

due Friday, Mar. 30, 2018, 11.59 pm

9 Homework lecture 9

9.1 Linear algebra: triangular systems

- **Question: The equations below are in upper triangular form.**
Solve the equations for the values of x_1 , x_2 and x_3 .

$$x_1 - 2x_2 + 3x_3 = 2, \quad (9.1.1a)$$

$$3x_2 + x_3 = 1, \quad (9.1.1b)$$

$$x_3 = 4. \quad (9.1.1c)$$

- **Question: The equations below are in lower triangular form.**
Solve the equations for the values of x_1 , x_2 and x_3 .

$$x_1 = 2, \quad (9.1.2a)$$

$$x_1 - 2x_2 = 4, \quad (9.1.2b)$$

$$x_1 - x_2 - 3x_3 = 8. \quad (9.1.2c)$$

- As experience has demonstrated, if you do not understand the above expressions/questions, **THEN ASK.**
- If you do not understand the words/sentences in Lecture 6, etc. **THEN ASK.**
- Send me an email, explain what you do not understand.
- Do not just keep quiet and produce nonsense in exams.

9.2 Linear algebra: inconsistency and linear dependence

- You are given the following equations

$$x_1 + 2x_2 + 3x_3 = 1, \quad (9.2.1a)$$

$$2x_1 + 2x_2 + x_3 = 1, \quad (9.2.1b)$$

$$3x_1 + 2x_2 - x_3 = 1. \quad (9.2.1c)$$

- **Question: Add or subtract multiples of eq. (9.2.1a) from eq. (9.2.1b) to eliminate x_1 from eq. (9.2.1b). Next add or subtract multiples of eq. (9.2.1a) from eq. (9.2.1c) to eliminate x_1 from eq. (9.2.1c).**
- Write the resulting equations in the following form.
(You have to calculate the values of a , b , c , d , r_2 and r_3 .)

$$x_1 + 2x_2 + 3x_3 = 1, \quad (9.2.2a)$$

$$ax_2 + bx_3 = r_2, \quad (9.2.2b)$$

$$cx_2 + dx_3 = r_3. \quad (9.2.2c)$$

- **Question: Are eqs. (9.2.2a) – (9.2.2c) inconsistent?**
- **Question: Are eqs. (9.2.2a) – (9.2.2c) not linearly independent?**
- **STOP HERE: You do **not** have to solve eqs. (9.2.2a) – (9.2.2c).**

9.3 Linear algebra: Gaussian elimination without pivoting

- You are given the following equations

$$x_1 + 2x_2 + x_3 = 1, \quad (9.3.1a)$$

$$2x_1 + 2x_2 - x_3 = 1, \quad (9.3.1b)$$

$$-3x_1 + 2x_2 + 3x_3 = 7. \quad (9.3.1c)$$

- Question: Add or subtract multiples of eq. (9.3.1a) from eq. (9.3.1b) to eliminate x_1 from eq. (9.3.1b). Next add or subtract multiples of eq. (9.3.1a) from eq. (9.3.1c) to eliminate x_1 from eq. (9.3.1c).**
- Write the resulting equations in the following form.
(You have to calculate the values of a , b , c , d , r_2 and r_3 .)

$$x_1 + 2x_2 + x_3 = 1, \quad (9.3.2a)$$

$$ax_2 + bx_3 = r_2, \quad (9.3.2b)$$

$$cx_2 + dx_3 = r_3. \quad (9.3.2c)$$

- Question: Add or subtract multiples of eq. (9.3.2b) from eq. (9.3.2c) to eliminate x_2 from eq. (9.3.2c).**
- Write the resulting equations in the following form.
(You have to calculate the values of d' , and r'_3 .)

$$x_1 + 2x_2 + x_3 = 1, \quad (9.3.3a)$$

$$ax_2 + bx_3 = r_2, \quad (9.3.3b)$$

$$d'x_3 = r'_3. \quad (9.3.3c)$$

- Question: Now eqs. (9.3.3a) – (9.3.3c) are in upper triangular form. Solve eqs. (9.3.3a) – (9.3.3c) by backsubstitution and calculate the values of x_1 , x_2 and x_3 .**

9.4 Linear algebra: Gaussian elimination & zero pivot

- You are given the following equations

$$x_1 + 2x_2 + x_3 = 1, \quad (9.4.1a)$$

$$2x_1 + 4x_2 - x_3 = 1, \quad (9.4.1b)$$

$$-3x_1 + 2x_2 + 3x_3 = 7. \quad (9.4.1c)$$

- **Question: Add or subtract multiples of eq. (9.4.1a) from eq. (9.4.1b) to eliminate x_1 from eq. (9.4.1b). Next add or subtract multiples of eq. (9.4.1a) from eq. (9.4.1c) to eliminate x_1 from eq. (9.4.1c).**
- Write the resulting equations in the following form.
(You have to calculate the values of a , b , c , d , r_2 and r_3 .)

$$x_1 + 2x_2 + x_3 = 1, \quad (9.4.2a)$$

$$ax_2 + bx_3 = r_2, \quad (9.4.2b)$$

$$cx_2 + dx_3 = r_3. \quad (9.4.2c)$$

- If you have done your work correctly, you should obtain $a = 0$ (zero pivot).
- **NOTE:** You do **not** have to solve eqs. (9.4.2a) – (9.4.2c).

9.5 Linear algebra: Gaussian elimination with partial pivoting

- We employ the equations from the previous section eqs. (9.4.1a) – (9.4.1c).
- **Question: Calculate the value of \hat{a}_i for $i = 1, 2, 3$ in eqs. (9.4.1a) – (9.4.1c).**
- **Question: Calculate the value of $|a_{i1}|/\hat{a}_i$ for $i = 1, 2, 3$ in eqs. (9.4.1a) – (9.4.1c).**
- **Question: Which pair of equations in eqs. (9.4.1a) – (9.4.1c) will be swapped?**
- Swap the relevant pair of equations and write the resulting equations in the following form.
(You have to calculate the values of a_{ij} , and b_i , for $i, j = 1, 2, 3$.)

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 , \quad (9.5.1a)$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 , \quad (9.5.1b)$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 . \quad (9.5.1c)$$

- **Question: Add or subtract multiples of eq. (9.5.1a) from eq. (9.5.1b) to eliminate x_1 from eq. (9.5.1b). Next add or subtract multiples of eq. (9.5.1a) from eq. (9.5.1c) to eliminate x_1 from eq. (9.5.1c).**
- Write the resulting equations in the following form.
(You have to calculate the values of a'_{ij} , and b'_j for $i, j = 2, 3$.)

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 , \quad (9.5.2a)$$

$$a'_{22}x_2 + a'_{23}x_3 = b'_2 , \quad (9.5.2b)$$

$$a'_{32}x_2 + a'_{33}x_3 = b'_3 . \quad (9.5.2c)$$

- If you have done your work correctly, you should obtain $a'_{22} \neq 0$ (nonzero pivot).
- **NOTE: You do not have to solve eqs. (9.5.2a) – (9.5.2c).**

9.6 Linear algebra: (optional)

- Solve eqs. (9.4.1a) – (9.4.1c).