

# **Introduction to multivariate analysis**

## **Outline**

- Why do a multivariate analysis
- Ordination, classification, model fitting
- Principal component analysis
- Discriminant analysis, quickly
- Species presence/absence data
- Distance data

## **Data are usually multivariate**

Typically we measure multiple variables on the populations, species, and ecosystems that we study.

This creates a challenge: how to display and analyze measurements on all those variables.

We need ways to make it easier to find the important patterns and relationships among the many variables.

## Ordination is what we usually want:

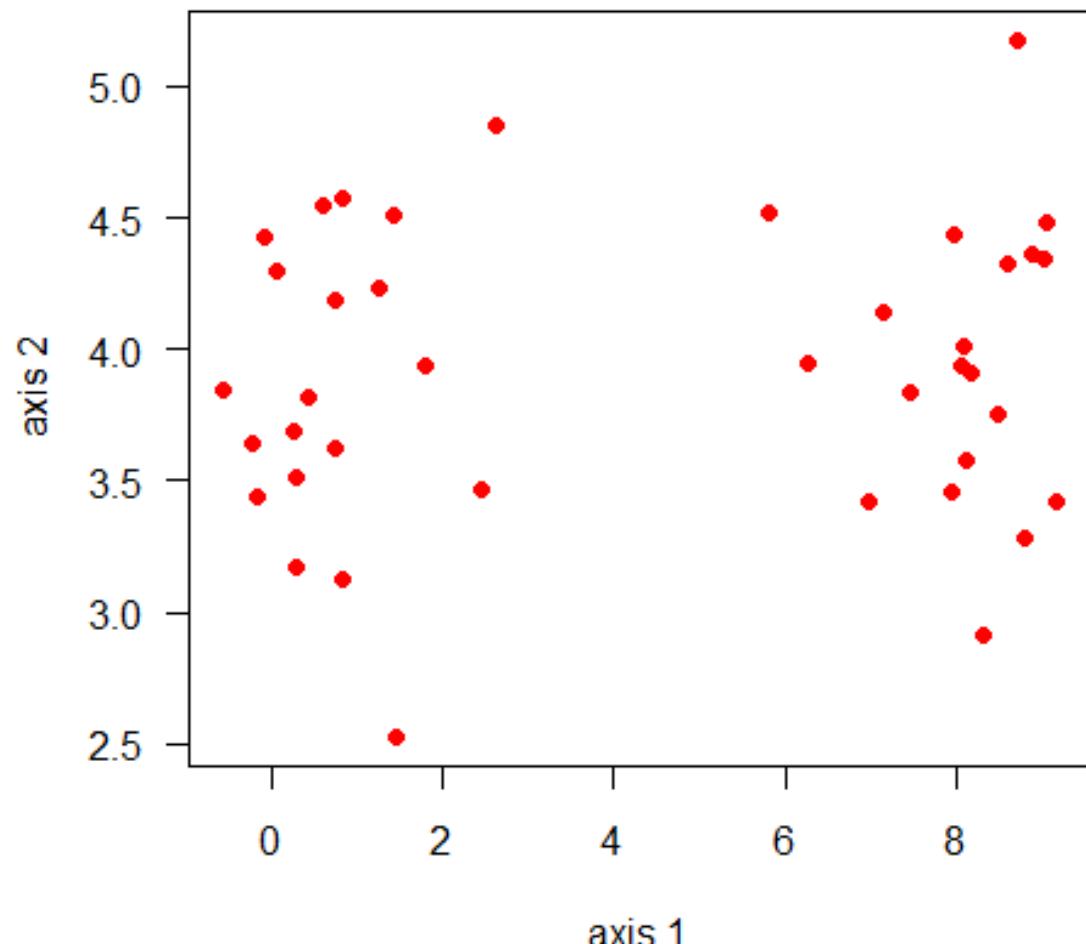
Arrange sampling units along gradients or according to combinations of variables

Goals:

- To visualize complex data in few dimensions
- To find meaningful combinations of the original variables that can be used in subsequent analyses

Large number  
of variables

*multivariate  
analysis*



## **Ordination, Classification, Model fitting**

Multivariate methods are also used for

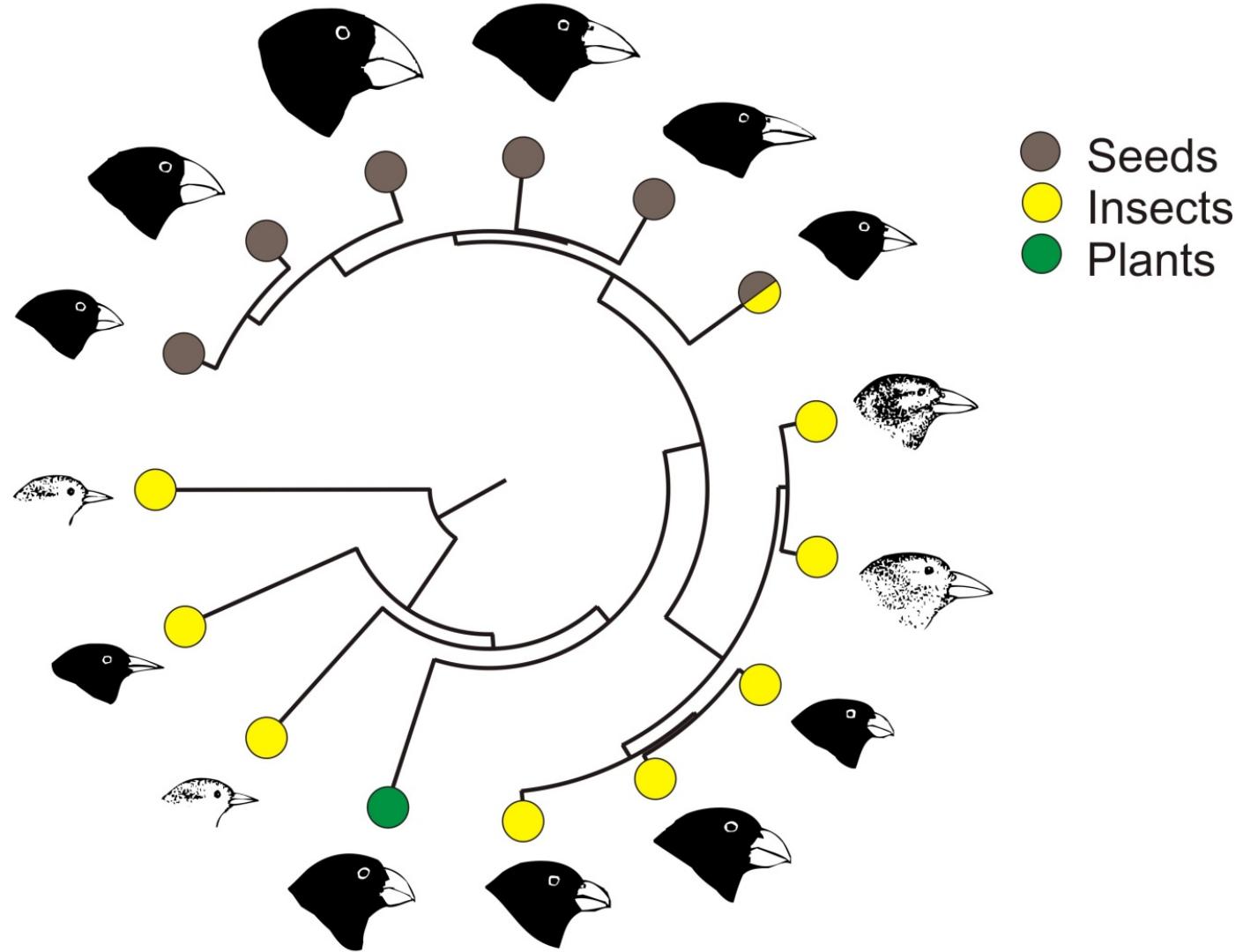
- Classification: place sampling units into groups
- Model fitting: multivariate analysis of variance; multiple regression

Today, we'll focus mainly on ordination.

We'll start with Principal Components Analysis because it is the most straightforward multivariate method.

# Principal Component Analysis

Example 1: Differences in beak and body dimensions of Darwin's finches



# Principal Component Analysis

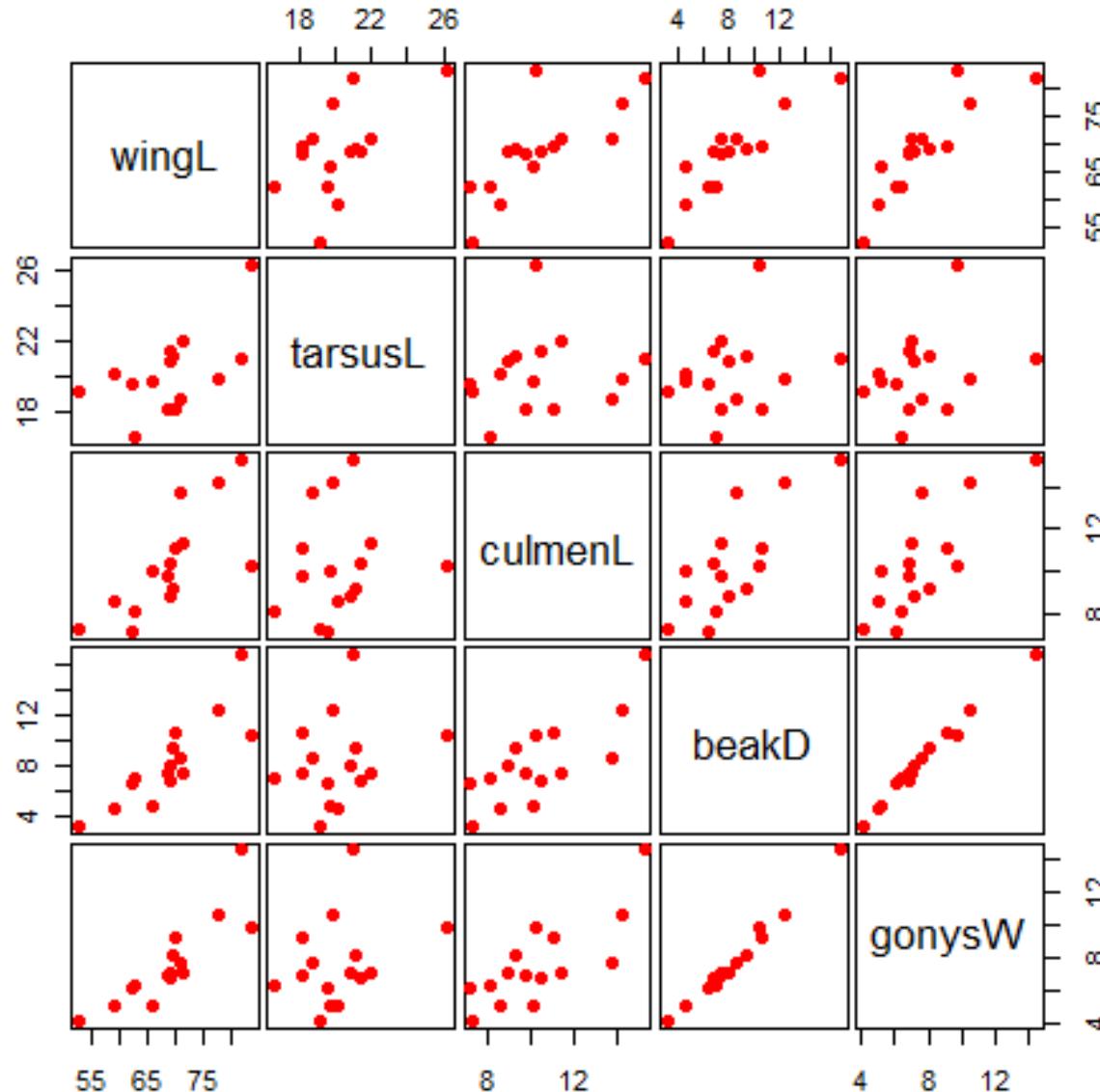
mydata: Data frame of means of measurements (5 variables) on 14 species

UNITS (species)		VARIABLES (traits)				
		wingL	tarsusL	culmenL	beakD	gonysW
	<i>C.heliobates</i>	68.79	21.35	10.45	6.75	6.78
	<i>C.pallida</i>	71.2	21.96	11.36	7.51	7.02
	<i>C.parvulus</i>	62.28	19.55	7.2	6.51	6.13
	<i>C.pauper</i>	68.89	20.82	8.91	7.95	7.11
	<i>C.psittacula</i>	69.34	21.11	9.25	9.34	8.06
UNITS (species)	<i>Certhidea.fusca</i>	59.02	20.04	8.57	4.56	5.04
	<i>Certhidea.olivacea</i>	52.48	19.1	7.3	3.17	4.11
	<i>G.conirostris</i>	77.47	19.77	14.22	12.35	10.59
	<i>G.difficilis</i>	68.31	18.15	9.75	7.47	6.89
	<i>G.fortis</i>	69.69	18.08	11.1	10.62	9.22
	<i>G.fuliginosa</i>	62.36	16.55	8.13	6.97	6.33
	<i>G.magnirostris</i>	81.79	20.88	15.25	16.84	14.53
	<i>G.scandens</i>	70.9	18.71	13.76	8.54	7.67
	<i>Pinaroloxias</i>	65.93	19.69	10.09	4.7	5.1
	<i>Platyspiza</i>	83.43	26.24	10.26	10.37	9.81

# Principal Component Analysis

The data have only 5 variables but visualizing them still represents a challenge.

`pairs(mydata)`



# Principal Component Analysis

How to visualize multivariate data?

These are “Chernoff faces”, which display multivariate data in the shape of a human face. The individual parts of the face represent values of the variables by their shape, size, placement and orientation. Humans are good at distinguishing faces.



