Queens College, CUNY, Department of Computer Science

**Computational Finance**

**CSCI 365 / 765**

**Spring 2018**

Instructor: Dr. Sateesh Mane



# Midterm 2 Spring 2018

## due Wednesday April 18, 2018 11:59 pm
## *** extra question Q. 7 added 4/9/2018 ***

- **NOTE: It is the policy of the Computer Science Department to issue a failing grade to any student who either gives or receives help on any test.**
- This is an **open-book** test.
- **Any problem to which you give two or more (different) answers receives the grade of zero automatically.**
- This is a **take home exam.**
  Please submit your solution via email, as a file attachment, to `Sateesh.Mane@qc.cuny.edu`.
  The file name should have either of the formats:

  `StudentId_first_last_CS365_midterm2_Apr2018`

  `StudentId_first_last_CS765_midterm2_Apr2018`

  Acceptable file types are txt, doc/docx, pdf (also cpp, with text in comment blocks).
- **In all questions where you are asked to submit programming code, programs which display any of the following behaviors will receive an automatic F:**
  1. Programs which do not compile successfully (compiler warnings which are not fatal are excluded, e.g. use of deprecated features).
  2. Array out of bounds. Dereferencing of uninitialized variables (including null pointers).
  3. Operations which yield NAN or infinity, e.g. divide by zero, square root of negative number, etc. *Infinite loops.*
  4. Programs which do NOT implement the public interface stated in the question.
- **In addition, note the following:**
  1. Programs which compile and run successfully but have memory leaks will receive a poor grade (but not F).
  2. All debugging and/or output statements (e.g. `cout` or `printf`) will be commented out.
  3. Program performance will be tested solely on function return values and the values of output variable(s) in the function arguments.
  4. In other words, program performance will be tested solely via the public interface presented to the calling application. (I will write the calling application.)

# 1 Question 1 no code

- Consider a yield curve with the two initial tenors $t_{0.5} = 0.5$ and $t_{1.0} = 1.0$.
- The corresponding spot rates are $r_{0.5} = 5.0 + \Delta r_{0.5}$ and $r_{1.0} = 5.0 + \Delta r_{1.0}$.
- The offsets $\Delta r_{0.5}$ and $\Delta r_{1.0}$ are obtained as follows:
  1. **Take the first 4 digits of your student id. Define:**

$$\Delta r_{0.5} = \frac{\text{first 4 digits of your student id}}{10^4}\,. \tag{1.1}$$

  2. **Take the last 4 digits of your student id. Define:**

$$\Delta r_{1.0} = \frac{\text{last 4 digits of your student id}}{10^4}\,. \tag{1.2}$$

  3. For example if your student id is 23054617, then

$$\Delta r_{0.5} = 0.2305\,, \qquad \Delta r_{1.0} = 0.4617\,. \tag{1.3}$$

- **Calculate the interpolated spot rate $r_{\text{cfr}}$ using constant forward rate interpolation for $t = 0.66$. State your answer as a percentage to 2 decimal places.**
- **(Optional/bonus) Prove that $5.0\% \leq r_{\text{cfr}} \leq 6.0\%$ for any student id.**

## 2 Question 2 no code

- Suppose the market price of a stock is $S_0$ at time $t_0$.
- The stock does not pay dividends.
- The interest rate is $r$ (a constant).
- All the options below have strike $K = 100$ and expiration time $T > t_0$.
- **You are also given that $r$ and $T$ are such that $e^{r(T-t_0)} < 100$.**

### 2.1 European call

- **Assume that the value of the stock price is in the interval $1 < S_0 < 100$.**
- The strike price of the option is $K = 100$.
- A European call trades today (time $t_0$) at a market price = 1.
- We formulate the following trading strategy:
  **(a) buy the call, (b) short sell one share of stock, (c) save money in a bank.**
- The initial value of our portfolio is zero.
- **Find a scenario where this strategy leads to a profit.**
- **Find a scenario where this strategy leads to a loss.**
- **Note: if we do not exercise the option, we must buy back the stock, to cover the short sale.**

