Queens College, CUNY, Department of Computer Science

Computational Finance

CSCI 365 / 765

Spring 2018

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Midterm 2 Spring 2018

- **NOTE: It is the policy of the Computer Science Department to issue a failing grade to any student who either gives or receives help on any test.**
- This is an **open-book** test.
- **Any problem to which you give two or more (different) answers receives the grade of zero automatically.**
- This is a **take home exam.**
  Please submit your solution via email, as a file attachment, to `Sateesh.Mane@qc.cuny.edu`.
  The file name should have either of the formats:

  `StudentId_first_last_CS365_midterm2_Apr2018`

  `StudentId_first_last_CS765_midterm2_Apr2018`

  Acceptable file types are txt, doc/docx, pdf (also cpp, with text in comment blocks).
- **In all questions where you are asked to submit programming code, programs which display any of the following behaviors will receive an automatic F:**
  1. Programs which do not compile successfully (compiler warnings which are not fatal are excluded, e.g. use of deprecated features).
  2. Array out of bounds. Dereferencing of uninitialized variables (including null pointers).
  3. Operations which yield NAN or infinity, e.g. divide by zero, square root of negative number, etc. *Infinite loops.*
  4. Programs which do NOT implement the public interface stated in the question.
- **In addition, note the following:**
  1. Programs which compile and run successfully but have memory leaks will receive a poor grade (but not F).
  2. All debugging and/or output statements (e.g. `cout` or `printf`) will be commented out.
  3. Program performance will be tested solely on function return values and the values of output variable(s) in the function arguments.
  4. In other words, program performance will be tested solely via the public interface presented to the calling application. (I will write the calling application.)

1 Question 1 no code

- Consider a yield curve with the two initial tenors $t_{0.5} = 0.5$ and $t_{1.0} = 1.0$.
- The corresponding spot rates are $r_{0.5} = 5.0 + \Delta r_{0.5}$ and $r_{1.0} = 5.0 + \Delta r_{1.0}$.
- The offsets $\Delta r_{0.5}$ and $\Delta r_{1.0}$ are obtained as follows:

  1. **Take the first 4 digits of your student id. Define:**
  $$\Delta r_{0.5} = \frac{\text{first 4 digits of your student id}}{10^4}\,. \tag{1.1}$$
  2. **Take the last 4 digits of your student id. Define:**
  $$\Delta r_{1.0} = \frac{\text{last 4 digits of your student id}}{10^4}\,. \tag{1.2}$$
  3. For example if your student id is 23054617, then
  $$\Delta r_{0.5} = 0.2305\,, \qquad \Delta r_{1.0} = 0.4617\,. \tag{1.3}$$

- **Calculate the interpolated spot rate r_{cfr} using constant forward rate interpolation for $t = 0.66$. State your answer as a percentage to 2 decimal places.**
- **(Optional/bonus) Prove that $5.0\% \le r_{\text{cfr}} \le 6.0\%$ for any student id.**

- **This was intended as a simple "gift question" which you would all pass easily.**
- **Nevertheless some of you managed to get it wrong and performed linear interpolation instead.**
- **Anyone who calculated linear interpolation received a straight F for this question.**
- **I have no sympathy.**
- **There were a few students who calculated cfr interpolation but made errors of algebra. They scored a B for this question.**
- **Those who answered this question correctly scored an A+ for this question.**
- **Nevertheless, the questions are not equally weighted, and some students who answered this question using linear interpolation obtained an averall score of A anwyay, for the full midterm (but not A+).**
- **The value of the interpolating fraction is**

$$\lambda = \frac{0.66 - 0.5}{1.0 - 0.5} = 0.32\,.$$

- **Then the cfr (constant forward rate) interpolated spot rate is given by**

$$r_{\rm cfr} = \frac{(1-\lambda)r_{0.5}t_{0.5} + \lambda r_{1.0}t_{1.0}}{t} = \frac{0.5\times(1-0.32)r_{0.5} + 1.0\times 0.32\,r_{1.0}}{0.66}\,.$$

- **Bonus: Since by construction** $0 \le \Delta r_{0.5} \le 1$ **and** $0 \le \Delta r_{1.0} \le 1$ **for any secid, then** $5.0 \le r_{0.5} \le 6.0$ **and** $5.0 \le r_{1.0} \le 6.0$**, the minimum values of** $r_{\rm cfr}$ **is**

$$r_{\rm cfr,\,min} = 5.0\,\frac{0.5\times(1-0.32) + 1.0\times 0.32}{0.66} = 5.0\,.$$

  The maximum values of $r_{\rm cfr}$ **is**

$$r_{\rm cfr,\,max} = 6.0\,\frac{0.5\times(1-0.32) + 1.0\times 0.32}{0.66} = 6.0\,.$$

2 Question 2 no code

- Suppose the market price of a stock is S_0 at time t_0.
- The stock does not pay dividends.
- The interest rate is r (a constant).
- All the options below have strike $K = 100$ and expiration time $T > t_0$.
- **You are also given that r and T are such that $e^{r(T-t_0)} < 100$.**

2.1 European call

- **Assume that the value of the stock price is in the interval $1 < S_0 < 100$.**
- The strike price of the option is $K = 100$.
- A European call trades today (time t_0) at a market price $= 1$.
- We formulate the following trading strategy:
  (a) buy the call, (b) short sell one share of stock, (c) save money in a bank.
- The initial value of our portfolio is zero.
- **Find a scenario where this strategy leads to a profit.**
- **Find a scenario where this strategy leads to a loss.**
- **Note: if we do not exercise the option, we must buy back the stock, to cover the short sale.**

- **At time t_0, we buy the call and short sell the stock.**
- **It is a short sale because we begin with zero, hence we do not own any stock before t_0.**
- **We pay 1 to buy the call and receive cash S_0 by selling the stock.**
- **Hence our initial cash equals $S_0 - 1$. This is a positive number because $S_0 > 1$.**
- **At the expiration time T, the cash will compound to $(S_0 - 1)e^{r(T-t_0)}$.**
- **Hence this is the amount of cash available to us to pay for expenses at time T.**
- ***Here is a subtlety. It is impossible for the value of $(S_0 - 1)e^{r(T-t_0)}$ exceed 100.***
  1. **Why is this?**
  2. **The answer comes from put call parity.**
  3. **The stock pays no dividends, hence put–call parity at time t_0 says**
  $$c - p = S_0 - Ke^{-r(T-t_0)}\,.$$
  4. **Rearrange terms to obtain**
  $$S_0 - c = Ke^{-r(T-t_0)} - p\,.$$
  5. **Now $p \geq 0$ hence $S_0 - c \leq Ke^{-r(T-t_0)}$. Multiply through by $e^{-r(T-t_0)}$ to obtain**
  $$(S_0 - c)e^{r(T-t_0)} \leq K\,.$$
  6. **Substitute $c = 1$ and $K = 100$ to obtain**
  $$(S_0 - 1)e^{r(T-t_0)} \leq 100\,.$$
- **Hence if the option is in the money at the expiration time T, and we exercise the option, *our cash amount of $(S_0 - 1)e^{r(T-t_0)}$ is not enough to pay the strike price.***
- ***Hence all scenarios where the option expires in the money always lead to loss.***
- **Several students submitted solutions to say that if $S_T > K$, then if $(S_0 - 1)e^{r(T-t_0)} > K(= 100)$, we exercise the option, pay the strike price, receive the stock and use it to close out our short stock position, leading to a profit of $(S_0 - 1)e^{r(T-t_0)} - K > 0$. Such a scenario is impossible.**
- **I did not penalize students who submitted the above as a "profit" scenario. In fact some of them scored A+ overall. I do not take off points for picky details. This is a hard class for you and I do not ask trick questions. The point is to make you learn, not to make you feel miserable.**
- **But it shows that finding the "general solution" to this question is complicated.**
- **Some students forgot that we must close out our short stock position, and that was a mistake which did cost points.**