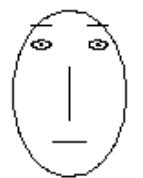
"C.heliobates"



"C.psittacula"



"G.difficilis"



"G.scandens"



"C.pallida"



"Certhidea.fusca"



"G.fortis"



"Pinaroloxias"



"C.parvulus"



"Certhidea.olivacea"



"G.fuliginosa"



"Platyspiza"



"C.pauper"



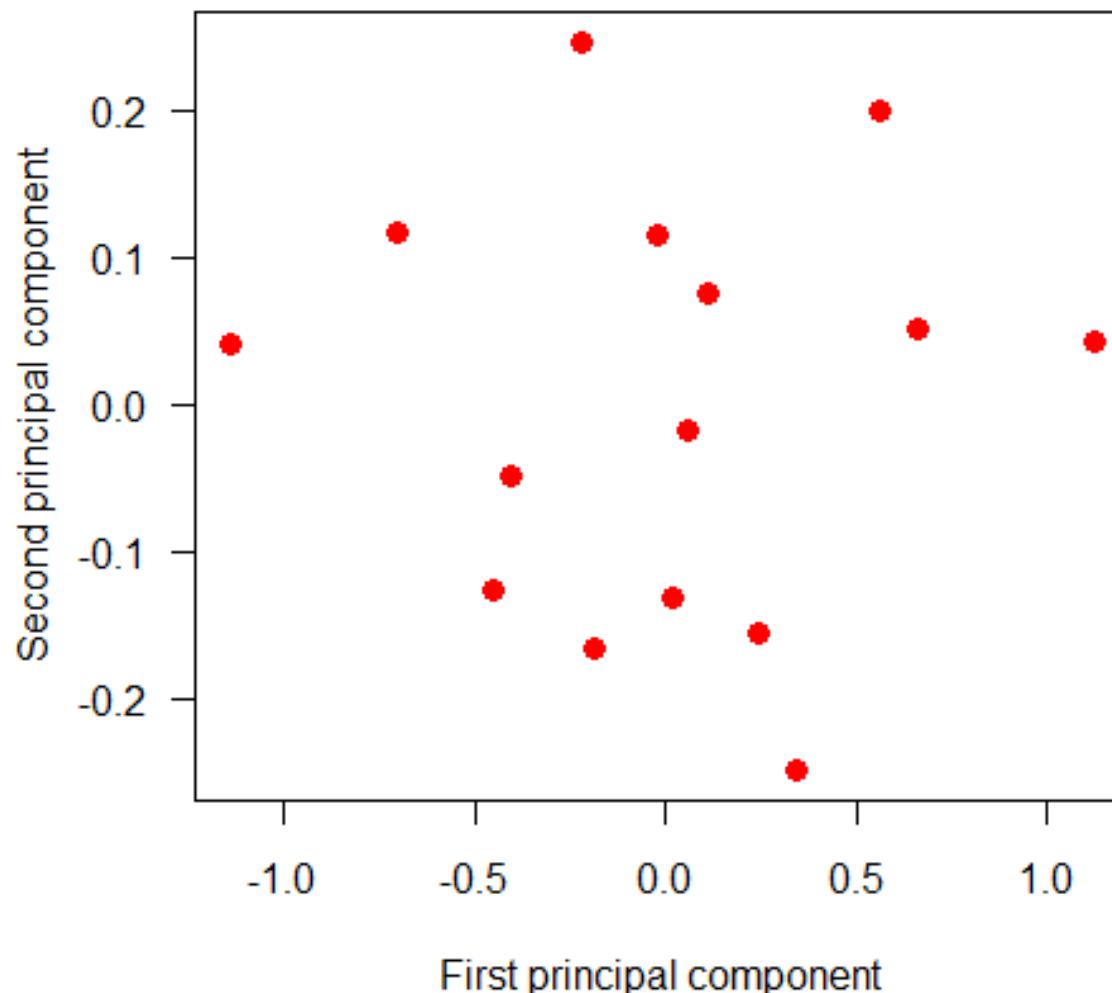
"G.conirostris"



"G.magnirostris"

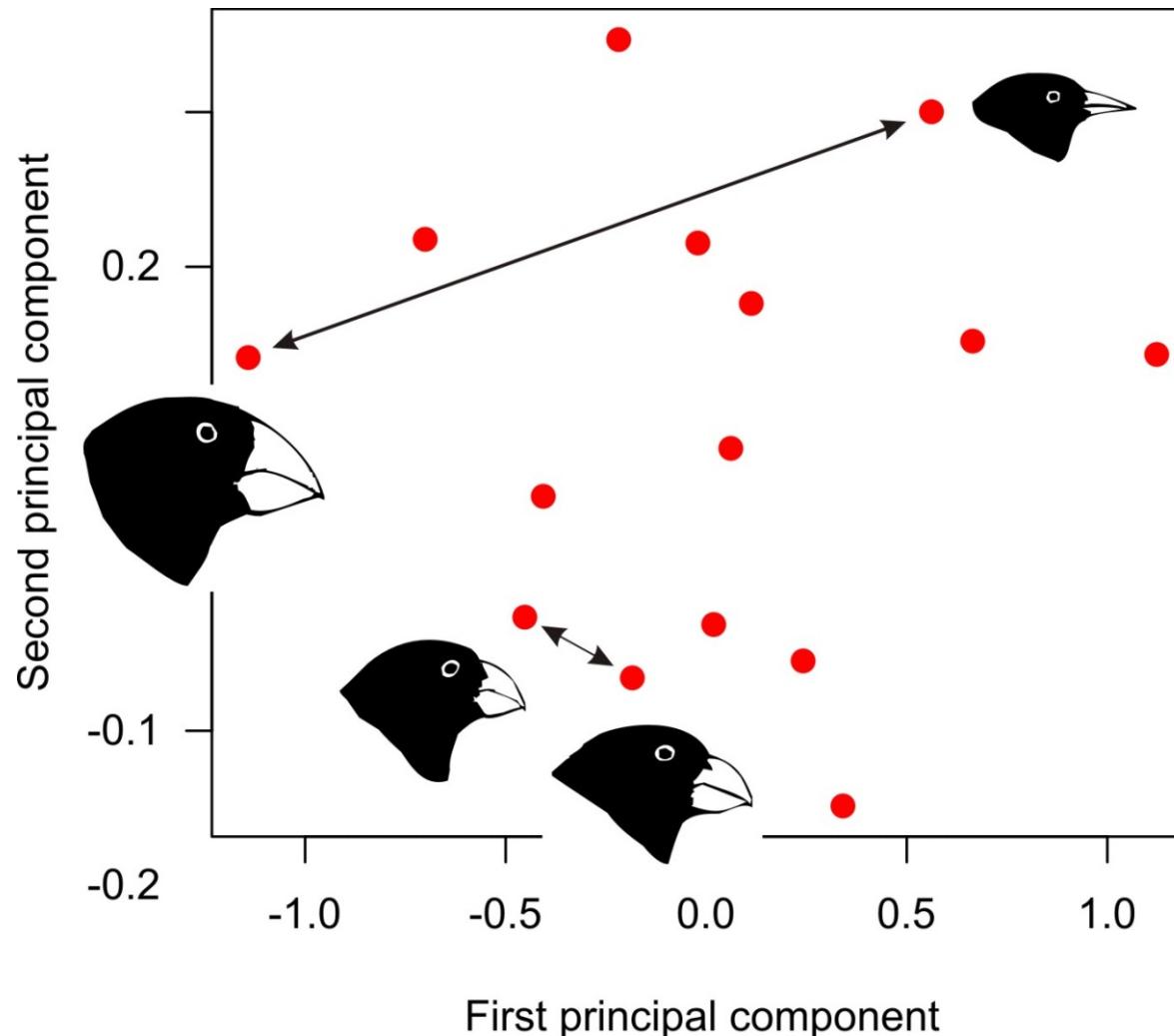
## Principal Component Analysis

96% of all the variation among the Darwin's finch species is in just 2 dimensions or principal components: PC1 and PC2. This is because the traits strongly co-vary among species. PC's are linear combinations of the original variables.



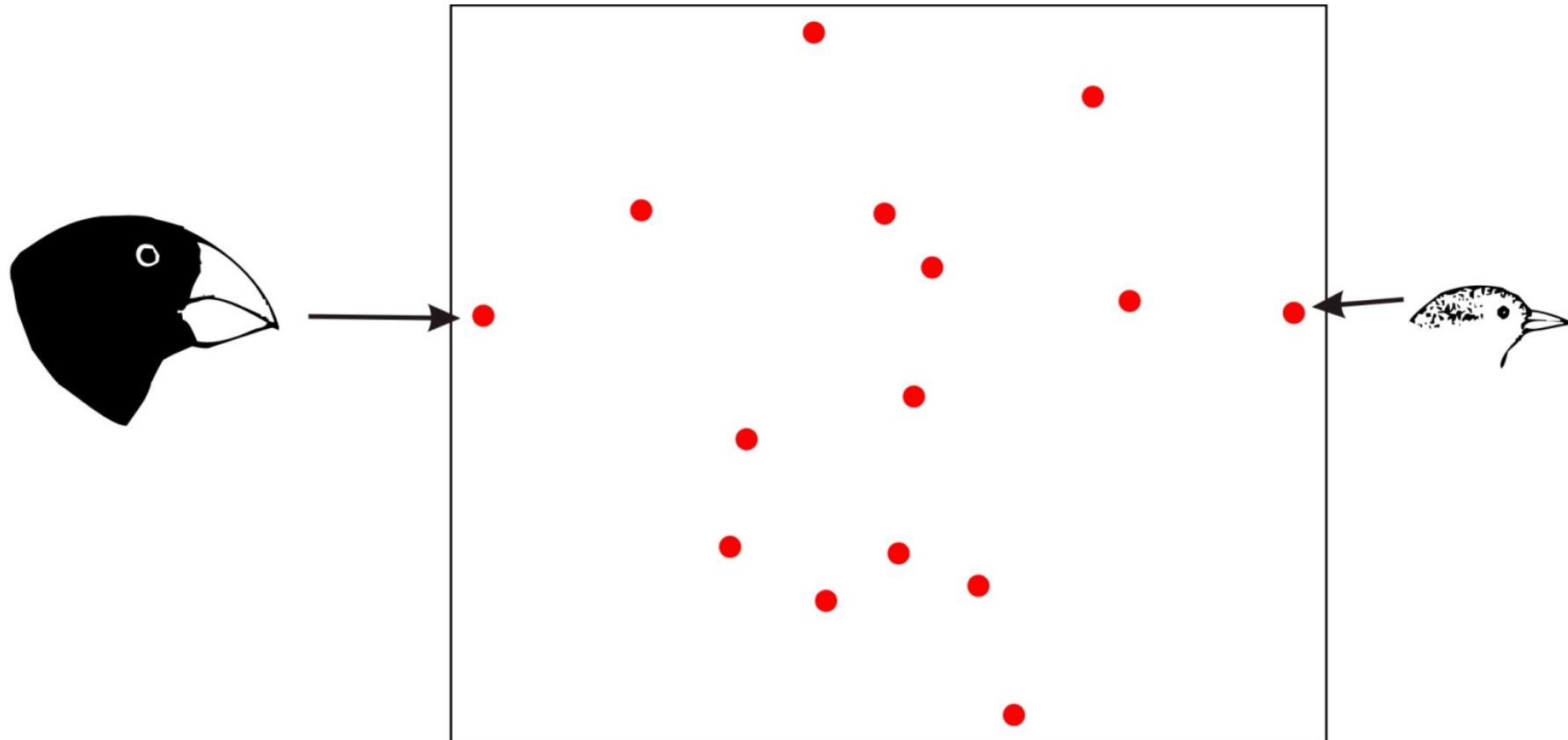
## Principal component analysis

Even though we're visualizing 5 dimensions of data with only the first 2, distances between species are approximately preserved. Points close together indicate species that are similar. Points far apart indicate species that are more different.



## Principal component analysis

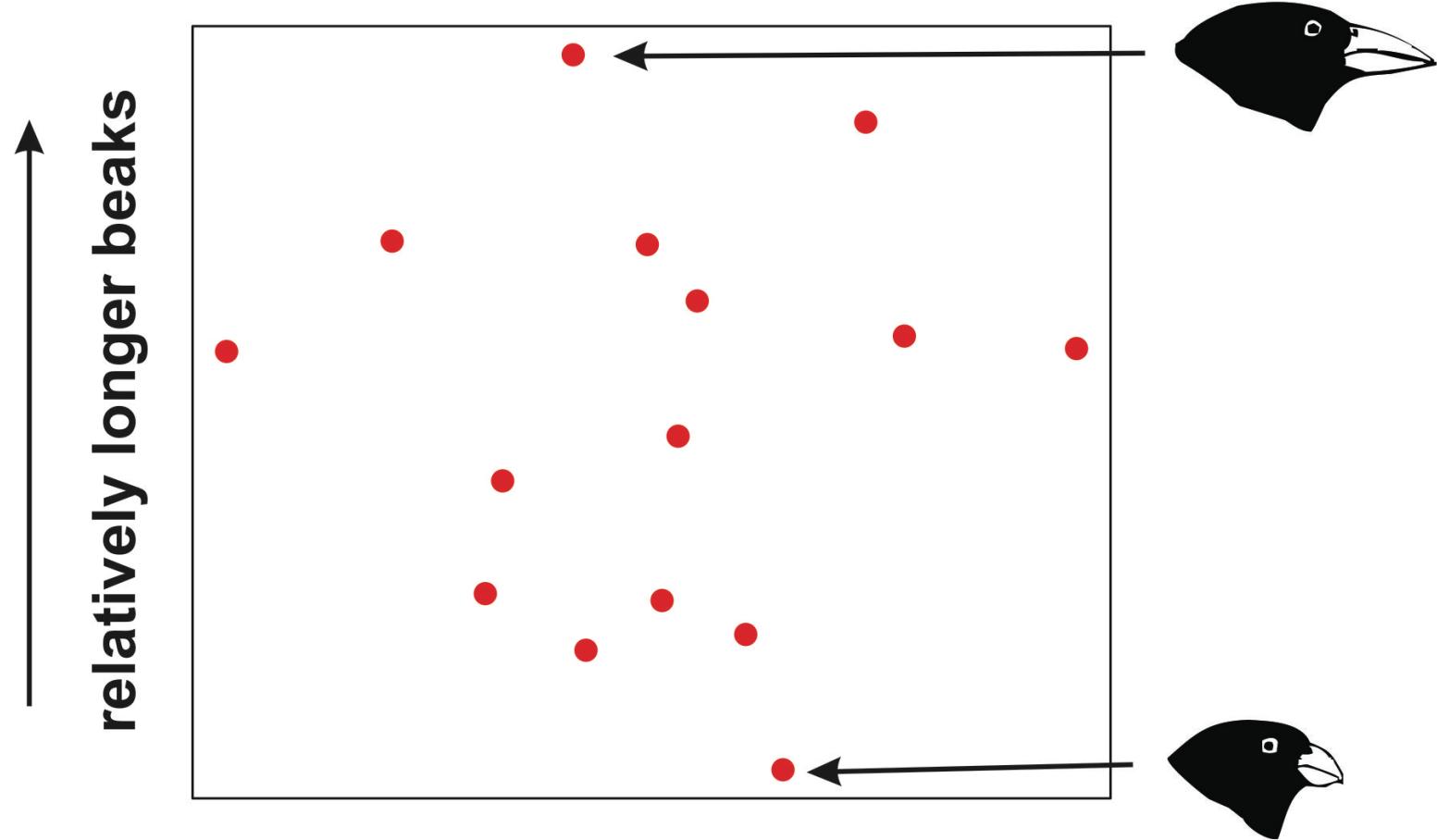
Even though they are composite variables, the axes in this case are interpretable. The first axis arranges the species according to differences in overall beak size.



deeper, longer, wider beaks

## Principal component analysis

The second axis arranges the species according to differences in beak length (relative to overall beak size). It represents an axis of beak shape.



