Queens College, CUNY, Department of Computer Science

Computational Finance

CSCI 365 / 765

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1 Homework 1

- Please email your solution, as a file attachment, to `Sateesh.Mane@qc.cuny.edu`.
- Please submit one zip archive with all your files in it.
  1. The zip archive should have either of the names (CS365 or CS765):

     `StudentId_first_last_CS365_hw1.zip`

     `StudentId_first_last_CS765_hw1.zip`
  2. The archive should contain one "text file" named "hw1.[txt/docx/pdf]" and one cpp file per question named "Q1.cpp" and "Q2.cpp" etc.
  3. Note that not all homework assignments may require a text file.
  4. Note that not all questions may require a cpp file.

1.1 Future value

- Here is a C++ function which inputs (i) today's cashflow F_0, (ii) today's time t_0, (iii) future time t_1, (iv) continuously compounded interest rate r. The value of r is expressed as a percentage, if the interest rate is 5% then $r = 5$.

```
double future_value(double F0, double t0, double t1, double r)
{
  double r_decimal = 0.01*r;
  double F1 = F0*exp(r_decimal*(t1-t0));
  return F1;
}
```

- **Compile and run this for yourself (you will need to write a main program).**
- Try a few input values. You should be able to implement a similar calculation in Excel and get the same answers.
- I say "future value" but note that the function will work even if $t_1 < t_0$.
- Sometimes when we need to baseline a set of cashflows to a common point in time, some cashflows may be in the past.

1.2 Discount factor

- **Write a function to do the inverse calculation.** (This should be easy.)
- The inputs are (i) today's cashflow F_0, (ii) future cashflow F_1, (iii) today's time t_0, (iv) future time t_1. The outputs are (v) discount factor d, (vi) continuously compounded interest rate r. As above, the value of r should be expressed as a percentage, if the interest rate is 5% then $r = 5$.
- The function signature is

```
int df_and_r(double F0, double F1, double t0, double t1, double & df, double & r);
```

- The return type is "int" because we want some validation checks.
- If $t_1 - t_0$ equals zero, then set $d = 0$ and $r = 0$ and exit with a return value -1.
- If $F_0 \leq 0$ or $F_1 \leq 0$, then set $d = 0$ and $r = 0$ and exit with a return value -2.
- If everything is fine, then exit with a return value 0.
- Hence your function should look like this

```
int df_and_r(double F0, double F1, double t0, double t1, double & df, double & r)
{
  if (t1-t0 == 0.0) {
    df = 0;
    r = 0;
    return -1;
  }
  if ((F0 < 0.0) || (F1 < 0.0)) {
    // *** you figure it out ***
  }
  // *** you have to write the rest ***

  return 0;
}
```

1.3 Bond price and yield

- Consider a newly issued bond (i.e. $t_0 = 0$) with a maturity of two years.
- Suppose the bond pays semiannual coupons (two coupons per year).
- Let the face be F and the annualized coupon rates be $c_1, \dots, c_4$ and the yield be y.
- The formula relating the bond price and yield is

$$B = \frac{\frac{1}{2}c_1}{1+\frac{1}{2}y} + \frac{\frac{1}{2}c_2}{(1+\frac{1}{2}y)^2} + \frac{\frac{1}{2}c_3}{(1+\frac{1}{2}y)^3} + \frac{F+\frac{1}{2}c_4}{(1+\frac{1}{2}y)^4}\,. \tag{1.3.1}$$

- We shall solve eq. (1.3.1) to obtain the exact solution y_{ex} in various scenarios.
- **Set $F = 100$ and $c_1 = \cdots = c_4 = 4$.**
  1. **Fill in the table below with the values of $B(y)$ (answers to two decimal places).**

y (%)	$B(y)$
0	(2 d.p.)
2	(2 d.p.)
4	(2 d.p.)
6	(2 d.p.)
8	(2 d.p.)

  2. Let the market price of the bond be $B_{\text{market}} = 100.5$.
  3. **State which pair $(y, y+2)$ gives a lower and upper bound for y_{ex}.**
  4. Call the values y_{low} and y_{high}, so $y_{\text{high}} = y_{\text{low}} + 2$ and define $y_{\text{mid}} = (y_{\text{low}} + y_{\text{high}})/2.0$.
  5. **Calculate the bond price $B(y_{\text{mid}})$.**
  6. **State the updated values of y_{low} and y_{high} for the next iteration step.**
  7. **Calculate the updated value of y_{mid} and the updated bond price $B(y_{\text{mid}})$.**
- **Next set $F = 100$ and $c_1 = 1$, $c_2 = 3$, $c_3 = 5$ and $c_4 = 7$.**
  1. **Fill in the table below with the values of $B(y)$ (answers to two decimal places).**

y (%)	$B(y)$
1	(2 d.p.)
3	(2 d.p.)
5	(2 d.p.)
7	(2 d.p.)
9	(2 d.p.)

  2. Let the market price of the bond be $B_{\text{market}} = 100$.
  3. **State which pair $(y, y+2)$ gives a lower and upper bound for y_{ex}.**
  4. Call the values y_{low} and y_{high}, so $y_{\text{high}} = y_{\text{low}} + 2$ and define $y_{\text{mid}} = (y_{\text{low}} + y_{\text{high}})/2.0$.
  5. **Calculate the bond price $B(y_{\text{mid}})$.**
  6. **State the updated values of y_{low} and y_{high} for the next iteration step.**
  7. **Calculate the updated value of y_{mid} and the updated bond price $B(y_{\text{mid}})$.**

1.4 Yield curve

- Consider only bonds with semiannual coupons (two coupons per year).
- The bonds all have face $F = 100$.
- Let us have three newly issued par bonds, with maturities of 0.5, 1.0, 1.5 years.
- **You are given the following values for the yields:**

$$y_{0.5} = 4.0\,\%\,, \qquad y_{1.0} = 4.2\,\%\,, \qquad y_{1.5} = 4.1\,\%\,. \tag{1.4.1}$$

- Use the formulas in the lectures to compute the values of the discount factors $d_{0.5}$, $d_{1.0}$ and $d_{1.5}$. State your answers to four decimal places.
- Also calculate the continuously compounded spot rates $r_{0.5}$, $r_{1.0}$ and $r_{1.5}$. State your answers as percentages, to two decimal places.
- This is an example of a **humped yield curve.** The yields go up, then down. A humped yield curve is rare, but can exist.