the point of demand is greater than the average arrival rate of supplies at the staging area, is continuous dispatching an optimal policy?*

Intuitively, one might expect continuous dispatching to achieve a minimal number of shortages if demands occur at a faster rate than supplies arrive. When demand is plentiful and supplies are scarce, there seems to be no benefit in delaying shipments when the goal is to minimize shortages. However, is such a policy optimal? What relationship must exist between the supply rate and demand rate in order for the CD policy to be an optimal policy?

Question 4. *In which situations is dispatching full truckloads an optimal policy?*

It is hard to imagine the FTD policy not being optimal when the vehicle capacity is prohibitively small (e.g., a vehicle capacity of one unit). However in this case, the FTD and CD policies would be equivalent, so this question really pertains to scenarios in which the vehicle capacity is reasonably accommodating. Question 3 implies that FTD is not optimal and does not outperform CD if the average demand rate is greater than the average arrival rate of supply, but if supplies are coming in faster than demands are occurring, would FTD be the better policy? Our initial guess is "no"; given the objective of minimizing unsatisfied demand, there seems to be no benefit to delaying shipments on a regular basis.

The remainder of the paper is organized as follows. Section 2 reviews the academic literature that is relevant to this study, followed by the development of the stochastic dynamic programming model in Section 3. Section 4 begins our findings, starting with analytical results then moving to computational results in Section 5. Finally we conclude with Section 6 to summarize and describe opportunities for future work.

2. Literature review

Any study in humanitarian relief must always look both at the disaster operations management literature and related works in the for-profit sector. On the humanitarian side, we review research that addresses last-mile distribution as well as studies that consider the effects of donations on the humanitarian relief chain. In addition, we examine literature related to vehicle dispatching policies in profit-driven supply chains, which is often referred to as shipment consolidation.

2.1. Last mile distribution

As mentioned in Section 1, last mile distribution in humanitarian relief concerns of the logistics of moving relief supplies from staging areas (SAs) to points of distribution (PODs) during the aftermath of disaster events. At the operational level, it involves scheduling vehicles to make deliveries, designing delivery routes, and allocating supplies among PODs (e.g. Balcik et al., 2008). Last-mile distribution has also been studied from the perspective of the tactical decision problem of where to locate staging areas and what capacities they should have (e.g. Noyan et al., 2016). Researchers have considered various combinations of the above-mentioned operational and tactical decisions with varying degrees of complexity. In one of the earliest studies in the area, Knott (1987) considers vehicle scheduling for delivering food from a distribution center to several demand points. Specifically, the author develops a linear programming model that specifies the minimum number of direct shipments such that demand at each demand point is satisfied based on a framework that assumes a one period planning horizon, a single commodity, and no forms of uncertainty. Subsequent studies have moved towards addressing more and more features of last-mile distribution observed in practice resulting in increasingly complex mathematical models. In this direction, extensions to the Knott (1987) model include (i) more than one of the operational and/or tactical decisions listed above; (ii) multiple commodity types; (iii) multiple period planning horizons; and (iv) uncertainty in supply, demand, and/or network reliability. As seen in Table 1, the majority of last-mile distribution studies address at least one of these four extensions.

The last-mile distribution literature is also characterized by diverse objective functions, modeling frameworks, and solution approaches. In general, atypical performance metrics is often what sets humanitarian logistics research apart from traditional operations management studies driven by commercial applications. While the objective of the latter is almost always to minimize cost or maximize profit, metrics related to service level are more critical in humanitarian applications. Measures of performance that have appeared in last mile distribution studies include minimizing unsatisfied demand (e.g. Özdamar et al., 2004), minimizing delays in satisfying demand (e.g. Barbarosoğlu et al., 2002), equitable distribution of supplies (e.g. Huang et al., 2012; Noyan et al., 2016), as well as minimizing logistics costs (e.g. Haghani and Oh, 1996). In reality, relief organizations commonly plan and execute logistics activities within the confines of a limited budget. As such, last mile distribution studies are often characterized by creative performance metrics that attempt to capture objectives related to both cost and service level (e.g. Balcik et al., 2008; Van Hentenryck et al., 2010; Huang et al., 2012; Rennemo et al., 2014; Ahmadi et al., 2015; Noyan et al., 2016; Vanajakumari et al., 2016). With regard to modeling framework, integer program-

Table 1
Characteristics of representative studies from the last mile distribution literature.

	Decisions				Multi-		Uncertain		
	Vehicle Routes	Vehicle Schedules	Allocate Sup	Ntwk Dsgn	Item	Period	Dmd	Sup	Ntwk
Knott (1987)	✓								
Haghani and Oh (1996)	✓	✓		✓					
Barbarosoğlu et al. (2002)	✓								
Barbarosoğlu and Arda (2004)	✓	✓			✓		✓	✓	
Özdamar et al. (2004)	✓		✓		✓	✓	✓	✓	
Balcik et al. (2008)	✓	✓	✓		✓	✓	✓	✓	
Van Hentenryck et al. (2010)			✓	✓			✓		✓
Huang et al. (2012)	✓	✓							
Rennemo et al. (2014)		✓		✓	✓		✓		✓
Ahmadi et al. (2015)	✓			✓					✓
Noyan et al. (2016)			✓	✓			✓		✓
Vanajakumari et al. (2016)	✓	✓	✓	✓					
This paper		✓				✓	✓	✓	

ming models (e.g. Balcik et al., 2008; Van Hentenryck et al., 2010; Huang et al., 2012; Vanajakumari et al., 2016) and multi-stage stochastic programming models (e.g. Rennemo et al., 2014; Ahmadi et al., 2015; Noyan et al., 2016) seem to be the most common. Consequently, solution methods include a combination of commercial optimization solvers (e.g. Huang et al., 2012; Rennemo et al., 2014; Vanajakumari et al., 2016) and heuristic methods (e.g. Huang et al., 2012; Van Hentenryck et al., 2010; Ahmadi et al., 2015; Noyan et al., 2016), both of which are sometimes applied in a rolling horizon framework to capture changing conditions in multi-period settings (e.g. Balcik et al., 2008; Vanajakumari et al., 2016).

This paper contributes to the last mile distribution literature in disaster operations management by considering the impact of material donations; specifically, how material donations precipitate as stochastic supply. Papers in disaster relief often consider demand uncertainty or the unavailability of network links Ahmadi et al. (2015), but the few papers who do include stochastic supply either assume that supply is known and use a rolling-horizon framework to account for changing conditions (Balcik et al., 2008), or they only consider a single decision period (Barbarosoğlu and Arda, 2004). To the authors' knowledge, we are the first study in last mile distribution to consider stochastic supply in a multi-period setting.

2.2. Donations in disaster relief

Donations are an important but, we believe, often ignored part of the humanitarian relief supply chain. The vast majority of aid is donated, and so it is important to consider the effects of the different categories of donations. The simplest and most broad delineation is between financial and in-kind (a.k.a. physical/non-financial) donations.

Burkart et al. (2017) provides a comprehensive survey of academic literature related to funding humanitarian operations. The authors categorize the studies into groups by the funding's nature (monetary or in-kind, further divisible into special cases like food or organ donations), source (private or institutional), restrictions (i.e. earmarking), and many others. Each of these classifications brings their own unique challenges, but the fledgling field does not adequately represent the complexity of real-world systems; for example, most studies assume that sufficient resources are available, yet in 2015, the UN received only 55% of its required aid, and that was a good year. Not only is the total amount of aid insufficient, but NGOs must often compete against each other to secure these scarce resources (Nagurney et al., 2016). These and other complications resulting from monetary and material donations make humanitarian logistics decisions extremely challenging (Oloruntoba and Gray, 2006).

In fairness, there often is an abundance of donations, but that doesn't mean the donations are all – or even mostly – useful. Holguín-Veras et al. (2012) cite several examples of a post-disaster phenomenon called *material convergence*, which refers to the emergence of supplies and equipment upon affected areas in overwhelming quantities. Even though it is often the case that the majority of these items are unusable, it is important to remember that a significant portion are either needed now or will be valuable later. The Pan American Health Organization uses a three-part scheme to classify donated items: urgent or high priority (HP) goods are needed now; non-urgent or low priority (LP) will be valuable later; and the rest are non-priority (NP) goods (PANO, 2001). The issue is that the relief organizations are unable to quickly identify, access, and disseminate the HP and LP donations because they are overwhelmed by NP donations. In addition, in-kind donations are troublesome in their inflexibility and variability (Burkart et al., 2017): extra socks may not be repurposed as diapers nor the other way around. Furthermore, the supply chain disruptions (see Snyder et al., 2016, for a comprehensive review of this topic) often associated with major disasters amplifies donation variability in the humanitarian relief chain. As such, decision-makers face substantial uncertainty in knowing when donations will arrive, and in what quantities. Moreover, in the absence of technologies such as radio frequency identification (RFID), decision-makers cannot be certain of which items are included when a truckload of donations arrives.

Unfortunately, analogous problems in the commercial context are limited, and there seems to be a gap in the humanitarian logistics research literature that deals with donations as a source of stochastic supply; this study attempts to fill that gap. This study does not, however, address material convergence directly. Rather, our research contributes to the literature by considering donations as an exogenous, stochastic source of supply without presuming there will be sufficient donations to meet demand, and investigate the logistics of distributing stochastic donations to beneficiaries in a disaster relief setting. It is worth noting at this point that part of the solution to material convergence is to prevent it from occurring in the first place; communicating with individuals and small organizations is difficult, but case studies like Arnette and Zobel (2016) are encouraging.