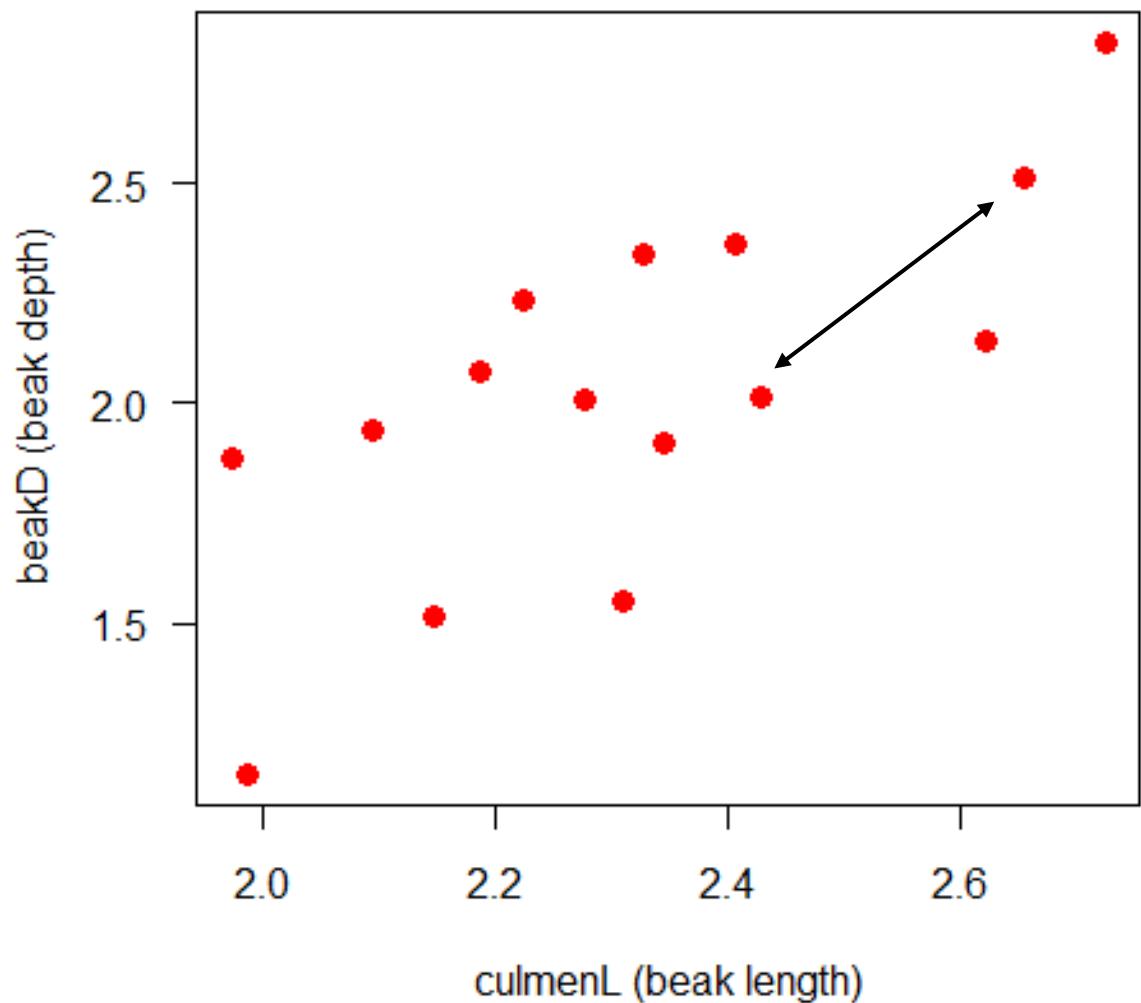
## Principal component analysis – how it works

The procedure amounts to nothing more than a rotation of the axes, allowing you to view most of the data in a small number of dimensions. To see this, imagine just two traits. I've log-transformed all the variables to help put on a similar scale.

The data can be represented by the Euclidean distances between all pairs of points (species).

“Euclidean distance” is the straight line distance between two points

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$



## Principal component analysis – how it works

Or the data can be represented by the associations between the variables (as illustrated by the ellipse)

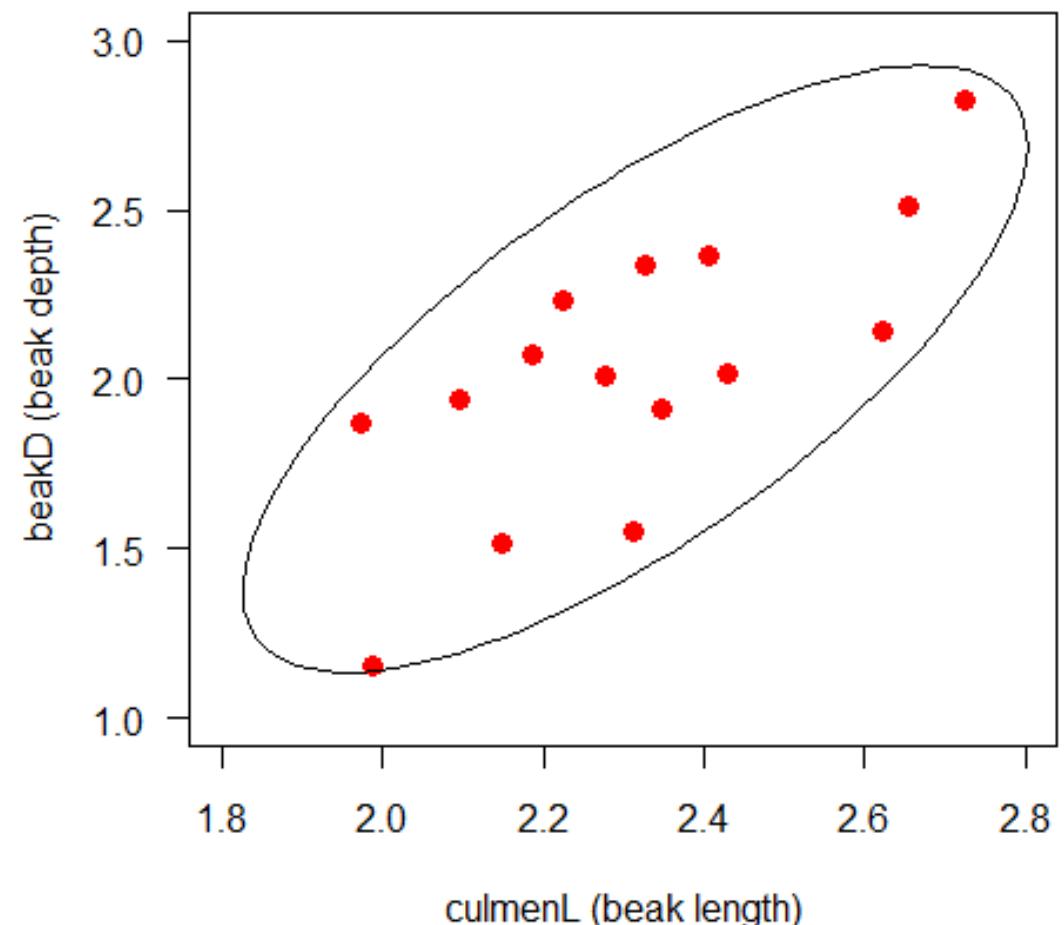
The elements of the covariance matrix

are:

$\text{var}(x_1)$	$\text{cov}(x_1, x_2)$
$\text{cov}(x_1, x_2)$	$\text{var}(x_2)$

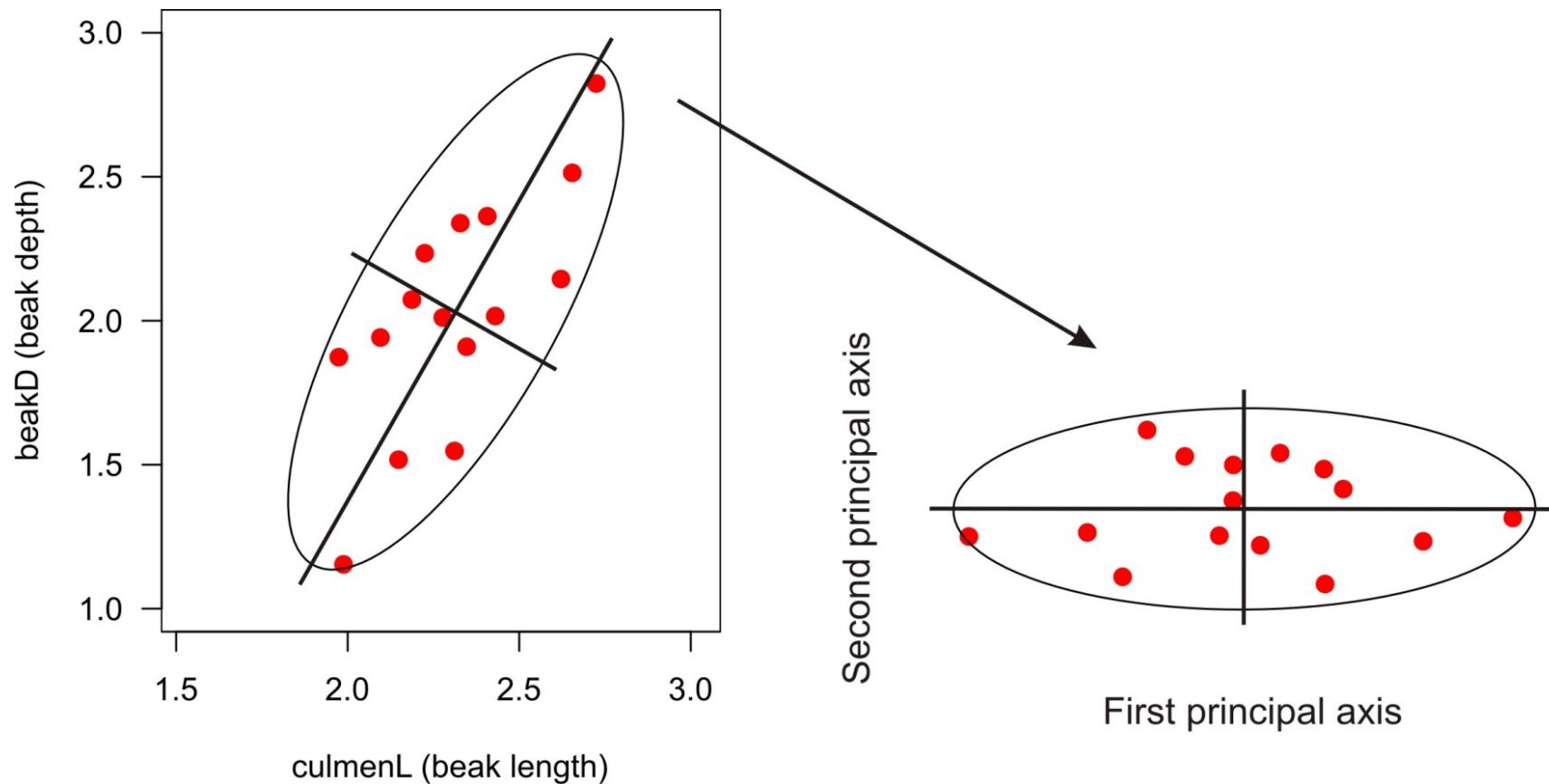
For the two finch variables:

	culmenL	beakD
culmenL	0.0518	0.0698
beakD	0.0698	0.1741



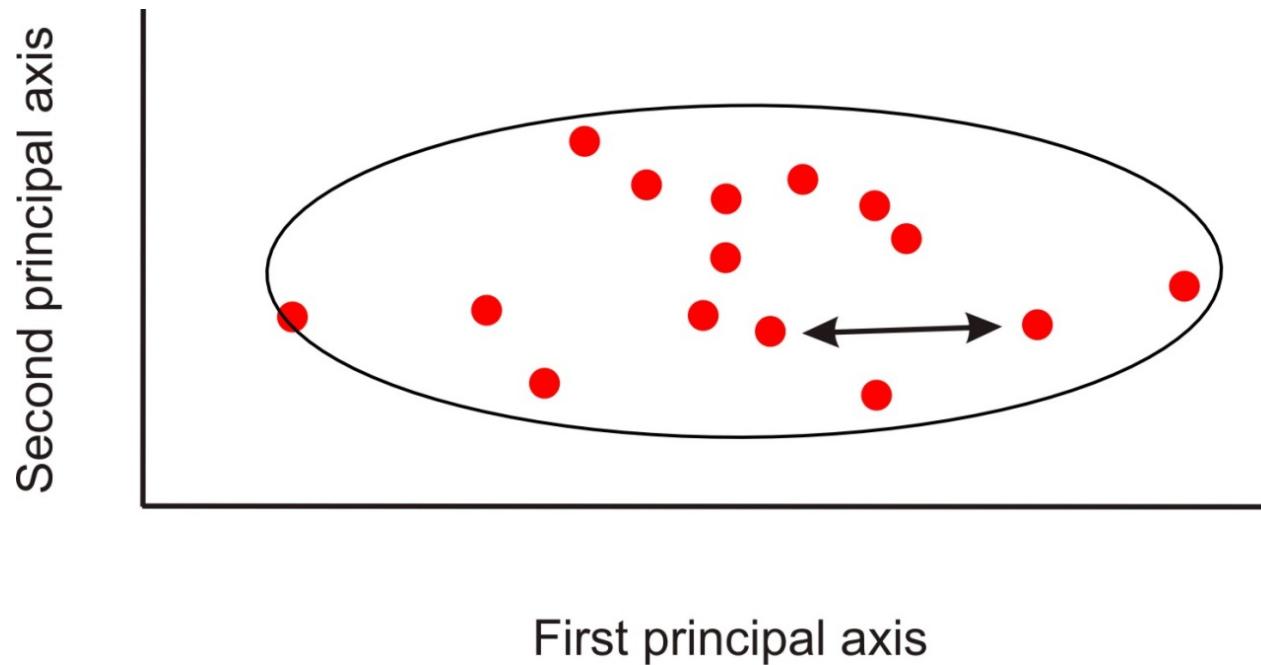
## Principal component analysis – how it works

Principal components analysis finds a linear transformation of the variables to make a composite variable having maximum possible variance. This is the first major axis, or principal component, PC1. A second linear transformation creates a variable having the maximum variance perpendicular (orthogonal) to PC1. (And so on, when there are more than 2 variables.) The result is simply a rotation.



