Queens College, CUNY, Department of Computer Science

Computational Finance

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19 Lecture 19a

Binomial model: worked examples

- We display worked examples to calculate option fair values using the **binomial model.**

  1. American call
  2. American put
  3. European up and out barrier call
  4. American up and out barrier call
  5. European binary call
  6. European binary put

- **There is no explicit mathematical probability theory in this lecture.**

19.8 Parameter values and binomial tree

- All of the examples below employ the following parameter values:

$$S = 100\,, \tag{19.8.1a}$$
$$K = 100\,, \tag{19.8.1b}$$
$$r = 0.1\,, \tag{19.8.1c}$$
$$q_{\rm div} = 0.1\,, \tag{19.8.1d}$$
$$\sigma = 0.5\,, \tag{19.8.1e}$$
$$T = 0.4\,, \tag{19.8.1f}$$
$$t_0 = 0\,. \tag{19.8.1g}$$

- The binomial tree is constructed with $n = 4$ steps.
- Then the values of the relevant parameters are as follows:

$$\Delta T = \frac{T - t_0}{n} \qquad =0.1\,, \tag{19.8.2a}$$
$$\text{discount factor} = e^{-r\Delta t} \qquad \simeq 0.99005\,, \tag{19.8.2b}$$
$$\text{growth factor} = e^{(r-q_{\rm div})\Delta t} \qquad =1\,, \tag{19.8.2c}$$
$$u = e^{\sigma\sqrt{\Delta t}} \qquad \simeq 1.1713\,, \tag{19.8.2d}$$
$$d = \frac{1}{u} \qquad \simeq 0.853753\,, \tag{19.8.2e}$$
$$p = \frac{e^{(r-q_{\rm div})\Delta t} - d}{u - d} \qquad \simeq 0.460554\,, \tag{19.8.2f}$$
$$q = \frac{u - e^{(r-q_{\rm div})\Delta t}}{u - d} \qquad \simeq 0.539446\,. \tag{19.8.2g}$$

- The binomial tree and the stock prices at the various nodes are shown in Fig. 1.

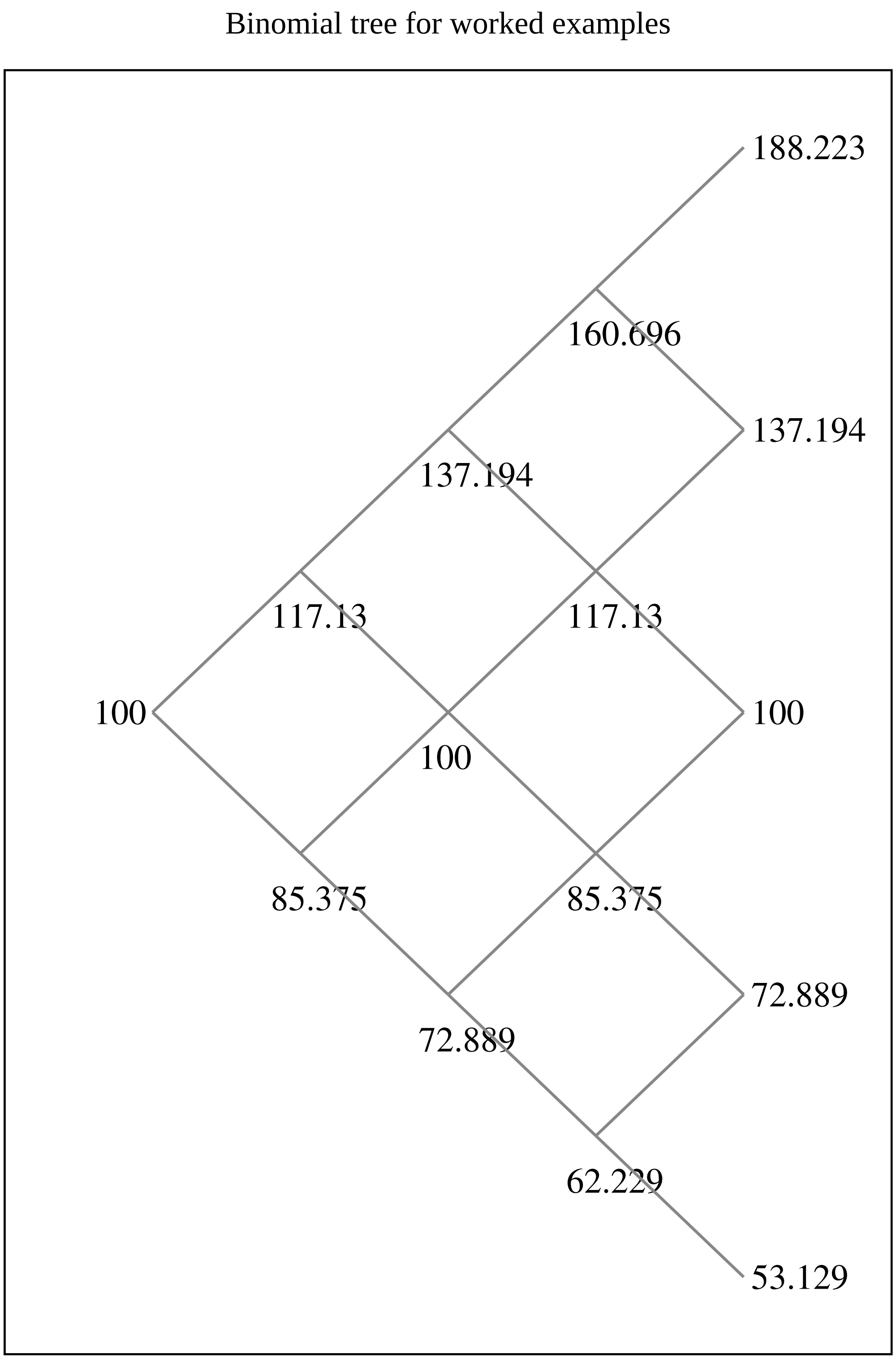

Figure 1: Binomial tree and stock prices for worked examples.

19.9 American call

- We calculate the fair value of an **American call.**
- The option valuation tree is shown in Fig. 2.

1. We begin by filling in the terminal payoff on the expiration date ($i = n$).
2. We then work backwards through the tree $i = n - 1, \ldots, 0$.
3. At every node at step i we calculate the discounted expected value from the previous step $i + 1$
$$V_{\text{disc exp}} = e^{-r\Delta t}(pV_u + qV_d)\,. \tag{19.9.1}$$
4. We then perform a valuation test (for early exercise). If the value of V is less than the intrinsic value at that node, we set V to the intrinsic value.
$$V = \max\{V_{\text{disc exp}}, \text{intrinsic value}\}\,. \tag{19.9.2}$$

- Step $i = n = 4$.