Scenario $S_T < (S_0 - 1)e^{r(T-t_0)}$

- **Such a scenario for S_T is always possible, because the right hand side is positive.**
- **Since $(S_0 - 1)e^{r(T-t_0)} \le K$, it means the option is out of the money.**
- **We buy the stock at the price S_T and close out our short stock position.**
- **This yields a profit of $(S_0 - 1)e^{r(T-t_0)} - S_T$.**
- **Therefore it is possible for the trading strategy in this question to yield a profit.**

Scenario $S_T = (S_0 - 1)e^{r(T-t_0)}$

- **We buy the stock at the price S_T and close out our short stock position.**
- **This scenario is break even, there is no profit or loss.**

Scenario $(S_0 - 1)e^{r(T-t_0)} < S_T < K$

- **Such a scenario for S_T can exist, because $(S_0 - 1)e^{r(T-t_0)} \le K$.**
- **Since $S_T < K$, it means the option is out of the money.**
- **We buy the stock at the price S_T and close out our short stock position.**
- **This yields a loss of $S_T - (S_0 - 1)e^{r(T-t_0)}$.**
- **Hence we have demonstrated that the trading strategy in this question guarantees neither profit nor loss.**

Scenario $(S_0 - 1)e^{r(T-t_0)} \le K \le S_T$

- **Such a scenario for S_T obviously is always possible, because $(S_0 - 1)e^{r(T-t_0)} \le K$.**
- **Since $S_T \ge K$, it means the option is in the money.**
- **We exercise the option.**
- **We buy the stock, *but pay the strike* K, and close out our short stock position.**
- **This yields a loss of $K - (S_0 - 1)e^{r(T-t_0)}$.**
- **Note that if we did not exercise the option, we would have to pay a higher price of S_T to buy the stock, which would lead to a bigger loss.**
- **Hence exercising the option caps our loss.**
- **Hence both our profit and loss amounts are capped.**
- **We do not have unlimited exposure.**

Impossible scenario

- **What would happen if $(S_0 - 1)e^{r(T-t_0)} > K$?**
- **Then if $S_T \geq K$, we exercise the option and have enough cash to pay the strike.**
- **Conversely, if $S_T < K$, we buy the stock and we have enough cash to do so.**
- **In both cases, we use the stock to close out our short stock position.**
- **In both cases, we have money left over.**
- ***Since we can calculate if the value of $(S_0 - 1)e^{r(T-t_0)}$ is $> K$ at time t_0, we can implement this trading strategy (buy the call and short the stock) at time t_0 and obtain a guaranteed profit at time T, i.e. arbitrage.***
- **This is an impossible scenario. It violates the rational option pricing inequalities.**
- **Obviously the students who submitted this scenario did not notice it is impossible.**
- **I accepted their solutions.**

Non-impossible scenario

- **One student asked "if we own an option, can we refuse to exercise it?"**
- **It is not a foolish question.**
- **Suppose we believe (at time t_0) that the stock price will go down at a later time.**
- **Hence we short the stock at the price S_0 at time t_0.**
- ***But what if the stock price increases instead, and goes to a very high level?***
- **We eventually have to buy the stock and close out our short stock position.**
- **By shorting the stock, *we have a risk exposure to an unlimited amount of loss.***
- **Buying a call option *caps our maximum loss.***
- **Hence look at it this way:**
  1. **Our real trading strategy is to take a gamble that the stock price will decline at $t > t_0$.**
  2. **Buying the call option is an insurance protection to cap our loss in case things go wrong.**
  3. **Buying the call costs some money, which is effectively the cost of that insurance.**
  4. ***And if the stock price declnes, as was hoped for, buy the stock at a low price $S_t < (S_0-1)e^{r(t-t_0)}$, close out our short stock position, and throw away the option.***
  5. **A better idea is to sell the option instead of throwing it away.**
  6. **As the student noted, just because we own an option, does not mean we have to exercise it.**
  7. **If the insurance protection is no longer required, *sell the option.***
- **Hence the general solution for a profit scenario is: find a time t (where $t_0 < t \le T$) such that $S_t < (S_0 - 1)e^{r(t-t_0)}$ (to a sufficient degree to satisfy our trading goal), close out our short stock position, *and sell the option.* If no such time occurs, then at time T we buy the stock anyway, for a loss, and the option *caps our loss.***
- **One student was puzzled that the loss scenario in this trading strategy occurs when the option is in the money, and the profit scenario occurs when the option is out of the money. The naïve belief is that buying an option yields a profit when the option expires in the money.**
- **However, that may not be the reason to buy an option. The option may be a protection to guard against unlimited loss. The real focus of the investor may be the stock, not the option.**
- **Of course one can also purchase a put option at time t_0. There is more than one way to trade on a belief that the stock price will decline.**

2.2 American call

- **Assume that the value of the stock price is in the interval $1 < S_0 < 100$.**
- The strike price of the option is $K = 100$.
- An American call trades today (time t_0) at a market price $= 1$.
- We formulate the following trading strategy:
  (a) buy the call, (b) short sell one share of stock, (c) save money in a bank.
- The initial value of our portfolio is zero.
- **Find a scenario where this strategy leads to a profit.**
- **Find a scenario where this strategy leads to a loss.**
- **Note: if we do not exercise the option, we must buy back the stock, to cover the short sale.**

- **The only difference in the analysis for an American call is the case of early exercise.**
- **If the stock pays no dividends, the value of an American call equals the value of a European call.**
- **Hence it is still true that** $(S_0 - 1)e^{r(T-t_0)} \le K(= 100)$**.**
- **At earlier times** $t_0 < t < T$**, therefore also** $(S_0 - 1)e^{r(t-t_0)} \le Ke^{-r(T-t)}$**.**
- **Hence if the option is in the money at any time** $t_0 < t < T$**, any attempt to exercise the option early will lead to loss.**
- **The amount of the loss will be** $K - (S_0 - 1)e^{r(t-t_0)}$**.**

2.3 European put

- **Assume that the value of the stock price is in the interval $100 < S_0 < 200$.**
- The strike price of the option is $K = 100$.
- A European put trades today (time t_0) at a market price $= 1$.
- We formulate the following trading strategy:
  (a) buy the put, (b) borrow money from a bank.
- The initial value of our portfolio is zero.
- **Find a scenario where this strategy leads to a profit.**
- **Find a scenario where this strategy leads to a loss.**
- **Note: if we exercise the put option, we must buy the stock at the market price on the exercise date, to deliver to the writer of the put.**

- **We are hoping the stock price will decline, but now we trade by buying a put.**
- **We pay 1 to buy the put, so we borrow 1 from the bank at time t_0.**
- **There is no impossible scenario here.**
- **We know that for a European put, we must have $p \le \mathbf{PV}(K)$.**
- **We are given that $e^{r(T-t_0)} < 100$, hence**

$$\mathbf{PV}(K) = Ke^{-r(T-t_0)} = 100\, e^{-r(T-t_0)} > 1\,.$$

- **We are given $p = 1$ so indeed $p < \mathbf{PV}(K)$.**
- **At the expiration time T, the loan will compound to $e^{r(T-t_0)}$.**
- **The loan must be repaid at the time T.**

Scenario $S_T < K - e^{r(T-t_0)}$

- **This scenario is always possible, because $K = 100$ so $K - e^{r(T-t_0)} > 0$.**
- **The put option is in the money.**
- **Purchase one share of stock at the price S_T (borrow cash equal to S_T).**
- **Exercise the put, deliver the stock to the option writer, receive cash of $K(=100)$, repay the loan to buy the option and the loan to buy the stock.**
- **This scenario yields a profit of $K - e^{r(T-t_0)} - S_T > 0$.**