## Principal component analysis

Because it is nothing more than a rotation of the axes, the Euclidean distances between pairs of species are unaltered by the transformation, when all the PC axes are included\*.



\*Warning: in some non-R programs the default procedure is to standardize the variables ("correlation matrix") before carrying out the analysis. Use the correlation matrix if variables don't have a common scale. Euclidean distances will then be based on standardized data, not the original measurements.

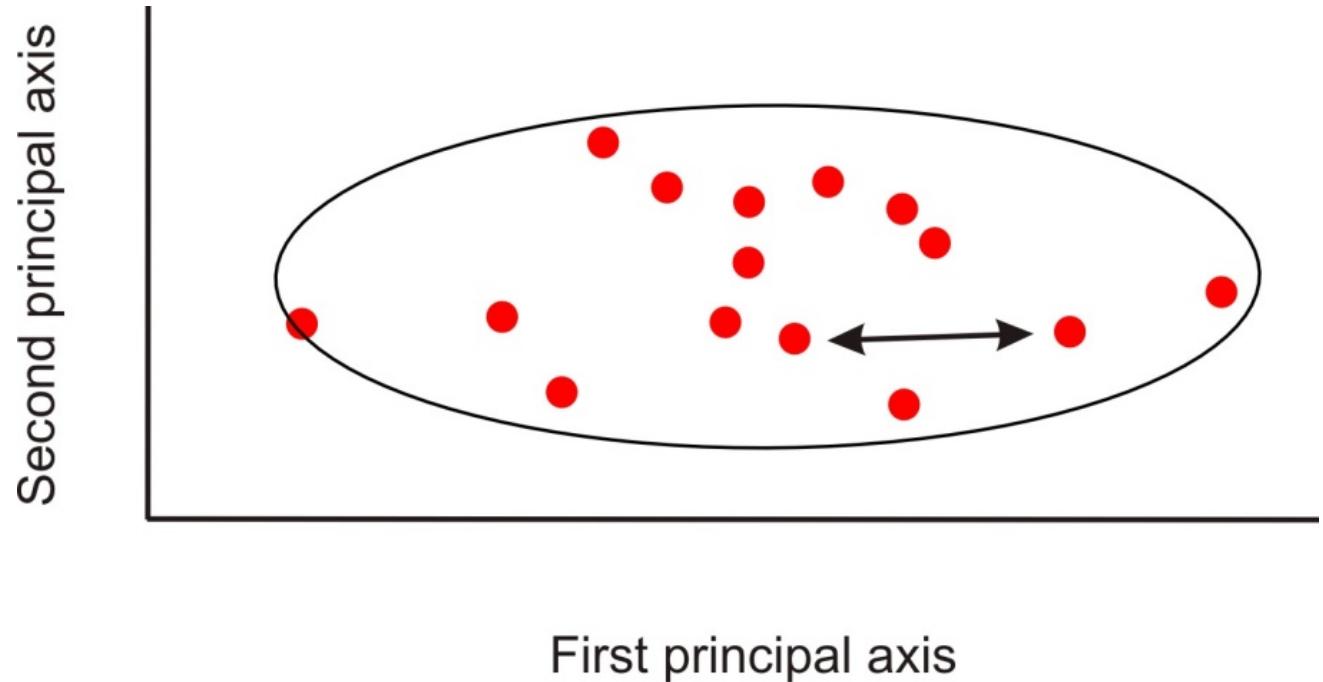
## Principal component analysis

The covariance matrix of the new, composite variables has variances on the diagonal and zeros off the diagonal. Principal component axes have zero covariance because we deliberately searched for orthogonal composite variables.

	pc1	pc2
pc1	0.192	0
pc2	0	0.019

These variances are called  
the **eigenvalues**

They sum to the same total  
as the sum of the variances  
of the original traits.

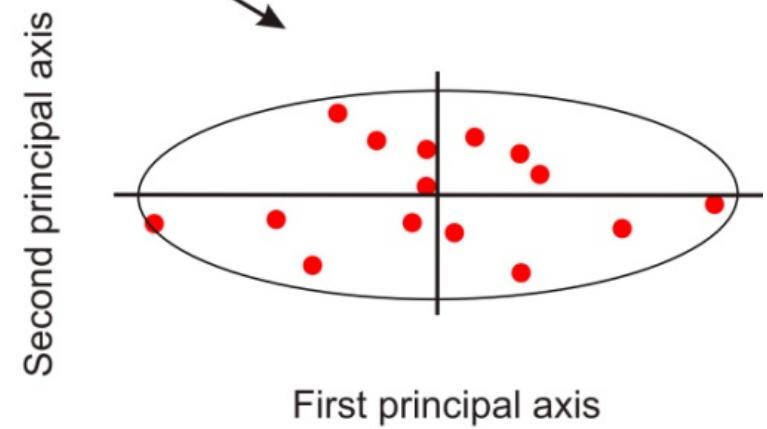
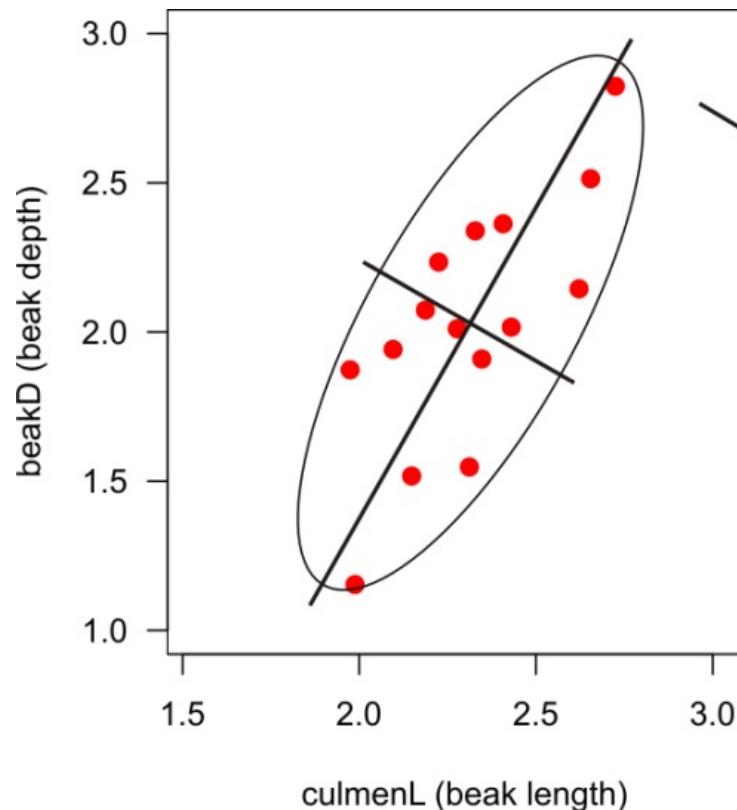


## Principal component analysis

The vectors that contain the constants for transforming the original variables into the principal components are called the **eigenvectors**.

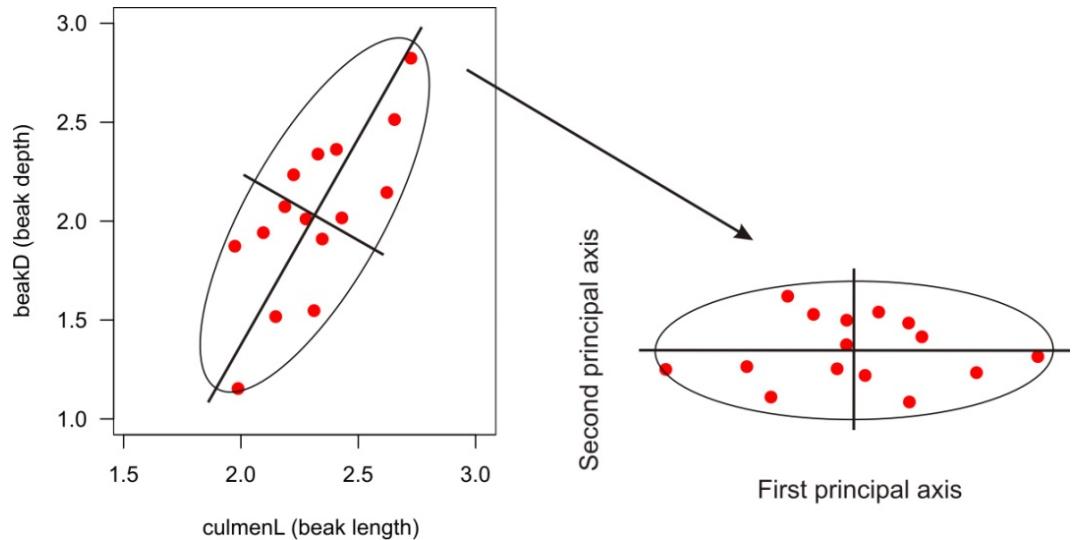
	eigen1	eigen2
culmenL	0.413	-0.911
beakD	0.911	0.413

These constants  
are called  
**loadings**.



## Principal component analysis

	eigen1	eigen2
culmenL	0.413	-0.911
beakD	0.911	0.413



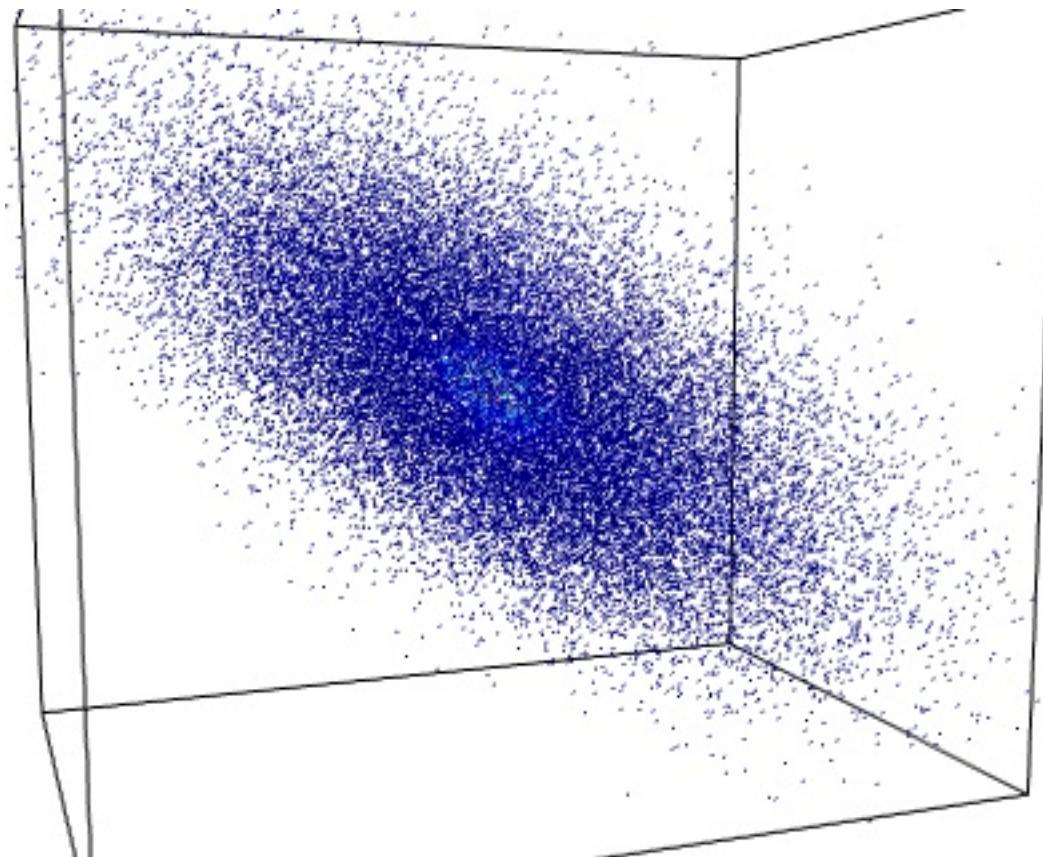
The constants are called **loadings** because they indicate the contribution of each variable to the principal component.

pc1 can be interpreted as “beak size” because both beak traits make contributions having the same sign (depth contributes more than length). The axis separates big-beaked birds at one extreme from small-beaked birds at the other.

pc2 is interpreted as “beak shape” because beak depth loads positively but beak length negatively. It separates short deep beaks at one end from long shallow beaks at the other.