1. Expiration. The option fair values at the nodes are:
$$V(Su^4) \simeq 88.2227\,, \tag{19.9.3a}$$
$$V(Su^2) \simeq 37.1943\,, \tag{19.9.3b}$$
$$V(S) = 0\,, \tag{19.9.3c}$$
$$V(Sd^2) = 0\,, \tag{19.9.3d}$$
$$V(Sd^4) = 0\,. \tag{19.9.3e}$$
2. The above values are obtained from the terminal payoff formula $V = \max\{S_T - K, 0\}$.

- We work backwards through the tree:

  1. Step $i = 3$:

$$\begin{aligned}
V(Su^3)_{\text{disc exp}} &= e^{-r\Delta t}(p \times 88.2227 + q \times 37.1943) &&\simeq 60.0917\,, && \text{(19.9.4a)}\\
\boldsymbol{V(Su^3)} &= \mathbf{max\{60.0917, 60.6956\}} &&= \mathbf{60.6956}\,, && \text{(19.9.4b)}\\
V(Su)_{\text{disc exp}} &= e^{-r\Delta t}(p \times 37.1943 + q \times 0) &&\simeq 16.9595\,, && \text{(19.9.4c)}\\
\boldsymbol{V(Su)} &= \mathbf{max\{16.9595, 17.13\}} &&= \mathbf{17.13}\,, && \text{(19.9.4d)}\\
V(Sd) &= e^{-r\Delta t}(p \times 0 + q \times 0) &&= 0\,, && \text{(19.9.4e)}\\
V(Sd^3) &= e^{-r\Delta t}(p \times 0 + q \times 0) &&= 0\,. && \text{(19.9.4f)}
\end{aligned}$$

  2. Step $i = 2$:

$$\begin{aligned}
V(Su^2)_{\text{disc exp}} &= e^{-r\Delta t}(p \times 60.6956 + q \times 17.13) &&\simeq 36.8242\,, && \text{(19.9.5a)}\\
\boldsymbol{V(Su^2)} &= \mathbf{max\{36.8242, 37.1943\}} &&= \mathbf{37.1943}\,, && \text{(19.9.5b)}\\
V(S) &= e^{-r\Delta t}(p \times 17.13 + q \times 0) &&\simeq 7.81077\,, && \text{(19.9.5c)}\\
V(Sd^2) &= e^{-r\Delta t}(p \times 0 + q \times 0) &&= 0\,. && \text{(19.9.5d)}
\end{aligned}$$

  3. Step $i = 1$:

$$\begin{aligned}
V(Su) &= e^{-r\Delta t}(p \times 37.1943 + q \times 7.81077) &&\simeq 21.1311\,, && \text{(19.9.6a)}\\
V(Sd) &= e^{-r\Delta t}(p \times 7.81077 + q \times 0) &&\simeq 3.56148\,. && \text{(19.9.6b)}
\end{aligned}$$

  4. Step $i = 0$:

$$V(S) = e^{-r\Delta t}(p \times 21.1311 + q \times 3.56148) \simeq 11.5373\,. \tag{19.9.7}$$

- Hence the American call fair value is, using a binomial model with $n = 4$,

$$V_{\text{Am call}} \simeq 11.5373\,. \tag{19.9.8}$$

Binomial tree for American call

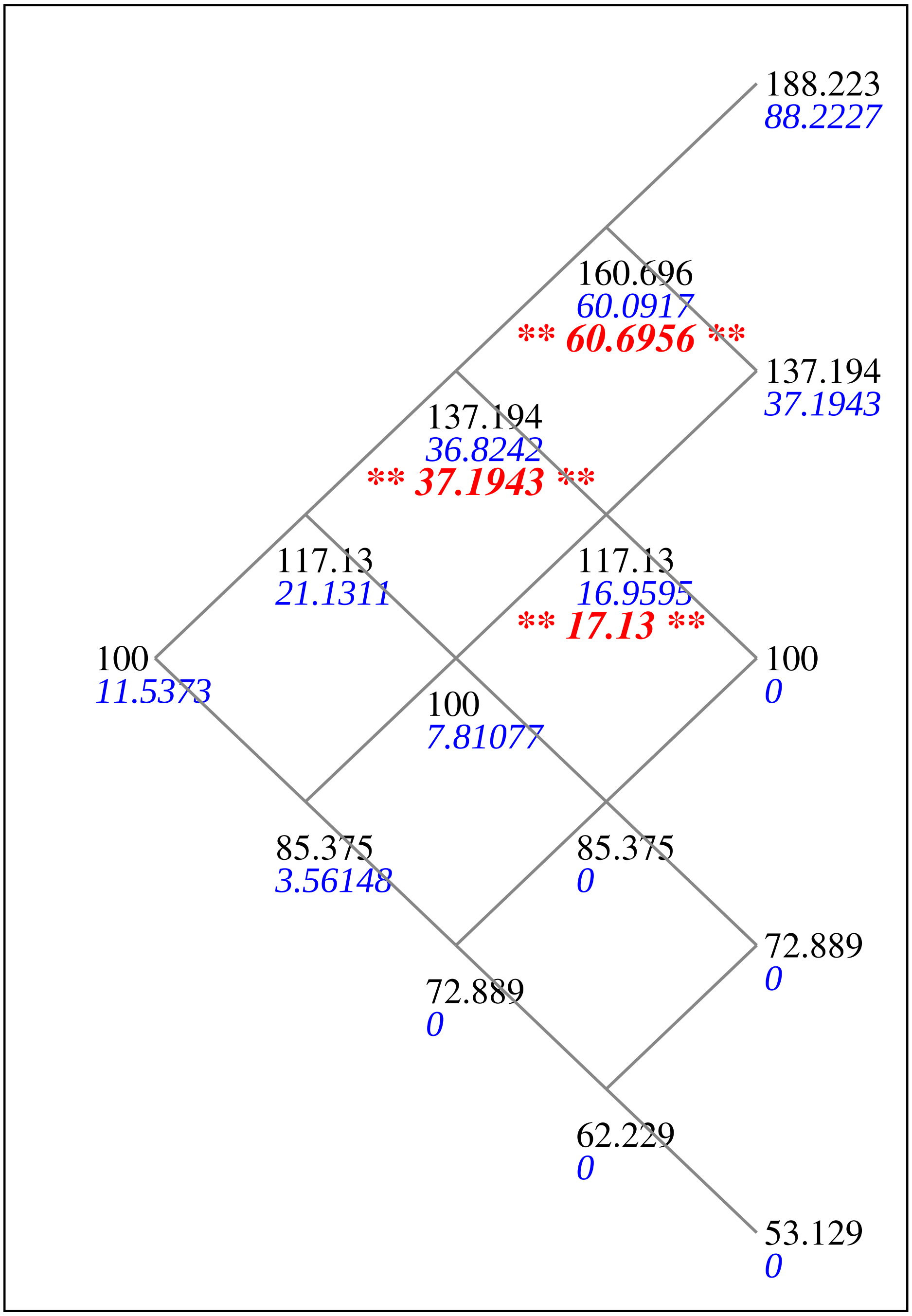

Figure 2: Valuation of American call option using the binomial tree in Fig. 1.