Scenario $S_T = K - e^{r(T-t_0)}$

- **This is break even.**
- **The put option is in the money.**
- **Purchase one share of stock at the price S_T (borrow cash equal to S_T).**
- **Exercise the put, deliver the stock to the option writer, receive cash of $K(=100)$, repay the loan to buy the option and the loan to buy the stock.**
- **This scenario a break even $K - e^{r(T-t_0)} - S_T = 0$.**

Scenario $K - e^{r(T-t_0)} < S_T \leq K$

- **This scenario is always possible.**
- **The put option is in the money.**
- **Purchase one share of stock at the price S_T (borrow cash equal to S_T).**
- **Exercise the put, deliver the stock to the option writer, receive cash of $K(=100)$, repay the loan to buy the option and the loan to buy the stock.**
- **This scenario yields a loss of $S_T + e^{r(T-t_0)} - K$.**
- **The maximum loss is when $S_T = K$, i.e. $e^{r(T-t_0)}$.**

Scenario $S_T \geq K$

- **This scenario is always possible.**
- **The put option is out of the money and expires worthless.**
- **We must repay the loan, which leads to a loss of $e^{r(T-t_0)}$.**
- **Hence our maximum loss is $e^{r(T-t_0)}$.**

Summary

- **If we believe the stock price will decline, purchasing a put option caps our loss at $e^{r(T-t_0)}$.**
- **In general, if the option price is p_0 at t_0, our loss is capped at $p_0 e^{r(T-t_0)}$.**

2.4 American put

- **Assume that the value of the stock price is in the interval $100 < S_0 < 200$.**
- The strike price of the option is $K = 100$.
- An American put trades today (time t_0) at a market price $= 1$.
- We formulate the following trading strategy:
  (a) buy the put, (b) borrow money from a bank.
- The initial value of our portfolio is zero.
- **Find a scenario where this strategy leads to a profit.**
- **Find a scenario where this strategy leads to a loss.**
- **Note: if we exercise the put option, we must buy the stock at the market price on the exercise date, to deliver to the writer of the put.**

- **The only difference in the analysis for an American put is the case of early exercise.**
- **If at any time $t_0 < t < T$, the stock price declines to a value S_t such that $S_t < K - e^{r(t-t_0)}$, then we can choose to exercise the option early at that time t.**
- **The put option is in the money.**
- **Purchase one share of stock at the price S_t (borrow cash equal to S_t).**
- **Exercise the put, deliver the stock to the option writer, receive cash of $K(= 100)$, repay the loan to buy the option and the loan to buy the stock.**
- **This scenario yields a profit of $K - e^{r(t-t_0)} - S_t > 0$.**
- **Note that just because $S_t < K - e^{r(t-t_0)}$, we do not *have* to exercise the put option.**
- **We can wait to see if we can obtain a larger profit.**

3 Question 3 no code

3.1 European call

- The current time is $t_0 = 0$.
- The value of the stock price today (time t_0) is $S_0 = 98$.
- The stock does not pay dividends.
- The interest rate is 5%.
- A European call option trades today (time t_0) at a market price $= 3$.
- The strike price of the option is $K = 100$.
- The option expiration time is 1 year from today ($T - t_0 = 1$).
- **We begin trading with a position of zero (no cash, stock, option, etc.).**
- We formulate the following trading strategy:
  (a) sell the call, (b) buy one share of stock, (c) borrow money from a bank.
- The initial value of our portfolio is zero.
- **Find a scenario where this strategy leads to a profit.**
- **Find a scenario where this strategy leads to a loss.**

- **We begin with zero, as stated.**
- **We sell the call and receive 3. We buy the stock and pay $S_0 = 98$.**
- **Hence overall we borrow money in the amount of $98 - 3 = 95$.**
  1. **Some students forgot that we receive money for selling the call and said the loan amount is 98. Bad mistake, which cost points.**
  2. **One student calculated $98 - 3 = 96$. *Don't go into business for yourself, is all I can say.***
- **At the expiration time, the loan amount compounds to:**

$$L = \textbf{loan amount at expiration} = 95\, e^{r(T-t_0)} = 95\, e^{0.05} \simeq 99.87\,.$$

- **At expiration, we must repay the loan.**
  1. **Some students forgot that we use continuous compounding and calculated**

$$L_{\rm alt} = \textbf{loan amount at expiration} = 95\,(1 + 0.05) = 99.75\,.$$

  2. **I accepted such a solution. I do not take points off for picky details.**

3.1.1 Scenario $S_T \geq K(= 100)$

- **The option expires in the money.**
- **The option holder exercises the option.**
- **Some students said that *we exercise the option.* Bad mistake, which cost points.**
- **Remember that we are *short* the option, we are *not the option holder.***
- **We deliver our share of stock and receive cash in the amount $K(= 100)$.**
- **We use the cash to repay our loan amount of 99.87.**
- **This scenario yields a profit of $100 - 99.87 \simeq 0.13$.**

3.1.2 Scenario $L < S_T < K(= 100)$

- **The option expires out of the money.**
- **The option holder does not exercise the option, it is worthless.**
- **We sell our share of stock and receive cash in the amount S_T.**
- **We use the cash to repay our loan amount of 99.87.**
- **This scenario yields a profit of $S_T - 99.87 > 0$.**

3.1.3 Scenario $S_T = L$

- **This is similar to the above but is break even.**
- **The option expires out of the money, not exercised, worthless.**
- **We sell our share of stock and receive cash in the amount S_T.**
- **We use the cash to repay our loan amount of 99.87.**
- **This scenario yields break even $S_T - 99.87 = 0$.**

3.1.4 Scenario $S_T < L$

- **This is similar to the above but leads to a loss.**
- **The option expires out of the money, not exercised, worthless.**
- **We sell our share of stock and receive cash in the amount S_T.**
- **We use the cash to repay our loan amount of 99.87, but the money is not enough.**
- **This scenario yields a loss of $99.87 - S_T$.**

3.2 European put

- The current time is $t_0 = 0$.
- The value of the stock price today (time t_0) is $S_0 = 99$.
- The stock does not pay dividends.
- The interest rate is 5%.
- A European put trades today (time t_0) at a market price = 2.
- The strike price of the option is $K = 100$.
- The option expiration time is 1 year from today ($T - t_0 = 1$).
- **We begin trading with a position of zero (no cash, stock, option, etc.).**
- We formulate the following trading strategy:
  (a) sell the put, (b) sell one share of stock, (c) save money in a bank.
- The initial value of our portfolio is zero.
- **Find a scenario where this strategy leads to a profit.**
- **Find a scenario where this strategy leads to a loss.**

- We begin with zero, as stated.
- We sell the put and receive 2. We sell the stock and receive $S_0 = 99$.
- Hence overall we receive money in the amount of $2 + 99 = 101$, which we save in a bank.
   1. Almost all students got this right.
   2. I thought it was strange that some students knew to add $2 + 99 = 101$ for the put, but forgot to subtract $98 - 3 = 95$ for the call.
- At the expiration time, the savings in the bank compounds to:

$$\text{Sav} = \text{savings at expiration} = 101\, e^{r(T-t_0)} = 101\, e^{0.05} \simeq 106.1784\,.$$

- At expiration, we use this money to pay expenses.

3.2.1 Scenario $S_T \leq K(= 100)$

- The option expires in the money.
- The option holder exercises the option.
- Once again, students who said we exercise the option lost points.
- We receive one share of stock and use it to close out our short stock position.
- We pay money to the option holder in the amount of $K(= 100)$.
- This scenario yields a profit of $106.1784 - 100 \simeq 6.1784$.