2.3. Shipment consolidation

In general, the shipment (or inventory) consolidation literature can be divided into one of two streams – (1) optimizing the shipping decision or (2) integrating inventory replenishment with shipping decisions – but both problems must overcome the challenges of holding costs and customer service (Çetinkaya and Bookbinder, 2003). Most papers in the inventory consolidation literature consider time-based (orders are shipped periodically) and/or quantity-based (wait for a predetermined load quantity) policies for their convenience and widespread use in industry. In practice, policies are usually a combination of both because time-and-quantity based models significantly improve customer service level from quantity-based models with only a slight increase in cost (Mutlu et al., 2010).

Obviously, consolidating multiple orders that arrive separately implies delaying the earlier orders until the others arrive, and there is a cost associated with doing so. Similarly, delayed orders lead to an increased lead time, not to mention variability in lead time – both of which impact the service level at the customer. This framework is similar to our problem, but the main difference is how late orders are counted. Çetinkaya et al. (2014), for example, measures time-based penalties like Maximum Waiting Time or Average Order Delay, but we are not interested in *how much* an order is late but *if* it is late at all. We assume that demand is not backordered, so if there is a shortage, it doesn't matter if the supplies show up a day or a week late because the supplies were not available when they were needed. Other papers such as Stenius et al. (2016) address unmet demand by constraining the customer service level to some lower bound and then optimizing the policy around the constraint based on cost of the policy.

Lai et al. (2016) addresses the issue of suppliers' or retailers' cooperation in inventory consolidation; specifically, they model a collaborative distribution system with multiple suppliers managed by a 3PL who can consolidate orders for the suppliers. This approach has some interesting applications to our problem setting, but is beyond the scope of this study; we relegate a discussion of this possibility to Section 6. We will also investigate Nguyen et al. (2014) which introduces the idea of supply perishability. This study differs from the above in that we are interested in minimizing shortages, not cutting costs, and we ask different questions, aiming to evaluate the effectiveness of common-sense heuristic policies.

3. Model formulation

In this section, we present a finite horizon stochastic dynamic programming model of the vehicle dispatching problem described in Section 1. Each time period $t \in \{1, \dots, T\}$ represents an opportunity to dispatch the vehicle from its current location, and length of each period can be thought of as the travel time between the point of demand (POD) and staging area (SA). The time it takes to load the vehicle at the SA and unload it at the POD are not considered explicitly in the model. Essentially, we assume that load/unload times are included in the travel time between the SA and POD, which means that we also assume fixed load/unload times that do not vary with the number of units being loaded/unloaded. This is one of the limitations of our model.

3.1. Sequence of events

Within each period t, the following sequence of events occurs: first, the state of the system (i_t, u_t, w_t) is observed, where i_t is the location of the vehicle, u_t is the number of units of supply available for shipment from the staging area, and w_t is the quantity of supply at the point of demand available for distribution to disaster survivors.[1] Each of these state variables reflects conditions at the very beginning of period t before any action is taken. Based on this information, an action a_t is then taken; for each state, exactly two actions are possible: dispatch the vehicle ($a_t = 1$), or do not dispatch the vehicle ($a_t = 0$). Either way, the action is implemented before any donors or beneficiaries arrive in that period. After the vehicle leaves – or doesn't leave – its current location, donations begin to accumulate at the SA while beneficiaries start arriving at the POD, continuing from then to the end of the period. At the end of the period, the actual number of donations that have accumulated at the SA (denoted y_t) is observed, and the actual amount of demand at the POD (denoted x_t) is also observed. Next, donation units are distributed among disaster survivors followed by the departure of all x_t survivors from the POD. The resulting number of shortages is then recorded, and the state variables are updated to begin the next period. This sequence of events is summarized in Fig. 2.

3.2. Model assumptions

The above sequence of events and stochastic dynamic programming formulation to come are based on the following simplifying assumptions, many of which are introduced in an effort to create a scenario where the optimal policy has a simple form such as one of the policies (continuous dispatching or full truckload dispatching) mentioned in Section 1.

<u>Summary of model assumptions:</u>

A 1. Dispatch decisions occur on a periodic basis and the time between decision epochs can be interpreted as the (one-way) travel time between the SA and POD. Furthermore, this travel time is always the same, regardless of the decision made. For example, if the travel time is 30 min, then a dispatch decision is made every 30 min, independent of whether or not the vehicle traveled during the last period.

This assumption allows us to model the system as a discrete time stochastic dynamic program. Although the real-world system evolves in continuous time, there are advantages to the discrete time formulation that emerges from this assumption. One practical benefit is that it won't be necessary to constantly process information and make dispatching decisions. During the early stages of relief efforts, first responders often face the challenge of volunteer and material convergence in which affected areas are overwhelmed by massive quantities of spontaneous volunteers and unsolicited material donations. In these situations, it would probably be easier for the first responders to evaluate inventory levels at regular intervals as opposed to constantly interrupting other duties each time a new donation, volunteer, or beneficiary arrives. Another advantage of the discrete time formulation relative to the alternative of a continuous time model has to do with the technical aspects of solving stochastic dynamic programming problems. In particular, the process of uniformization (e.g., Bertsekas,

[1] A complete list of notations is shown in Table 2.

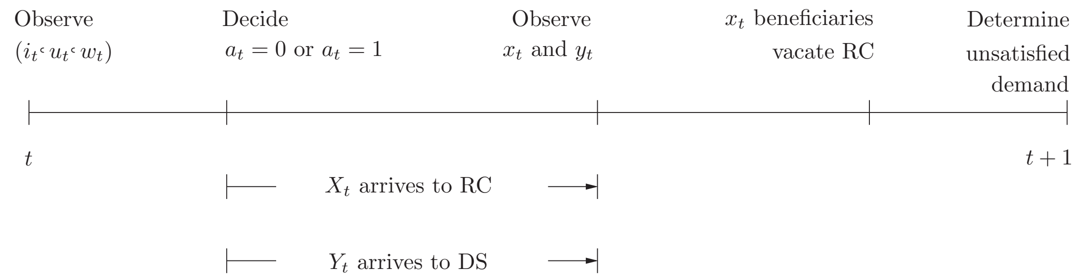

Fig. 2. Sequence of events in each period.

Table 2
List of notations.

State variables:	i_t:	Current location of vehicle;	$i_t = 0 \rightarrow$ POD; $i_t = 1 \rightarrow$ SA.
	u_t:	Beginning inventory level at SA;	
	w_t:	Beginning inventory level at POD;	
Decision variable:	a_t:	Dispatch decision;	$a_t(i_t) = 0 \rightarrow i_{t+1} = i_t$. $a_t(i_t) = 1 \rightarrow i_{t+1} = 1 - i_t$.
Random variables:	X_t:	Demand at POD;	(occurs at end of period).
	Y_t:	Supply accumulation at SA;	(occurs at end of period).
	x_t, y_t	Realizations of X_t and Y_t;	
Other:	C:	Capacity of the vehicle;	
	q_t:	Quantity shipped upon dispatch from SA to POD;	$q_t = \min\{C, u_t\}$.
	$\pi_t(i_t, u_t, w_t)$	Decision rule to determine an action depending on state variables;	
	$s_t(i_t, u_t, w_t, a_t)$	Single period expected shortage.	
	$V_t(i_t, u_t, w_t)$	Optimal value in period t;	(shortage to go).
	$v_t(i_t, u_t, w_t, a_t)$	expected shortage in period t onward if decision a is chosen now and optimal decisions chosen in periods $t+1$ onward.	

2005), in which a continuous time model is transformed into its discrete time equivalent, is a common approach used to solve stochastic dynamic programming models. The discrete time formulation bypasses this intermediate step. These advantages come at the expense of sacrificing some realism. For example, if dispatch is delayed at a particular decision epoch and a major change (such as a sudden increase in inventory level at the SA from a large donation) occurs shortly thereafter, then the vehicle cannot be dispatched until the next decision epoch. In reality, the manager would likely dispatch the vehicle immediately after a surge in donations if there is a need at the POD. This limitation can potentially be overcome in practice with a less than rigid implementation that allows "emergency dispatches", but including this feature in the stochastic model would compromise the prospect of characterizing the optimal policy analytically, or even computationally for that matter.

A 2. The time it takes to load the vehicle at the SA and unload it at the POD is included in the travel time. As mentioned in Assumption 1, this travel time remains the same each period. Thus, load/unload times are independent of the number of units being loaded or unloaded.

In reality, load/unload times are highly dependent upon the number of units involved. However, allowing load times and unload times to vary across periods would mean that the time between decision epochs would also vary, which in turn would require a continuous time formulation (because the time between periods would be random). Based on the explanations presented in the discussion that follows Assumption 1, we chose a discrete time modeling framework in lieu of a continuous time alternative.

A 3. Donation heterogeneity is not considered. Each donation that arrives at the SA can be thought of as a basket of goods, and each beneficiary at the POD requires exactly one of these. We also assume that all donations are usable when they arrive and can potentially be distributed to beneficiaries.

Examples of items that are donated in response to disaster events include clothing, food, water, flashlights, batteries, blankets, and a host of other household and personal items. Some or all of these items are likely to be included in the "basket of goods" mentioned in Assumption 3. Although the distribution of organized care packages to recipients may sometimes occur in practice, the donations usually don't arrive that way. Such care packages have to be assembled after donations have been sorted and perhaps stored on a temporary basis. While the first part of Assumption 3 ensures that each donated unit is identical in terms of its intended use, the latter part speaks to the homogeneity of donated items with respect to quality. All donations are assumed to be of an acceptable quality level suitable for distribution to the public so that no quality inspection is necessary. Our model does not include this level of granularity. Doing so would make the model substantially more complicated; supply and demand would each have to be represented as separate multi-dimensional stochastic processes, possibly with correlations among the variables that make up their respective dimensions. Furthermore, a multi-item model with several quality categories for each item would increase the dimension of the state space in the resulting Markov decision problem and consequently exacerbate the infamous curse of dimensionality associated with the dynamic programming methodology.