### 2.2 American call

- **Assume that the value of the stock price is in the interval $1 < S_0 < 100$.**
- The strike price of the option is $K = 100$.
- An American call trades today (time $t_0$) at a market price = 1.
- We formulate the following trading strategy:
  **(a) buy the call, (b) short sell one share of stock, (c) save money in a bank.**
- The initial value of our portfolio is zero.
- **Find a scenario where this strategy leads to a profit.**
- **Find a scenario where this strategy leads to a loss.**
- **Note: if we do not exercise the option, we must buy back the stock, to cover the short sale.**

### 2.3 European put

- **Assume that the value of the stock price is in the interval** $100 < S_0 < 200$**.**
- The strike price of the option is $K = 100$.
- A European put trades today (time $t_0$) at a market price $= 1$.
- We formulate the following trading strategy:
  **(a) buy the put, (b) borrow money from a bank.**
- The initial value of our portfolio is zero.
- **Find a scenario where this strategy leads to a profit.**
- **Find a scenario where this strategy leads to a loss.**
- **Note: if we exercise the put option, we must buy the stock at the market price on the exercise date, to deliver to the writer of the put.**

### 2.4 American put

- **Assume that the value of the stock price is in the interval** $100 < S_0 < 200$**.**
- The strike price of the option is $K = 100$.
- An American put trades today (time $t_0$) at a market price $= 1$.
- We formulate the following trading strategy:
  **(a) buy the put, (b) borrow money from a bank.**
- The initial value of our portfolio is zero.
- **Find a scenario where this strategy leads to a profit.**
- **Find a scenario where this strategy leads to a loss.**
- **Note: if we exercise the put option, we must buy the stock at the market price on the exercise date, to deliver to the writer of the put.**

# 3 Question 3 no code

## 3.1 European call

- The current time is $t_0 = 0$.
- The value of the stock price today (time $t_0$) is $S_0 = 98$.
- The stock does not pay dividends.
- The interest rate is 5%.
- A European call option trades today (time $t_0$) at a market price = 3.
- The strike price of the option is $K = 100$.
- The option expiration time is 1 year from today ($T - t_0 = 1$).
- **We begin trading with a position of zero (no cash, stock, option, etc.).**
- We formulate the following trading strategy:
  **(a) sell the call, (b) buy one share of stock, (c) borrow money from a bank.**
- The initial value of our portfolio is zero.
- **Find a scenario where this strategy leads to a profit.**
- **Find a scenario where this strategy leads to a loss.**

## 3.2 European put

- The current time is $t_0 = 0$.
- The value of the stock price today (time $t_0$) is $S_0 = 99$.
- The stock does not pay dividends.
- The interest rate is 5%.
- A European put trades today (time $t_0$) at a market price = 2.
- The strike price of the option is $K = 100$.
- The option expiration time is 1 year from today ($T - t_0 = 1$).
- **We begin trading with a position of zero (no cash, stock, option, etc.).**
- We formulate the following trading strategy:
  **(a) sell the put, (b) sell one share of stock, (c) save money in a bank.**
- The initial value of our portfolio is zero.
- **Find a scenario where this strategy leads to a profit.**
- **Find a scenario where this strategy leads to a loss.**

# 4 Question 4 no code

- **Ignore interest rate compounding for all profit/loss calculations in this question.**
- A stock trades today ($t_0 = 0$) at a price of $S_0 = 100.0$.
- A futures contract on the stock trades today at a price $F_0 = 103.25$.
- The futures contract expiration date is 5 days from today.
- Every day for $t_i = i$, $i = 1, 2, 3, 4, 5$, the stock price is $S_1, S_2, S_3, S_4, S_5$.
- The corresponding futures price every day is $F_1, F_2, F_3, F_4, F_5$.
- On the expiration day, $F_5 = S_5$ (the futures price converges to the stock price).
- **The stock pays a dividend of** $0.07$ **on day 3.**
- Here is a list of the stock and futures prices, for $i = 0, 1, 2, 3, 4, 5$.