3.2.2 Scenario $K(= 100) < S_T <$ Sav

- The option expires out of the money.
- The option holder does not exercise the option, it is worthless.
- We use our cash to buy one share of stock to close out our short stock position.
- Students who forgot that we must close out our short stock position lost points.
- This scenario yields a profit of $106.1784 - S_T > 0$.

3.2.3 Scenario $S_T =$ Sav

- This is similar to the above but is break even.
- The option expires out of the money, not exercised, worthless.
- We use our cash to buy one share of stock to close out our short stock position.
- This scenario yields break even $106.1784 - S_T = 0$.

3.2.4 Scenario $S_T >$ Sav

- **This is similar to the above but leads to a loss.**
- **The option expires out of the money, not exercised, worthless.**
- **We use our cash to buy one share of stock to close out our short stock position.**
- **However we do not have enough money to pay to buy the stock.**
- **This scenario yields a** loss **of** $S_7 - 106.1784$**.**

4 Question 4 no code

- **Ignore interest rate compounding for all profit/loss calculations in this question.**
- A stock trades today ($t_0 = 0$) at a price of $S_0 = 100.0$.
- A futures contract on the stock trades today at a price $F_0 = 103.25$.
- The futures contract expiration date is 5 days from today.
- Every day for $t_i = i$, $i = 1, 2, 3, 4, 5$, the stock price is S_1, S_2, S_3, S_4, S_5.
- The corresponding futures price every day is F_1, F_2, F_3, F_4, F_5.
- On the expiration day, $F_5 = S_5$ (the futures price converges to the stock price).
- **The stock pays a dividend of** 0.07 **on day 3.**
- Here is a list of the stock and futures prices, for $i = 0, 1, 2, 3, 4, 5$.

i	S_i	F_i	dividend
0	100.0	103.25	
1	98.75	101.55	
2	S_2	F_2	
3	S_3	F_3	0.07
4	S_4	F_4	
5	S_5	F_5	

- **The values of $S_2, \ldots, S_5$ are arbitrary, but satisfy the following inequalities:**

$$98.75 < S_2 < S_3 < S_4 < S_5\,. \tag{4.1}$$

- **The values of $F_2, \ldots, F_5$ are arbitrary, but satisfy the following inequalities:**

$$101.55 < F_2 < F_3 < F_4 < F_5 (= S_5)\,. \tag{4.2}$$

See next page

4.1 Investor A

- **Investor A goes long one share of stock on day 0.**
- Investor A sells the stock on day 5.
- **Calculate (or state) the money paid/received by A every day, starting from day 0, until A's portfolio is closed out.**
  (Note that in some cases your answer may be a formula not a dollar number.)
- **Calculate the total profit/loss for A after selling the stock.**
- **State on which day A makes that profit/loss.**

4.2 Investor B

- **Investor B goes long one futures contract on day 1.**
- Investor B holds the futures contract to expiration.
- **Calculate the money paid/received every day in B's mark to market account, until B's portfolio is closed out.**
  (Note that in some cases your answer may be a formula not a dollar number.)
- **Calculate the total money paid by B after closing the futures contract.**
- **State what B receives in exchange for closing the futures contract.**

4.3 Investor C

- **Investor C goes short a forward contract on day 0.**
- The forward price is $F_{\text{fwd}} = 103.25$ and the expiration time is 5 days.
- **From the data on day 1, it is possible for C to lock in a guaranteed profit?**
  1. **If yes, state the strategy to lock in a guaranteed profit.**
  2. **If no, explain why not.**
- **State all the trades performed on the day C's portfolio is closed out.**
- **Calculate the total profit/loss for C (if any).**
- **State on which day C makes that profit/loss (if any).**

4.4 Investor D

- **Investor D goes long one futures contract on day 1.**
- Investor D sells the futures contract on day 3.
- **Calculate the money paid/received every day in D's mark to market account, until D's portfolio is closed out.**
  (Note that in some cases your answer may be a formula not a dollar number.)
- **Calculate the total profit/loss for D after selling the futures contract.**
- **State on which day D makes that profit/loss.**
- **State the trades (stock/cash/futures) which happen for D on day 5 (futures expiration).**

Investor A

- **A pays 100 on day 0.**
- **A owns the stock hence collects the dividend on day 3, cash =** 0.07**.**
- **A sells the stock on day 5, at the price** S_5**.**
- **There are no other cashflows for A.**
  1. **Some students calculated a gain/loss for A every day.**
  2. **While it is true that the value of the stock changes every day, there are no actual cashflows (except the dividend).**
  3. **But the students (mostly) calculated the correct final result.**
- **Because we neglect interest rate compounding, A makes a profit on day 5 of**

$$\textbf{profit} = S_5 + 0.07 - 100 = S_5 - 99.93\,.$$

- **This will be a loss if** $S_5 < 99.93$**.**
- **That was the answer I had in mind and I accepted it as correct.**
- **I forgot (but some students pointed out) that** $S_5 = F_5$ **and the information in the question says** $F_5 \geq 101.55$**.**
- **Hence in actuality A makes a profit. The minimum profit amount is** $101.55-99.3 = 1.62$**.**
- **But any student who derived** $S_5 - 99.93$ **received full credit.**

Investor B

- **B begins trading on day 1.**
- **The money paid in B's mark to market account every day is**

day 1	**n/a**
day 2	$101.55 - F_2$
day 3	$F_2 - F_3$
day 4	$F_3 - F_4$
day 5	$F_4 - F_5$

- **B is not a shareholder, therefore does not receive the dividend.**
- **Students who said B receives the dividend lost points.**
- **The total money paid by** B **is**

$$\textbf{money paid by B} = S_5 + (F_4 - F_5) + (F_3 - F_4) + (F_2 - F_3) + (F_1 - F_2) = F_1 = 101.55\,.$$

- **Therefore on day 5, B pays a total amount of** $F_1 = 101.55$**.**
  1. **I have explained many times how to calculate this.**
  2. **Students who said B starts trading on day 0 and overall pays** $F_0 = 103.25$ **lost points.**
- **On day 5, B takes delivery of one share of stock.**

Investor C

- **This question was intended as a puzzle, but I bungled it somewhat.**
- **First of all, note that a forward is a private contract, hence it cannot be bought and sold publicly (unlike a futures).**
- **Hence C must hold the short forward position till expiration (day 5).**
- **The question/puzzle is: what can C do on day 1?**
  1. **The answer I had in mind is that C goes long a futures contract on day 1.**
  2. **Then, on day 5, C nets the forward and futures against each other.**
  3. **On day 5, C pays 101.55 (overall) for the futures and receives one share of stock.**
  4. **On day 5, C receives 103.25 for the forward and delivers one share of stock.**
  5. **Hence the stock cancels out and C makes a profit of $103.25 - 101.55 = 1.7$.**
  6. **C makes the profit on day 5.**
- **However, I bungled this question.**
  1. **Several students pointed out that because we neglect interest rate compounding, C can buy one share of stock on day 0 itself (borrow 100). Collect the dividend on day 3. Then on day 5, receive cash 103.25, deliver the stock, repay the loan of 100 and make a profit of $0.07 + 103.25 - 100 = 3.32$.**
  2. **Other students pointed out that because we neglect interest rate compounding, C can buy one share of stock on day 1 (borrow 98.75). Collect the dividend on day 3. Then on day 5, receive cash 103.25, deliver the stock, repay the loan of 98.75 and make a profit of $0.07 + 103.25 - 98.75 = 4.57$.**
  3. **All such answers were accepted as correct.**
- **I should have realized that if we neglect interest rate compounding, then the numbers I displayed have arbitrage opportunities.**