A 4. Inspecting, sorting, and other material handling activities associated with receiving donations at the staging area are not considered explicitly in the model.

Although y_t is defined as the number of donation arrivals in period t, it can be thought of as the number of donations that have been sorted, inspected, and available for transport by the end of period t. The items included in y_t may have arrived during period t or earlier. So essentially, this assumption does not widen the gap between the mathematical model and the real world problem; it simply provides an alternative interpretation of the variable y_t that captures additional characteristics of the system.

A 5. Regarding the calculation of unsatisfied demand, it is assumed that each of the x_t beneficiaries who arrive during period t will depart the POD at the end of period t whether or not his demand requirements have been met. Thus, beneficiaries do not wait from one period to the next for donations to arrive, which is analogous to assuming that all shortages result in lost sales similar to a commercial supply chain context. Accordingly, a beneficiary who arrives during period t will wait at most until the end of period t for donations to arrive before departing the POD.

To a certain degree, this behavior is consistent with what happens in practice, at least based on the authors' observations following the response to a major disaster. In that situation, the process of distributing relief supplies at the POD was a queuing system in which recipients were served on a first-come, first-serve basis. When a recipient reached the front of the beneficiary queue, he or she received relief supplies (with assistance from a volunteer or professional responder) from the POD's on-hand inventory at that precise time. If sufficient inventory was available during a recipient's service experience, that recipient collected the supplies distributed to him or her and then immediately exited the system. If, on the other hand, an item was out of stock while a recipient was being served, that recipient collected other items he or she needed, but still immediately exited the POD afterwards. The latter constitutes unsatisfied demand, which this paper aims to minimize. As an example, consider a beneficiary who needs food, water, and flashlights. If there are no flashlights in stock when this beneficiary reaches the front of the queue, she will receive the food and water from available inventory and then immediately exit the system without the flashlights. If this beneficiary decides to return to the POD at a later time in search of flashlights, she would be treated as an entirely new customer in our model, but the original lost demand would still be applied to the model's objective function. The primary limitation of Assumption 5 is more so a consequence of the discrete time modeling framework resulting from Assumption 1. Recipients do not actually wait from one period to the next; they leave immediately after service is complete. However in a discrete time framework, events are only acknowledged at the beginning of each period; thus the discrete time equivalent of a customer departing the system immediately after completing service is departing the system at the end of the current period, just before the start of the next period.

3.3. Optimality equations

The optimality equations describe the minimum expected shortage from the current period through the end of the finite horizon. Let $V_t(i_t, u_t, w_t)$ represent the optimal value of state (i_t, u_t, w_t) in period t, which is equivalent to the minimum expected shortage over periods t through T if the state at the beginning of period t is (i_t, u_t, w_t). Then the optimality equations are

$$V_t(i_t, u_t, w_t) = \min_{a_t \in \{0,1\}} s_t(i_t, u_t, w_t, a_t) + \mathsf{E}[V_{t+1}(i_{t+1}, u_{t+1}, w_{t+1})], \quad t = 1, \ldots, T \tag{1}$$

where $s_t(i_t, u_t, w_t, a_t)$ is the expected shortage incurred in period t based on action a_t, and the expectation in Eq. (1) is with respect to both X_t and Y_t (this will be apparent once the transition equations for i_t, u_t, and w_t are shown). The expected shortage in period t can be expressed as follows. Let $i_t = 0$ indicate that the location of the vehicle at the beginning of period t is the point of demand (POD), and $i_t = 1$ that the location is the staging area (SA). Note that 0 and 1 have different meanings for i_t and for a_t; recall that $a_t = 0$ refers to a "do not dispatch" decision and $a_t = 1$ is a "dispatch" decision. Additionally, let q_t represent the quantity of units shipped from the SA to the POD if $a_t(1) = 1$ (i.e., the vehicle is dispatched from the SA to the POD). Then

$$q_t = \min\{u_t, C\}, \tag{2}$$

and

$$s_t(i_t, u_t, w_t, a_t) = \mathsf{E}[X_t - w_t - i_t \cdot a_t(i_t) \cdot q_t]^+, \tag{3}$$

where $z^+ := \max\{z, 0\}$. The transition equations for $t = 1, \ldots, T-1$ are

$$i_{t+1} = \begin{cases} 0, & \text{for } (i_t, a_t) = (0,0) \text{ or } (1,1) \\ 1, & \text{for } (i_t, a_t) = (0,1) \text{ or } (1,0) \end{cases} \tag{4}$$

$$u_{t+1} = \begin{cases} y_t + [u_t - C]^+, & \text{for } (i_t, a_t) = (1,1) \\ y_t + u_t, & \text{otherwise} \end{cases} \tag{5}$$

$$w_{t+1} = \begin{cases} w_t + [u_t - C]^+ - x_t, & \text{for } (i_t, a_t) = (1,1) \\ w_t - x_t, & \text{otherwise.} \end{cases} \tag{6}$$

Eq. (4) shows how the starting position of the vehicle evolves from one period to the next. In particular, the vehicle will begin the next period at the POD if either its current position is the POD and it is not dispatched, or its current position is the SA and it is dispatched (i.e., $i_{t+1} = 0$ if $(i_t, a_t) = (0,0)$ or $(1,1)$). Similarly, the vehicle's next position will be the SA if either its current position is the SA and it is not dispatched, or its current position is the POD and it is dispatched (i.e., $i_{t+1} = 1$ if $(i_t, a_t) = (0,1)$ or $(1,0)$). The next equation, Eq. (5), describes how the inventory level at the SA transitions from one period to the next. Specifically, Eq. (5) says that unless the vehicle is dispatched from the SA, no inventory is moved from the SA to the POD. For all such cases $(i_t, a_t) = (0,0), (0,1)$, and $(1,0)$, the inventory level at the SA is simply increased by the accumulation in that period. If inventory is moved, however, then the inventory at SA is decreased by the lesser of the inventory on hand (before accumulation) and the vehicle capacity. Finally, Eq. (6) reflects the other half of the inventory exchange; if no inventory is moved, the inventory level at the POD simply decreases by the demand that period. If a dispatch decision is made from the SA, then the number of units shipped, $\min\{u_t, C\}$, is added to the inventory at the POD before demand is realized. These transition equations can be written more compactly as follows:

$$i_{t+1} = i_t[1 - a_t(i_t)] + (1 - i_t) \cdot a_t(i_t) \tag{7}$$

$$u_{t+1} = y_t + i_t \cdot \{a_t(i_t) \cdot [u_t - C]^+ + [1 - a_t(i_t)] \cdot u_t\} + (1 - i_t) \cdot u_t \tag{8}$$

$$w_{t+1} = [w_t - x_t - i_t \cdot a_t(i_t) \cdot q_t]^+. \tag{9}$$

The solution to this stochastic dynamic programming model is a sequence of decisions $a_1^*, a_2^*, \ldots, a_T^*$ that satisfy

$$V_1(i_1, u_1, w_1) = \min_{a_1,\ldots,a_T} \sum_{t=1}^{T} \mathrm{E}[s_t(i_t, u_t, w_t, a_t)], \tag{10}$$

where the equivalence of $V_1(i_1, u_1, w_1)$ and the right hand side of Eq. (10) is guaranteed to hold in general based on finite horizon Markov decision process (MDP) theory (e.g., Bertsekas, 2005). More generally, the optimal policy is a sequence of functions $\pi_t^* : (i_t, u_t, w_t) \mapsto \{0, 1\}$ such that $a_t^* = \pi_t^*(i_t, u_t, w_t)$. In the next section, we identify characteristics of optimal policies, which in turn narrows down the number of policies that need to be considered when seeking optimal decisions.

4. Analytical results

There are three special cases in which we are able to prove the structure of the optimal policy analytically, the first of which pertains to Question 1, and the second to Question 2. The third result examines the optimality of the continuous dispatching policy (CD) when an unlimited number of vehicles is available to travel between the staging area and point of distribution.

4.1. Dispatching from the point of demand

Question 1 asks whether or not dispatching from the point of demand is always an optimal decision. According to Proposition 1, the answer is "yes", which confirms our intuition:

Proposition 1. *$a_t^* = \pi^*(0, u_t, w_t) = 1$ is an optimal action for $t = 1, \ldots, T$.*

The proof for Proposition 1 presented below follows directly from a corollary to the following technical result.

Lemma 1. *$V_t(1, u_t, w_t) = \min\left\{V_t(0, u_t, w_t), \mathrm{E}[X_t - u_t - w_t]^+ + \mathrm{E}\left\{V_{t+1}\left(0, Y_{t+1}, [u_t + w_t - X_t]^+\right)\right\}\right\}$ for $t = 1, \ldots, T-1$.*

A proof for Lemma 1 is shown in Appendix A, and the aforementioned corollary that facilitates the proof of Proposition 1 is as follows.

Corollary 1. *$V_t(1, u_t, w_t) \leqslant V_t(0, u_t, w_t)$ for $t = 1, \ldots, T$.*

Proof. The result for $t = 1, \ldots, T-1$ follows immediately from Lemma 1. Specifically, if the minimum of the two terms on the right hand side of the equation in Lemma 1 is $V_t(0, u_t, w_t)$, then $V_t(1, u_t, w_t) = V_t(0, u_t, w_t)$. Otherwise the second term is smaller and $V_t(1,u_t,w_t) < V_t(0,u_t,w_t)$. For $t = T$, we have $V_T(1,u_T,w_T) = \mathsf{E}[X_T - u_T - w_T]^+$ and $V_T(0,u_T,w_T) = \mathsf{E}[X_T - w_T]^+$. Thus $V_T(1,u_T,w_T) \leqslant V_T(0,u_T,w_T)$ since $u_T \geqslant 0$. □

According to Corollary 1, the optimal expected shortage from period t onward is lower if the vehicle's position at the beginning of period t is the staging area as opposed to the relief center. This property is intuitive: if, at any decision epoch t, the vehicle is located at the staging area, a dispatch decision results in the transport of u_t units of inventory that can be used to satisfy demand in period t. On the other hand, a dispatch decision from the point of demand represents an empty vehicle traveling back to the staging area, and hence does not reduce shortages during period t. From this perspective, dispatching from the point of demand is only beneficial from the standpoint of potentially reducing expected shortages in future periods. With this result, Proposition 1 is easily proven.