| $i$ | $S_i$ | $F_i$ | dividend |
|---|---|---|---|
| 0 | 100.0 | 103.25 | |
| 1 | 98.75 | 101.55 | |
| 2 | $S_2$ | $F_2$ | |
| 3 | $S_3$ | $F_3$ | 0.07 |
| 4 | $S_4$ | $F_4$ | |
| 5 | $S_5$ | $F_5$ | |

- **The values of $S_2, \ldots, S_5$ are arbitrary, but satisfy the following inequalities:**

$$98.75 < S_2 < S_3 < S_4 < S_5\,. \tag{4.1}$$

- **The values of $F_2, \ldots, F_5$ are arbitrary, but satisfy the following inequalities:**

$$101.55 < F_2 < F_3 < F_4 < F_5 (= S_5)\,. \tag{4.2}$$

See next page

### 4.1 Investor A

- **Investor A goes long one share of stock on day 0.**
- Investor A sells the stock on day 5.
- **Calculate (or state) the money paid/received by A every day, starting from day 0, until A's portfolio is closed out.**
  (Note that in some cases your answer may be a formula not a dollar number.)
- **Calculate the total profit/loss for A after selling the stock.**
- **State on which day A makes that profit/loss.**

### 4.2 Investor B

- **Investor B goes long one futures contract on day 1.**
- Investor B holds the futures contract to expiration.
- **Calculate the money paid/received every day in B's mark to market account, until B's portfolio is closed out.**
  (Note that in some cases your answer may be a formula not a dollar number.)
- **Calculate the total money paid by B after closing the futures contract.**
- **State what B receives in exchange for closing the futures contract.**

### 4.3 Investor C

- **Investor C goes short a forward contract on day 0.**
- The forward price is $F_{\text{fwd}} = 103.25$ and the expiration time is 5 days.
- **From the data on day 1, it is possible for C to lock in a guaranteed profit?**
  1. **If yes, state the strategy to lock in a guaranteed profit.**
  2. **If no, explain why not.**
- **State all the trades performed on the day C's portfolio is closed out.**
- **Calculate the total profit/loss for C (if any).**
- **State on which day C makes that profit/loss (if any).**

### 4.4 Investor D

- **Investor D goes long one futures contract on day 1.**
- Investor D sells the futures contract on day 3.
- **Calculate the money paid/received every day in D's mark to market account, until D's portfolio is closed out.**
  (Note that in some cases your answer may be a formula not a dollar number.)
- **Calculate the total profit/loss for D after selling the futures contract.**
- **State on which day D makes that profit/loss.**
- **State the trades (stock/cash/futures) which happen for D on day 5 (futures expiration).**

## 5 Question 5 no code

- **For each case below, you are the trader on the opposite side of the trade with me at time $t_0(=0)$.**
- **For each case below, state the trade you will perform with me at time $t_0(=0)$.**
- **For each case, you must explain the reason for your trading decision.**
- **Note:**
  **You are not permitted to say that you do not want to trade.**
  **You must go either long or short.**
- In all cases, both you and I begin with a portfolio of zero (no cash, no stock, options, etc.).
  1. I also act as the bank.
  2. Money required to buy things is borrowed from me. Interest must be paid on the loan.
  3. Money received by selling things is loaned to me. I will pay you interest on the savings.
  4. The borrow and lend interest rates are equal. Both are equal to the risk-free rate.
  5. There is no limit on the quantity to buy or sell anything, including short selling of stock.
- In all the cases below:
  1. The current value of the stock price is $S_0 = 100$.
  2. The stock does not pay dividends.
  3. For questions involving a stock index, the current value of the index is 1000 points.
  4. The stock index has a continuous dividend yield of 1.5%.
  5. The interest rate is 5% (borrow and lend rates are equal).
  6. All the options have the same expiration time of 1 year.
  7. All the forward/futures contracts have the same expiration time of 1 year.