Investor D

- **D goes long the a futures contract on day 1 and sells (unwinds) the position on day 3.**
- **Therefore D's position is closed out on day 3.**
- **This is also a calculation I have explained many times.**
- **The money paid in D's mark to market account every day is**

day 1	**n/a**
day 2	$101.55 - F_2$
day 3	$F_2 - F_3$

- **D is not a shareholder, therefore does not receive the dividend.**
- **Students who said D receives the dividend lost points.**
- **D makes a profit/loss on day 3.**
- **The total money paid by D is**

$$\textbf{money paid by D} = (F_2 - F_3) + (F_1 - F_2) = F_1 - F_3 = 101.55 - F_3\,.$$

- **Therefore D makes a profit if $F_3 > 101.55$ and D makes a loss if $F_3 < 101.55$.**
- **The profit/loss money comes from the money in the mark to market account.**
- **On day 5, nothing happens to D.**
- **D's position was closed out on day 3, hence on day 5 D no longer has a position in the futures.**

5 Question 5 no code

- **For each case below, you are the trader on the opposite side of the trade with me at time $t_0(=0)$.**
- **For each case below, state the trade you will perform with me at time $t_0(=0)$.**
- **For each case, you must explain the reason for your trading decision.**
- **Note:**
  You are not permitted to say that you do not want to trade.
  You must go either long or short.
- In all cases, both you and I begin with a portfolio of zero (no cash, no stock, options, etc.).
  1. I also act as the bank.
  2. Money required to buy things is borrowed from me. Interest must be paid on the loan.
  3. Money received by selling things is loaned to me. I will pay you interest on the savings.
  4. The borrow and lend interest rates are equal. Both are equal to the risk-free rate.
  5. There is no limit on the quantity to buy or sell anything, including short selling of stock.
- In all the cases below:
  1. The current value of the stock price is $S_0 = 100$.
  2. The stock does not pay dividends.
  3. For questions involving a stock index, the current value of the index is 1000 points.
  4. The stock index has a continuous dividend yield of 1.5%.
  5. The interest rate is 5% (borrow and lend rates are equal).
  6. All the options have the same expiration time of 1 year.
  7. All the forward/futures contracts have the same expiration time of 1 year.

See next page

5.1

- Recall the stock pays no dividends, the interest rate is 5% and the expiration time is 1 year.
- A portfolio consists of:
  (a) short one European call,
  (b) long one European put,
  (c) long one forward contract.
- **Both the call and the put have the same strike price of 99.**
- **The forward price also equals 99.**
- The portfolio trades at a price of 1.
- **Note the following:**
  1. The portfolio is a package.
  2. The above statement "trades at a price of 1" means the price of **the entire package.**
  3. **The "price" is the price of the entire package.**
  4. ***You must go long/short the entire package.***
  5. You cannot say "I will go long only the forward contract but I do not want the options" etc.
  6. **The same policy applies to all the other portfolios below.**

- **State which position (long/short) you will take, to trade this portfolio with me at time t_0.**
- **Explain the reason for your trading decision.**

Solution 5.1

- **The payoff of the forward at expiration is $S_T - F = S_T - 99$.**
- **The stock does not pay dividends, so we use put-call parity without dividends.**
- **Using put-call parity, the value of short call and long put at expiration is**

$$-c + p = -(S_T - K) = -S_T + 99\,.$$

- **Hence the value of the portfolio at expiration is**

$$-c + p + F = -(S_T - K) + (S_T - F) = K - F = 99 - 99 = 0\,.$$

- **The portfolio is worth zero at expiration, hence it is worth zero at time t_0.**
- **Therefore the portfolio is overpriced.**
- **Short the portfolio.**
- **Sell the portfolio for 1.**
- **The money compounds to $1 \times e^{r(T-t_0)} = e^{0.05} \simeq 1.051271$ at time T.**

5.2

- Recall the stock pays no dividends, the interest rate is 5% and the expiration time is 1 year.
- A portfolio consists of:
  (a) long one European call,
  (b) short one European put,
  (c) short one forward contract.
- **Both the call and the put have the same strike price of $K = 101$.**
- **The forward price is $F = K + 1 = 102$.**

5.2.1

- The portfolio trades at a price of 1.
- **State which position (long/short) you will take, to trade this portfolio with me at time t_0.**
- **Explain the reason for your trading decision.**

5.2.2

- The portfolio trades at a price of 0.9.
- **State which position (long/short) you will take, to trade this portfolio with me at time t_0.**
- **Explain the reason for your trading decision.**

5.2.3

- The portfolio trades at a price of 1.1.
- **State which position (long/short) you will take, to trade this portfolio with me at time t_0.**
- **Explain the reason for your trading decision.**

Solution 5.2

- **The payoff of the forward at expiration is $S_T - F$.**
- **The stock does not pay dividends, so we use put-call parity without dividends.**
- **Using put-call parity, the value of long call and short put at expiration is**

$$c - p = S_T - K\,.$$

- **Hence the value of the portfolio at expiration is**

$$c - p - F = (S_T - K) - (S_T - F) = -K + F = -101 + 102 = 1\,.$$

- **The portfolio is worth 1 at expiration.**
- **Hence it is worth PV(1) at time t_0, i.e.** $1 \times e^{-r(T-t_0)} \simeq 0.951229$**.**

Case 5.2.1

- **The portfolio trades at 1 at time t_0, which ia greater than PV(1).**
- **Therefore the portfolio is overpriced.**
- **Short the portfolio.**
- **Sell the portfolio for 1.**
- **The cash will compound to** $1 \times e^{r(T-t_0)} = e^{0.05} \simeq 1.051271$ **at time T.**
- **Pay 1 at time T to close out the portfolio.**
- **This yields a net profit of** $e^{0.05} - 1 \simeq 0.051271$ **at time T.**

Case 5.2.2

- **The portfolio trades at 0.9 at time t_0, which ia less than PV(1).**
- **Therefore the portfolio is underpriced.**
- **Long the portfolio.**
- **Buy the portfolio for 0.9 (borrow 0.9 from bank).**
- **The loan will compound to** $0.9 \times e^{r(T-t_0)} = 0.9e^{0.05} \simeq 0.946144$ **at time T.**
- **Receive 1 at time T to close out the portfolio.**
- **This yields a net profit of** $1 - 0.9e^{0.05} \simeq 0.053856$ **at time T.**

Case 5.2.3

- **The portfolio trades at 1.1 at time t_0, which ia greater than PV(1).**
- **Therefore the portfolio is overpriced.**
- **Short the portfolio.**
- **Similar logic to above, the profit at time T is** $1.1e^{0.05} - 1 \simeq 0.156398$**.**

5.3

- Recall the stock pays no dividends, the interest rate is 5% and the expiration time is 1 year.

5.3.1

- A portfolio consists of:
  (a) long one **European** call with strike 99.5,
  (b) long one **European** put with strike 100.5.
- The portfolio trades at a price of 0.9.
- **State which position (long/short) you will take, to trade this portfolio with me at time t_0.**
- **Explain the reason for your trading decision.**

5.3.2

- A portfolio consists of:
  (a) long one **American** call with strike 99.5,
  (b) long one **American** put with strike 100.5.
- The portfolio trades at a price of 0.94.
- **State which position (long/short) you will take, to trade this portfolio with me at time t_0.**
- **Explain the reason for your trading decision.**
- **If you buy the portfolio, state the trades you will perform with it and when you will execute those trades.**

5.3.3

- A portfolio consists of:
  (a) long one **American** call with strike 99.5,
  (b) long one **American** put with strike 100.5.
- The portfolio trades at a price of 0.98.
- **State which position (long/short) you will take, to trade this portfolio with me at time t_0.**
- **Explain the reason for your trading decision.**
- **If you buy the portfolio, state the trades you will perform with it and when you will execute those trades.**