## Principal component analysis

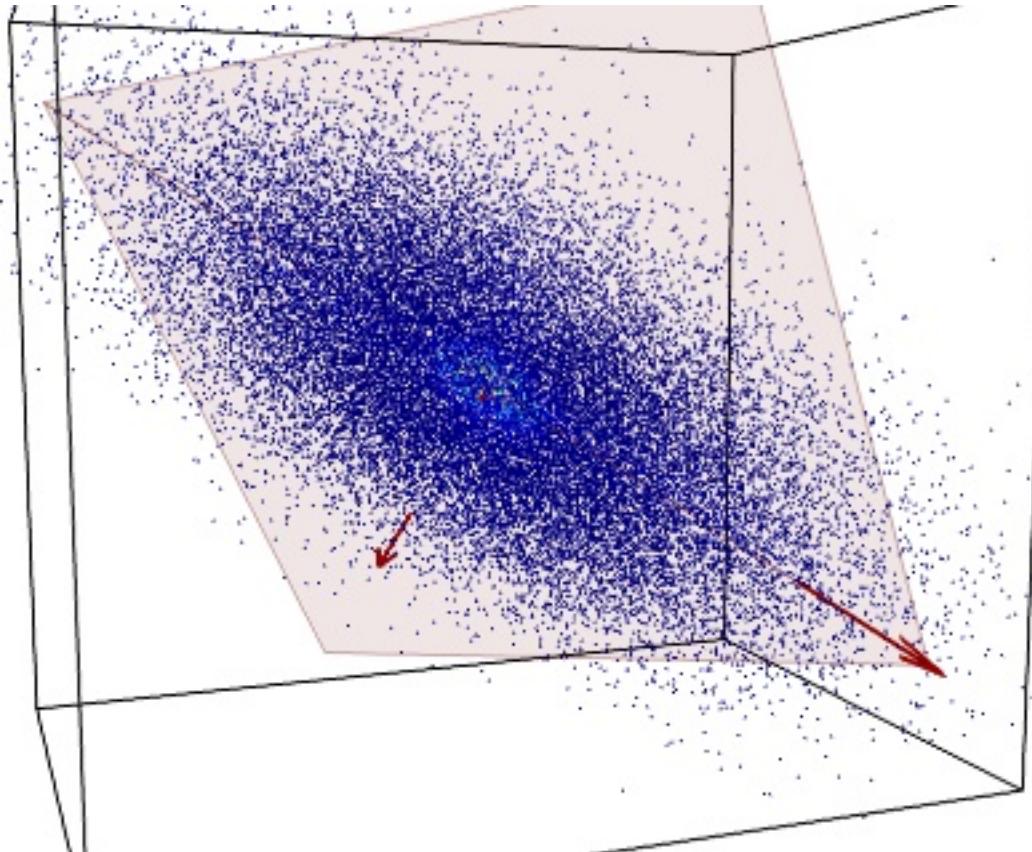
We can still visualize this process with 3 variables instead of 2 (it gets hard to visualize with more than 3 variables)



[http://gael-varoquaux.info/science/ica\\_vs\\_pca.html](http://gael-varoquaux.info/science/ica_vs_pca.html)

## Principal component analysis

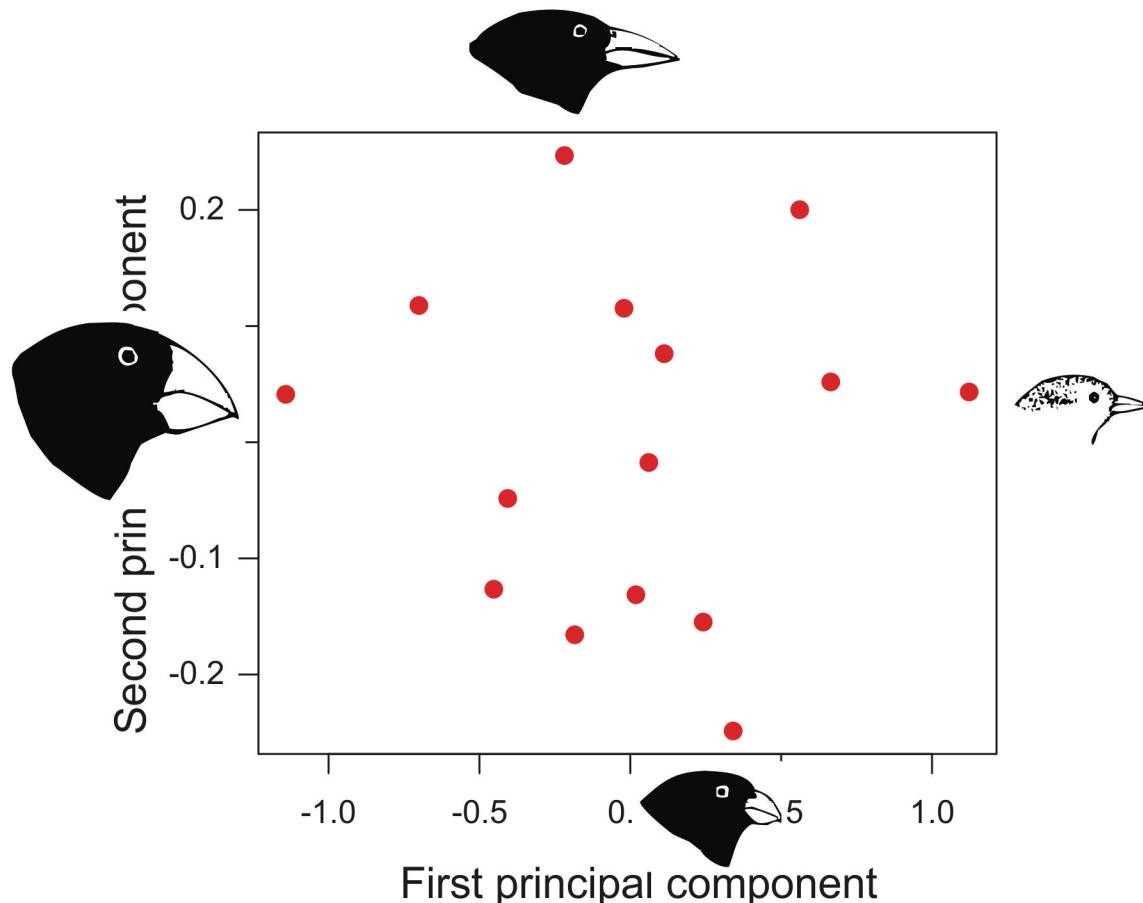
PC1 is the long axis of the ellipsoid. PC2 is perpendicular to it in the next most variable direction.



[http://gael-varoquaux.info/science/ica\\_vs\\_pca.html](http://gael-varoquaux.info/science/ica_vs_pca.html)

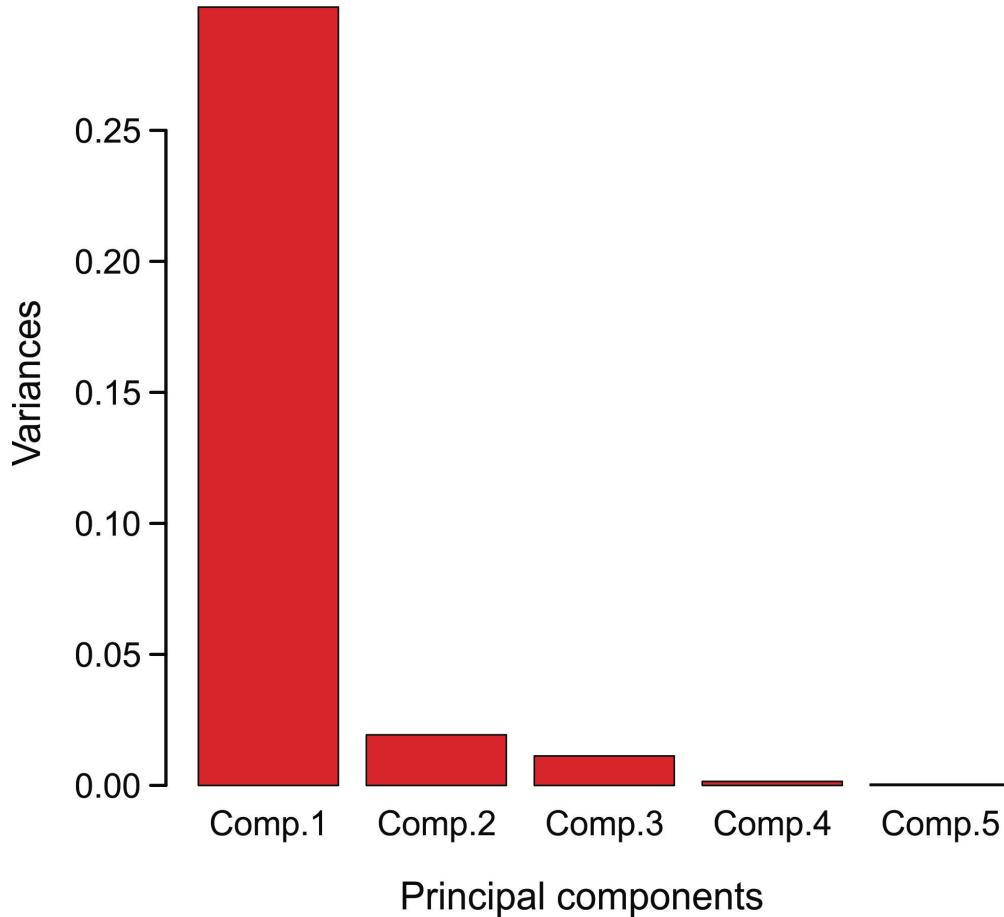
## Principal component analysis

The idea is the same with 5, or any number of variables. The plot below is of the first two PC axes from the PC analysis of all 5 variables for the 15 Darwin's finch species. The only difference is that when we look at only the first two principal components we aren't seeing all the differences among the species.



## Principal component analysis

The eigenvalues tell you how much of the total variation is captured by the first two principal components. These can be visualized in a “scree plot”



## Principal component analysis

The eigenvectors indicate the loadings.

For the Darwin's finch data:

PC1 can be interpreted as “beak size”.

PC2 can be interpreted as “beak shape”

PC3 is mainly one trait, tarsus length

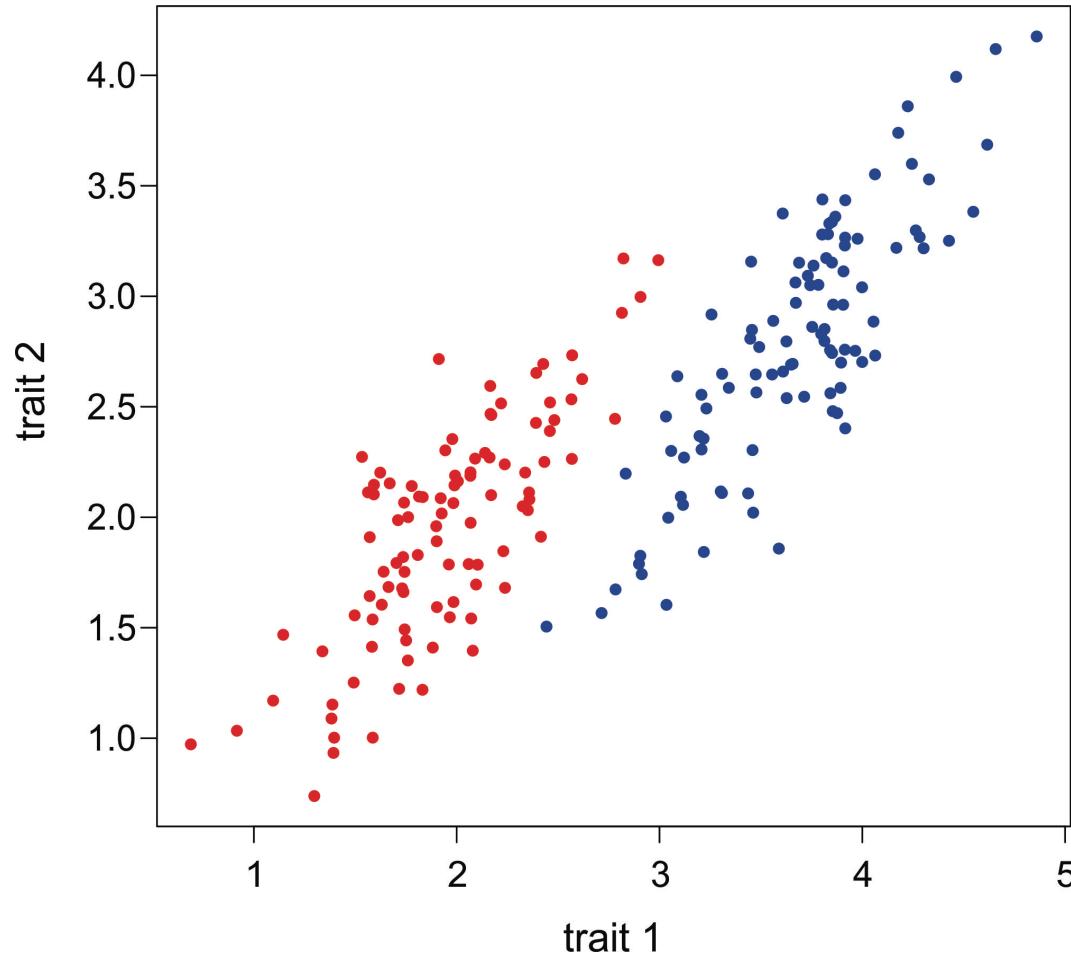
Etc...