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**5.1**

- Recall the stock pays no dividends, the interest rate is 5% and the expiration time is 1 year.
- A portfolio consists of:
  (a) short one European call,
  (b) long one European put,
  (c) long one forward contract.
- **Both the call and the put have the same strike price of 99.**
- **The forward price also equals 99.**
- The portfolio trades at a price of 1.
- **Note the following:**
  1. The portfolio is a package.
  2. The above statement "trades at a price of 1" means the price of **the entire package.**
  3. **The "price" is the price of the entire package.**
  4. ***You must go long/short the entire package.***
  5. You cannot say "I will go long only the forward contract but I do not want the options" etc.
  6. **The same policy applies to all the other portfolios below.**

- **State which position (long/short) you will take, to trade this portfolio with me at time $t_0$.**
- **Explain the reason for your trading decision.**

### 5.2

- Recall the stock pays no dividends, the interest rate is 5% and the expiration time is 1 year.
- A portfolio consists of:
  (a) long one European call,
  (b) short one European put,
  (c) short one forward contract.
- **Both the call and the put have the same strike price of $K = 101$.**
- **The forward price is $F = K + 1 = 102$.**

#### 5.2.1

- The portfolio trades at a price of 1.
- **State which position (long/short) you will take, to trade this portfolio with me at time $t_0$.**
- **Explain the reason for your trading decision.**

#### 5.2.2

- The portfolio trades at a price of 0.9.
- **State which position (long/short) you will take, to trade this portfolio with me at time $t_0$.**
- **Explain the reason for your trading decision.**

#### 5.2.3

- The portfolio trades at a price of 1.1.
- **State which position (long/short) you will take, to trade this portfolio with me at time $t_0$.**
- **Explain the reason for your trading decision.**

### 5.3

- Recall the stock pays no dividends, the interest rate is 5% and the expiration time is 1 year.

#### 5.3.1

- A portfolio consists of:
  (a) long one **European** call with strike 99.5,
  (b) long one **European** put with strike 100.5.
- The portfolio trades at a price of 0.9.
- **State which position (long/short) you will take, to trade this portfolio with me at time $t_0$.**
- **Explain the reason for your trading decision.**

#### 5.3.2

- A portfolio consists of:
  (a) long one **American** call with strike 99.5,
  (b) long one **American** put with strike 100.5.
- The portfolio trades at a price of 0.94.
- **State which position (long/short) you will take, to trade this portfolio with me at time $t_0$.**
- **Explain the reason for your trading decision.**
- **If you buy the portfolio, state the trades you will perform with it and when you will execute those trades.**

#### 5.3.3

- A portfolio consists of:
  (a) long one **American** call with strike 99.5,
  (b) long one **American** put with strike 100.5.
- The portfolio trades at a price of 0.98.
- **State which position (long/short) you will take, to trade this portfolio with me at time $t_0$.**
- **Explain the reason for your trading decision.**
- **If you buy the portfolio, state the trades you will perform with it and when you will execute those trades.**

### 5.4

- A stock index has a current value of 1000 index points.
- The interest rate is 5% and the index has a dividend yield of 1.5%.
- The volatility of the index is 30%.

#### 5.4.1

- An American call option on the stock index has a strike of 920 and expiration of 1 year.
- The option is **cash settled** with a multiplier of $1 for every index point that the option is in the money.
- The option is currently trading at a value of 65 index points.
  - **That means, if you buy the option you must pay $65.**
  - **If you sell the option you receive $65.**
- **State which position (long/short) you will take, to trade this portfolio with me at time $t_0$.**
- **Explain the reason for your trading decision.**
- **If you buy the option, state the trades you will perform with it, and when you will execute those trades.**

#### 5.4.2

- An American put option on the stock index has a strike of 1050 and expiration of 1 year.
- The option is **cash settled** with a multiplier of $1 for every index point that the option is in the money.
- The option is currently trading at a value of 45 index points.
  - **That means, if you buy the option you must pay $45.**
  - **If you sell the option you receive $45.**
- **State which position (long/short) you will take, to trade this portfolio with me at time $t_0$.**
- **Explain the reason for your trading decision.**
- **If you buy the option, state the trades you will perform with it, and when you will execute those trades.**