Solution 5.3

- **A graph of the payoff of the portfolio is shown in Fig. 1.**
- **For the case 5.3.1 (European options) it applies only at expiration.**
- **For the cases 5.3.2 and 5.3.3, (American options) it applies at all times.**
- **The payoff function is**

$$\text{payoffQ5.3} = \max(S - 99.5, 0) + \max(100.5 - S, 0)\,.$$

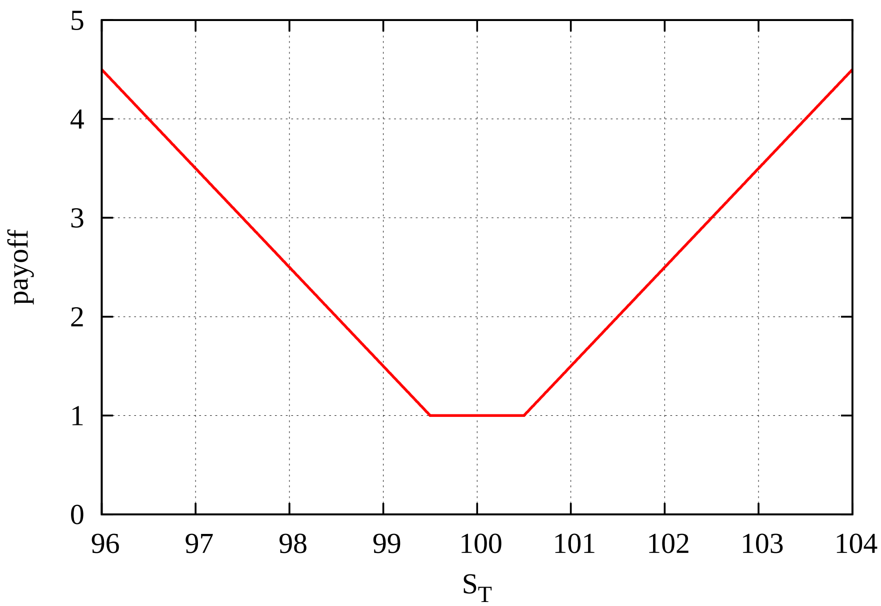

Figure 1: Graph of the terminal payoff of the portfolio in Question 5.3.

Case 5.3.1

- **The portfolio is worth a minimum value of 1 at expiration.**
- **Therefore it is worth a minimum value of PV(1) at time t_0, i.e.** $1 \times e^{-r(T-t_0)} = e^{-0.05} \simeq 0.951229$**.**
- **The portfolio trades at a price of 0.9, hence it is underpriced.**
- **Long the portfolio.**
- **Borrow cash today of 0.9, at expiration the loan amount will compound to** $0.9e^{0.05} \simeq 0.946144$**.**
- **The portfolio always yields a payoff more than this at expiration.**
- **The minimum profit is** $1 - e^{0.05} \simeq 0.053856$**.**

Case 5.3.2

- **The options are American, hence the portfolio can be exercised for a minimum value of 1 at any time.**
- **The portfolio trades at a price of 0.94, hence it is underpriced.**
- **Long the portfolio.**
- **Borrow cash today of 0.94.**
- **At expiration the loan amount will compound to $0.94e^{0.05} \simeq 0.988195$, which is less than 1.**
- **Hence we observe the stock price movements in the time interval $t_0 \leq t \leq T$, and if the payoff is to our liking (large enough to satisfy us), we can exercise at any time in the interval $t_0 \leq t \leq T$.**

Case 5.3.3

- **The options are American, hence the portfolio can be exercised for a minimum value of 1 at any time.**
- **The portfolio trades at a price of 0.98, hence it is underpriced.**
- **Long the portfolio.**
- **Borrow cash today of 0.98.**
- **However, in this case the loan amount at expiration will compound to $0.98e^{0.05} \simeq 1.030246$, which is *more than 1.***
- **Hence we must exercise this portfolio immediately.**
- **If we wait, the loan amount may increase to more than 1 and we might incur a loss.**
- **The profit if we exercise immediately is $1 - 0.98 = 0.02$.**
- **In principle, we can wait to exercise, as long as the loan amount is less than 1, to see if the payoff increases and our profit increases.**

5.4

- A stock index has a current value of 1000 index points.
- The interest rate is 5% and the index has a dividend yield of 1.5%.
- The volatility of the index is 30%.

5.4.1

- An American call option on the stock index has a strike of 920 and expiration of 1 year.
- The option is **cash settled** with a multiplier of $1 for every index point that the option is in the money.
- The option is currently trading at a value of 65 index points.
  - **That means, if you buy the option you must pay $65.**
  - **If you sell the option you receive $65.**
- **State which position (long/short) you will take, to trade this portfolio with me at time t_0.**
- **Explain the reason for your trading decision.**
- **If you buy the option, state the trades you will perform with it, and when you will execute those trades.**

5.4.2

- An American put option on the stock index has a strike of 1050 and expiration of 1 year.
- The option is **cash settled** with a multiplier of $1 for every index point that the option is in the money.
- The option is currently trading at a value of 45 index points.
  - **That means, if you buy the option you must pay $45.**
  - **If you sell the option you receive $45.**
- **State which position (long/short) you will take, to trade this portfolio with me at time t_0.**
- **Explain the reason for your trading decision.**
- **If you buy the option, state the trades you will perform with it, and when you will execute those trades.**

Solution 5.4

- **Both options are American and trading below intrinsic value.**
- **Buy the option in both cases and exercise immediately.**
- **The call will yield a profit of** $(S-K)-C = (1000-920)-65 = 80-65 = 15$**.**
- **The put will yield a profit of** $(K-S)-P = (1050-1000)-45 = 50-45 = 5$**.**
- **The interest rate, dividend yield, volatility and time to expiration are not relevant.**
- **I included that (irrelevant) information just to make the point that problems (or arbitrage opportunities) in real life do not come with labels attached ("this information is relevant but that is not").**
- **It is your responsibility to figure out what is relevant and what is not.**

6 Question 6 no code

- **Note: In this question, the underlying stock does not pay dividends.**
- The terminal payoff diagrams of five financial derivatives are plotted in Fig. 2.
- *All of them describe real derivatives which trade in the financial markets, except maybe (e).*
- The horizontal axis shows the stock price at expiration S_T, for $0 \le S_T \le 100$.
- The vertical axis shows the payoff at expiration $V(S_T)$, call them $(V_a, V_b, V_c, V_d, V_e)$.
- **For values $S_T > 100$, the functions $(V_a, V_b, V_c, V_d, V_e)$ continue in straight lines.**
- **Your task is to construct combinations of options today (i.e. at time t_0), whose payoffs at expiration (time T) match those displayed in Fig. 2.**
- **You must use combinations of European puts and calls only.**
- You are ***not permitted to use stock and/or cash (bonds).***
- The functions $(V_a, V_b, V_c, V_d, V_e)$ have the following values in Fig. 2:

S_T	V_a	V_b	V_c	V_d	V_e
0	40	20	40	20	20
20					20
40	40	20	40	20	40
60					40
80			80	80	
100	100	80	80	80	100

- **Note the following (very important):**
  1. **There may be more than one way to match the terminal payoffs.**
  2. **All correct solutions will be accepted, if they use not more than six different strikes.**
  3. You may need to use **fractional numbers of calls and/or puts.**
     (In real life the derivatives are expressed using fractional numbers of calls and puts.)
  4. Some of the (fractional) numbers of calls and/or puts **may be negative**.
     (In fact I think it is impossible to do them all using only positive numbers.)