Proof of Proposition 1. The value of actions $a_t = 0$ and $a_t = 1$ are

$$\begin{aligned} v_t(0,u_t,w_t,0) =& \mathsf{E}[X_t - w_t]^+ + \mathsf{E}[V_{t+1}(0,u_{t+1},w_{t+1})] \\ v_t(0,u_t,w_t,1) =& \mathsf{E}[X_t - w_t]^+ + \mathsf{E}[V_{t+1}(1,u_{t+1},w_{t+1})]. \end{aligned}$$

Since $V_{t+1}(1,u_{t+1},w_{t+1}) \leqslant V_{t+1}(0,u_{t+1},w_{t+1})$ by Corollary 1, $v_t(0,u_t,w_t,1) \leqslant v_t(0,u_t,w_t,0)$ so that $\pi_t(0,u_t,w_t) = 1$ is always an optimal action (although not necessarily uniquely optimal). □

4.2. Dispatching from the staging area

Similar to Question 1, Question 2 concerns optimal dispatching decisions from a specific location. While Question 1 addresses the optimality of a dispatch decision from the point of demand, Question 2 investigates the conditions under which delaying dispatch from the staging area is an optimal decision. One situation where postponing dispatch is likely to be an optimal decision is when there is no inventory at the staging area. The next result confirms that this is true.

Proposition 2. *If $u_t = 0$, then $a_t^* = \pi_t(1,u_t,w_t) = 0$ is an optimal action for $t = 1,\dots,T$.*

Proof. This result also follows from Corollary 1. The values of actions $a_t(1) = 0$ and $a_t(1) = 1$ are

$$v_t(1,u_t,w_t,0) = \mathsf{E}[X_t - w_t]^+ + \mathsf{E}[V_{t+1}(1,U_{t+1},W_{t+1})] \tag{11}$$

$$v_t(1,u_t,w_t,1) = \mathsf{E}[X_t - w_t - q_t]^+ + \mathsf{E}[V_{t+1}(0,U_{t+1},W_{t+1})]. \tag{12}$$

For Eq. (11), the transition equations (8) and (9) result in $U_{t+1} = u_t + Y_t$ and $W_{t+1} = (w_t - X_t)^+$. Similarly, the transition equations for Eq. (12) are $U_{t+1} = Y_t$ and $W_{t+1} = (q_t + w_t - X_t)^+$. When $u_t = 0$ (which, from Eq. (2), implies that $q_t = 0$), equations (11) and (12) become

$$\begin{aligned} v_t(1,u_t,w_t,0) =& \mathsf{E}[X_t - w_t]^+ + \mathsf{E}\left[V_{t+1}(1,Y_t,(w_t - X_t))^+\right] \\ v_t(1,u_t,w_t,1) =& \mathsf{E}[X_t - w_t]^+ + \mathsf{E}\left[V_{t+1}(0,Y_t,(w_t - X_t))^+\right]. \end{aligned}$$

It follows from Corollary 1 that $V_{t+1}(1,Y_t,(w_t - X_t)^+) \leqslant V_{t+1}(0,Y_t,(w_t - X_t))^+$. Therefore, $V_t(0,u_t,w_t) = v_t(0,u_t,w_t,0)$ and $a_t^*(0) = 0$ whenever $u_t = 0$. □

On the other hand, our intuition tells us that we *do* want to ship when we have a full vehicle. The argument is summarized as follows. A shipment of a fixed amount is more valuable now then it would be delayed, because either it wouldn't matter or there would be uncaptured demand. We begin the proof with the following proposition.

Proposition 3. *The value of a shipment is higher now than it will be if delayed.*

If we have a fixed amount to ship A and we can either ship it now or in the next period, and we assume for the time being that no more shipments will come in, our unmet demand is

$$\mathsf{E}_1 = \left(\sum_{t \in T} X_t - A\right)^+$$

if we dispatch in this period or

$$\mathsf{E}_2 = X_1 + \left(\sum_{t=2}^{T} X_t - A\right)^+$$

if we wait until $t = 2$. These statements are equivalent unless $A > \sum_{t=2}^{T} X_t$ and $X_1 > 0$, in which case the leftover A that could have been used to reduce X_1 is wasted, so to speak, so $\mathsf{E}_1 \leqslant \mathsf{E}_2$□

The argument for Proposition 3 can easily be extended to include additional shipments in the future, and since a shipment can never exceed C by definition, once u_t reaches C, Proposition 3 tells us that it is an optimal decision to immediately dispatch.

4.3. Optimality of continuous dispatching

As described in Section 1, the continuous dispatching (CD) policy dispatches the vehicle at each decision epoch $t \in \{1, \ldots, T\}$ of the finite horizon, no matter its location at the beginning of each period. Consequently, the CD policy, denoted π^{CD}, is a function such that $\pi^{CD} : (i, u, w) \mapsto \{1\}$, i.e., $a_t = \pi^{CD}(i, u, w) = 1$ for all states (i, u, w) and periods t. We know from Proposition 2 that CD is not an optimal policy in general. However, CD is optimal for all states with $i = 0$ according to Proposition 1. This begs the question of whether or not a modified version of the CD policy is optimal, where the modification is such that the vehicle is dispatched from the SA whenever there is inventory there ready to be transported to the POD. Based on the results of our computational experimentation presented in the next section (Section 5), the answer to this question is "no". On the other hand, if we consider a variation of the original dispatching problem introduced in Section 1 with multiple vehicles, then CD is actually an optimal policy. More specifically, suppose a fleet of two or more vehicles travels back and forth between the SA and POD in Fig. 1 as opposed to only a single vehicle. Then the optimal policy is to continuously dispatch the vehicles in such a way that at the beginning of each period, there is at least one vehicle at both the SA and the POD. The formal statement of this result is presented below as Corollary 2, which follows from this next proposition that speaks to the optimality of continuous dispatching when there is an unlimited number of vehicles that can be dispatched between the SA and POD.

Proposition 4. *Let $\pi_t^{*,\infty}(i, u, w)$ denote the optimal decision rule when in state (i, u, w) at epoch t when there is an infinite supply of vehicles at the staging area. Then $\pi_t^{*,\infty}(1, u, w) = 1$ for $t = 1, \ldots, T$.*

The proof for Proposition 4, which proceeds based on mathematical induction, is somewhat lengthy and is therefore relegated to Appendix A. However, we do point out here an important difference between the single vehicle problem, which is the primary focus of this paper, and the unlimited vehicles version that is considered in Proposition 4. The distinction is that continuous dispatching potentially results in items being shipped from the staging area during each period $t = 1, \ldots, T$ when there is an infinite number of vehicles, but items are shipped at most every other period under the continuous dispatching policy when there is only a single vehicle. This next result indicates that CD is an optimal policy if the fleet of vehicles that travels between the SA and POD consists of two or more vehicles.

Corollary 2. *If two or more vehicles travel between the staging area and point of distribution, then the optimal policy is stationary and $\pi^{*,(2,\infty)}(i, u, w) = \pi^{CD,(2,\infty)}(i, u, w)$, where $\pi^{*,(2,\infty)}(i, u, w)$ and $\pi^{CD,(2,\infty)}(i, u, w)$ are the optimal and continuous dispatching polices, respectively, when the fleet size is two or more.*

Instead of justifying Corollary 2 mathematically, we provide the following logical explanation. First, Proposition 1 indicates that if a vehicle is at the point of distribution, it is always optimal to dispatch. Similarly, Proposition 4 says that dispatching a vehicle from the staging area is always an optimal action. So together, Propositions 1 and 4 reveal that continuous dispatching is an optimal policy when there are two or more vehicles. Furthermore, an implication of Proposition 4 is that it is always optimal to have a vehicle at the staging area, and this can be achieved by continuously dispatching two vehicles between the two locations. Otherwise, there would exist at least one t with $\pi_t^{*,\infty}(1, u, w) = 0$.

In summary, we have shown that for the original vehicle dispatching problem in which a single vehicle is dispatched between the SA and POD as shown in Fig. 1, the CD policy is not optimal. However, if we consider a different problem in which a fleet of two or more vehicles can be dispatched between the SA and POD, then the optimal policy is to continuously dispatch vehicles so that there is always a vehicle at each location at the beginning of each period.

5. Computational results

Up to this point we have presented analytical results that address the first two research questions presented in Section 1, but we have not addressed the latter two. We know from Proposition 2 that if $u_t = 0$, dispatching from the staging area is never optimal, and from Proposition 3 that dispatching is always optimal whenever $u_t \geqslant C$, but what about $0 < u_t < C$? In this case, the dispatch decision depends on the available inventory at the point of demand, w_t; the number of remaining periods, $T - t$; and the ratio of supply and demand rates, $r = X/Y$. As w_t increases, we have less urgency to ship, but it is also less of a problem to dispatch a less-than-full vehicle as there will be time to make more round-trips before w_t drops to 0. If there is considerably more supply than demand, w_t will rise over time, but if the end of the planning horizon is near, long-term behavior is less relevant. With these situations, the vehicle's capacity can force the decision to dispatch, so it is important to consider the interaction between vehicle capacity and the other factors. To answer Questions 3 and 4, we turn to a modification of the traditional 2^k Factorial Design to help isolate these effects. Note that all of the computational experiments presented in this section pertain to the original vehicle dispatching problem introduced in Section 1 in which a single vehicle is dispatched between the staging area and point of distribution. Because we have shown in Section 4.3 that CD is always an optimal policy for the multiple vehicle extension to the original model, there is no need to include the multiple vehicle model in computational tests designed for the purpose of assessing the performance of the heuristic policies (CD and FTD) relative to the optimal policy.

