Queens College, CUNY, Department of Computer Science

Numerical Methods
CSCI 361 / 761
Summer 2018

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Final Part 4

Due Monday August 13, 2018 at 11.59 pm

- **NOTE: It is the policy of the Computer Science Department to issue a failing grade to any student who either gives or receives help on any test.**
- **A student caught cheating on any question in an exam, project or quiz will fail the entire course.**
- **Any problem to which you give two or more (different) answers receives the grade of zero automatically.**
- *This is a take home exam. Answers should be typed in a file. See below for instructions.*
- Please submit your solution via email, as a file attachment, to `Sateesh.Mane@qc.cuny.edu`.
- Please submit one zip archive with all your files in it.
  1. The zip archive should have either of the names (CS361 or CS761):

     `StudentId_first_last_CS361_final_pt4_Aug2018.zip`
     `StudentId_first_last_CS761_final_pt4_Aug2018.zip`
  2. The archive should contain one "text file" named "Final_pt4.[txt/docx/pdf]" and one cpp file per question named "Q1.cpp" and "Q2.cpp" etc.
  3. Note that text answers may not be required for all questions.
  4. Note that not all questions may require a cpp file.
- **In all questions where you are asked to submit programming code, programs which display any of the following behaviors will receive an automatic F:**
  1. Programs which do not compile successfully (non-fatal compiler warnings are excluded).
  2. Array out of bounds, reading of uninitialized variables (including null pointers).
  3. Operations which yield NAN or infinity, e.g. divide by zero, square root of negative number, etc. *Infinite loops.*
  4. Programs which do NOT implement the public interface stated in the question.
- **In addition, note the following:**
  1. All debugging statements (for your personal testing) should be commented out.
  2. Program performance will be graded solely on the public interface stated in the questions.

General information

- **The statements below are for general information only.**
- Ignore them if they are not relevant for the exam questions below.
- The questions in this exam do not involve problems of overflow or underflow.
- Solutions involving the writing of algorithms will not be judged if they work on a 64–bit instead of a 32–bit computer.
- **Value of π to machine precision on any computer.**
  1. Some compilers support the constant `M_PI` for π, in which case you can write

     ```
     const double pi = M_PI;
     ```
  2. If your compiler does not support `M_PI`, the value of π can be computed via

     ```
     const double pi = 4.0*atan2(1.0,1.0);
     ```

3 Question 3

- Define parameter values α and β as follows.
  1. Take the first four digits of your student id and multiply by 10^{-4}.
  2. Take the last four digits of your student id and multiply by 10^{-4}.
  3. Then α and β are given as follows.

  $$\alpha = (\text{first four digits of id}) \times 10^{-4}\,, \qquad \beta = (\text{last four digits of id}) \times 10^{-4}\,.$$

  4. For example if your student id is 23054617, then $\alpha = 0.2305$ and $\beta = 0.4617$.
  5. **Solutions which employ $\alpha = 0.2305$ and $\beta = 0.4617$ below will score zero.**

- You are given the following equations, to solve for unknowns x_1, x_2 and x_3.

$$\alpha x_1 + x_2 + 2x_3 = -1\,, \tag{3.1}$$
$$\beta x_1 + 3x_2 + 4x_3 = \boldsymbol{r_2}\,, \tag{3.2}$$
$$\boldsymbol{\gamma} x_1 + 5x_2 + 6x_3 = -3\,, \tag{3.3}$$

- **Find the value of γ such that the solution of the equations encounters a zero pivot, which cannot be avoided by swapping, etc.**
  1. In other words, the equations are either inconsistent or not linearly independent.
  2. If there is more than one solution for γ, **any valid value is acceptable.**
  3. Denote your solution for γ by γ_0.
  4. **Calculate the value of γ_0 to 4 decimal places.**

- **Using the value γ_0, find the value of r_2 such that the equations are consistent but not linearly independent.**
  1. In other words, set $\gamma = \gamma_0$ and then find conditions to make the equations consistent but not linearly independent.
  2. **Calculate the value of r_2 to 4 decimal places.**
  3. If there is more than one solution for r_2, **any valid value is acceptable.**

- **Note: do NOT attempt to solve the resulting equations.**

4 Question 4

- You are given the following matrix

$$A = \begin{pmatrix} 1 & -2 & 1 & -2 \\ 1 & -2 & 1 & 0 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} . \tag{4.1}$$

- The initial array of the swap indices is $S = (1, 2, 3, 4)$.
- It is obvious to a human that this is a lower triangular matrix in disguise, but a computer does not know that.
- **Perform the LU decomposition for the matrix A in eq. (4.1).**
  1. Display the steps in your calculation.
  2. **Calculate the matrix elements of L and U to 4 decimal places.**
- **Write down the final array of the swap indices.**
- **Calculate the determinant of the matrix A.**
  Hint: You should be able to calculate the determinant without LU decomposition.
- **Find the matrix X which is the solution of the following equation:**

$$AX = \tfrac{1}{2}(A + A^T) . \tag{4.2}$$

  1. **To answer this part of the question, you are permitted to use the functions displayed in the online lectures, for LU decomposition and backsubstitution.**
  2. You do **NOT** need to display all the backsubstitution steps.
  3. Just state the answer.
  4. **Calculate the matrix elements of X to 4 decimal places.**

5 Question 5

- Let μ be a real number and let T be the following tridiagonal matrix:

$$T = \begin{pmatrix} 3+\mu^2 & 1+\mu & 0 & 0 & 0 \\ 1+\mu & 3+\mu^2 & 1+\mu & 0 & 0 \\ 0 & 1+\mu & 3+\mu^2 & 1+\mu & 0 \\ 0 & 0 & 1+\mu & 3+\mu^2 & 1+\mu \\ 0 & 0 & 0 & 1+\mu & 3+\mu^2 \end{pmatrix} \tag{5.1}$$

- **Find the values of μ such that the matrix T in eq. (5.1) is strongly diagonally dominant.**

- **Find the values of μ such that the matrix T in eq. (5.1) is weakly diagonally dominant.**

- **Find the values of μ such that the matrix T in eq. (5.1) is not diagonally dominant.**

  1. You will need to consider the cases $\mu \geq -1$ and $\mu < -1$ separately.
  2. If $\mu \geq -1$ then $|1+\mu| = 1+\mu$.
  3. If $\mu < -1$ then $|1+\mu| = -(1+\mu)$.
  4. **Remember to pay attention to the special cases in the first and last rows.**

- **Solve the following matrix equation for the unknowns x_1, x_2, x_3, x_4, x_5, for $\mu = 0$:**

$$T_{(\mu=0)} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} . \tag{5.2}$$

  1. **To answer this part of the question, you are permitted to use the functions displayed in the online lectures, for tridiagonal matrices.**
  2. You do **NOT** need to display all the forward elimination and backsubstitution steps.
  3. **Calculate the values of x_1, x_2, x_3, x_4, x_5 to 4 decimal places.**