## 6 Question 6 no code

- **Note: In this question, the underlying stock does not pay dividends.**
- The terminal payoff diagrams of five financial derivatives are plotted in Fig. 1.
- *All of them describe real derivatives which trade in the financial markets, except maybe (e).*
- The horizontal axis shows the stock price at expiration $S_T$, for $0 \leq S_T \leq 100$.
- The vertical axis shows the payoff at expiration $V(S_T)$, call them $(V_a, V_b, V_c, V_d, V_e)$.
- **For values $S_T > 100$, the functions $(V_a, V_b, V_c, V_d, V_e)$ continue in straight lines.**
- **Your task is to construct combinations of options today (i.e. at time $t_0$), whose payoffs at expiration (time $T$) match those displayed in Fig. 1.**
- **You must use combinations of European puts and calls only.**
- You are ***not permitted to use stock and/or cash (bonds).***
- The functions $(V_a, V_b, V_c, V_d, V_e)$ have the following values in Fig. 1:

| $S_T$ | $V_a$ | $V_b$ | $V_c$ | $V_d$ | $V_e$ |
|---|---|---|---|---|---|
| 0 | 40 | 20 | 40 | 20 | 20 |
| 20 | | | | | 20 |
| 40 | 40 | 20 | 40 | 20 | 40 |
| 60 | | | | | 40 |
| 80 | | | 80 | 80 | |
| 100 | 100 | 80 | 80 | 80 | 100 |

- **Note the following (very important):**
  1. **There may be more than one way to match the terminal payoffs.**
  2. **All correct solutions will be accepted, if they use not more than six different strikes.**
  3. You may need to use **fractional numbers of calls and/or puts.**
     (In real life the derivatives are expressed using fractional numbers of calls and puts.)
  4. Some of the (fractional) numbers of calls and/or puts **may be negative**.
     (In fact I think it is impossible to do them all using only positive numbers.)
- **Express your answers in the following form.**
  1. For each case (a)–(e), write your answer in the form:
  $$V(S_0, t_0) = a_1 c(K_1) + b_2 p(K_2) + a_3 c(K_3) + b_4 p(K_4) + \cdots \tag{6.1}$$
  2. Here '$c$' denotes a European call and '$p$' denotes a European put.
  3. The coefficients $a_1, b_2, a_3, b_4, \ldots$ are constants (possibly fractional and/or negative).
  4. The parameters $K_1, K_2, \ldots$ are the strikes of the corresponding options.
  5. **You can leave out '$(S_0, t_0)$' from the argument list of the calls and puts, because it is the same for all.**