- **Express your answers in the following form.**
  1. For each case (a)–(e), write your answer in the form:
$$V(S_0, t_0) = a_1 c(K_1) + b_2 p(K_2) + a_3 c(K_3) + b_4 p(K_4) + \cdots \tag{6.1}$$
  2. Here 'c' denotes a European call and 'p' denotes a European put.
  3. The coefficients $a_1, b_2, a_3, b_4, \ldots$ are constants (possibly fractional and/or negative).
  4. The parameters $K_1, K_2, \ldots$ are the strikes of the corresponding options.
  5. **You can leave out '(S_0, t_0)' from the argument list of the calls and puts, because it is the same for all.**

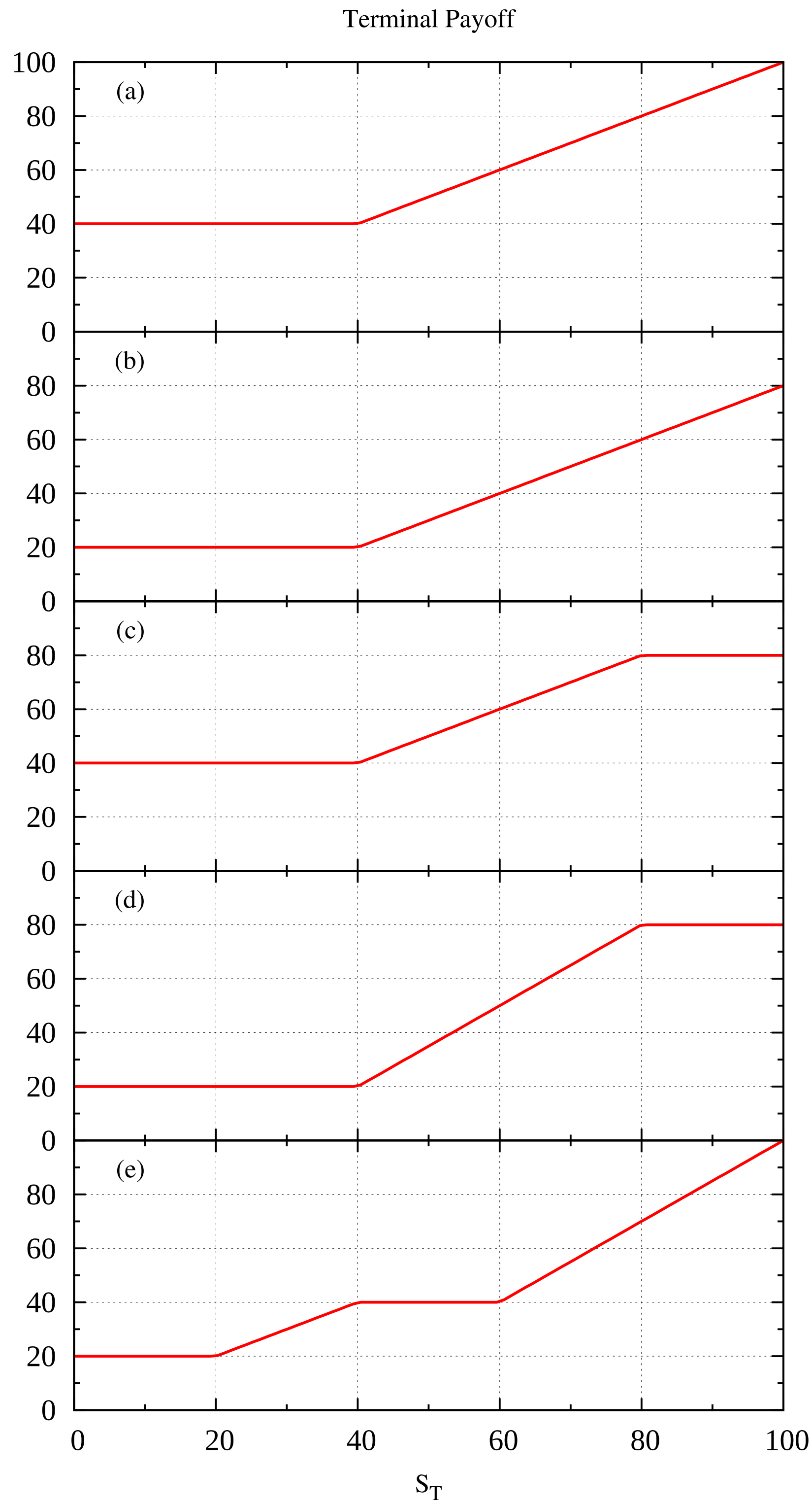

Figure 2: Graph of the terminal payoffs of derivatives listed in Question 6.

- **There are many possible answers.**
- **I received and accepted all of the following correct solutions.**

$$\begin{aligned}
V_a &= c(0) + p(40) \\
&= 2c(40) - c(80) + p(80) - p(40) \\
&= 2 * p(40) - 2 * p(20) + 2 * c(20) - c(40)
\end{aligned}$$

$$\begin{aligned}
V_b &= 0.5c(0) + 0.5p(40) + 0.5c(40) \\
&= p(40) + c(20) - p(20) \\
&= c(0) - c(20) + p(20) + c(40)
\end{aligned}$$

$$\begin{aligned}
V_c &= c(0) + p(40) - c(80) \\
&= 2p(40) - 2p(20) + 2c(20) - c(40) - c(80) \\
&= 3c(40) - 2c(60) - c(80) + 2p(60) - 2p(40)
\end{aligned}$$

$$\begin{aligned}
V_d &= 0.5c(0) + 0.5p(40) + 1.0c(40) - 1.5c(80) \\
&= c(20) - p(20) + p(40) + 0.5c(40) - 1.5c(80) \\
&= c(0) + p(20) - c(20) + 1.5c(40) - 1.5c(80) \\
&= c(0) - 0.5p(80) + 1.5p(40) - c(80) \\
&= 2c(40) - 2c(80) + 0.5p(80) - 0.5p(40)
\end{aligned}$$

$$\begin{aligned}
V_e &= c(0) + p(20) - c(40) + 1.5c(60) \\
&= 0.5c(0) + 0.5p(40) + c(20) - 1.5c(40) + 1.5c(60) \\
&= 2c(20) - p(20) + p(40) - 2c(40) + 1.5c(60) \\
&= c(20) - p(40) + 0.5c(60) + p(60) \\
&= 2c(40) - 0.5c(60) + 2p(60) - 3p(40) + p(20)
\end{aligned}$$

7 Question 7 *** added 4/9/2018 *** no code

- The market price of a stock is $S_0 = 100$ at time $t_0 = 0$.
- The stock does not pay dividends.
- **All the options below have strike $K = 100$ and expiration time $T - t_0 = 1$ year.**
- **The interest rate is r (a constant) and $e^{-r(T-t_0)} = 0.99$.**

7.1 American call

- An American call trades today (time $t_0 = 0$) at a market price $= 2$.
- We formulate the following trading strategy:
  (a) buy the call, (b) short sell one share of stock, (c) save money in a bank.
- The initial value of our portfolio is zero.
- **Find the general solution where this strategy leads to a profit, in the time interval $t_0 \le t \le T$.**
- **Find the general solution where this strategy leads to a loss, in the time interval $t_0 \le t \le T$.**
- ***** Your solution must take into account the possibility of early exercise. *****
- **It is not permitted to wait indefinitely to cover the short sale. The short stock position must be covered at a time $t \le T$.**

7.2 American put

- An American put trades today (time $t_0 = 0$) at a market price $= 2$.
- We formulate the following trading strategy:
  (a) buy the put, (b) buy one share of stock, (c) borrow money from a bank.
- The initial value of our portfolio is zero.
- **Find the general solution where this strategy leads to a profit, in the time interval $t_0 \le t \le T$.**
- **Find the general solution where this strategy leads to a loss, in the time interval $t_0 \le t \le T$.**
- ***** Your solution must take into account the possibility of early exercise. *****
- **It is not permitted to wait indefinitely and refuse to repay the loan. The loan must be repaid at a time $t \le T$.**

