Queens College, CUNY, Department of Computer Science

Numerical Methods

CSCI 361 / 761

Spring 2019

Instructor: Dr. Sateesh Mane

Quiz 1

Thursday March 7, 2019 (in class)

Wednesday March 13, 2019 12:00 noon (take home)

- **NOTE: It is the policy of the Computer Science Department to issue a failing grade to any student who either gives or receives help on any test.**
- **A student caught cheating on any question in an exam, project or quiz will fail the entire course.**
- The in-class version of this quiz is an **open-book** test.
- Once you leave the classroom, you cannot come back to the test.
- **Any problem to which you give two or more (different) answers receives the grade of zero automatically.**
- Submit your solution in the envelope provided, with your name and student id on the cover.
- Write your name and student id on the cover of the solution blue book provided to you.
- Write the question number clearly at the start of each solution to a problem.
- **Answers must be written in legible handwriting.**
- **A failing grade will be awarded if the examiner is unable to decipher your handwriting.**
- **Students may submit a take home solution for a grade boost.**
- For the take home version of this quiz, plesse submit your solution via email, as a zip archive, to `Sateesh.Mane@qc.cuny.edu`.
  The zip archive should have either of the naming formats:

  `studentid_first_last_CS361_quiz1_Spring2019.zip`
  `studentid_first_last_CS761_quiz1_Spring2019.zip`

  Acceptable file types are txt/docx/pdf.

1 Question 1

1.1 Part 1

- **Ignore roundoff error and assume infinite precision in all computations in this question.**
- We employ the bisection algorithm to find a root of the function $f(x)$ graphed in Fig. 1 below.
    1. **DO NOT ATTEMPT to use a ruler to "measure the graph"** to determine the values of the roots, etc.
    2. Solutions which attempt to "measure the graph" **will not be considered valid.**
- We are given the following information about the function $f(x)$:
    1. The function $f(x)$ is positive and never equals zero for $x \leq 0$.
    2. The function $f(x)$ equals zero but does not cross zero at $x = \alpha$, where $2 < \alpha < 3$.
    3. The function $f(x)$ is continuous everywhere except at $x = 5$.
    4. The function $f(x)$ is discontinuous and changes sign across $x = 5$ $(= \beta)$. Also $f(x) \to \infty$ as $x \to 5$ from the left.
    5. The function $f(x)$ has a root in the interval $6 < x < 7$. Denote this root by γ below.
    6. The function $f(x)$ equals zero but does not cross zero at $x = 8$ $(= \delta)$.
    7. The function $f(x)$ has a root in the interval $9 < x < 10$. Denote this root by ε below.
    8. The function $f(x)$ is negative and never equals zero for $x \geq 10$.
- **For each case below, state what will be the next step of the bisection algorithm.**

    It is essential that you answer this part of the question.

    You will score zero for the whole question if you do not answer this part of the question.

    1. $x_{\text{low}} = 1.0$, $x_{\text{high}} = 11.5$.
    2. $x_{\text{low}} = 5.5$, $x_{\text{high}} = 10.0$.
    3. $x_{\text{low}} = 4.0$, $x_{\text{high}} = 11.0$.
    4. $x_{\text{low}} = 0.0$, $x_{\text{high}} = 9.0$.
    5. $x_{\text{low}} = 3.0$, $x_{\text{high}} = 8.0$.
    6. $x_{\text{low}} = 5.5$, $x_{\text{high}} = 7.5$.
    7. $x_{\text{low}} = 4.5$, $x_{\text{high}} = 7.0$.
    8. $x_{\text{low}} = -3000.0$, $x_{\text{high}} = 9.0$ ***(x_{low} not shown in Fig. 1)***.
    9. $x_{\text{low}} = 4.5$, $x_{\text{high}} = 60.5$ ***(x_{high} not shown in Fig. 1)***.
- **For each case above which yields a valid initial bracket, to which root/discontinuity will the bisection algorithm converge?**
- "Convergence" is defined as follows in this question.
    1. The tolerance for the bisection algorithm in this question is **zero.**
    2. State the root/discontinuity the iterates will approach in the limit of an infinite number of iterations.

1.2 Part 2

- In this problem, the initial value of the low iterate is set to $x_{\text{low}} = 4.0$.
- We wish to find a value for x_{high} such that the bisection algorithm converges to the root ε. We know from Fig. 1 that ε lies in the interval $9 < \varepsilon < 10$.