19.10 American put

- We calculate the fair value of an **American put.**
- The option valuation tree is shown in Fig. 3.

1. We begin by filling in the terminal payoff on the expiration date ($i = n$).
2. We then work backwards through the tree $i = n-1, \ldots, 0$.
3. At every node at step i we calculate the discounted expected value from the previous step $i+1$
$$V_{\text{disc exp}} = e^{-r\Delta t}\big(pV_u + qV_d\big)\,. \tag{19.10.1}$$
4. We then perform a valuation test (for early exercise). If the value of V is less than the intrinsic value at that node, we set V to the intrinsic value.
$$V = \max\{V_{\text{disc exp}},\, \text{intrinsic value}\}\,. \tag{19.10.2}$$

- Step $i = n = 4$.

1. Expiration. The option fair values at the nodes are:
$$V(Su^4) = 0\,, \tag{19.10.3a}$$
$$V(Su^2) = 0\,, \tag{19.10.3b}$$
$$V(S) = 0\,, \tag{19.10.3c}$$
$$V(Sd^2) \simeq 27.1107\,, \tag{19.10.3d}$$
$$V(Sd^4) \simeq 46.8714\,. \tag{19.10.3e}$$
2. The above values are obtained from the terminal payoff formula $V = \max\{K - S_T, 0\}$.

- We work backwards through the tree:

1. Step $i = 3$:

$$\begin{aligned}
V(Su^3) &= e^{-r\Delta t}(p \times 0 + q \times 0) && = 0\,, && (19.10.4a) \\
V(Su) &= e^{-r\Delta t}(p \times 0 + q \times 0) && = 0\,, && (19.10.4b) \\
V(Sd)_{\text{disc exp}} &= e^{-r\Delta t}(p \times 0 + q \times 27.1107) && \simeq 14.4792\,, && (19.10.4c) \\
\boldsymbol{V(Sd)} &= \mathbf{max\{14.4792, 14.6247\}} && = \mathbf{14.6247}\,, && (19.10.4d) \\
V(Sd^3)_{\text{disc exp}} &= e^{-r\Delta t}(p \times 27.1107 + q \times 46.8714) && \simeq 37.3947\,, && (19.10.4e) \\
\boldsymbol{V(Sd^3)} &= \mathbf{max\{37.3947, 37.7705\}} && = \mathbf{37.7705}\,. && (19.10.4f)
\end{aligned}$$

2. Step $i = 2$:

$$\begin{aligned}
V(Su^2) &= e^{-r\Delta t}(p \times 0 + q \times 0) && = 0\,, && (19.10.5a) \\
V(S) &= e^{-r\Delta t}(p \times 0 + q \times 14.6247) && \simeq 7.81077\,, && (19.10.5b) \\
V(Sd^2)_{\text{disc exp}} &= e^{-r\Delta t}(p \times 14.6247 + q \times 37.7705) && \simeq 26.8409\,, && (19.10.5c) \\
\boldsymbol{V(Sd^2)} &= \mathbf{max\{26.8409, 27.1107\}} && = \mathbf{27.1107}\,. && (19.10.5d)
\end{aligned}$$

3. Step $i = 1$:

$$\begin{aligned}
V(Su) &= e^{-r\Delta t}(p \times 0 + q \times 7.81077) && \simeq 4.17156\,, && (19.10.6a) \\
V(Sd) &= e^{-r\Delta t}(p \times 7.81077 + q \times 27.1107) && \simeq 18.0407\,. && (19.10.6b)
\end{aligned}$$

4. Step $i = 0$:

$$V(S) = e^{-r\Delta t}(p \times 4.17156 + q \times 18.0407) \simeq 11.5373\,. \tag{19.10.7}$$

- Hence the American put fair value is, using a binomial model with $n = 4$,

$$V_{\text{Am put}} \simeq 11.5373\,. \tag{19.10.8}$$

1. Because $S = K$ and $r = q_{\text{div}}$ and we are valuing the options using Geometric Brownian Motion, the option prices satisfy $V_{\text{call}} = V_{\text{put}}$.
2. The valuation using the binomial tree satisfies this symmetry, because the binomial model implements Geometric Brownian Motion.
3. Hence the fair value of the American put is the same as that of the American call in eq. (19.9.8).