Before presenting the details of our 2^k factorial design, let us first discuss how we determine the supply and demand rates. We would like to use a Poisson distribution, but this would lead to an infinite state space if we don't restrict the support of the distribution. Additionally, allowing either supply or demand to be negative could have meaningful interpretations – spoilage or returned items, respectively – but also increases the possible states at the POD. In our experimentation, we found that the computation time was highly dependent on how many values we allowed the distributions to take as per the curses of dimensionality. So, before considering the ratio $r = X/Y$, we define both variables as Poisson ($\lambda = 5$), but only allow them to take the values between 0, 2, and 4, then we calculate the probability of a Poisson variable taking each of those values. Finally, we normalize each of those probabilities so as to make a valid probability distribution function. The result of this process is shown in Table 3.

Without changing the associated probabilities, r is determined as $r \cdot X$ if $r > 1$ or Y/r if $r < 1$, which is necessary to keep the support of both distributions integral. Two examples are given in Table 4.

5.1. Experimental design

Following the line of reasoning above, we perform a sensitivity analysis with respect to vehicle capacity, the ratio of supply and demand rates, and the length of the planning horizon. First, we choose high and low values (henceforth called outer or extreme values) like in the standard 2^k design (Table 5), but we also select intermediate values for each parameter in order to test over a demonstrative range of values. An experiment consists of two parameters fixed at an extreme value (either high or low) like a standard 2^k factorial design, then we test all the values, outer and intermediate, for the third parameter. We repeat this process for all combinations of outer parameter values as shown in Table 6, resulting in a total of 88 problem instances. Note, however, that 16 are duplicate instances; $2^3 = 8$ but there are 24 instances of just high or low values. We present it in this way so as to include even duplicated values in experiments in which they are relevant as shown in Tables 5–7.

We consider the shortages scaled *per period* so as to avoid solution inflation as the length of the planning horizon increases. For each of the 12 experiments shown in Table 7, we produce 2 graphs comparing the heuristics' "optimality-gaps", or the difference between a heuristic solution's objective function value and that of the MDP-optimal solution. One graph compares the absolute differences and the other displays the relative; note that the optimal solution does not appear on the relative charts since it is used in calculating the values for the heuristics. In many cases, the relative graphs are more informative, but they can also be misleading as in Fig. 3. In this case, the two heuristics are equivalent with small values of r (supply > demand), but then diverges to an optimality gap of about 2 units per period, even as r grows large (demand > supply). As a consequence, the relative chart has a steep peak with tails that asymptotically approach 0 as r approaches

Table 3
Example of supply and demand probability distributions.

x	0	2	4	$\sum$
$p(x)$	0.0067	0.0842	0.1755	0.2664
$\hat{p}(x)$	0.0253	0.3161	0.6586	1.00

Table 4
Examples of r with X and Y.

x	$\hat{p}(x)$
$0 \cdot r = 0(2) = 0$	0.0253
$2 \cdot r = 2(2) = 4$	0.3161
$4 \cdot r = 4(2) = 8$	0.6586
$\sum$	1.00
y	$\hat{p}(y)$
$0 \cdot 1/r = 0(1/1/2) = 0(2) = 0$	0.0253
$2 \cdot r = 2(2) = 4$	0.3161
$4 \cdot r = 4(2) = 8$	0.6586
$\sum$	1.00

Table 5
2^k Experimental design.

Parameter	C	$X = Y$	T
(Low, High)	(2, 10)	(0.2, 5)	(3, 10)

Table 6
Sensitivity analysis.

Parameter	Values
C	{2, 4, 6, 8, 10}
$X = Y$	{0.2, 0.4, 0.6, 0.8, 1, 2, 3, 4, 5}
T	{3, 4, 5, 6, 7, 8, 9, 10}

Table 7
Modified 2^k factorial design.

Exp. #	Vehicle Capacity	$\frac{\text{Rate of Demand}}{\text{Rate of Supply}}$	# of Time Periods
1	L	L	Varied
2	L	H	Varied
3	H	L	Varied
4	H	H	Varied
5	Varied	L	L
6	Varied	H	L
7	Varied	L	H
8	Varied	H	H
9	L	Varied	L
10	H	Varied	L
11	L	Varied	H
12	H	Varied	H

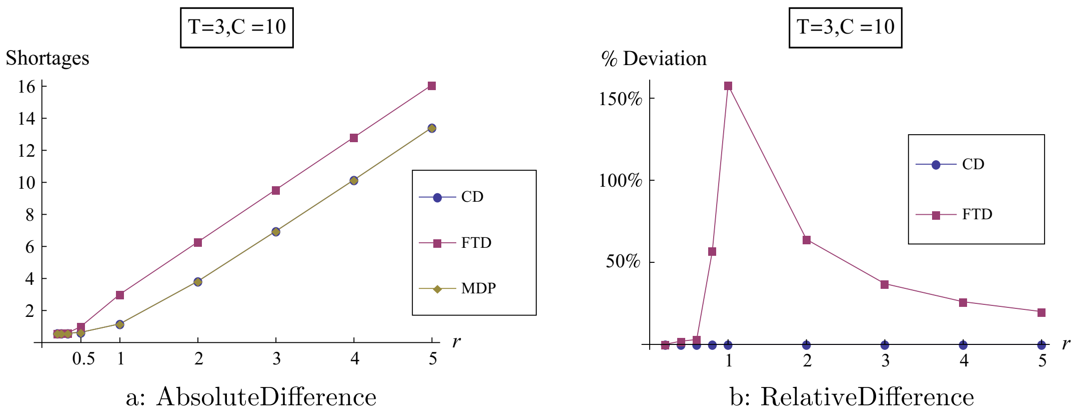

Fig. 3. Optimality gap for Experiment 9 (varying r).

$\pm\infty$. Note that for all of the figures in Section 5, the two fixed parameters are shown above the chart and the varied parameter is in the figure name. Not all 24 graphs are shown, but those chosen are representative.

5.2. Results

5.2.1. Optimality of continuous dispatching

Given that CD is an optimal policy when the size of the vehicle fleet is two or more (according to Corollary 2), it seems intuitive that CD would also be optimal when there is only a single vehicle. That is, given a more restricted number of vehicles, the urgency of the dispatch decision would appear to be more pronounced. However, it turns out that this is not the case; the CD rule is not necessarily an optimal policy. That being said, CD is an optimal policy for many of the experimental conditions that we tested in our computational study, and was usually pretty close when not.

The CD policy consistently outperformes the FTD policy; in all 12 of our experimental settings, we found that CD was either equivalent or superior to FTD. The charts in Fig. 4 illustrate the optimality gap comparing each heuristic to the MDP optimal solution, where again r is the variable name for the ratio of demand to supply.

Observe that in experiment 3 (Fig. 4a) where there is 5 times as much supply as demand, FTD and CD are equivalent, but in experiment 4 (Fig. 4b) CD outperforms FTD overall. In fact, this relationship holds true in all of our experiments: when r is large (supply $\gg$ demand), FTD = CD, but when r is small (supply $\ll$ demand), CD outperforms FTD.

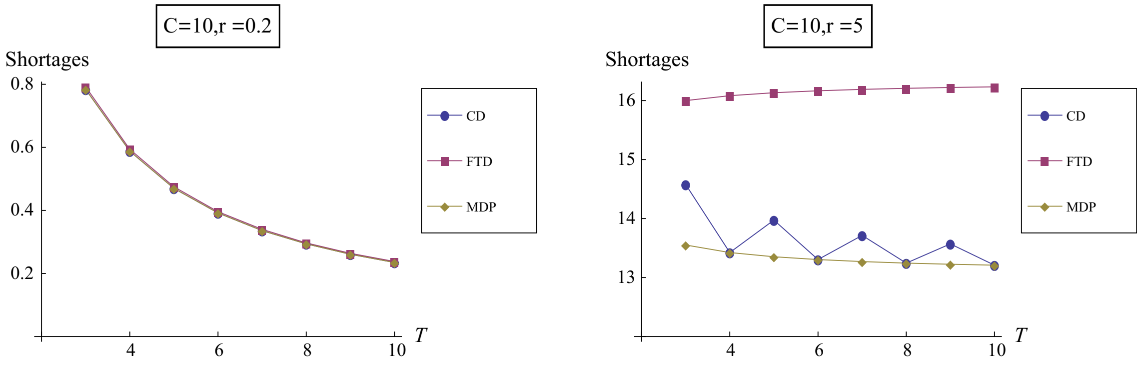

Fig. 4. Optimality gap for experiments 3 and 4 (varying *T*).

5.2.2. Vehicle capacity

We can see from Fig. 5 that the optimality gap grows with the vehicle capacity. When capacity is prohibitively small, all solutions are bad, including the optimal. That's why MDP, CD, and FTD are the same: full truckloads are dispatched continuously when there is limited capacity. However, when there is sufficient capacity, the solutions generated by MDP, CD, and FTD diverge. We observe here that CD is not optimal but outperforms FTD. Thus, vehicle capacity is an important factor influencing choice of dispatching policy.

5.2.3. Optimality gap per period

As mentioned previously, all experiments are displaying shortages per period. In dynamic programming models like this where the problem size increases exponentially with T, it is important to choose a sufficiently large T to faithfully represent the entire behavior of the system as it ages without sacrificing computational tractability. Since all experiments that show the effects of changing T (experiments 1–4) exhibit asymptotic behavior, we can be confident that the solutions are "stable" relative to the planning horizon and that we are not missing anything interesting at larger values of T. Experiments 3 and 4 are shown above in Fig. 4 while experiments 1 and 2 are shown in Fig. 6.