	PC1	PC2	PC3	PC4	PC5
wingL	-0.195	0.062	-0.335	0.577	0.716
tarsusL	-0.052	-0.043	-0.919	-0.071	-0.383
culmenL	-0.326	0.932	0.039	0.002	-0.153
beakD	-0.733	-0.330	0.193	0.395	-0.400
gonyW	-0.562	-0.127	-0.073	-0.711	0.397

## Discriminant function analysis, quickly

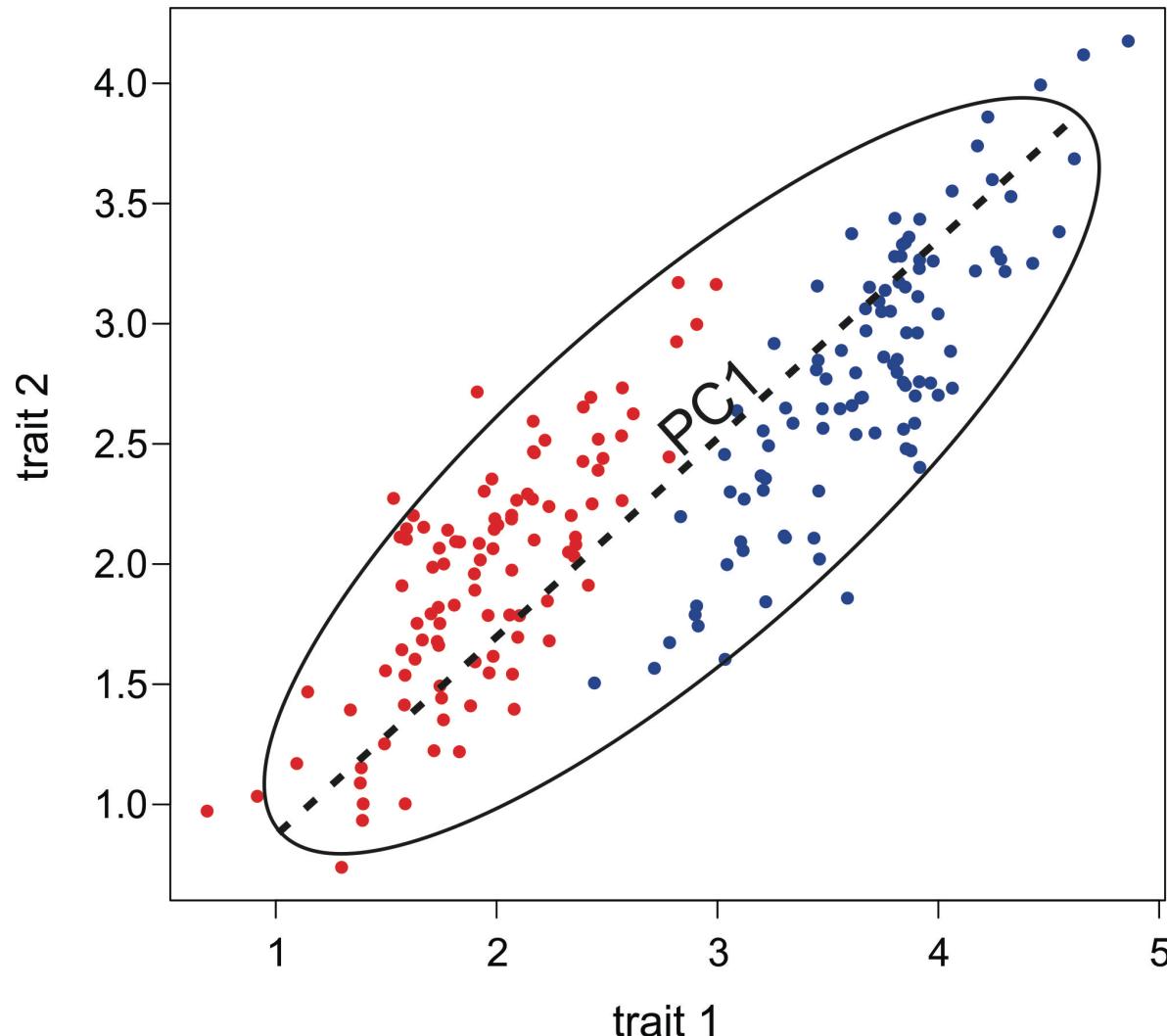
*Discriminant function analysis* is for classification rather than ordination.

It finds axes that maximally separate two or more previously identified groups. It finds axes that maximize variation among groups relative to variation between groups. These axes can then be used to classify new observations into the same groups.



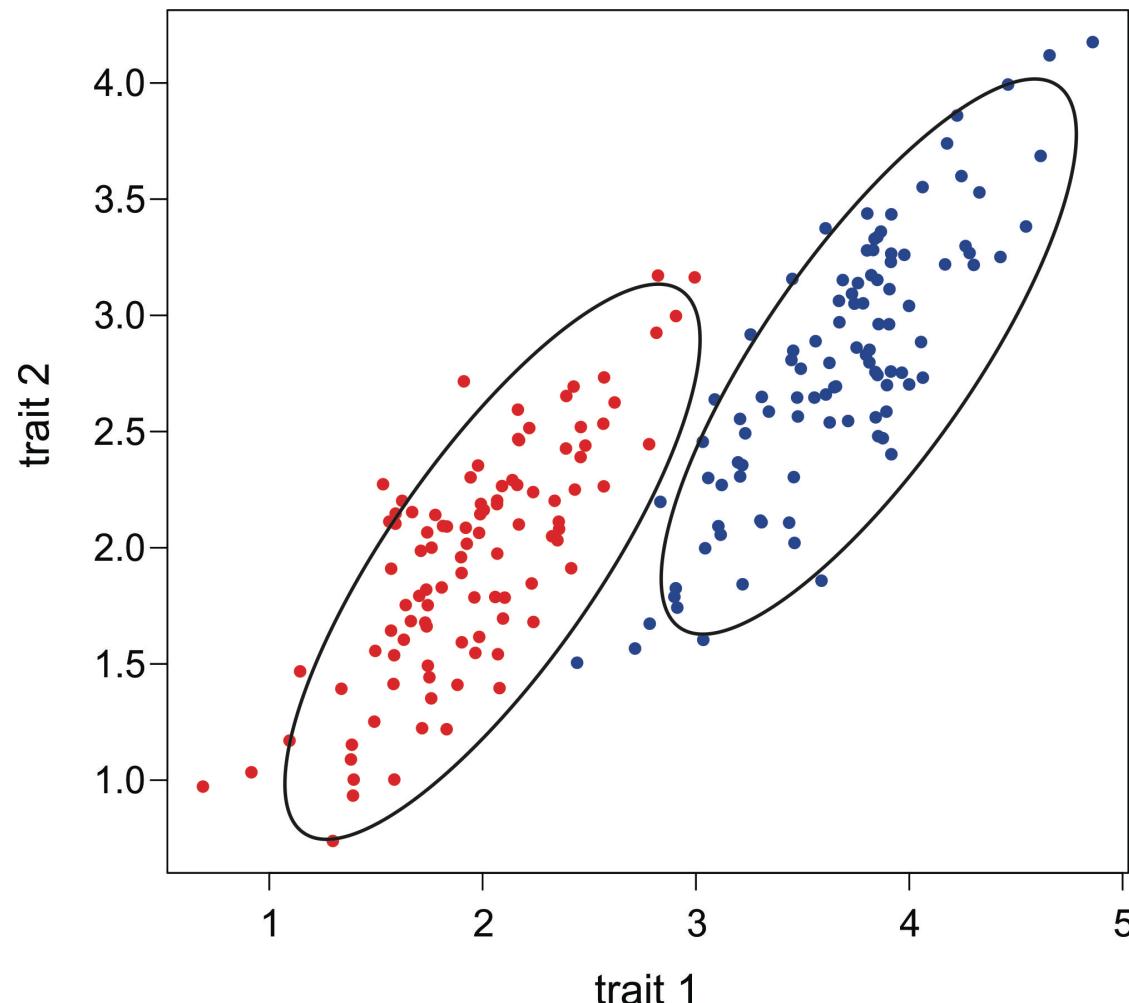
## Discriminant function analysis

In contrast, principal component analysis ignores the groups, and only finds the directions of maximum total variance,



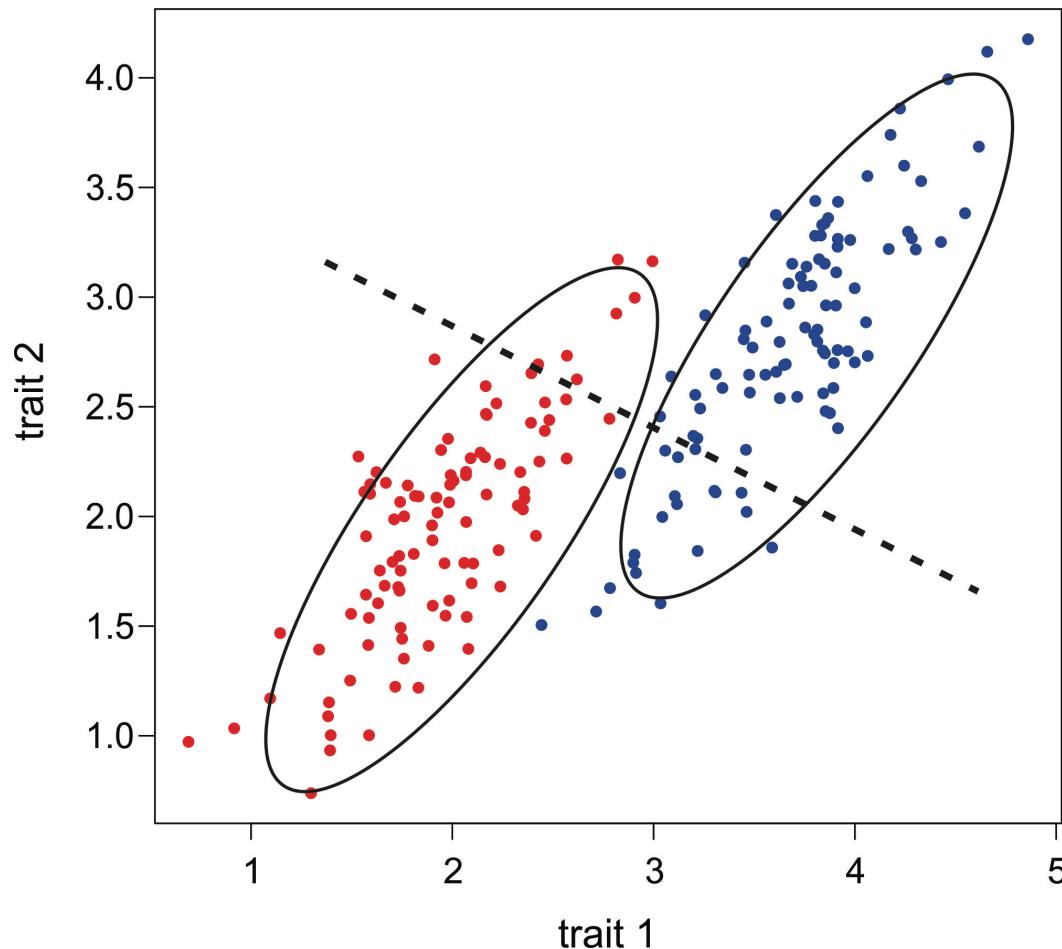
## Discriminant function analysis

Discriminant function analysis explicitly finds the axes that best separate the groups. It doesn't do this simply by finding the direction of the biggest difference between means.



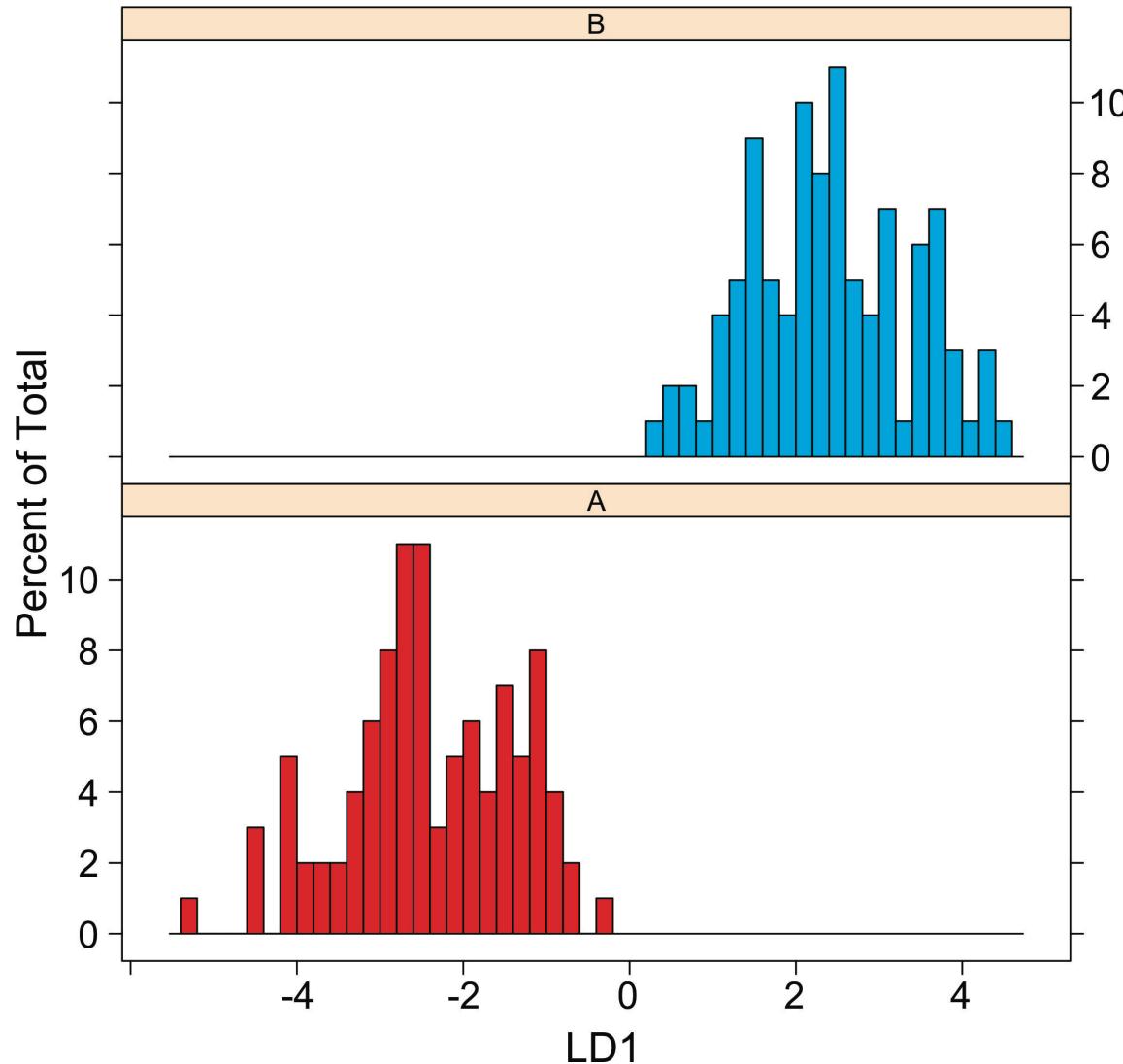
## Discriminant function analysis

Instead, it finds the axes that best separate groups relative to within-group variation.