Binomial tree for American put

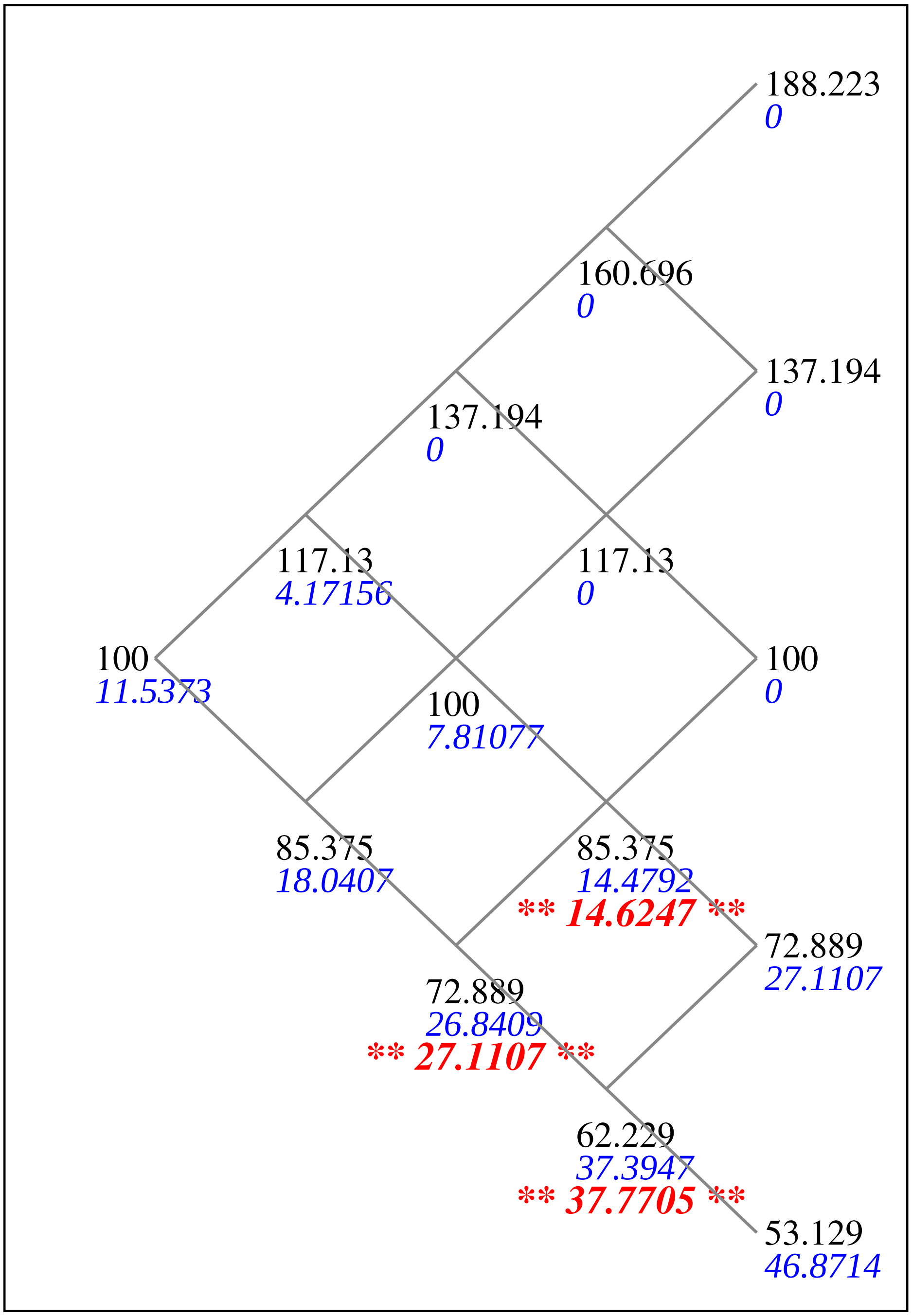

Figure 3: Valuation of American put option using the binomial tree in Fig. 1.

19.11 European up and out barrier call

- We calculate the fair value of a **European up and out barrier call.**
- The barrier level is $B = 130$.
- The options knocks out if $S_t \ge B$ at any time $t_0 \le t \le T$.
- The option pays a rebate $R = B - K = 30$ if the options knocks out.
- The option valuation tree is shown in Fig. 4.
  1. We begin by filling in the terminal payoff on the expiration date ($i = n$).
  2. We then work backwards through the tree $i = n-1, \dots, 0$.
  3. At every node at step i we calculate the discounted expected value from the previous step $i+1$
  $$V_{\text{disc exp}} = e^{-r\Delta t}\big(pV_u + qV_d\big)\,. \tag{19.11.1}$$
  4. We then perform a valuation test to see if the option knocks out because of the barrier:
  $$V = B - K \qquad\qquad (S_{\text{node}} \ge B)\,. \tag{19.11.2}$$
  5. Because this a European option, there is no test for early exercise.
- Step $i = n = 4$.
  1. Expiration. The option fair values at the nodes are:
  $$V(Su^4) = \mathbf{30}\,, \tag{19.11.3a}$$
  $$V(Su^2) = \mathbf{30}\,, \tag{19.11.3b}$$
  $$V(S) = 0\,, \tag{19.11.3c}$$
  $$V(Sd^2) = 0\,, \tag{19.11.3d}$$
  $$V(Sd^4) = 0\,. \tag{19.11.3e}$$
  2. The above values are obtained from the terminal payoff formula
  $$V_{\text{up out barrier call}}(S_T, T) = \begin{cases} 0 & S_T < K\,, \\ S_T - K & K \le S_T < B\,, \\ B - K & S_T \ge B\,. \end{cases} \tag{19.11.4}$$

- We work backwards through the tree:

1. Step $i = 3$:

$$\begin{aligned}
V(Su^3)_{\text{disc exp}} &= e^{-r\Delta t}(p \times 30 + q \times 30) && \simeq 29.7015\,, && (19.11.5a)\\
\boldsymbol{V(Su^3)} &= \boldsymbol{(S \geq B)} && = \mathbf{30}\,, && (19.11.5b)\\
V(Su) &= e^{-r\Delta t}(p \times 30 + q \times 0) && \simeq 13.67913\,, && (19.11.5c)\\
V(Sd) &= e^{-r\Delta t}(p \times 0 + q \times 0) && = 0\,, && (19.11.5d)\\
V(Sd^3) &= e^{-r\Delta t}(p \times 0 + q \times 0) && = 0\,. && (19.11.5e)
\end{aligned}$$

2. Step $i = 2$:

$$\begin{aligned}
V(Su^2)_{\text{disc exp}} &= e^{-r\Delta t}(p \times 30 + q \times 13.67913) && \simeq 20.98487\,, && (19.11.6a)\\
\boldsymbol{V(Su^2)} &= \boldsymbol{(S \geq B)} && = \mathbf{30}\,. && (19.11.6b)\\
V(S) &= e^{-r\Delta t}(p \times 13.67913 + q \times 0) && \simeq 6.237288\,, && (19.11.6c)\\
V(Sd^2) &= e^{-r\Delta t}(p \times 0 + q \times 0) && = 0\,. && (19.11.6d)
\end{aligned}$$

3. Step $i = 1$:

$$\begin{aligned}
V(Su) &= e^{-r\Delta t}(p \times 30 + q \times 6.237288) && \simeq 17.01034\,, && (19.11.7a)\\
V(Sd) &= e^{-r\Delta t}(p \times 6.237288 + q \times 0) && \simeq 2.844023\,. && (19.11.7b)
\end{aligned}$$

4. Step $i = 0$:

$$V(S) = e^{-r\Delta t}(p \times 17.01034 + q \times 2.844023) \simeq 9.275155\,. \qquad (19.11.8)$$

