Queens College, CUNY, Department of Computer Science

Computational Finance

CSCI 365 / 765

Fall 2017

Instructor: Dr. Sateesh Mane

March 27, 2018

7 Lecture 7b

Rational option pricing: Worked examples

- This lecture contains worked examples of arbitrage strategies to derive some of the rational option pricing inequalities in Lecture 7.
- The relevant concept here is the **intrinsic value** of an option.

7.1 Intrinsic value

- Consider options on a stock S. It does not matter if the stock pays dividends.
- The option strike is K.
- The **intrinsic value** of a call option (American or European) is defined as

$$\text{intrinsic value (call option)} \quad = \quad \max(S-K,0) = \begin{cases} 0 & S < K\,, \\ S-K & S \geq K\,. \end{cases} \tag{7.1.1}$$

- The intrinsic value of a put option (American or European) is defined as

$$\text{intrinsic value (put option)} \quad = \quad \max(K-S,0) = \begin{cases} K-S & S \leq K\,, \\ 0 & S > K\,. \end{cases} \tag{7.1.2}$$

- For an American option (put or call), the intrinsic value is what the option would be worth if it were exercised immediately.
- In other words, the intrinsic value is the payoff of an American option if the option were exercised immediately.
- European options can only be exercised at expiration, hence for European options the intrinsic value is a mathematical or artificial definition.
- On the expiration date, the value of an option (American or European) equals its intrinsic value (which culd be zero).

7.2 Rational option pricing

- An American option must always be worth $\geq$ its intrinsic value.
- Let the stock price be S and the strike price be K.
- Let the value of an American call and put option be C and P, respectively.
- Then we must have

$$C \geq \max(S-K,0)\,, \qquad P \geq \max(K-S,0)\,. \tag{7.2.1}$$

- The proof is easy. If an American option is trading at less than its intrinsic value, we buy it and exercise it immediately. This immediately yields an arbitrage profit.
- The worked examples below will demonstrate the idea.

7.3 Arbitrage

7.3.1 American call option

- An American call option has a strike of 100.
- The stock price is 105 today.
- The market price of the option is 4.5.
- **Formulate an arbitrage strategy to make a risk-free profit using this option.**
- **Solution**
  1. **Buy the option.**
  2. This costs cash $= 4.5$, which we must borrow.
  3. **Exercise the option immediately.**
  4. We must pay the strike price $= 100$, and we must also borrow the money to do so.
  5. Our total borrowing is therefore 104.5 $(= C + K)$.
  6. We receive one share of stock, whose market price is 105.
  7. **Sell the stock immediately.**
  8. The sale of the stock give us cash of 105.
  9. We immediately repay our loan of 104.5.
  10. There is no interest because we repay immediately.
  11. Hence we start with zero and we end with a risk free profit of 0.5.
- Therefore this is an arbitrage trade.
- Our total profit is given by

$$\text{profit} = S - K - C\,. \tag{7.3.1}$$

7.3.2 American put option

- An American put option has a strike of 100.
- The stock price is 96 today.
- The market price of the option is 3.25.
- **Formulate an arbitrage strategy to make a risk-free profit using this option.**
- **Solution**
  1. **Buy the option.**
  2. This costs cash = 3.25, which we must borrow.
  3. **Exercise the option immediately.**
  4. Because this is a put, we must deliver one share of the stock.
  5. We buy one share of stock, at the price 96. We must borrow money to do this.
  6. Our total borrowing is therefore 99.25 $(= P + S)$.
  7. We receive the strike price $K = 100$ in cash by exercising the put.
  8. We immediately repay our loan of 99.25.
  9. There is no interest because we repay immediately.
  10. Hence we start with zero and we end with a risk free profit of 0.75.
- Therefore this is an arbitrage trade.
- Our total profit is given by

$$\text{profit} = K - S - P \,. \tag{7.3.2}$$

7.3.3 Cash settled American call option

- Suppose the option is cash settled.
- Let us use the same example as before, but instead of a stock, the underlying is a stock index.
- The dollar multiplier per index point is M.
- An American call option has a strike of 100 index points.
- The stock index is 105 index points today.
- The market price of the option is 4.5 index points.
- **Formulate an arbitrage strategy to make a risk-free profit using this option.**
- **Solution**
  1. **Buy the option.**
  2. This costs cash $= 4.5M$, which we must borrow.
  3. **Exercise the option immediately.**
  4. The option is cash settled, hence we receive $(105 - 100)M = 5M$ cash.
  5. We immediately repay our loan of $4.5M$.
  6. There is no interest because we repay immediately.
  7. Hence we start with zero and we end with a risk free profit of $0.5M$.
- Therefore this is an arbitrage trade.
- Our total profit is given by

$$\text{profit} = (S - K - C)M\,. \tag{7.3.3}$$

7.3.4 Cash settled American put option

- Once again, we employ the same example as above, but replace the stock by a stock index.
- The dollar multiplier per index point is M.
- An American put option has a strike of 100 index points.
- The stock index is 96 index points today.
- The market price of the option is 3.25 index points.
- **Formulate an arbitrage strategy to make a risk-free profit using this option.**
- **Solution**
  1. **Buy the option.**
  2. This costs cash $= 3.25M$, which we must borrow.
  3. **Exercise the option immediately.**
  4. Because the option is cash settled, we receive $(100 - 96)M = 4M$ cash.
  5. We immediately repay our loan of $3.25M$.
  6. There is no interest because we repay immediately.
  7. Hence we start with zero and we end with a risk free profit of $0.75M$.
- Therefore this is an arbitrage trade.
- Our total profit is given by

$$\text{profit} = (K - S - P)M \,. \tag{7.3.4}$$

7.4 European options

- It is possible for a European option to trade below its intrinsic value.
- A European option can only be exercised at expiration, hence unlike the case for American options, one cannot formulate a set of trades which all happen simultaneously, to formulate an arbitrage strategy.
- If the interest rate is positive (which is almost always true), then a European put option which is sufficiently deep in the money will trade below its intrinsic value.