Queens College, CUNY, Department of Computer Science

Numerical Methods

CSCI 361 / 761

Summer 2018

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Final Part 3

Due Thursday August 9, 2018 at 11.59 pm

- **NOTE: It is the policy of the Computer Science Department to issue a failing grade to any student who either gives or receives help on any test.**
- **A student caught cheating on any question in an exam, project or quiz will fail the entire course.**
- **Any problem to which you give two or more (different) answers receives the grade of zero automatically.**
- *This is a take home exam. Answers should be typed in a file. See below for instructions.*
- Please submit your solution via email, as a file attachment, to `Sateesh.Mane@qc.cuny.edu`.
- Please submit one zip archive with all your files in it.
  1. The zip archive should have either of the names (CS361 or CS761):

     `StudentId_first_last_CS361_final_pt3_Aug2018.zip`
     `StudentId_first_last_CS761_final_pt3_Aug2018.zip`
  2. The archive should contain one "text file" named "Final_pt3.[txt/docx/pdf]" and one cpp file per question named "Q1.cpp" and "Q2.cpp" etc.
  3. Note that text answers may not be required for all questions.
  4. Note that not all questions may require a cpp file.
- **In all questions where you are asked to submit programming code, programs which display any of the following behaviors will receive an automatic F:**
  1. Programs which do not compile successfully (non-fatal compiler warnings are excluded).
  2. Array out of bounds, reading of uninitialized variables (including null pointers).
  3. Operations which yield NAN or infinity, e.g. divide by zero, square root of negative number, etc. *Infinite loops.*
  4. Programs which do NOT implement the public interface stated in the question.
- **In addition, note the following:**
  1. All debugging statements (for your personal testing) should be commented out.
  2. Program performance will be graded solely on the public interface stated in the questions.

General information

- **The statements below are for general information only.**
- Ignore them if they are not relevant for the exam questions below.
- The questions in this exam do not involve problems of overflow or underflow.
- Solutions involving the writing of algorithms will not be judged if they work on a 64–bit instead of a 32–bit computer.
- **Value of π to machine precision on any computer.**
  1. Some compilers support the constant `M_PI` for π, in which case you can write

     ```
     const double pi = M_PI;
     ```

  2. If your compiler does not support `M_PI`, the value of π can be computed via

     ```
     const double pi = 4.0*atan2(1.0,1.0);
     ```

3 Question 3

- Define parameter values α and β as follows.
    1. Take the first four digits of your student id and multiply by 10^{-4}.
    2. Take the last four digits of your student id and multiply by 10^{-4}.
    3. Then α and β are given as follows.

    $$\alpha = (\text{first four digits of id}) \times 10^{-4}\,, \qquad \beta = (\text{last four digits of id}) \times 10^{-4}\,.$$

    4. For example if your student id is 23054617, then $\alpha = 0.2305$ and $\beta = 0.4617$.
    5. **Solutions which employ $\alpha = 0.2305$ and $\beta = 0.4617$ below will score zero.**
- Bessel's equation is a linear homogeneous second order ordinary differential equation:

$$x^2\,\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} + (x^2 - \nu^2)y = 0\,. \tag{3.1}$$

- Here ν can be any number, even complex. **We shall use $\nu = 1.0$ in this question.**
- Define a set of $n+1$ equally spaced points with x_i via $x_0 = 1$ and $x_n = 20$.
- Hence the interval size we shall employ in this question is $h = 19/n$.
- **Express $y'(x_i)$ and $y''(x_i)$ using centered finite differences, for $i = 1, \dots, n-1$.**
- In addition, the boundary conditions at x_0 and x_n are as follows.

$$y_0 = \alpha\,, \qquad \left[\frac{dy}{dx}\right]_{x=x_n} = \beta\,.$$

- **Formulate the solution of eq. (3.1) as a set of tridiagonal matrix equations.**
- **State your expressions for a_i, b_i and c_i, for $i = 0, \dots, n$.**
- Remember the end points $i = 0$ and $i = n$ are special cases.
- *The resulting tridiagonal matrix is NOT diagonally dominant. Do not worry.*
- **Solve the tridiagonal matrix equations using $n = 19000$.**
- **Plot a graph of $y(x)$ for $1 \le x \le 20$.** (A hand-drawn sketch is acceptable.)
- The value of $y(x)$ oscillates and equals zero multiple times in the interval $1 < x < 20$.
- **Use your solution of the tridiagonal equations to determine where $y(x)$ equals zero in the interval $1 < x < 20$. State your answers to a tolerance (in the value of x) of $\Delta x = 0.001$.**