- Hence the European up and out barrier call fair value is, using a binomial model with $n = 4$,

$$V_{\text{Eur up out barrier call}} \simeq 9.275155\,. \qquad (19.11.9)$$

Binomial tree for European up out barrier call

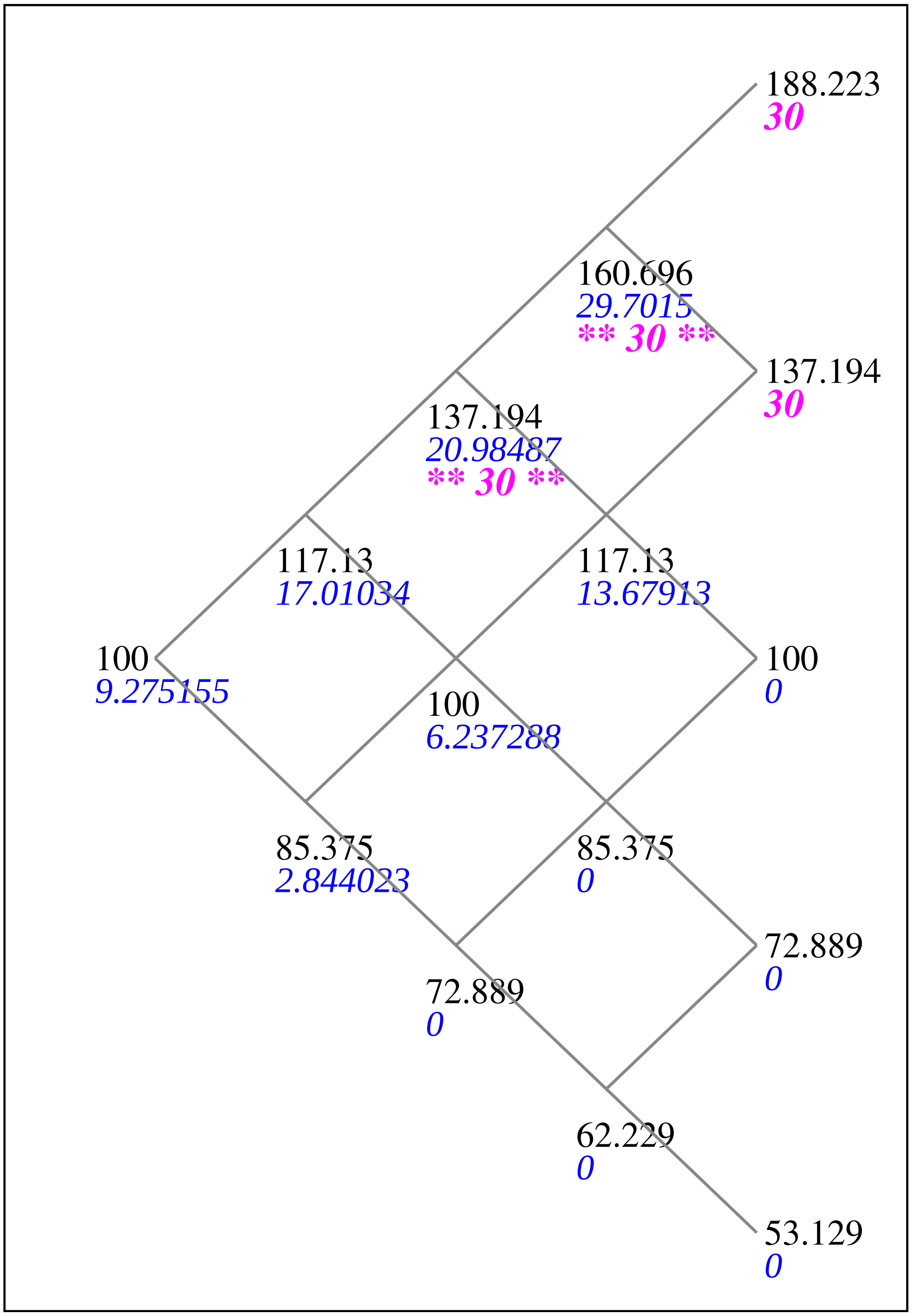

Figure 4: Valuation of European up and out barrier call option using the binomial tree in Fig. 1.

19.12 American up and out barrier call

- We calculate the fair value of an **American up and out barrier call.**
- The barrier level is $B = 130$.
- The options knocks out if $S_t \ge B$ at any time $t_0 \le t \le T$.
- The option pays a rebate $R = B - K = 30$ if the options knocks out.
- The option valuation tree is shown in Fig. 5.
  1. We begin by filling in the terminal payoff on the expiration date ($i = n$).
  2. We then work backwards through the tree $i = n-1, \dots, 0$.
  3. At every node at step i we calculate the discounted expected value from the previous step $i+1$
  $$V_{\text{disc exp}} = e^{-r\Delta t}(pV_u + qV_d)\,. \tag{19.12.1}$$
  4. We then perform a valuation test to see if the option knocks out because of the barrier:
  $$V = B - K \qquad\qquad (S_{\text{node}} \ge B)\,. \tag{19.12.2}$$
  5. Because this an American option, we also test for early exercise.
  6. **For stock prices in the interval $K \le S < B$,** if the value of V is less than the intrinsic value at that node, we set V to the intrinsic value.
  $$V = \max\{V_{\text{disc exp}}, \text{intrinsic value}\} \qquad\qquad (K \le S_{\text{node}} < B)\,. \tag{19.12.3}$$

- Step $i = n = 4$.
  1. Expiration. The option fair values at the nodes are:
  $$V(Su^4) = \mathbf{30}\,, \tag{19.12.4a}$$
  $$V(Su^2) = \mathbf{30}\,, \tag{19.12.4b}$$
  $$V(S) = 0\,, \tag{19.12.4c}$$
  $$V(Sd^2) = 0\,, \tag{19.12.4d}$$
  $$V(Sd^4) = 0\,. \tag{19.12.4e}$$
  2. The above values are obtained from the terminal payoff formula
  $$V_{\text{up out barrier call}}(S_T, T) = \begin{cases} 0 & S_T < K\,, \\ S_T - K & K \le S_T < B\,, \\ B - K & S_T \ge B\,. \end{cases} \tag{19.12.5}$$

- We work backwards through the tree:

1. Step $i = 3$:

$$V(Su^3)_{\text{disc exp}} = e^{-r\Delta t}(p \times 30 + q \times 30) \simeq 29.7015\,, \tag{19.12.6a}$$

$$\boldsymbol{V(Su^3) = (S \geq B)} = \mathbf{30}\,, \tag{19.12.6b}$$

$$V(Su)_{\text{disc exp}} = e^{-r\Delta t}(p \times 30 + q \times 0) \simeq 13.67913\,, \tag{19.12.6c}$$

$$\boldsymbol{V(Su) = \max\{13.67913, 17.13\}} = \mathbf{17.13}\,, \tag{19.12.6d}$$

$$V(Sd) = e^{-r\Delta t}(p \times 0 + q \times 0) = 0\,, \tag{19.12.6e}$$

$$V(Sd^3) = e^{-r\Delta t}(p \times 0 + q \times 0) = 0\,. \tag{19.12.6f}$$

2. Step $i = 2$:

$$V(Su^2)_{\text{disc exp}} = e^{-r\Delta t}(p \times 30 + q \times 17.13) \simeq 22.8279\,, \tag{19.12.7a}$$

$$\boldsymbol{V(Su^2) = (S \geq B)} = \mathbf{30}\,, \tag{19.12.7b}$$

$$V(S) = e^{-r\Delta t}(p \times 17.13 + q \times 0) \simeq 7.810785\,, \tag{19.12.7c}$$

$$V(Sd^2) = e^{-r\Delta t}(p \times 0 + q \times 0) = 0\,. \tag{19.12.7d}$$

3. Step $i = 1$:

$$V(Su) = e^{-r\Delta t}(p \times 30 + q \times 7.810785) \simeq 17.85071\,, \tag{19.12.8a}$$

$$V(Sd) = e^{-r\Delta t}(p \times 7.810785 + q \times 0) \simeq 3.561492\,. \tag{19.12.8b}$$

