

Sub: Task 3

Day

Time:

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(i) $f(z) = \log_e(1+z)$

where,

$$z = x^T x$$

$$= [x_1, x_2, \dots, x_d] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$$

$$= [x_1^2 + x_2^2 + \dots + x_d^2]$$

now,

$$\frac{df}{dx} = \frac{d}{dx} \log_e(1+z)$$

$$= \frac{1}{1+z} \frac{d}{dx} (1+z)$$

$$= \frac{1}{1+z} \frac{d}{dx} (z)$$

$$= \frac{1}{1+z} \frac{d}{dx} [x_1^2 + x_2^2 + \dots + x_d^2]$$

$$= \frac{1}{1+z} \frac{d}{dx} \sum_{i=1}^d x_i^2$$

$$= \frac{1}{1+z} \cdot 2 \cdot \sum_{i=1}^d x_i$$

$$= \frac{2}{1+x^T x} \sum_{i=1}^d x_i \quad (\text{Ans}).$$

$$(ii) f(z) = e^{-\frac{z}{2}} \quad (1/2 + 0i + 1/2 i) \quad \text{real} = 1/2$$

Using the chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \times \frac{dz}{dy} \times \frac{dy}{dx}$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dz} (e^{-\frac{z}{2}}) \cdot \frac{d}{dy} (y^T S^{-1} y) \cdot \frac{d}{dx} (x - \mu)$$

Computing the derivatives,

$$\frac{d}{dz} (e^{-\frac{z}{2}}) = -\frac{1}{2} e^{-\frac{z}{2}}$$

$$\frac{d}{dx} (x - \mu) = 1$$

$$\begin{aligned} \frac{d}{dy} (y^T S^{-1} y) &= \lim_{h \rightarrow 0} \frac{g(y+h) - g(y)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(y^T + h^T) S^{-1} (y+h) - y^T S^{-1} y}{h} \\ &= \lim_{h \rightarrow 0} \frac{y^T S^{-1} h + h^T S^{-1} y + h^T S^{-1} h}{h} \\ &= \lim_{h \rightarrow 0} \frac{h (y^T S^{-1} + S^{-1} y + S^{-1} h)}{h} \end{aligned}$$

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$$= \lim_{h \rightarrow 0} (y^T s^{-1} + s^{-1} y + s^{-1} h)$$

$$= y^T s^{-1} + s^{-1} y$$

above marks are

$$\frac{x b}{y b} \times \frac{s b}{y b} \times \frac{z b}{s b} =$$

So,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx} =$$

$$= -\frac{1}{2} e^{-\frac{z}{2}} \cdot (y^T s^{-1} + s^{-1} y) +$$

$$= -\frac{e^{-\frac{z}{2}}}{2} (y^T s^{-1} + s^{-1} y) \quad (\text{Ans})$$

$$I = (u -$$

$$\frac{(b)g - (n+b)g}{n}$$

mil
 $0 \leftarrow n$

$$= (b' z^T$$

$$\frac{b' z^T b - (n+b)' z (n+b)}{n}$$

mil
 $0 \leftarrow n$

=

$$n' z + b' z n + n' z^T b$$

mil