5.2.4. Effects of model parameters

To recap, we saw in Fig. 3 that excess supply ($r < 1$) leads to good results regardless of the policy, but CD dominates FTD when the demand rate surpasses the supply rate ($r > 1$). This is the opposite of what we predicted, but we will defer further discussion until we revisit the research questions in Section 5.3. As for vehicle capacity, Fig. 5 shows that more capacity leads to fewer shortages, but also greater opportunities for suboptimal decisions. At one extreme, if the vehicle capacity is only 1 unit, then the CD and FTD policies are equivalent (assuming there is at least a single unit of inventory at the SA). On the other hand, an extremely large vehicle might not ever fill up during a short planning horizon, resulting in the worst possible solution under the FTD policy. So as the vehicle's capacity grows, the performance of FTD deteriorates relative to the optimal and CD policies, and the deviation between the optimal and CD policies also increases.

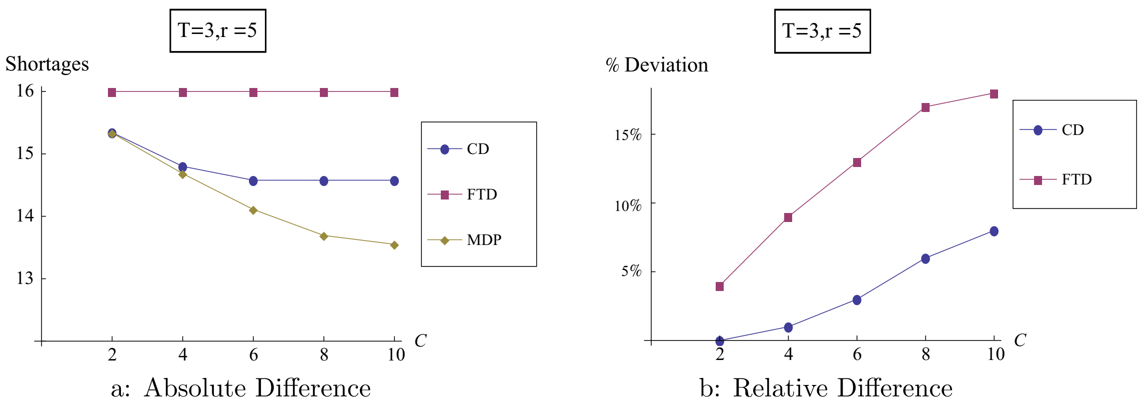

Fig. 5. Absolute and relative optimality gap for Experiment 6 – (varying *C*).

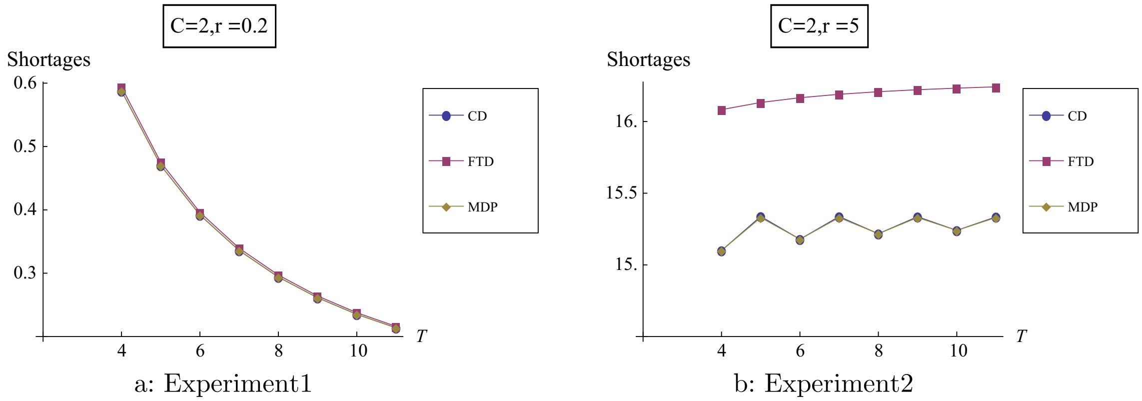

Fig. 6. Absolute and relative optimality gap for Experiments 1 and 2 – varying T.

Fig. 4 shows how expected shortages change as the length of the planning horizon, T, changes. In general, the relationship exhibits a decreasing trend, but the policies stay equivalent if there is sufficient supply (Fig. 4a, Experiment 3). Fig. 4(b) (Experiment 4), on the other hand, shows a far more peculiar relationship, namely that the CD policy leads to optimal decisions when the planning horizon consists of an even number of periods T, but suboptimal decisions when T is odd. Recall that all three policies are equivalent when there is more supply than demand (i.e., $r < 1$), so this only happens when the demand rate is greater than the supply rate (Fig. 4b). In the other case, $r > 1$, the CD policy exhibits a "zig-zag" pattern that bifurcates between optimal and suboptimal decisions as T changes. This happens because we assume (arbitrarily) that the location of the vehicle at the beginning of the planning horizon is at the POD, which means that two periods are required to complete each delivery under the CD policy (POD → SA in Period 1, SA → POD in Period 2, and so on). As a result, an even number T allows for the vehicle to stay in motion under the CD policy and make its final delivery to the POD in the last period. On the other hand, the CD policy would result in the last delivery being made in the penultimate period if T is odd, resulting in a suboptimal policy. To see this, consider $T = 3$; then using the CD policy, the vehicle goes from the POD to the SA in Period 1, SA → POD in Period 2, and POD → SA in Period 3. Thus at most one delivery can be made during a $T = 3$ period horizon. Instead of continuously dispatching the vehicle, it would make more sense to postpone dispatch from Period 2 until Period 3; that way, a larger shipment can be made from the SA to the POD given there is only one delivery opportunity associated with $T = 3$. In general, the optimal policy calls for deferring dispatch exactly once during a planning horizon that has (i) an odd number of periods and (ii) the vehicle initially located at the POD. Similarly, it would be optimal to delay dispatch during exactly one period if the vehicle starts out at the SA and the planning horizon consists of an odd number of periods.[2] This explains the CD policy's deviation from the optimal policy during odd periods in Fig. 4(b). Note, however, that this behavior is completely artificial from a practical standpoint because in reality, time flows continuously. Thus T would a real number, not necessarily integer, that won't be even or odd (justification for the discrete time model is presented in Section 3.2).

5.3. Research questions revisited

To conclude our discussion of our results, let us return to our original questions and summarize our findings.

Question 1. *If the current position of the vehicle is at the point of demand, is dispatching the vehicle to the staging area always an optimal decision?*

In a word, yes. While the proof is a bit lengthy (Proposition 1), the basic argument is that if you stay, you would need 2 periods to make a delivery (one from POD to SA and one back) as opposed to one if you immediately return to the SA.

Question 2. *If the vehicle is at the staging area, are there any situations in which deferring dispatch to a future period is an optimal decision?*

We can say for sure that when there is no inventory at the SA, it doesn't make sense to send the vehicle to the POD (Proposition 2), and when the vehicle is already full ($u_t \geqslant C$) there is no advantage to waiting (Proposition 3). Beyond that, all we can say is there is a complex relationship as described in Section 5.

Question 3. *If the average rate of demand at the point of demand is greater than the average arrival rate of supplies at the staging area, is continuous dispatching an optimal policy?*

[2] We confirmed this by rerunning our experiments with the initial position of the vehicle being the SA, in which case the CD policy turned out to be optimal for odd T and suboptimal for even T.

Actually, the opposite is true. As we saw in Section 5.2.1, CD is optimal when the average rate of supply exceeds that of demand, not the other way around. If the rate of supply is larger than demand, then inventory will build up at the POD as long as the vehicle doesn't sit idle, so CD is an optimal solution. If the rate of supply is great enough, then the vehicle won't be able to "keep up", and will continually dispatch full, and CD, FTD, and the MDP solution will all be identical.

Question 4. *In which situations is dispatching full truckloads an optimal policy?*
The only times FTD is optimal is when it is equivalent to CD, either arising from a sufficiently small vehicle capacity or sufficiently large supply relative to demand.

6. Conclusion and future work

The study considers the impact of material donations on last mile distribution in a two-stage supply chain consisting of one staging area (SA) and one point of demand (POD). Donations arrive over time at the SA in uncertain quantities, and a single vehicle is dispatched periodically to meet stochastic demand at the POD. We test the effectiveness of two common-sense policies and compare within the problem setting considered in this study. We expected to find that a simple policy such as continuous dispatching (CD) would be optimal. However, the optimal policy does not appear to exhibit any distinctive patterns, which is likely a consequence of the fact that both supply and demand are stochastic. Although CD is either optimal or close to optimal in most cases, CD is only an optimal policy in general when there are two or more vehicles. These findings are important. Since CD is not optimal for the basic problem introduced in this paper, it is unlikely that it will be in more complex settings. However, it would be interesting to see if CD would be close to optimal in other settings as it is in this study. Possible extensions include collaborative integration of last-mile distribution decisions among multiple agents, perishable inventory, and multiple objectives that consider measures related to both cost and service level. An investigation that considers a network with multiple SAs, PODs, and vehicles would also be of significant practical value.