4. Step $i = 0$:

$$V(S) = e^{-r\Delta t}(p \times 17.85071 + q \times 3.561492) \simeq 10.04152\,. \tag{19.12.9}$$

- Hence the American up and out barrier call fair value is, using a binomial model with $n = 4$,

$$V_{\text{Am up out barrier call}} \simeq 10.04152\,. \tag{19.12.10}$$

Binomial tree for American up out barrier call

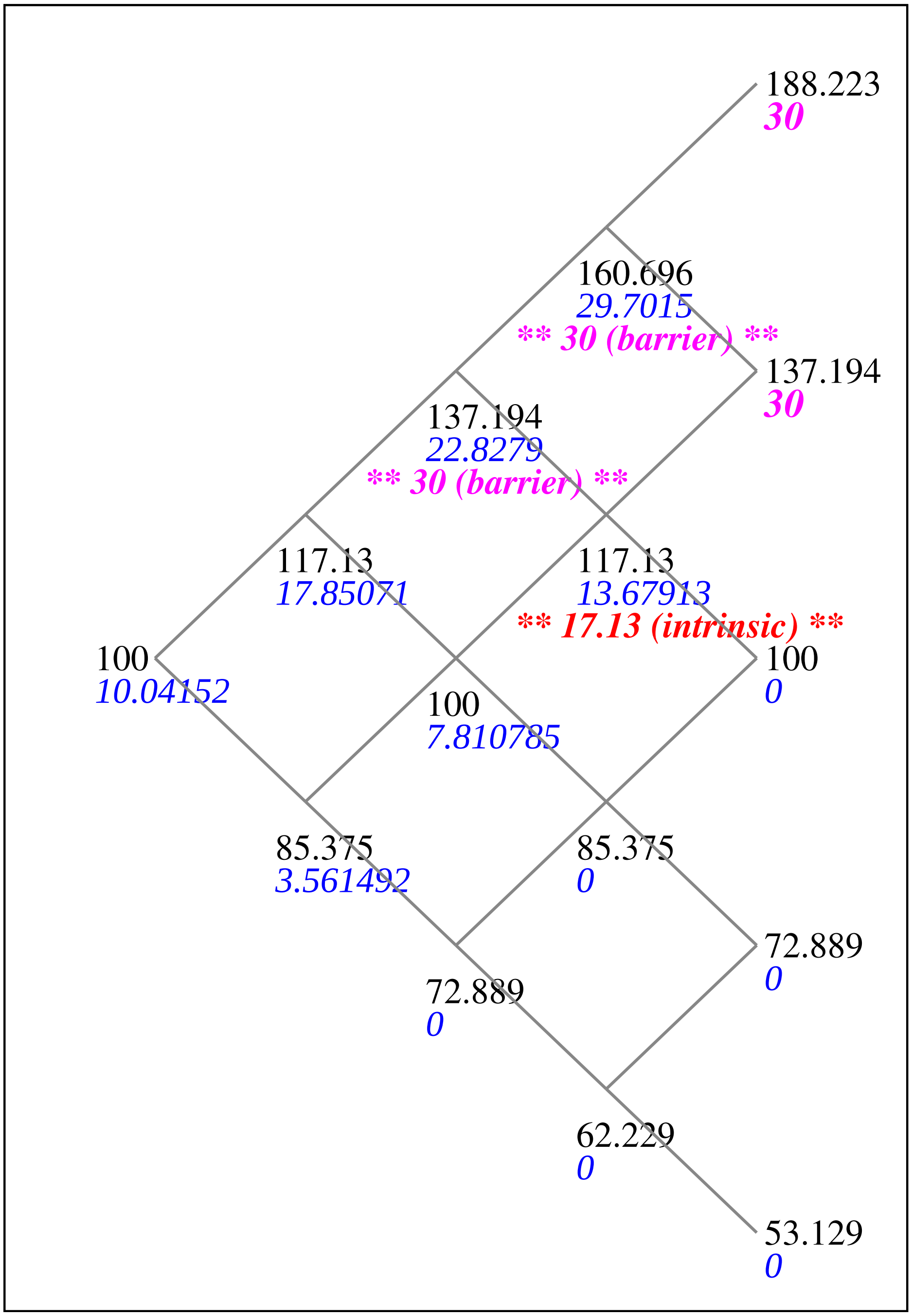

Figure 5: Valuation of American up and out barrier call option using the binomial tree in Fig. 1.

19.13 European binary call

- We calculate the fair value of a **European binary call.**
- At expiration, the option pays \$1 if $S_T \ge K$ and 0 if $S_T < K$.
- The option valuation tree is shown in Fig. 6.
  1. We begin by filling in the terminal payoff on the expiration date ($i = n$).
  2. We then work backwards through the tree $i = n-1, \dots, 0$.
  3. At every node at step i we calculate the discounted expected value from the previous step $i+1$
  $$V_{\text{disc exp}} = e^{-r\Delta t}\bigl(pV_u + qV_d\bigr)\,. \tag{19.13.1}$$
  4. Because this a European option, there are no early exercise tests.
- Step $i = n = 4$.
  1. Expiration. The option fair values at the nodes are:
  $$V(Su^4) = \mathbf{1}\,, \tag{19.13.2a}$$
  $$V(Su^2) = \mathbf{1}\,, \tag{19.13.2b}$$
  $$V(S) = \mathbf{1}\,, \tag{19.13.2c}$$
  $$V(Sd^2) = 0\,, \tag{19.13.2d}$$
  $$V(Sd^4) = 0\,. \tag{19.13.2e}$$
  2. The above values are obtained from the terminal payoff formula
  $$V_{\text{binary call}}(S_T, T) = \begin{cases} 0 & S_T < K\,, \\ 1 & S_T \ge K\,. \end{cases} \tag{19.13.3}$$

- We work backwards through the tree:

  1. Step $i = 3$:

$$
\begin{aligned}
V(Su^3) &= e^{-r\Delta t}(p \times 1 + q \times 1) && \simeq 0.99005\,, && (19.13.4a)\\
V(Su) &= e^{-r\Delta t}(p \times 1 + q \times 1) && \simeq 0.99005\,, && (19.13.4b)\\
V(Sd) &= e^{-r\Delta t}(p \times 1 + q \times 0) && = 0.455971\,, && (19.13.4c)\\
V(Sd^3) &= e^{-r\Delta t}(p \times 0 + q \times 0) && = 0\,. && (19.13.4d)
\end{aligned}
$$