## Discriminant function analysis

The resulting axes best separate the groups in the data, and can be used for classification of new observations.



## Correspondence Analysis

An ordination method to visualize spatial associations between species based on their presence/absence (or abundance) at sites. Equivalently, the method visualizes differences among sites in the extent to which they share the same species.

The method is like principal component analysis.

Instead of using a covariance matrix describing association between pairs of traits (as in PCA), correspondence analysis uses a contingency table to measure association between all pairs of species based on their frequency of co-occurrence (measured relative to the co-occurrence expected under independence).

Equivalently, it measured association between all pairs of sites based on the number of shared species (relative to the random expectation).

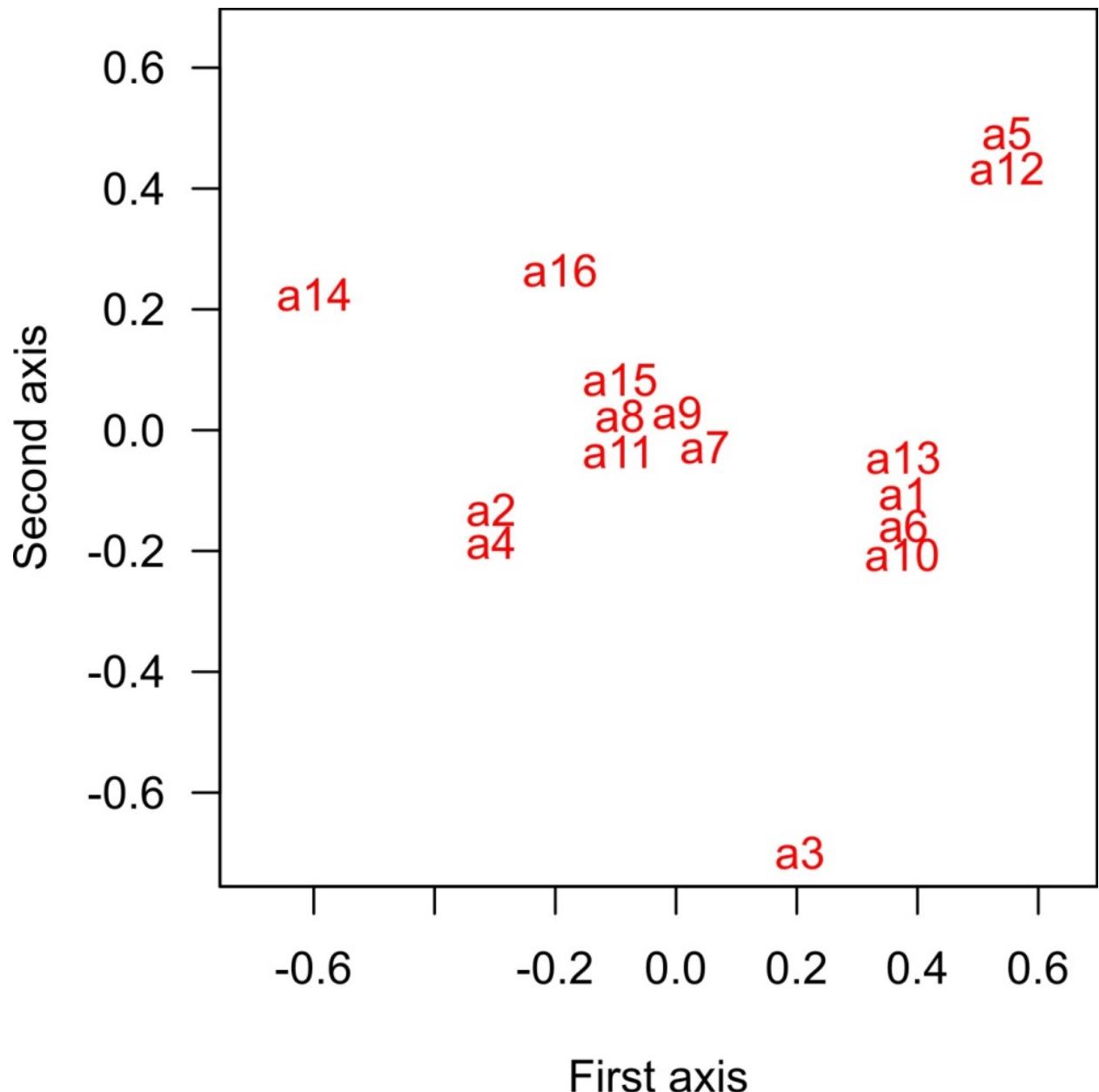
## Correspondence Analysis

Example: Presence/absence of 16 ant species in 4 geographic regions (Gotelli and Ellison 2004). In this matrix, the sites are the spatial units, and the species are the variables. The data are 1 or 0 (a species is present or absent in a site).

		VARIABLES (ant species)															
Site		a1	a2	a3	a4	a5	a6	a7	a8	a9	a10	a11	a12	a13	a14	a15	a16
UNITS	1. CT	0	1	0	1	0	0	0	1	1	0	1	0	0	0	1	1
	2. MA.mainland	1	1	1	1	0	1	1	1	1	1	1	0	1	0	1	0
	3. MA.islands	1	0	0	0	1	1	1	1	1	1	1	1	0	1	1	
	4. VT	0	1	0	1	0	0	1	1	1	0	1	0	0	1	1	

## Correspondence Analysis

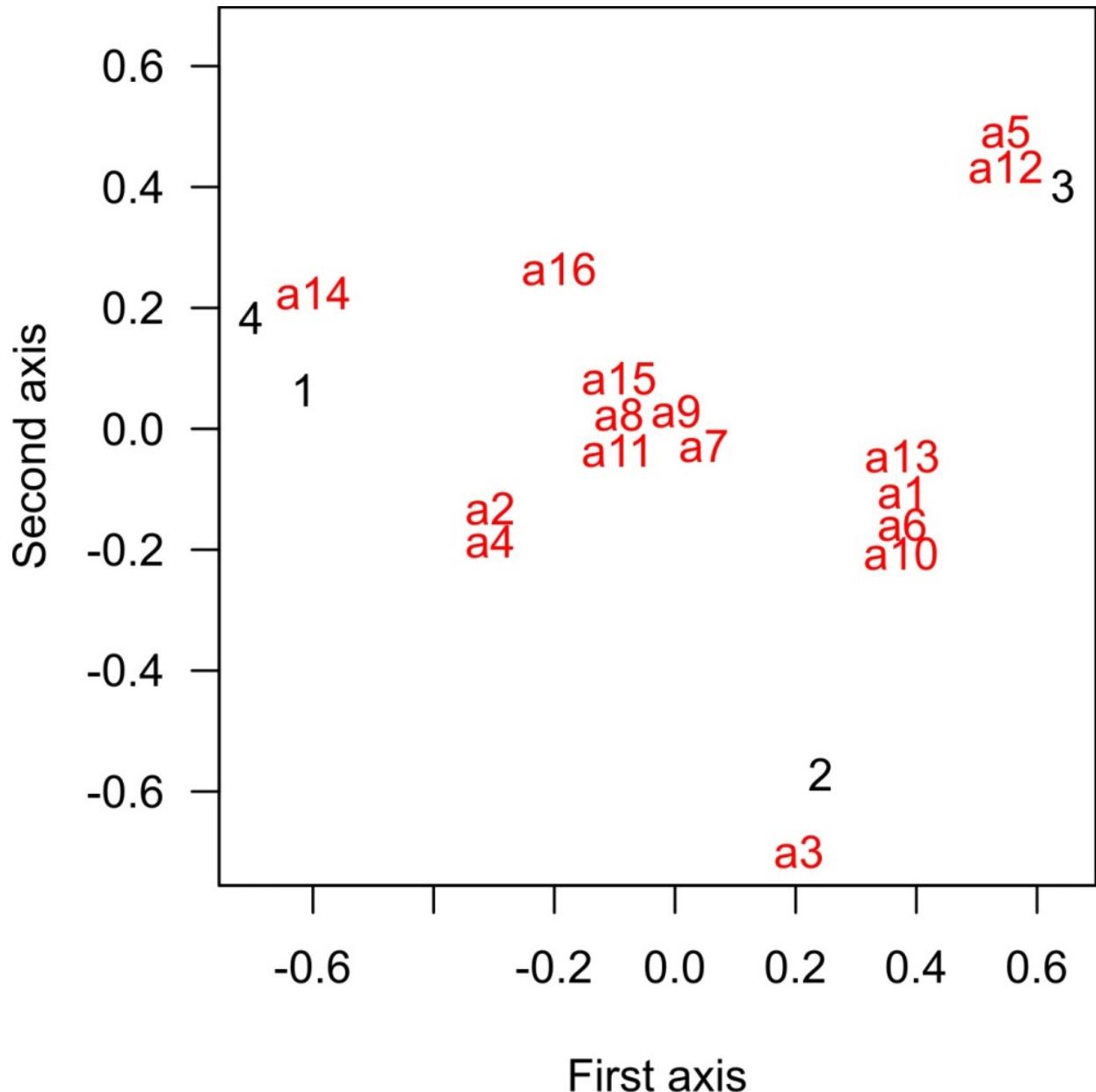
Applied to the ants,  
correspondence analysis  
produces a plot in which points  
close together on the axes  
indicate species that often co-  
occur at sites. Points far apart  
indicate species that occur  
infrequently at the same sites.



## Correspondence Analysis

Likewise, the sites can be compared by the observed and expected numbers of species shared. The method can thus ordinate sites and species simultaneously on the same axes.

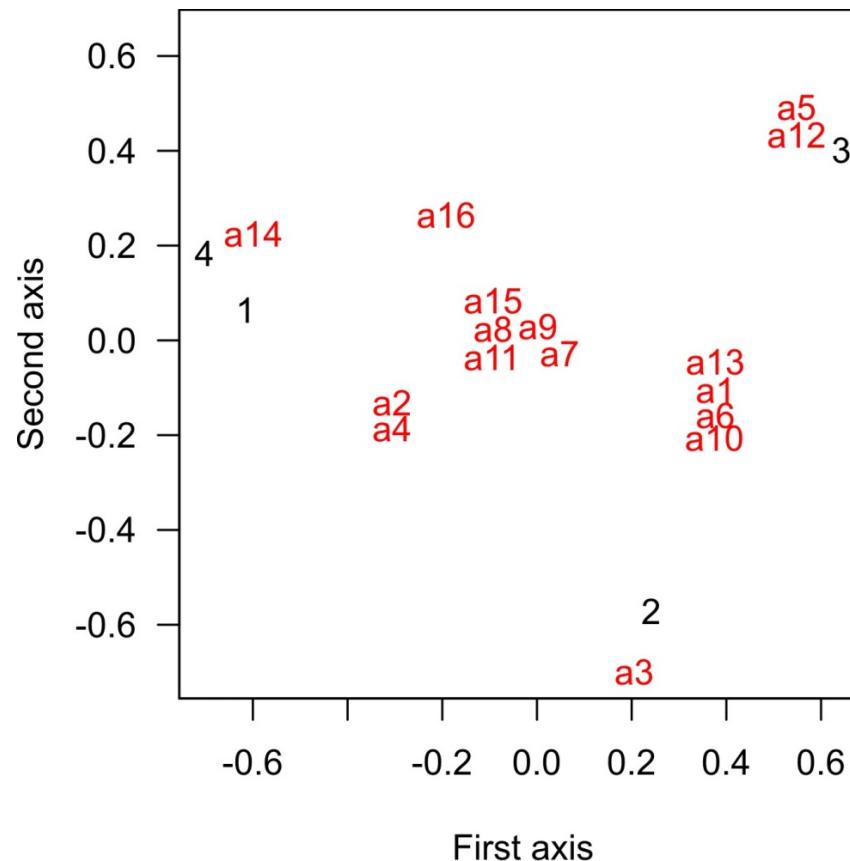
Species next to sites in the plot indicate species that occur predominantly there, whereas species falling between site points are shared among sites.



## Correspondence Analysis

By rearranging the site and ant species by their order along the first axis, we can see the “correspondence” between sites and species.

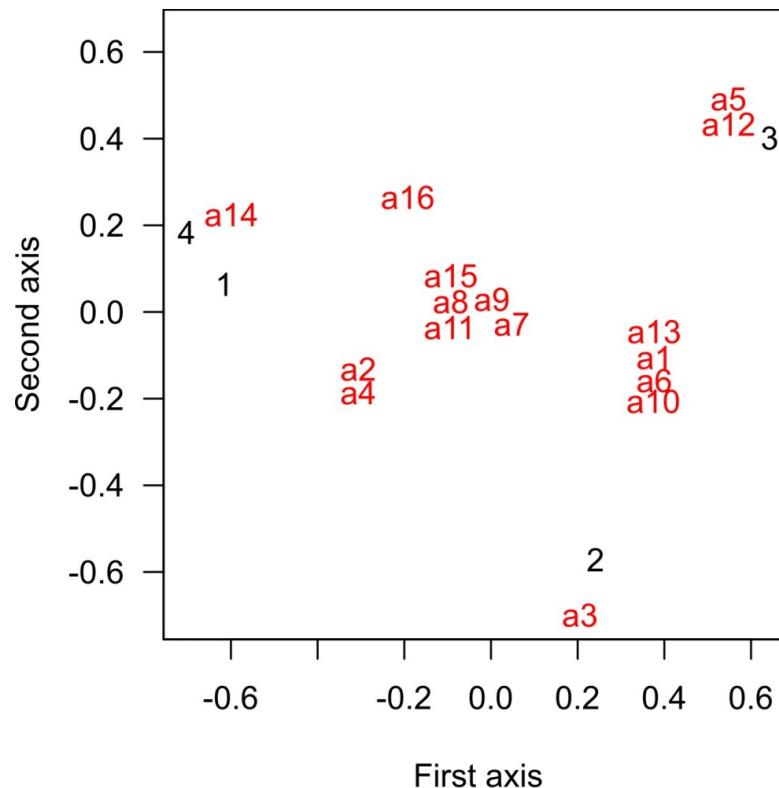
Site	a14	a2	a4	a16	a15	a8	a11	a9	a7	a3	a1	a13	a6	a10	a5	a12
UNITS	4. VT	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
	1. CT	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0
	2. MA.mainland	0	1	1	0	1	1	1	1	1	1	1	1	1	0	0
	3. MA.islands	0	0	0	1	1	1	1	1	0	1	1	1	1	1	1



## Correspondence Analysis

This first axis behaves like an ecological gradient, with species found at all sites clustered in the middle, and species with non-overlapping distributions at either end.