One of the many difficulties of disaster relief is that there are many agents working towards the same goal without collaboration, and we would like to investigate the benefits of integrated decision-making as described in Lai et al. (2016). Another interesting extension comes from Nguyen et al. (2014); in our current model, we assume that all units are homogeneous packages of supplies which do not expire, but there are plenty of examples where relief items could be perishable in the short term such as food, medicine, or blood stores. Additionally, there is a growing interest in multi-objective optimization, often taking the form of Huang et al. (2012)'s "equity, efficiency, and efficacy". Essentially, this means that decision-makers must not only avoid needless expenses, but they should help all people equally and use their available resources well. This study focuses on the efficacy, but we would like to investigate incorporating them in the future. As you can see, last mile distribution is still in its infancy and has a plethora of opportunities for growth. We see the potential for many more years of exciting and practical research in last mile distribution and humanitarian logistics as a whole.

Appendix A

Proof of Lemma 1. We use induction to prove Lemma 1, that is, the following equation holds for $t = 1, \ldots, T-1$:

$$V_t(1, u_t, w_t) = \min\left\{V_t(0, u_t, w_t), \mathrm{E}[X_t - u_t - w_t]^+ + \mathrm{E}\left\{V_{t+1}\left(0, Y_{t+1}, [u_t + w_t - X_t]^+\right)\right\}\right\}. \tag{13}$$

We first show that Eq. (13) is true for $t = T-1$. Since

$$\begin{aligned} V_T(1, u_T, w_T) &= \mathrm{E}[X_T - u_T - w_T]^+ \\ V_T(0, u_T, w_T) &= \mathrm{E}[X_T - w_T]^+, \end{aligned} \tag{14}$$

it follows that

$$\begin{aligned} v_{T-1}(0, u_{T-1}, w_{T-1}, 0) &= \mathrm{E}[X_{T-1} - w_{T-1}]^+ + \mathrm{E}[X_T - W_T] \\ v_{T-1}(0, u_{T-1}, w_{T-1}, 1) &= \mathrm{E}[X_{T-1} - w_{T-1}]^+ + \mathrm{E}[X_T - U_T - W_T], \end{aligned} \tag{15}$$

where $U_T = u_{T-1} + Y_{T-1}$ and $W_T = (w_{T-1} - X_{T-1})^+$ are due to the transition Eqs. (8) and (9), respectively. Also note that U_T and W_T are random variables because at the beginning of period $T-1$, X_{T-1} and Y_{T-1} are random variables. Now since $U_T \geqslant 0$, it is evident from Eqs. (14) and (15) above that $\mathrm{E}[V_T(1, u_T, w_T)] \leqslant \mathrm{E}[V_T(0, u_T, w_T)]$ and $v_{T-1}(0, u_{T-1}, w_{T-1}, 1) \leqslant v_{T-1}(0, u_{T-1}, w_{T-1}, 0)$ so that

$$\begin{aligned} V_{T-1}(0, u_{T-1}, w_{T-1}) &= v_{T-1}(0, u_{T-1}, w_{T-1}, 1) \\ &= \mathrm{E}[X_{T-1} - w_{T-1}]^+ + \mathrm{E}\left[X_T - (u_{T-1} + Y_{T-1}) - (w_{T-1} - X_{T-1})^+\right]^+. \end{aligned} \tag{16}$$

For $i_{T-1} = 1$ and $a_{T-1}(1, u_{T-1}, w_{T-1}) = 0$, we have $U_T = u_{T-1} + Y_{T-1}, W_T = (w_{T-1} - X_{T-1})^+$, and

$$\begin{aligned}v_{T-1}(1,u_{T-1},w_{T-1},0) =& \mathrm{E}[X_{T-1}-w_{T-1}]^+ + \mathrm{E}[V_T(1,U_T,W_T)]\\ =& \mathrm{E}[X_{T-1}-w_{T-1}]^+ + \mathrm{E}[X_T-U_T-W_T]^+\\ =& \mathrm{E}[X_{T-1}-w_{T-1}]^+ + \mathrm{E}[X_T-(u_{T-1}+Y_{T-1})-(w_{T-1}-X_{T-1})^+]^+\\ =& V_{T-1}(0,u_{T-1},w_{T-1}).\end{aligned} \tag{17}$$

Note that the last equality in Eq. (17) follows from Eq. (16).

When $i_{T-1}=1$ and $a_{T-1}(1,u_{T-1},w_{T-1})=1$, the transition Eqs. (8) and (9) become $U_T=Y_{T-1}$ and $W_T=(u_{T-1}+w_{T-1}-X_{T-1})^+$. The resulting value of action $a_{T-1}(1,u_{T-1},w_{T-1})=1$ is

$$v_{T-1}(1,u_{T-1},w_{T-1},1)=\mathrm{E}[X_{T-1}-u_{T-1}-w_{T-1}]^+ + \mathrm{E}[V_T(0,U_T,W_T)]. \tag{18}$$

Therefore when $i_{T-1}=1$, the optimal value of state $(i_{T-1},u_{T-1},w_{T-1})$ is the minimum of Eqs. (17) and (18):

$$\begin{aligned}V_{T-1}(1,u_{T-1},w_{T-1}) &= \min\{v_{T-1}(1,u_{T-1},w_{T-1},0), v_{T-1}(1,u_{T-1},w_{T-1},1)\} \\ &= \min\{V_{T-1}(0,u_{T-1},w_{T-1}), \mathrm{E}[X_{T-1}-u_{T-1}-w_{T-1}]^+ + \mathrm{E}[V_T(0,U_T,W_T)]\}.\end{aligned} \tag{19}$$

Eq. (19) shows that the result holds for $t=T-1$. We now formulate the inductive hypothesis as follows:

$$\begin{aligned}V_{t+1}(1,u_{t+1},w_{t+1}) = \min\{&V_{t+1}(0,u_{t+1},w_{t+1}),\\ &\mathrm{E}[X_{t+1}-u_{t+1}-w_{t+1}]^+ + \mathrm{E}\{V_{t+2}(0,Y_{t+1},[u_{t+1}+w_{t+1}-X_{t+1}]^+)\}\}.\end{aligned} \tag{20}$$

For $i_t=0$, the values of actions $a_t(0,u_t,w_t)=0$ and $a_t(0,u_t,w_t)=1$ are

$$\begin{aligned}v_t(0,u_t,w_t,0) =& \mathrm{E}[X_t-w_t]^+ + \mathrm{E}[V_{t+1}(0,U_{t+1},W_{t+1})]\\ v_t(0,u_t,w_t,1) =& \mathrm{E}[X_t-w_t]^+ + \mathrm{E}[V_{t+1}(1,U_{t+1},W_{t+1})].\end{aligned}$$

From Eq. (20), the inductive hypothesis, $V_{t+1}(1,u_{t+1},w_{t+1}) \leqslant V_{t+1}(0,u_{t+1},w_{t+1})$, which implies that $\mathrm{E}[V_{t+1}(1,U_{t+1},W_{t+1})] \leqslant \mathrm{E}[V_{t+1}(0,U_{t+1},W_{t+1})]$. Consequently, $v_t(0,u_t,w_{,t}1) \leqslant v_t(0,u_t,w_{,t},0)$ and

$$V_t(0,u_t,w_t) = v_t(0,u_t,w_t,1) = \mathrm{E}[X_t-w_t]^+ + \mathrm{E}[V_{t+1}(1,U_{t+1},W_{t+1})].$$

At $i_t=1$, the value of actions $a_t(1,u_t,w_t)=0$ and $a_t(1,u_t,w_t)=1$ are

$$\begin{aligned}v_t(1,u_t,w_t,0) =& \mathrm{E}[X_t-w_t]^+ + \mathrm{E}[V_{t+1}(1,U_{t+1},W_{t+1})] = V_t(0,u_t,w_t)\\ v_t(1,u_t,w_t,1) =& \mathrm{E}[X_t-u_t-w_t]^+ + \mathrm{E}[V_{t+1}(0,U_{t+1},W_{t+1})]\end{aligned}$$

Therefore,

$$\begin{aligned}V_t(1,u_t,w_t) &= \min\{v_t(1,u_t,w_t,0), v_t(1,u_t,w_t,1)\}\\ &= \min\{V_t(0,u_t,w_t), \mathrm{E}[X_t-u_t-w_t]^+ + \mathrm{E}[V_{t+1}(0,U_{t+1},W_{t+1})].\end{aligned} \tag{21}$$

Since $U_{t+1}=Y_t$ and $W_{t+1}=(w_t+u_t-X_t)^+$ (because of the transition Eqs. (8) and (9), respectively), Eq. (21) becomes

$$V_t(1,u_t,w_t) = \min\{V_t(0,u_t,w_t), \mathrm{E}[X_t-u_t-w_t]^+ + \mathrm{E}\{V_{t+1}(0,Y_t,[u_t+w_t-X_t]^+)\}\}, \tag{22}$$

which is the same as Eq. (13). □

Proof of Proposition 4. The following technical results will be used to prove Proposition 4.

Lemma 2. *Let Z and $W \geqslant 0$ be random variables. Then* $\mathrm{E}(W)+\mathrm{E}(Z)^+ \geqslant \mathrm{E}(W+Z)^+$.