  2. Step $i = 2$:

$$
\begin{aligned}
V(Su^2) &= e^{-r\Delta t}(p \times 0.99005 + q \times 0.99005) && \simeq 0.980199\,, && (19.13.5a)\\
V(S) &= e^{-r\Delta t}(p \times 0.99005 + q \times 0.455971) && \simeq 0.694959\,, && (19.13.5b)\\
V(Sd^2) &= e^{-r\Delta t}(p \times 0.455971 + q \times 0) && = 0.20791\,. && (19.13.5c)
\end{aligned}
$$

  3. Step $i = 1$:

$$
\begin{aligned}
V(Su) &= e^{-r\Delta t}(p \times 0.980199 + q \times 0.694959) && \simeq 0.818105\,, && (19.13.6a)\\
V(Sd) &= e^{-r\Delta t}(p \times 0.694959 + q \times 0.20791) && \simeq 0.427922\,. && (19.13.6b)
\end{aligned}
$$

  4. Step $i = 0$:

$$
V(S) = e^{-r\Delta t}(p \times 0.818105 + q \times 0.427922) \simeq 0.601576\,. \qquad (19.13.7)
$$

- Hence the European binary call fair value is, using a binomial model with $n = 4$,

$$
V_{\text{Eur binary call}} \simeq 0.601576\,. \qquad (19.13.8)
$$

Binomial tree for European binary call

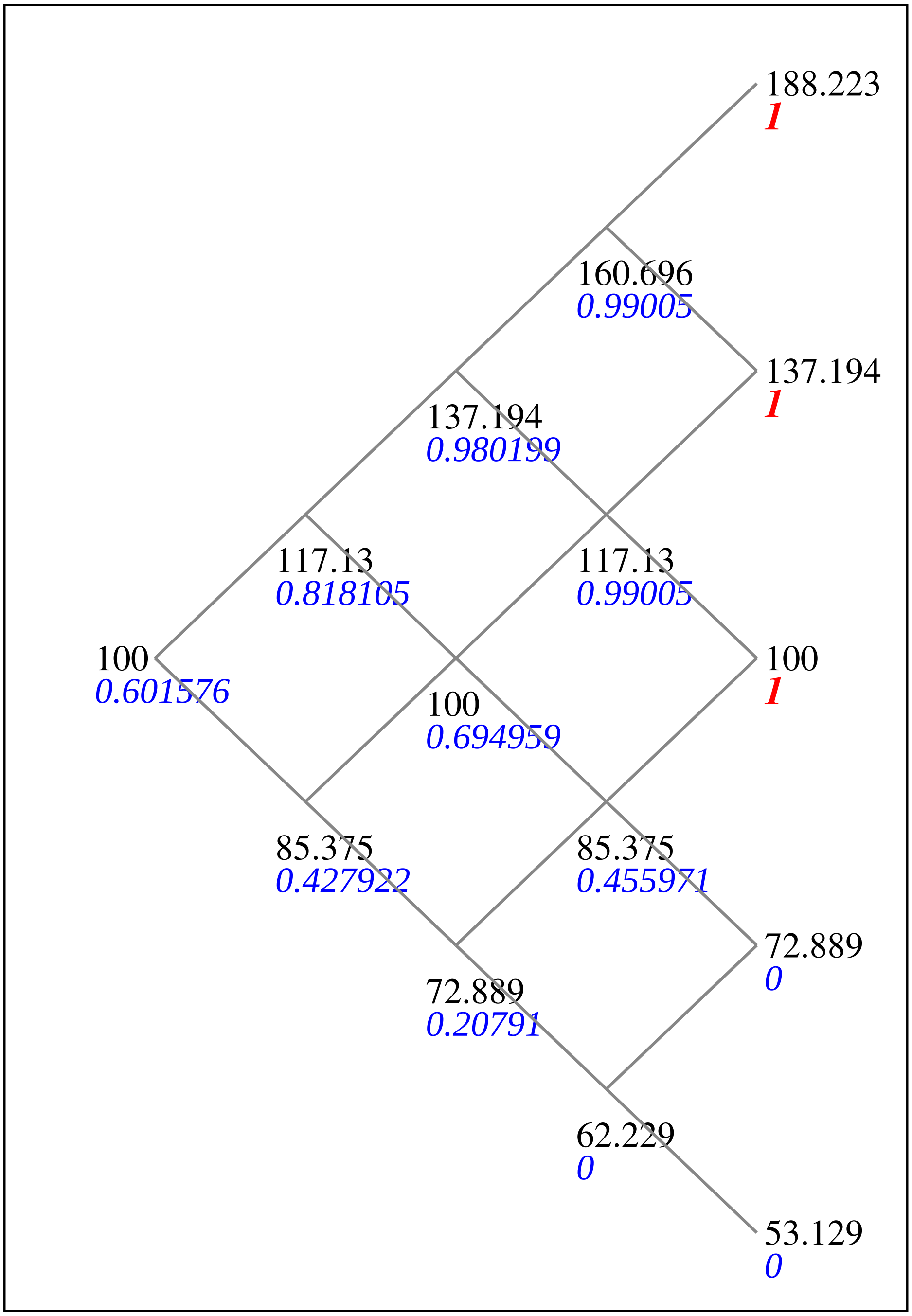

Figure 6: Valuation of European binary call option using the binomial tree in Fig. 1.

19.14 European binary put

- We calculate the fair value of a **European binary put.**
- At expiration, the option pays \$1 if $S_T < K$ and 0 if $S_T \geq K$.
- The option valuation tree is shown in Fig. 7.
  1. We begin by filling in the terminal payoff on the expiration date ($i = n$).
  2. We then work backwards through the tree $i = n - 1, \ldots, 0$.
  3. At every node at step i we calculate the discounted expected value from the previous step $i + 1$
  $$V_{\text{disc exp}} = e^{-r\Delta t}\big(pV_u + qV_d\big)\,. \tag{19.14.1}$$
  4. Because this a European option, there are no early exercise tests.
- Step $i = n = 4$.
  1. Expiration. The option fair values at the nodes are:
  $$V(Su^4) = 0\,, \tag{19.14.2a}$$
  $$V(Su^2) = 0\,, \tag{19.14.2b}$$
  $$V(S) = 0\,, \tag{19.14.2c}$$
  $$V(Sd^2) = \mathbf{1}\,, \tag{19.14.2d}$$
  $$V(Sd^4) = \mathbf{1}\,. \tag{19.14.2e}$$
  2. The above values are obtained from the terminal payoff formula
  $$V_{\text{binary put}}(S_T, T) = \begin{cases} 1 & \qquad S_T < K\,, \\ 0 & \qquad S_T \geq K\,. \end{cases} \tag{19.14.3}$$

