Queens College, CUNY, Department of Computer Science

Numerical Methods

CSCI 361 / 761

Spring 2019

Instructor: Dr. Sateesh Mane

Quiz 2

Thursday March 28, 2019

Sunday March 31, 2019 11:59 pm (take home)

- For the take home version of this quiz, plesse submit your solution via email, as a zip archive, to `Sateesh.Mane@qc.cuny.edu`.
  The zip archive should have either of the naming formats:

  `studentid_first_last_CS361_quiz2_Spring2019.zip`
  `studentid_first_last_CS761_quiz2_Spring2019.zip`

  Acceptable file types are docx/pdf.

- **<u>NOTE</u>: It is the policy of the Computer Science Department to issue a failing grade to any student who either gives or receives help on any test.**
- **A student caught cheating on any question in an exam, project or quiz will fail the entire course.**
- This is an **open-book** test.
- Once you leave the classroom, you cannot come back to the test.
- **Any problem to which you give two or more (different) answers receives the grade of zero automatically.**
- Submit your solution in the envelope provided, with your name and student id on the cover.
- Write your name and student id on the cover of the solution blue book provided to you.
- Write the question number clearly at the start of each solution to a problem.
- **Answers must be written in legible handwriting.**
- **A failing grade will be awarded if the examiner is unable to decipher your handwriting.**

1 Question 1

- You are given the following integral (β is a real number):

$$I(\beta) = \int_0^1 \frac{1}{\sqrt{1-\beta x^2}}\,dx\,.$$

- Calculate the value of $I(\beta)$ numerically using Simpson's rule and $n = 10$ steps.

- **Find a value β_* such that:**

$$0.69 < I(\beta_*) < 0.71\,.$$

- **State your value for β_* to one decimal place.**

- **State your value for $I(\beta_*)$ to three decimal places.**

- *You may employ any method you wish to find the value of β_*.*
  You are not obligated to use bisection or Newton-Raphson, etc.

- *You may code using C++ or Java or employ Excel, etc.*
  You are NOT required to submit your code as part of your answer.

2 Question 2

- You are given a function $f(x)$ of a real-valued variable x.
- You are also given a real number $h > 0$.
- Define three values $x_{-h} = -h$, $x_0 = 0$ and $x_h = h$.
- Let the corresponding values of $f(x)$ be $f(-h)$, $f(0)$ and $f(h)$, respectively.
- Define a quadratic function $q(x)$ as follows (a, b and c are constants):

$$q(x) = ax^2 + bx + c\,.$$

- Let the quadratic $q(x)$ be equal to $f(x)$ at the three values x_{-h}, x_0 and x_h:

$$\begin{aligned} q(-h) &= f(-h)\,, \\ q(0) &= f(0)\,, \\ q(h) &= f(h)\,. \end{aligned}$$

- **Derive expressions for a, b, c in terms of h and $f(-h)$, $f(0)$, $f(h)$.**

$$\begin{aligned} a &= \{\text{expression in terms of } h \text{ and } f(-h),\ f(0),\ f(h)\}\,, \\ b &= \{\text{expression in terms of } h \text{ and } f(-h),\ f(0),\ f(h)\}\,, \\ c &= \{\text{expression in terms of } h \text{ and } f(-h),\ f(0),\ f(h)\}\,. \end{aligned}$$

- **Derive an expression for the value of the integral of the quadratic in terms of h and a, b, c.**

$$I_q = \int_{-h}^{h} q(x)\,dx \;=\; \{\text{expression in terms of } h \text{ and } a,\ b,\ c\}\,.$$

- **Using the expressions for a, b and c in terms of h and $f(-h)$, $f(0)$, $f(h)$ that you derived above, derive an expression for the value of the integral of the quadratic in terms of h and $f(-h)$, $f(0)$ and $f(h)$.**

$$I_q = \{\text{expression in terms of } h \text{ and } f(-h),\ f(0),\ f(h)\}\,.$$

3 Question 3

- You are given a function $f(x)$ of a real-valued variable x.
- You are also given steps h_1 and h_2, where $h_1 \neq h_2$ in general.
- **Derive a finite difference approximation for the first derivative $f'(x)$ in terms of $f(x+h_1)$ and $f(x-h_2)$, such that the leading order error term is $O(f'''(x))$.**

$$f'(x) = \{\text{function of } f(x+h_1) \text{ and } f(x-h_2)\} + O(f'''(x)) + \cdots \qquad (\text{no term in } f''(x)).$$

- **Derive a finite difference approximation for the second derivative $f''(x)$ in terms of $f(x+h_1)$ and $f(x-h_2)$, such that the leading order error term is $O(f'''(x))$.**

$$f''(x) = \{\text{function of } f(x+h_1) \text{ and } f(x-h_2)\} + O(f'''(x)) + \cdots$$