- **I assigned this question because I essentially gave away the solution to Question 6 in the worked examples in class.**
- **In Question 2, it was satisfactory to give individual scenarios of profit and loss.**
- **Here I asked for the general solution.**
- **I gave a specific value for $e^{-r(T-t_0)} = 0.99$ to avoid the complication "$(S-1)e^{r(T-t_0)}$ cannot exceed 100" in Question 2. That would be very hard for you to deduce.**
- **The solution of Question 2 essentially contains the general solution.**
- **What follows below is very similar.**

American call

- **At time t_0, we buy the call and short the stock and save cash $S_0 - C = 100 - 2 = 98$ in the bank.**
- **By the expiration time, the savings will compound to a value of $98/0.99 \simeq 98.9899$, which is less than $K = 100$.**
- **As in Question 2, this strategy is really shorting the stock and using the call option to cap the loss.**
- **The option will only be exercised in a scenario to cap our loss.**

Scenario $S_t < (S_0 - C)e^{r(t-t_0)}$

- **Because $(S_0 - C)e^{r(T-t_0)} < K$ at expiration, it follows that at any time $t \le T$, we also have $(S_0 - C)e^{r(t-t_0)} < K$.**
- **Hence the option is out of the money.**
- **We buy the stock at the price S_t and close out our short stock position.**
- **This yields a profit of $(S_0 - C)e^{r(t-t_0)} - S_t$.**
- **If $t < T$, the option is now useless and we sell the option (for extra profit).**
- ***We do not exercise the option at any time $t < T$.***
- **Note that we do not have to buy back the stock as soon as $S_t = (S_0 - C)e^{r(t-t_0)}$. We can wait and see if the stock price declines some more.**

Scenario $(S_0 - C)e^{r(T-t_0)} < S_T < K$ (expiration)

- **We have reached expiration and have not closed our short stock position.**
- **The option has expired out of the money.**
- **We buy the stock at the price S_T and close out our short stock position.**
- **This yields a loss of $S_T - (S_0 - C)e^{r(T-t_0)}$.**

Scenario $S_T \ge K$ (expiration)

- **We have reached expiration and have not closed our short stock position.**
- **The option has expired in the money.**
- **We exercise the option, pay the strike K and receive one share of stock, which we use to close out our short stock position.**
- **This yields a loss of $K - (S_0 - C)e^{r(T-t_0)}$.**
- **The amount of this loss is $100 - 98/0.99 \simeq 1.0101$.**

American put

- **At time t_0, we buy the put and the stock, hence we borrow $S_0 + P = 100 + 2 = 102$ from the bank.**
- **This amount is more than the strike price, and increases with time.**
- **It we exercise the put at any time, we receive 100 which is not enough to repay our loan.**
- **In this scenario, we are long the stock and we have purchased the put to cap our loss if the stock price declines.**
- **If at any time $t_0 < t \leq T$ we find that $S_t > (S_0 + P)e^{r(t-t_0)}$, we sell the stock.**
  1. **We pay back our loan and obtain a profit of $S_t - (S_0 + P)e^{r(t-t_0)}$.**
  2. **If $t < T$, we sell the put, which is now of no interest to us, for extra profit.**
  3. **Note that we do not have to sell the stock as soon as $S_t = (S_0 + P)e^{r(t-t_0)}$.**
  4. **We can wait and see of the stock price goes higher.**
- **If we reach expiration without selling the stock, we must repay our loan (the question says so).**
- **If at expiration we find that $K < S_T \leq (S_0 + P)e^{r(t-t_0)}$, we sell the stock.**
  1. **We pay back our loan but there is not enough money to do so and we suffer a loss of $(S_0 + P)e^{r(T-t_0)} - S_T$.**
  2. **The option expires out of the money and is worthless.**
- **If at expiration we find that $S_T \leq K$, we exercise the put option.**
  1. **We deliver our stock to close out the put option.**
  2. **We receive cash of the strike $K = 100$.**
  3. **We pay back our loan but there is not enough money to do so and we suffer a loss of $(S_0 + P)e^{r(T-t_0)} - K$.**
  4. **The amount of this loss is $102/0.99 - 100 \simeq 3.0303$.**

Statistics

- **There are 78 registered students, and the breakdown of grades is as follows.**

A+	**21**
A	**20**
A-	**3**
B+	**6**
B	**17**
B-	**8**
C	**2**
D	**1**

- **A histogram of the grades is plotted in Fig. 3.**
- **The contents are informative.**
- **I was concerned that the students in Section 1 (3:10 pm) were understanding the class material less well than the students in section 2 (5 pm).**
- **The results indicate that students in both sections performed equally well.**
- **My concerns about the students in Section 1 are unfounded.**
- **Approximately half the students scored A or A+ (41 out of 78).**
- **It was about the same in Fall 2017 (8 A+ and 5 A, total 13 out of 28 students).**
- **The grade distribution of the rest of the students forms a bell-shaped curve.**
- **I do not "grade on a curve" and the evidence indicates I do not need to.**
- **The students form the curve by themselves.**
- **Basically, there is a core consisting of about half the students who are clearly enthusiastic about this class.**
- **It was the same in Fall 2017.**
- **They form a clear spike of A and A+ grades.**

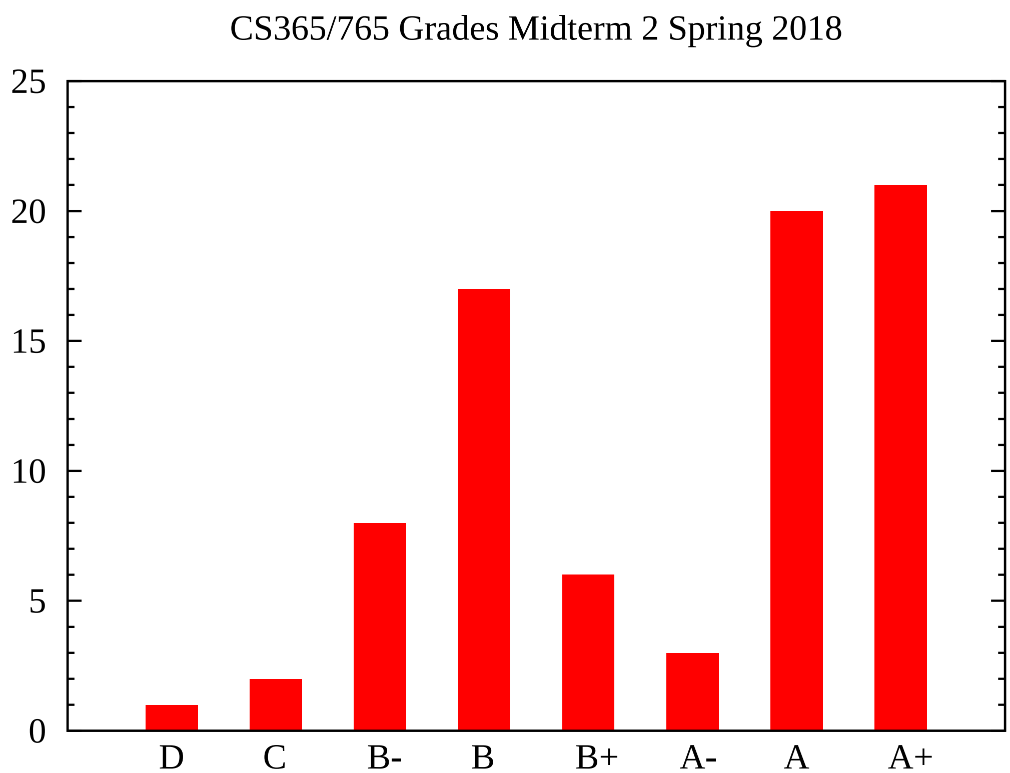

Figure 3: Histogram of grades for CS365/765 midterm 2 Spring 2018.