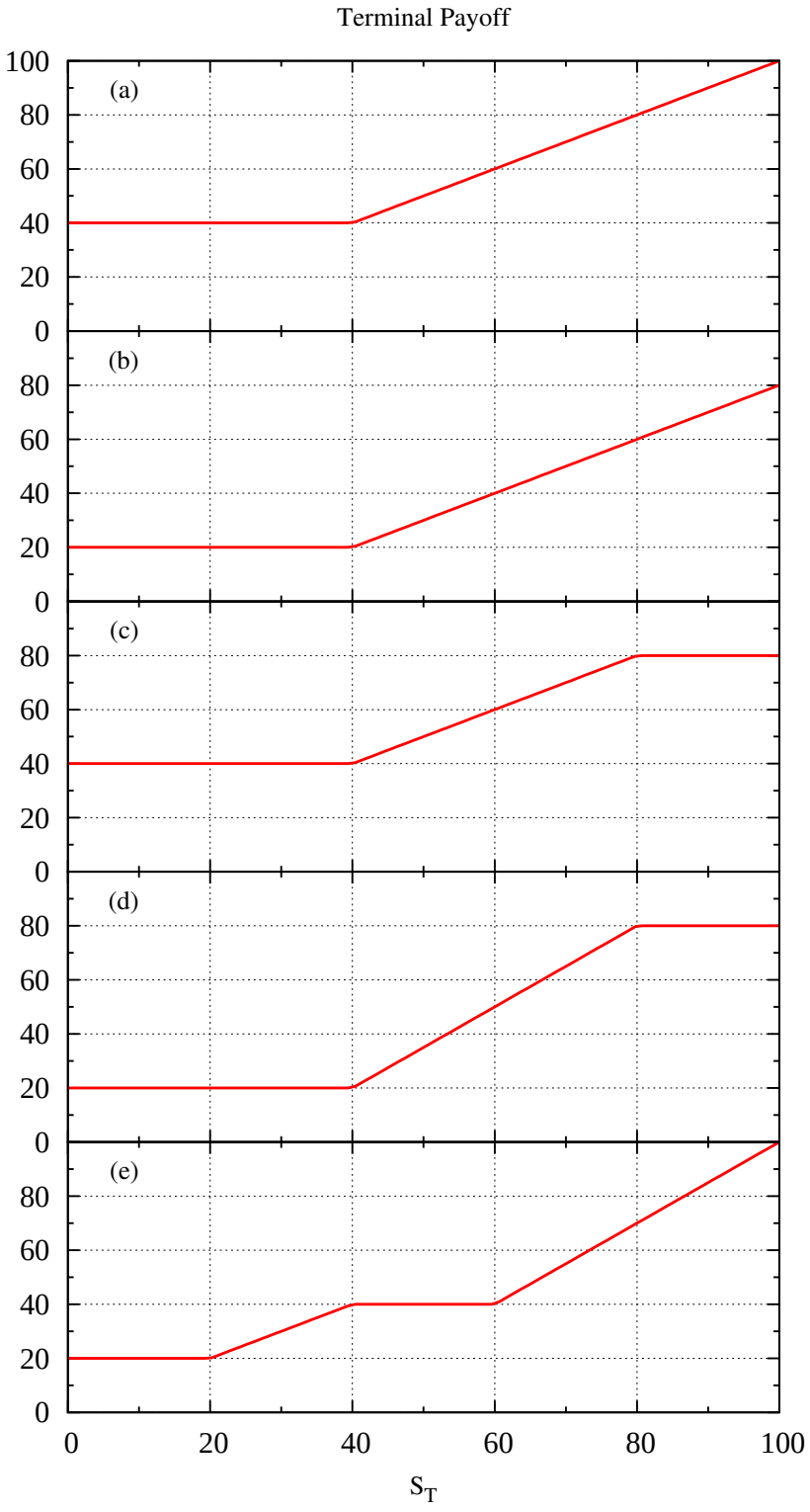


Figure 1: Graph of the terminal payoffs of derivatives listed in Question 6.

# 7 Question 7 *** added 4/9/2018 *** no code

- The market price of a stock is $S_0 = 100$ at time $t_0 = 0$.
- The stock does not pay dividends.
- **All the options below have strike $K = 100$ and expiration time $T - t_0 = 1$ year.**
- **The interest rate is $r$ (a constant) and $e^{-r(T-t_0)} = 0.99$.**

## 7.1 American call

- An American call trades today (time $t_0 = 0$) at a market price = 2.
- We formulate the following trading strategy:
  **(a) buy the call, (b) short sell one share of stock, (c) save money in a bank.**
- The initial value of our portfolio is zero.
- **Find the general solution where this strategy leads to a profit, in the time interval $t_0 \le t \le T$.**
- **Find the general solution where this strategy leads to a loss, in the time interval $t_0 \le t \le T$.**
- ***** Your solution must take into account the possibility of early exercise. *****
- **It is not permitted to wait indefinitely to cover the short sale. The short stock position must be covered at a time $t \le T$.**

## 7.2 American put

- An American put trades today (time $t_0 = 0$) at a market price = 2.
- We formulate the following trading strategy:
  **(a) buy the put, (b) buy one share of stock, (c) borrow money from a bank.**
- The initial value of our portfolio is zero.
- **Find the general solution where this strategy leads to a profit, in the time interval $t_0 \le t \le T$.**
- **Find the general solution where this strategy leads to a loss, in the time interval $t_0 \le t \le T$.**
- ***** Your solution must take into account the possibility of early exercise. *****
- **It is not permitted to wait indefinitely and refuse to repay the loan. The loan must be repaid at a time $t \le T$.**