

**due Sunday, Feb 11, 2018, 11.59 pm**

## 1 Homework set 1

### 1.1 Future value

- Here is a C++ function which inputs (i) today's cashflow  $F_0$ , (ii) today's time  $t_0$ , (iii) future time  $t_1$ , (iv) continuously compounded interest rate  $r$ . The value of  $r$  is expressed as a percentage, if the interest rate is 5% then  $r = 5$ .

```
double future_value(double F0, double t0, double t1, double r)
{
    double r_decimal = 0.01*r;
    double F1 = F0*exp(r_decimal*(t1-t0));
    return F1;
}
```

- **Compile and run this for yourself (you will need to write a main program).**
- Try a few input values. You should be able to implement a similar calculation in Excel and get the same answers.
- I say “future value” but note that the function will work even if  $t_1 < t_0$ .
- Sometimes when we need to baseline a set of cashflows to a common point in time, some cashflows may be in the past.

## 1.2 Discount factor

- **Write a function to do the inverse calculation.** (This should be easy.)
- The inputs are (i) today's cashflow  $F_0$ , (ii) future cashflow  $F_1$ , (iii) today's time  $t_0$ , (iv) future time  $t_1$ . The outputs are (v) discount factor  $d$ , (vi) continuously compounded interest rate  $r$ . As above, the value of  $r$  should be expressed as a percentage, if the interest rate is 5% then  $r = 5$ .
- The function signature is

```
int df_and_r(double F0, double F1, double t0, double t1, double & df, double & r);
```

- The return type is "int" because we want some validation checks.
- If  $t_1 - t_0$  equals zero, then set  $d = 0$  and  $r = 0$  and exit with a return value  $-1$ .
- If  $F_0 \leq 0$  or  $F_1 \leq 0$ , then set  $d = 0$  and  $r = 0$  and exit with a return value  $-2$ .
- If everything is fine, then exit with a return value  $0$ .
- Hence your function should look like this

```
int df_and_r(double F0, double F1, double t0, double t1, double & df, double & r)
{
    if (t1-t0 == 0.0) {
        df = 0;
        r = 0;
        return -1;
    }
    if ((F0 < 0.0) || (F1 < 0.0)) {
        // *** you figure it out ***
    }
    // *** you have to write the rest ***

    return 0;
}
```

### 1.3 Bond price and yield

- For simplicity, let today's time be  $t_0 = 0$ .
- Consider a newly issued bond with a maturity of two years.
- Suppose the bond pays semiannual coupons (two coupons per year).
- Let the face be  $F$  and the annualized coupon rates be  $c_1, \dots, c_4$  and the yield be  $y$ .
- The formula relating the bond price and yield is

$$B = \frac{\frac{1}{2}c_1}{1 + \frac{1}{2}y} + \frac{\frac{1}{2}c_2}{(1 + \frac{1}{2}y)^2} + \frac{\frac{1}{2}c_3}{(1 + \frac{1}{2}y)^3} + \frac{F + \frac{1}{2}c_4}{(1 + \frac{1}{2}y)^4}. \quad (1.3.1)$$

- Set  $F = 100$  and  $c_1 = \dots = c_4 = 4$ .

1. **Fill in the table below with the values of  $B(y)$  (answers to two decimal places).**

$y$ (%)	$B(y)$
0	(2 d.p.)
2	(2 d.p.)
4	(2 d.p.)
6	(2 d.p.)
8	(2 d.p.)

2. Let the market price of the bond be  $B_{\text{market}} = 100.5$ .
3. **State which pair  $(y, y + 2)$  gives a lower and upper bound for the true yield.**
4. Call the values  $y_{\text{low}}$  and  $y_{\text{high}}$ , so  $y_{\text{high}} = y_{\text{low}} + 2$  and define  $y_{\text{mid}} = (y_{\text{low}} + y_{\text{high}})/2.0$ .
5. **Calculate the bond price  $B(y_{\text{mid}})$ .**
6. **State the updated values of  $y_{\text{low}}$  and  $y_{\text{high}}$  for the next iteration step.**
7. **Calculate the updated value of  $y_{\text{mid}}$  and the updated bond price  $B(y_{\text{mid}})$ .**

- Next set  $F = 100$  and  $c_1 = 1, c_2 = 3, c_3 = 5$  and  $c_4 = 7$ .

1. **Fill in the table below with the values of  $B(y)$  (answers to two decimal places).**

$y$ (%)	$B(y)$
1	(2 d.p.)
3	(2 d.p.)
5	(2 d.p.)
7	(2 d.p.)
9	(2 d.p.)

2. Let the market price of the bond be  $B_{\text{market}} = 100$ .
3. **State which pair  $(y, y + 2)$  gives a lower and upper bound for the true yield.**
4. Call the values  $y_{\text{low}}$  and  $y_{\text{high}}$ , so  $y_{\text{high}} = y_{\text{low}} + 2$  and define  $y_{\text{mid}} = (y_{\text{low}} + y_{\text{high}})/2.0$ .
5. **Calculate the bond price  $B(y_{\text{mid}})$ .**
6. **State the updated values of  $y_{\text{low}}$  and  $y_{\text{high}}$  for the next iteration step.**
7. **Calculate the updated value of  $y_{\text{mid}}$  and the updated bond price  $B(y_{\text{mid}})$ .**

## 1.4 Yield curve

- Consider only bonds with semiannual coupons (two coupons per year).
- The bonds all have face  $F = 100$ .
- Let us have three newly issued par bonds, with maturities of 0.5, 1.0, 1.5 years.
- **You are given the following values for the yields:**

$$y_{0.5} = 4.0\%, \quad y_{1.0} = 4.2\%, \quad y_{1.5} = 4.1\%. \quad (1.4.1)$$

- Use the formulas in the lectures to compute the values of the discount factors  $d_{0.5}$ ,  $d_{1.0}$  and  $d_{1.5}$ . State your answers to four decimal places.
- Also calculate the continuously compounded spot rates  $r_{0.5}$ ,  $r_{1.0}$  and  $r_{1.5}$ . State your answers as percentages, to two decimal places.
- This is an example of a **humped yield curve**. The yields go up, then down. A humped yield curve is rare, but can exist.