  1. Note that if we select $x_{\text{high}} = \varepsilon$, there will be no need for a bisection calculation. However, since we do not know the value of ε in advance, this requires a lucky guess.
  2. **Explain why all choices for x_{high} such that $5 \le x_{\text{high}} < 9$ can be rejected instantly, regardless if we bracket a root/discontinuity or not.**
  3. Also explain why all choices for x_{high} such that $4 < x_{\text{high}} < 5$ can be rejected instantly, regardless if we bracket a root/discontinuity or not.
  4. Show that if we select $x_{\text{high}} \ge 10$, we will bracket a root or discontinuity, and *there is a possibility* that the bisection algorithm will converge to the root ε.
  5. Next select x_{high} such that $\mathbf{13 \le x_{\text{high}} \le 14}$ and let $x_{\text{mid}} = (x_{\text{low}} + x_{\text{high}})/2$.
     (a) **Calculate the range of values x_{mid} takes**, for $x_{\text{low}} = 4$ and $13 \le x_{\text{high}} \le 14$.
     (b) **Which value of x_{low} or x_{high} will be updated?**
     (c) **How many roots of $f(x)$ will be contained in the updated interval?**
  6. Explain why if we select x_{high} such that $13 \le x_{\text{high}} \le 14$ (and $x_{\text{low}} = 4$), the bisection algorithm will converge to the root ε.

- "Convergence" is defined as follows in this question.
  1. The tolerance on the bisection algorithm is zero.
  2. State the root/discontinuity the iterates will approach in the limit of an infinite number of iterations.

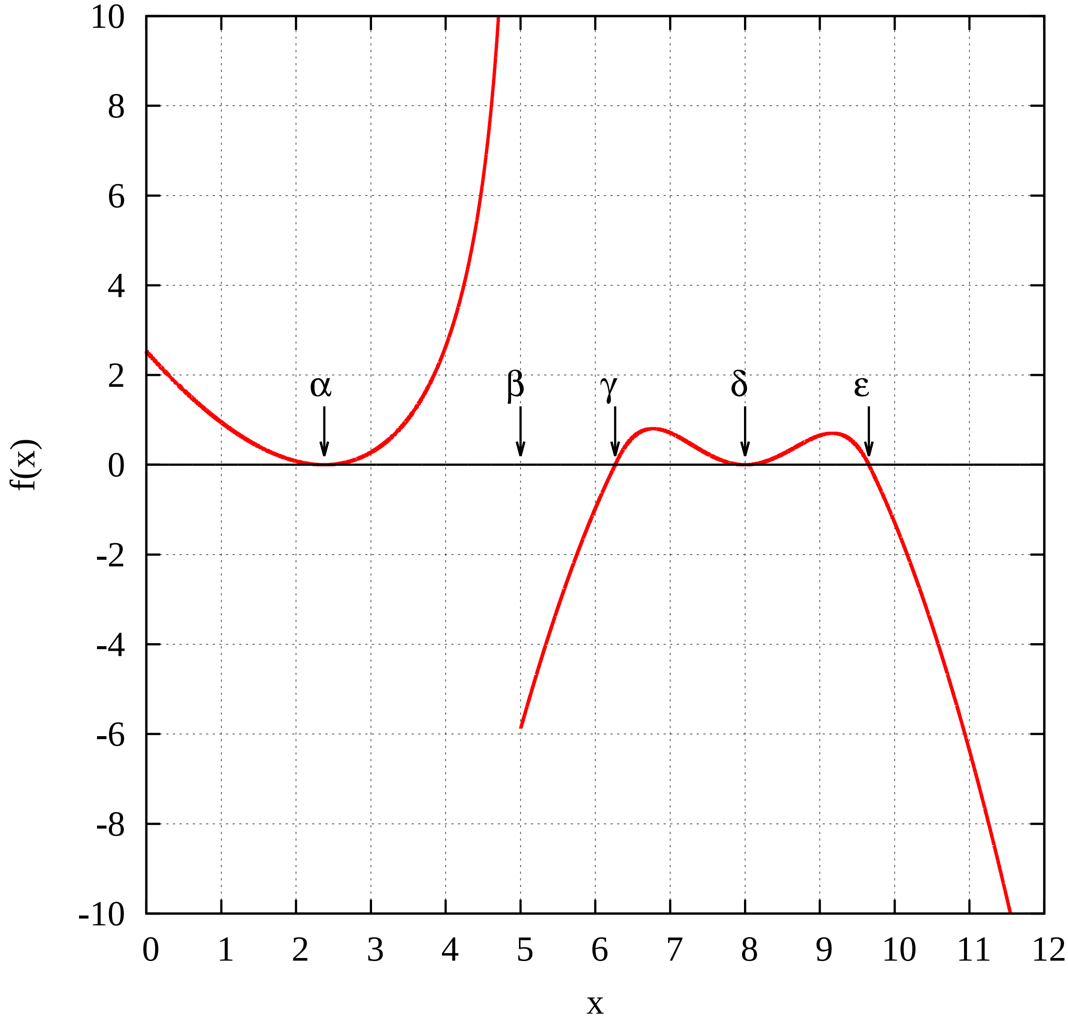

Figure 1: Graph of $f(x)$ in Question 1.