- We work backwards through the tree:

  1. Step $i = 3$:

$$\begin{aligned} V(Su^3) &= e^{-r\Delta t}(p \times 0 + q \times 0) && =0\,, && (19.14.4a) \\ V(Su) &= e^{-r\Delta t}(p \times 0 + q \times 0) && =0\,, && (19.14.4b) \\ V(Sd) &= e^{-r\Delta t}(p \times 0 + q \times 1) && =0.534079\,, && (19.14.4c) \\ V(Sd^3) &= e^{-r\Delta t}(p \times 1 + q \times 1) && =0.99005\,. && (19.14.4d) \end{aligned}$$

  2. Step $i = 2$:

$$\begin{aligned} V(Su^2) &= e^{-r\Delta t}(p \times 0 + q \times 0) && =0\,, && (19.14.5a) \\ V(S) &= e^{-r\Delta t}(p \times 0 + q \times 0.534079) && \simeq 0.28524\,, && (19.14.5b) \\ V(Sd^2) &= e^{-r\Delta t}(p \times 0.534079 + q \times 0.99005) && =0.772289\,. && (19.14.5c) \end{aligned}$$

  3. Step $i = 1$:

$$\begin{aligned} V(Su) &= e^{-r\Delta t}(p \times 0 + q \times 0.28254) && \simeq 0.152341\,, && (19.14.6a) \\ V(Sd) &= e^{-r\Delta t}(p \times 0.28254 + q \times 0.772289) && \simeq 0.542524\,. && (19.14.6b) \end{aligned}$$

  4. Step $i = 0$:

$$V(S) = e^{-r\Delta t}(p \times 0.152341 + q \times 0.542524) \simeq 0.359214\,. \tag{19.14.7}$$

- Hence the European binary put fair value is, using a binomial model with $n = 4$,

$$V_{\text{Eur binary put}} \simeq 0.359214\,. \tag{19.14.8}$$

- The European binary call and put option fair values satisfy the model-independent relation

$$c_{\text{bin}} + p_{\text{bin}} = e^{-r(T-t_0)}\,. \tag{19.14.9}$$

- Using eqs. (19.13.8) and (19.14.8), we obtain

$$\begin{aligned} V_{\text{Eur binary call}} + V_{\text{Eur binary put}} \simeq 0.601576 + 0.359214 \simeq 0.96079\,, \\ e^{-r(T-t_0)} = e^{-0.04} \simeq 0.960789\,. \end{aligned} \tag{19.14.10}$$

- Hence the binomial model satisfies $V_{\text{Eur binary call}} + V_{\text{Eur binary put}} = e^{-r(T-t_0)}$, up to roundoff.

Binomial tree for European binary put

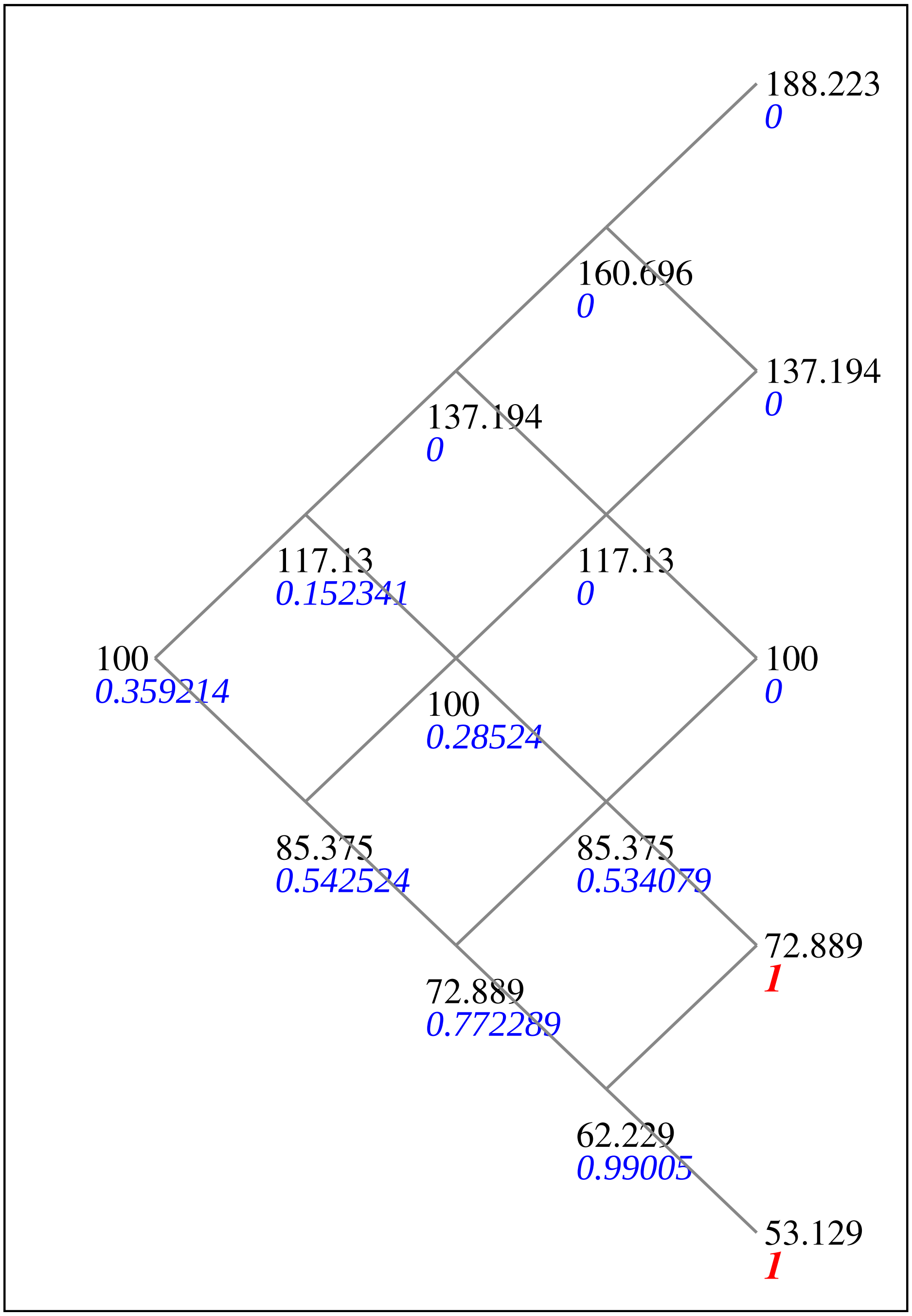

Figure 7: Valuation of European binary put option using the binomial tree in Fig. 1.