Queens College, CUNY, Department of Computer Science

Computational Finance

CSCI 365 / 765

Summer 2018

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Due date n/a

The material in this homework assignment will be used for questions in exams.

6 Homework 6a (Lecture 12): Black–Scholes–Merton formula for options

- The symbols S, K, r, q, σ, t_0 and T have theit usual meanings.
- Define the variables d_1 and d_2 as follows:

$$\begin{aligned} d_1 &= \frac{\ln(S/K) + r(T-t_0)}{\sigma\sqrt{T-t_0}} + \tfrac{1}{2}\sigma\sqrt{T-t_0}\,, \\ d_2 &= d_1 - \sigma\sqrt{T-t_0}\,. \end{aligned} \tag{6.1}$$

- The Black–Scholes–Merton formula for the fair value of a European call c and a European put p, and the corresponding Delta Δ_c and Δ_p, is

$$\begin{aligned} c &= Se^{-q(T-t_0)}\,N(d_1) - Ke^{-e(T-t_0)}\,N(d_2)\,, \\ p &= Ke^{-e(T-t_0)}\,N(-d_2) - Se^{-q(T-t_0)}\,N(-d_1)\,, \\ \Delta_c &= Se^{-q(T-t_0)}\,N(d_1)\,, \\ \Delta_p &= -Se^{-q(T-t_0)}\,N(-d_1)\,. \end{aligned} \tag{6.2}$$

- The **cumulative normal function** $N(x)$ is given by

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2}\,du\,. \tag{6.3}$$

- Fortunately C++ provides a function `erf(x)` which can be used to compute $N(x)$:

$$N(x) = \frac{1 + \mathrm{erf}(x/\sqrt{2})}{2}\,. \tag{6.4}$$

- You may use the C++ function below to compute the value of $N(x)$.

```
double cum_norm(double x)
{
  const double root = sqrt(0.5);
  return 0.5*(1.0 + erf(x*root));
}
```

- **Write functions to calculate the fair value and Delta of European call and put options using the Black–Scholes–Merton formula.**
- You will require the above functions to answer questions in exams.