Proof. Let $I(A)$ represent an indicator function; i.e.,

$$I(A) = \begin{cases} 1, & \text{if } A \text{ is true}\\ 0, & \text{if otherwise.}\end{cases}$$

Then

$$\begin{aligned}\mathrm{E}(W)+\mathrm{E}(Z)^+ =& \mathrm{E}[W\cdot I(Z\geqslant 0)] + \mathrm{E}[W\cdot I(Z<0)] + \mathrm{E}[Z\cdot I(Z\geqslant 0)]\\ \mathrm{E}(W+Z)^+ =& \mathrm{E}[(W+Z)\cdot I(Z\geqslant 0)] + \mathrm{E}\big[(W+Z)^+ I(Z<0)\big],\end{aligned}$$

and

$$\begin{aligned}\mathsf{E}(W)+\mathsf{E}(Z)^{+}-\mathsf{E}(W+Z)^{+} =&\mathsf{E}[I(Z\geqslant 0)]\{\mathsf{E}(W)+\mathsf{E}(Z)-\mathsf{E}(W+Z)\}\\ &+\mathsf{E}[I(Z<0)]\{\mathsf{E}(W)-\mathsf{E}(W+Z)^{+}\}\\ =&0+\left[\mathsf{E}(W)-\mathsf{E}(W+Z)^{+}\right]\cdot\mathsf{E}[I(Z<0)]\\ =&\mathsf{E}[W\cdot I(Z<-W)]+\mathsf{E}[(W-W-Z)\cdot I(-W\leqslant Z<0)].\end{aligned}$$

In the last line, the first term is nonnegative since $W \geqslant 0$ is a stated condition of Lemma 2; the second term, $\mathsf{E}(-Z)$, is also nonnegative since $Z < 0$ is the applicable range for Z of that term. Thus the last line is nonnegative since it is the sum of two nonnegative terms. □

Lemma 3. *Let Z be a random variable and $u \geqslant 0$ a constant. Then* $\mathsf{E}(Z)^{+} - u \leqslant \mathsf{E}(Z-u)^{+}$.

The proof for Lemma 3 is the same as the proof for Lemma 2 with $-u$ replacing W.

Proof of Proposition 4. To distinguish between the single and infinite vehicle models, let $V_t^{\infty}(i,u,w)$ denote the optimal value (i.e., minimum expected shortage) from period t onward when there is an infinite number of vehicles. Then

$$V_T^{\infty}(1,u,w)=\mathsf{E}(X_T-u-w)^{+}. \tag{23}$$

To prove the result (that CD is an optimal policy when the number of vehicles is infinite), we must show that $v_t^{\infty}(1,u,w,0)-v_t^{\infty}(1,u,w,1)\geqslant 0$ for all $(1,u,w)$ and $t=1,\ldots,T$, where $v_t^{\infty}(i,u,w,a)$ is the value of action $a\in\{0,1\}$ from period t onward when the state at decision epoch t is (i,u,w). We first show that the result holds for period $T-1$, and use induction to show that it holds for all t. Again, let $I(\cdot)$ denote the indicator function. Then

$$\begin{aligned}v_{T-1}^{\infty}(1,u,w,0) =&\mathsf{E}(X_{T-1}-w)^{+}+\mathsf{E}\left[V_T\left(u+Y,[w-X_{T-1}]^{+}\right)\right]\\ =&\mathsf{E}\{[0+V_T(u+Y,w-X_{T-1})]\cdot I(X_{T-1}<w)\}\\ &+\mathsf{E}\{[(X_{T-1}-w)+V_T(u+Y,0)]\cdot I(w\leqslant X_{T-1}<u+w)\}\\ &+\mathsf{E}\{[(X_{T-1}-w)+V_T(u+Y,0)]\cdot I(X_{T-1}\geqslant u+w)\}\\ =&\mathsf{E}\left[(X_T-u-Y-w+X_{T-1})^{+}\cdot I(X_{T-1}<w)\right]\\ &+\mathsf{E}\left\{\left[(X_{T-1}-w)+(X_T-u-Y)^{+}\cdot I(w\leqslant X_{T-1}<u+w)\right]\right\}\\ &+\mathsf{E}\{[(X_{T-1}-w)+(X_T-u-Y)\cdot I(X_{T-1}\geqslant u+w)]\};\end{aligned} \tag{24}$$

and,

$$\begin{aligned}v_{T-1}^{\infty}(1,u,w,1) =&\mathsf{E}(X_{T-1}-u-w)^{+}+\mathsf{E}\left[V_T\left(Y,[u+w-X_{T-1}]^{+}\right)\right]\\ =&\mathsf{E}\{[0+V_T(Y,u+w-X_{T-1})]\cdot I(X_{T-1}<w)\}\\ &+\mathsf{E}\{[0+V_T(Y,u+w-X_{T-1})]\cdot I(w\leqslant X_{T-1}<u+w)\}\\ &+\mathsf{E}\{[(X_{T-1}-u-w)+V_T(Y,0)]\cdot I(X_{T-1}\geqslant u+w)\}.\\ =&\mathsf{E}\left[(X_T-Y-u-w+X_{T-1})^{+}\cdot I(X_{T-1}<w)\right]\\ &+\mathsf{E}\left\{\left[(X_T-Y-u-w+X_{T-1})^{+}\cdot I(w\leqslant X_{T-1}<u+w)\right]\right\}\\ &+\mathsf{E}\left\{\left[(X_{T-1}-u-w)+(X_T-Y)^{+}\cdot I(X_{T-1}\geqslant u+w)\right]\right\}.\end{aligned} \tag{25}$$

Let $\Delta v_t^{\infty}=v_t^{\infty}(1,u,w,0)-v_t^{\infty}(1,u,w,1), W=X_{T-1}-w$, and $Z=X_T-Y$ (these substitutions are purely cosmetic and are intended to make the equations shorter and easier to read). Then

$$\begin{aligned}\Delta v_{T-1}^{\infty} =&\mathsf{E}\left\{\left[(W+Z-u)^{+}-(W+Z-u)^{+}\right]\cdot I(W<0)\right\} && (26)\\ &+\mathsf{E}\left\{\left[W+(Z-u)^{+}-(W+Z-u)^{+}\right]\cdot I(0\leqslant W<u)\right\} && (27)\\ &+\mathsf{E}\left\{\left[W+(-u+Z)^{+}-(W-u)+Z^{+}\right]\cdot I(W\geqslant u)\right\}. && (28)\end{aligned}$$

Eq. (26) reduces to zero. Eq. (27) is nonnegative due to Lemma 2, and Eq. (28) is nonnegative because of Lemma 3. Hence, $\Delta v_{T-1}^{\infty}\geqslant 0$, which shows that the result stated in Proposition 4 holds for $t=T-1$. In order to state the inductive hypothesis, we introduce some additional notation, including $W=X-w$, and $Z=X-Y$. Let

$$\begin{aligned}\Delta v_t^{\infty}(W<0) =&\mathsf{E}\{[V_t(u+Y,-W)-V_t(Y,u-W)]\cdot I(W<0)\} && (29)\\ \Delta v_t^{\infty}(0\leqslant W<u) =&\mathsf{E}\{[W+V_t(u+Y,0)-V_t(Y,u-W)]\cdot I(0\leqslant W<u)\} && (30)\\ \Delta v_t^{\infty}(W\geqslant u) =&\mathsf{E}\{[W+V_t(u+Y,0)-(W-u)-V_t(Y,0)]\cdot I(W\geqslant u)\}. && (31)\end{aligned}$$

Then the inductive hypothesis can be stated as follows:

$$\begin{aligned}\Delta v_t^{\infty}(W<0) =&0 && (32)\\ \Delta v_t^{\infty}(0\leqslant W<u) \geqslant&0 && (33)\\ \Delta v_t^{\infty}(W\geqslant u) \geqslant&0. && (34)\end{aligned}$$

Observe that the sum of Eqs. (29)–(31) is Δv_t^∞. Thus Eqs. (26)–(28) necessarily imply that $\Delta v_t^\infty \geqslant 0$, which in turn implies that $V_t^\infty(1,u,w) = v_t^\infty(1,u,w,1)$. Given this inductive hypothesis, we need to show that Eqs. (26)–(28) hold for $t = t-1$. For $t-1$, we have

$$\begin{aligned} v_{t-1}^\infty(1,u,w,0) =& \mathsf{E}(W)^+ + \mathsf{E}\{V_t(u+Y,[-W]^+)\} \\ =& \mathsf{E}\{[0+V_t(u+Y,-W)]\cdot I(W<0)\} \\ &+\mathsf{E}\{[W+V_t(u+Y,0)]\cdot I(0\leqslant W<u)\} \\ &+\mathsf{E}\{[W+V_t(u+Y,0)]\cdot I(W\geqslant u)\} \end{aligned} \tag{35}$$

$$\begin{aligned} v_{t-1}^\infty(1,u,w,1) =& \mathsf{E}(W-u)^+ + \mathsf{E}\{V_t(Y,[u-W]^+)\} \\ =& \mathsf{E}\{[0+V_t(Y,u-W)]\cdot I(W<0)\} \\ &+\mathsf{E}\{[W+V_t(Y,u-W)]\cdot I(0\leqslant W<u)\} \\ &+\mathsf{E}\{[(W-u)+V_t(Y,0)]\cdot I(W\geqslant u)\}. \end{aligned} \tag{36}$$

From Eqs. (35) and (36), we have

$$\Delta v_{t-1}^\infty(W<0) = \mathsf{E}\{[V_t(u+Y,-W)-V_t(Y,u-W)]\cdot I(W<0)\} \tag{37}$$

$$\Delta v_{t-1}^\infty(0\leqslant W<u) = \mathsf{E}\{[W+V_t(u+Y,0)-V_t(Y,u-W)]\cdot I(0\leqslant W<u)\} \tag{38}$$

$$\Delta v_{t-1}^\infty(W\geqslant u) = \mathsf{E}\{[W+V_t(u+Y,0)-(W-u)-V_t(Y,0)]\cdot I(W\geqslant u)\}. \tag{39}$$

It directly follows from the inductive hypothesis given by Eqs. (26)–(28) that $\Delta v_{t-1}^\infty(W<0)=0, \Delta v_{t-1}^\infty(0\leqslant W<u)\geqslant 0$, and $\Delta v_{t-1}^\infty(W\geqslant u)\geqslant 0$. Therefore, $\Delta v_t^\infty \geqslant 0$ for all t, which means that $V_t^\infty(1,u,w) = v_t^\infty(1,u,w,1)$ and $\pi^{*,\infty}(1,u,w) = \pi^{CD,\infty}(1,u,w) = 1$. □

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