Queens College, CUNY, Department of Computer Science

Numerical Methods

CSCI 361 / 761

Spring 2018

Instructor: Dr. Sateesh Mane

due Friday May 4, 2018, 11.59 pm

21 Homework lecture 21

- **As experience has demonstrated, if you do not understand the above expressions/questions, THEN ASK.**
- **If you do not understand the words/sentences in the lectures, THEN ASK.**
- **Send me an email, explain what you do not understand.**
- **Do not just keep quiet and produce nonsense in exams.**

21.1 Linear and nonlinear ordinary differential equations

- **State which of the following differential are linear.**
- **If the equation is linear, state if it is homogeneous.**

$$\frac{dy}{dx} + y = 0\,. \tag{21.1.1}$$

$$\frac{dy}{dx} + y = 1\,. \tag{21.1.2}$$

$$x^2\,\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} + (x^2-1)y = 0\,. \tag{21.1.3}$$

$$(1-x^2)\,\frac{d^2y}{dx^2} - 2x\,\frac{dy}{dx} + \left[6 - \frac{1}{1-x^2}\right] y = 0\,. \tag{21.1.4}$$

$$\frac{d^2y}{dx^2}\frac{dy}{dx} + y = 1\,. \tag{21.1.5}$$

$$3x^2\left(\frac{dy}{dx}\right)^3 + 2x\left(\frac{dy}{dx}\right)^2 - y^4 = 0\,. \tag{21.1.6}$$

$$3x^2\frac{d^3y}{dx^3} + 2x\frac{d^2y}{dx^2} - \frac{dy}{dx} = e^x\,. \tag{21.1.7}$$

21.2 Linear ordinary differential equations

21.2.1 Homogeneous equation

- You are given the differential equation

$$\frac{d^2y}{dx^2} + 4\,\frac{dy}{dx} + 3y = 0\,. \tag{21.2.1}$$

- **Use Euler substitutions to find the general solution.**
- **Calculate the solution if $y = 1$ and $dy/dx = 0$ at $x = 0$.**

21.2.2 Inhomogeneous equation

- You are given the differential equation

$$\frac{d^2y}{dx^2} + 4\,\frac{dy}{dx} + 3y = 1 + x\,. \tag{21.2.2}$$

- **Find the particular solution.**
- **Write down the general solution.**

21.2.3 Inhomogeneous equation

- You are given the differential equation

$$\frac{d^2y}{dx^2} + 4\,\frac{dy}{dx} + 3y = e^x + 2e^{-2x}\,. \tag{21.2.3}$$

- **Find the particular solution.**
- **Write down the general solution.**

21.3 Linear ordinary differential equations

21.3.1 Homogeneous equation

- You are given the differential equation

$$x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = 0\,. \tag{21.3.1}$$

- **Use a power law substitution to find the general solution.**
- **Calculate the solution if $y = 1$ and $dy/dx = 0$ at $x = 1$.**

21.3.2 Inhomogeneous equation

- You are given the differential equation

$$x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = 1 + 2x + 3x^2\,. \tag{21.3.2}$$

- **Find a particular solution by trying $y = k_0 + k_1 x + k_2 x^2$.**
- **Write down the general solution.**

21.4 Linear ordinary differential equations

21.4.1

- You are given the differential equation

$$\frac{dy}{dx} + y = 1\,. \tag{21.4.1}$$

- **Solve the equation using an integrating factor, where $y = 0$ at $x = 0$.**

21.4.2

- You are given the differential equation

$$\frac{dy}{dx} + \frac{y}{x} = 1\,. \tag{21.4.2}$$

- **Use $x_0 = 1$ and show that the integrating factor is $G(x) = x$.**
- **Show the following:**

$$\frac{d(xy)}{dx} = x\,. \tag{21.4.3}$$

- **Use $y = 1$ at $x_0 = 1$ and derive the following:**

$$xy - 1 = \frac{x^2 - 1}{2}\,. \tag{21.4.4}$$

- **Derive the solution for $y(x)$.**