Sub: Task 3

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(i) $f(z) = \log_e(1+z)$

where,

$$z = x^T x$$

$$= [x_1, x_2, \dots x_d] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$$

$$= [x_1^2 + x_2^2 \dots x_d^2]$$

now,

$$\frac{df}{dx} = \frac{d}{dx} \log_e(1+z)$$

$$= \frac{1}{1+z} \frac{d}{dx}(1+z)$$

$$= \frac{1}{1+z} \frac{d}{dx}(z)$$

$$= \frac{1}{1+z} \frac{d}{dx}[x_1^2 + x_2^2 + \dots + x_d^2]$$

$$= \frac{1}{1+z} \frac{d}{dx} \sum_{i=1}^{d} x_i^2$$

$$= \frac{1}{1+z} \cdot 2 \cdot \sum_{i=1}^{d} x_i$$

$$= \frac{2}{1+x^T x} \sum_{i=1}^{d} x_i \text{ (Ans)}.$$

(ii) $f(z) = e^{-\frac{z}{2}}$

Using the chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \times \frac{dz}{dy} \times \frac{dx}{dy}$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dz}\left(e^{-\frac{z}{2}}\right) \cdot \frac{d}{dy}\left(y^T s^{-1} y\right) \cdot \frac{d}{dx}(x-\mu)$$

Computing the derivatives,

$$\frac{d}{dz}\left(e^{-\frac{z}{2}}\right) = -\frac{1}{2} e^{-\frac{z}{2}}$$

$$\frac{d}{dx}(x-\mu) = 1$$

$$\frac{d}{dy}\left(y^T s^{-1} y\right) = \lim_{h \to 0} \frac{g(y+h) - g(y)}{h}$$

$$= \lim_{h \to 0} \frac{(y^T + h) s^{-1} (y+h) - y^T s^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{y^T s^{-1} h + h s^{-1} y + s^{-1} h^2}{h}$$

$$= \lim_{h \to 0} \frac{h\left(y^T s^{-1} + s^{-1} y + s^{-1} h\right)}{h}$$

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$$= \lim_{h \to 0} \left(y^T S^{-1} + S^{-1} y + S^{-1} h \right)$$

$$= y^T S^{-1} + S^{-1} y$$

So,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= -\frac{1}{2} e^{-\frac{z}{2}} \cdot \left(y^T S^{-1} + S^{-1} y \right)$$

$$= -\frac{e^{-\frac{z}{2}}}{2} \left(y^T S^{-1} + S^{-1} y \right) \text{ (Ans).}$$