Site	a14	a2	a4	a16	a15	a8	a11	a9	a7	a3	a1	a13	a6	a10	a5	a12
UNITS	4. VT	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
	1. CT	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0
	2. MA.mainland	0	1	1	0	1	1	1	1	1	1	1	1	1	0	0
	3. MA.islands	0	0	0	1	1	1	1	1	0	1	1	1	1	1	1



## Correspondence Analysis

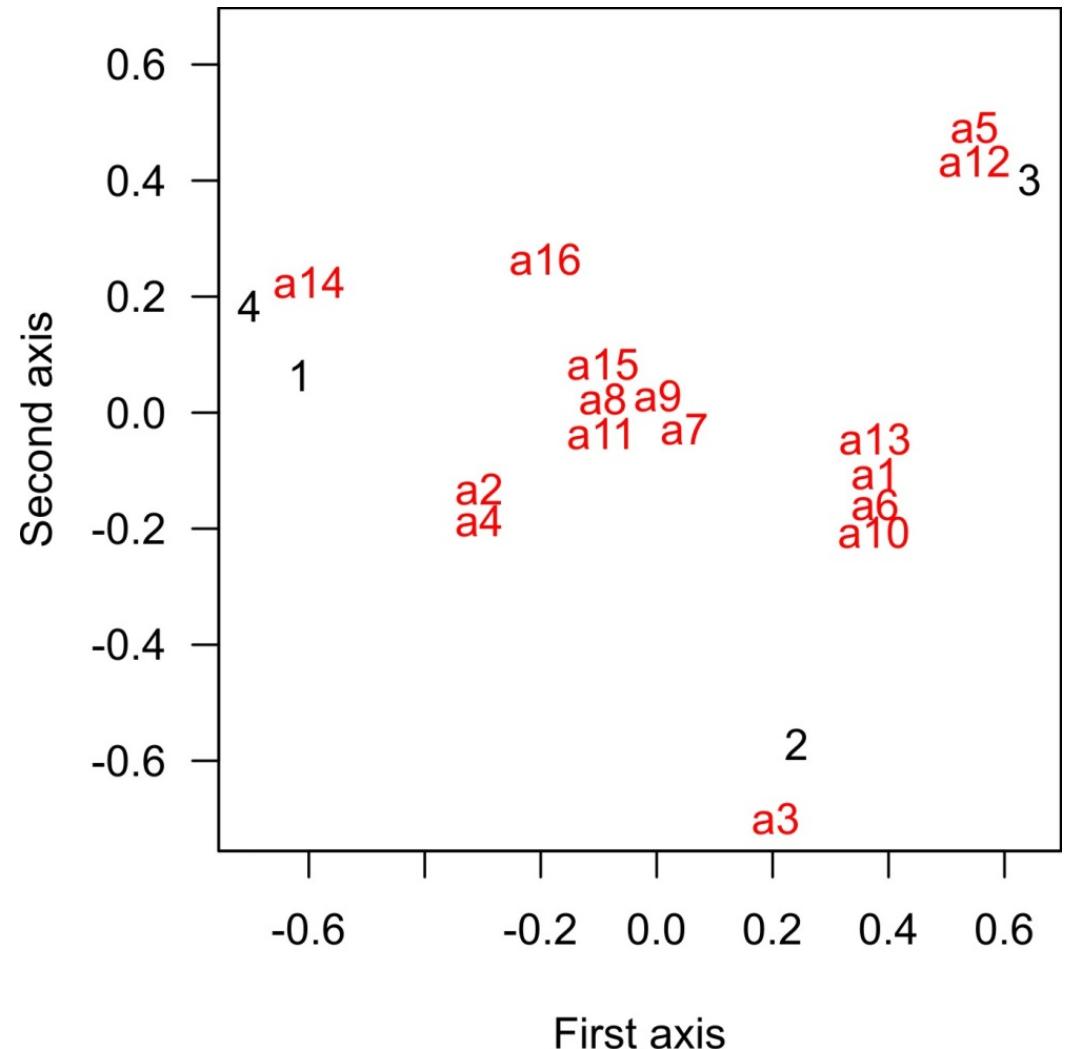
The first axis explains most of the variation in the data. It also maximizes the association between units (sites, rows) and variables (species, columns).

Eigenvalues for each axis indicate the correlation between species and site scores.

For the ant data:

Axis 1: 0.56

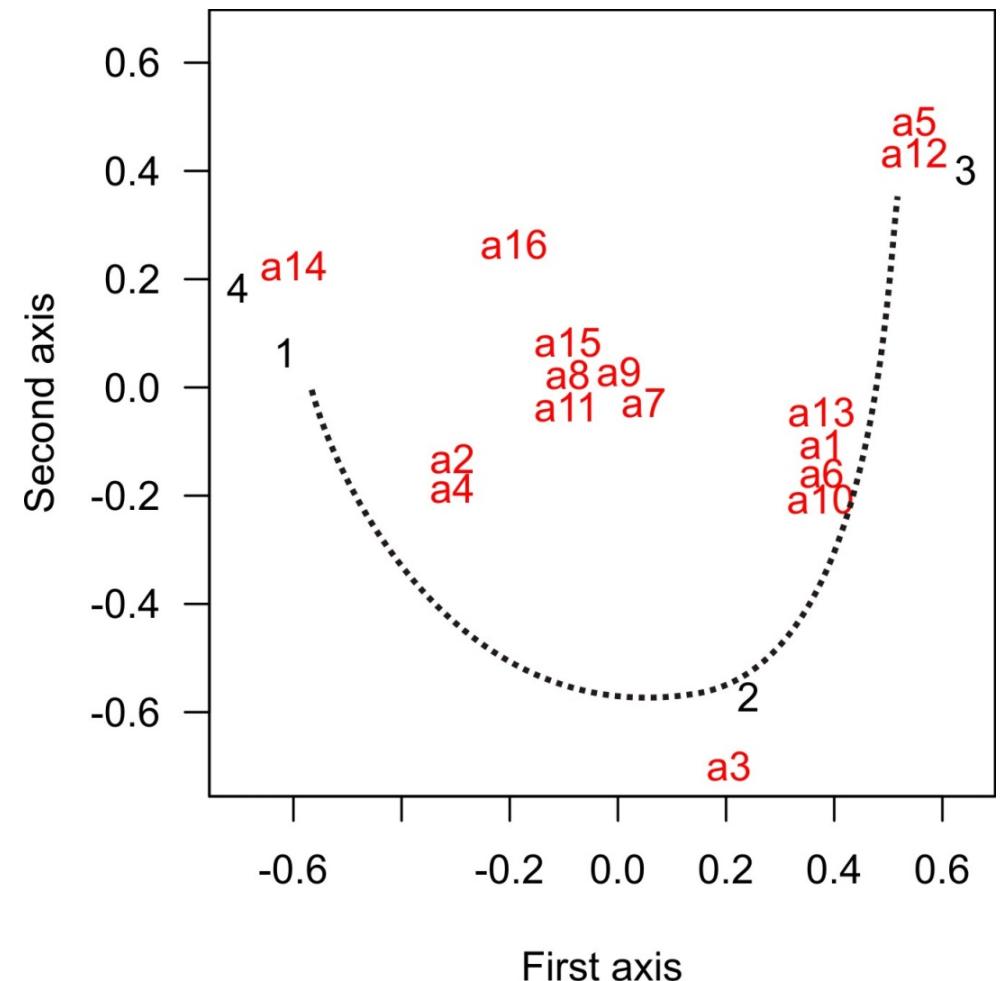
Axis 2: 0.39



## Correspondence Analysis

A worry with CA in general is the “arch” or, more seriously, a “horseshoe” in the ordination. When sites at the ends of ecological gradients have few species in common, they may be arranged such that they appear “similar” on axis 2 only because they are missing some of the same species.

It arises because the measure of distance between assemblages is not linear and doesn't increase with increasing “ecological distance”. The problem is most acute when spatial turnover of species (beta diversity) is high.



## Distance data

Can be analyzed similarly.

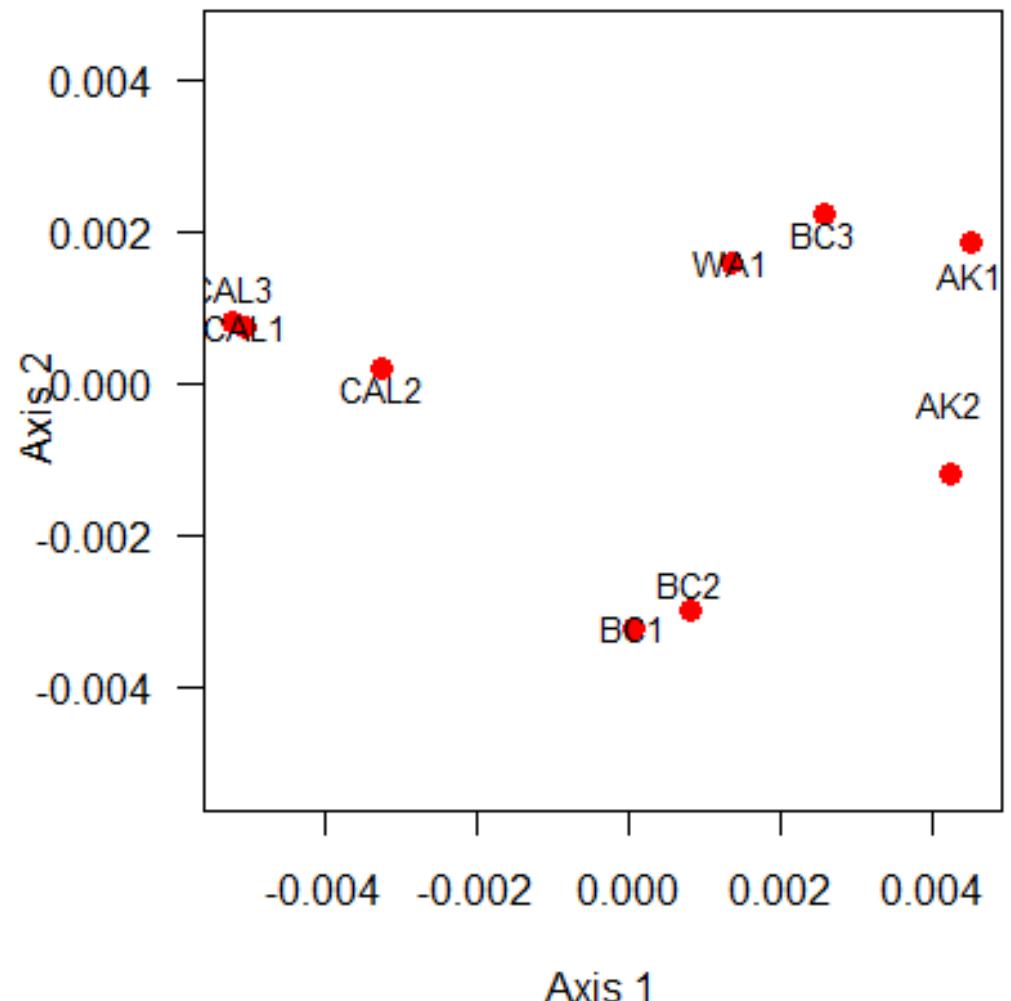
Example: Percent sequence divergence between Pacific Ocean basin freshwater stickleback populations in over 1000 bp of the Eda gene.

	BC1	BC2	WA1	CAL1	AK1	AK2	BC3	CAL2	CAL3
BC1	0.00000	0.00076	0.00454	0.00683	0.00680	0.00454	0.00605	0.00531	0.00681
BC2	0.00076	0.00000	0.00530	0.00758	0.00605	0.00530	0.00529	0.00607	0.00757
WA1	0.00454	0.00530	0.00000	0.00683	0.00302	0.00378	0.00151	0.00530	0.00681
CAL1	0.00683	0.00758	0.00683	0.00000	0.00988	0.00988	0.00834	0.00531	0.00379
AK1	0.00680	0.00605	0.00302	0.00988	0.00000	0.00302	0.00151	0.00833	0.00983
AK2	0.00454	0.00530	0.00378	0.00988	0.00302	0.00000	0.00530	0.00835	0.00985
BC3	0.00605	0.00529	0.00151	0.00834	0.00151	0.00530	0.00000	0.00682	0.00832
CAL2	0.00531	0.00607	0.00530	0.00531	0.00833	0.00835	0.00682	0.00000	0.00378
CAL3	0.00681	0.00757	0.00681	0.00379	0.00983	0.00985	0.00832	0.00378	0.00000

## Distance data

Various forms of “multidimensional scaling” use distance measurements. The overall goal is to produce a plot in which data points that are similar are close to one another in the plot, whereas points apart from one another are more similar.

Distances may not be preserved, eg in non-metric scaling only the rank order of distances between points is maintained.



**Discussion paper next week:**

Jetz et al. Conservation of phylogenetic distinctness in birds

Presenters: Tianyi and Tor

Moderators: