Queens College, CUNY, Department of Computer Science

Computational Finance

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8 Lecture 8

8.1 Spreads & combinations: option strategies using multiple options

- There is still much we can say about options, independent of any specific probability model for the stock price movements.
  1. It is easy to fall into the trap of fancy mathematics.
  2. It is important to learn which properties of options do not depend on a specific probability model for the stock price movements.
  3. From a programming perspective, such model-independent properties can serve as validation checks for a mathematical pricing formula.
  4. But there are more important points of view, as we shall see below.
- We do not need to buy or sell only one option. We can construct different payoff functions using multiple options. We shall study various option trading strategies below, to buy and sell combinations of call and put options.
- **All of the examples below are real option trading strategies and are really used in trading.**
- It is important to understand in each case why investors wish to trade such option strategies.
- A **spread** is an option trading strategy using two or more options of the same type (all calls or all puts).
- A **combination** is an option trading strategy using a mixture of puts and calls.
- Just to avoid silliness, let us be clear that all the option trading strategies below involve options on the **same stock**.

8.2 Spreads

- There are various classifications to describe spreads.

  1. A **vertical spread** involves multiple options with different strikes, but all with the same expiration date.
  2. Logically, if there are vertical spreads, there should also be horizontal spreads.
     Why not?
     We say **calendar spread**.
  3. **Actually a student found an online reference which says that the terms 'horizontal spread' and 'calendar spread' and 'time spread' are all used and they all mean the same thing.**
  4. A **calendar spread** involves multiple options with different expirations, but all with the same strike.
  5. Why should a calendar spread use options with the same strike?
     There are other kinds of spreads.
  6. A **diagonal spread** involves multiple options with different expirations and different strikes.

- A **long position** in a spread means we buy the spread.

- A **short position** in a spread means we sell the spread.

- In addition to all of the above, there are **bull spreads** and **bear spreads**.

  1. A bull spread is a trading strategy which makes a profit if the stock price goes up.
     A long position in a bull spread means the investor thinks the stock price will go up.
  2. A bear spread is a trading strategy which makes a profit if the stock price goes down.
     A long position in a bear spread means the investor thinks the stock price will go down.
  3. Hence there can be long/short bull vertical spreads and long/short bear vertical spreads, etc.

8.3 Payoff diagrams

- In all of the examples below, we shall describe the payoff for a **long position in the spread**.
- Otherwise there will be too many graphs and it will be confusing.

8.4 Bull call spread (Part 1)

- A **bull call spread**, is *long a call with a strike* K_1 *and short a call with a higher strike* K_2.
- This is a vertical spread. Most people leave out the word "vertical" and just call it a bull call spread.
- A graph of the terminal payoff of a long position in a bull call spread is shown in Fig. 1.
- The high strike call (K_2) costs less than the low strike call (K_1). This is obvious because if the high strike call is in the money, the low strike call must also be in the money. Hence the low strike call delivers everything the high strike call does, and more. Hence it costs more. Therefore a bull call spread has a positive cost.
- In Fig. 2, we display the same payoff as in Fig. 1, but the blue dotdash curve shows the profit after taking into account the cost to buy the bull call spread.
- Hence, because of the initial cost to buy the spread, the stock price at expiration must go above a level higher than the low strike K_1, for the overall trade to make a profit.

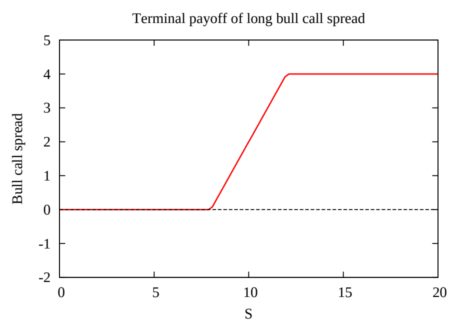

Figure 1: Graph of the terminal payoff of a long position in a bull call spread. The strike of the long call is $K_1 = 8$ and the strike of the short call is $K_2 = 12$.

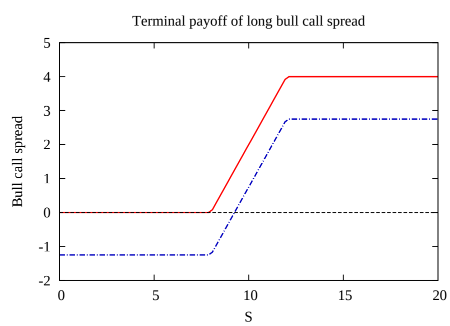

Figure 2: The same as Fig. 1 (bull call spread), but the dotdash curve displays the profit after taking into account the cost to buy the bull call spread.

8.5 Bull call spread (Part 2)

- Why do investors buy bull call spreads?
- They do it if they think that the stock price at expiration will go in the money (above the low strike K_1), but not so high that it will go above K_2.
- Hence this is a bullish trade, but not strongly bullish.
- The purpose of shorting the call struck at K_2 is that a bull call spread is cheaper to buy than a single option (struck at K_1). Selling the call struck at K_2 reduces the price of the trade.
- In exchange for paying less to buy a bull call spread, the investor accepts the fact that the upside payoff from the options is capped at $K_2 - K_1$. If the stock price at expiration is above K_2, the high strike option will be exercised against the investor and the payoff from the options will have a maximum value of $K_2 - K_1$.
- Obviously, if $K_2 \to \infty$, the high strike call will never be exercised and the spread reduces to a single call option struck at K_1.
- Conversely, if $K_2 \to K_1$, the payoff from the spread decreases to zero and the cost of the spread decreases to zero. *Let us revisit this issue when we discuss digital options (also known as binary options) later in these lectures.*
- To summarize:
  1. An investor buys a bull call spread because of a bullish view of the stock, that at expiration the stock price will be above K_1.
  2. However, the investor's view is not strongly bullish, the view is that at expiration the stock price will not be above K_2.
  3. The cost of buying a bull call spread is lower than the cost of buying only one option, struck at K_1.
  4. In exchange for paying less to buy the bull call spread, the investor acceps the fact that the spread has a cap on its upside profit.

8.6 Bull call spread (Part 3)

- **This is important: How is a spread traded in the financial markets?**
- A bull call spread is performed as **one trade**.
  The two options are part of one package, which is the spread.
- The same is true for any spread trade involving multiple options.
- If the investor sells the spread (prior to expiration), the entire combination is sold, as one package.
- The investor cannot sell only part of the spread and keep the rest. The whole spread is traded as one package.
  1. Nevertheless, to simplify the mathematics, **we shall calculate the cost of the spread by summing the costs of the individual options in the spread.**
  2. We shall do this for all the other types of option trading strategies, to be described below.

8.7 Bull call spread (Part 4)

- Let us return to rational option pricing theory and a no-arbitrage argument.
- Let us analyze the fair value of a long call spread. Let us denote the price of a European call with strike K by $c(S, K, t)$, with an obvious notation. Then the price of a bull call spread is

$$(\text{bull call spread}) = c(S, K_1, t) - c(S, K_2, t) \,. \tag{8.7.1}$$

- We saw from Fig. 1 (and can easily verify) that at the expiration time T, the maximum payoff of a bull call spread is $K_2 - K_1$. Hence for any value of the stock price S_T at the time T,

$$c(S_T, K_1, T) - c(S_T, K_2, T) \le K_2 - K_1 \,. \tag{8.7.2}$$

- This means that at any earlier time $t \le T$, we must have

$$c(S, K_1, t) - c(S, K_2, t) \le \mathrm{PV}(K_2 - K_1) \,. \tag{8.7.3}$$

- Hence overall we have the inequalities

$$0 \quad \le \quad c(S, K_1, t) - c(S, K_2, t) \quad \le \quad \mathrm{PV}(K_2 - K_1) \,. \tag{8.7.4}$$

- If either of these inequalities is not satisfied, then we can formulate an arbitrage argument.
- The same no-arbitrage concepts apply to other spreads also.

8.8 Bull call spread (Part 5): American options

- It was implicitly assumed above that the bull call spread consists of European options.
- However, any option strategy can be constructed from American options also.
- Since an American option can be exercised at any time on or before the expiration date, it is important to remember that a spread is traded as **one unit**. If one of the American options in the spread is exercised, **the entire spread terminates at that time.**
- Hence the no-arbitrage inequalities for American options are different from those for European options. The best we can say is (using C to denote American options)

$$0 \quad \leq \quad C(S, K_1, t) - C(S, K_2, t) \quad \leq \quad K_2 - K_1\,. \tag{8.8.1}$$

8.9 Bull put spread

- We can also construct a bull spread using put options.
- A bull put spread is long a put option at the low strike K_1 and short a put option at the high strike K_2.
- The high strike put at K_2 is worth more than the low strike put at K_1 because the high strike put is always in the money if the low strike put is in the money. Hence the high strike put does everything the low strike put does, and more. Hence it is worth more.
- This means that the buyer (long position) in a bull put spread ***receives money up front.***
- The profit at expiration is plotted in Fig. 3. The red solid curve plots the terminal payoff of the put options and the blue dotdash curve shows the profit after taking into account the income from the initial trade to buy the bull put spread.
- Clearly, the stock price at expiration must go above a level higher than the low strike K_1, for the overall trade to make a profit.
- Note that if the stock price goes down, the investor may suffer a loss (although the amount of loss is capped).
- The profit (if the stock price goes up) comes from the initial income when the bull put spread was purchased.
- The amount of profit using a bull put spread is not as great as the profit from a bull call spread with the same strikes.
- Nevertheless, a bull spread option trade *can* be done using put options.

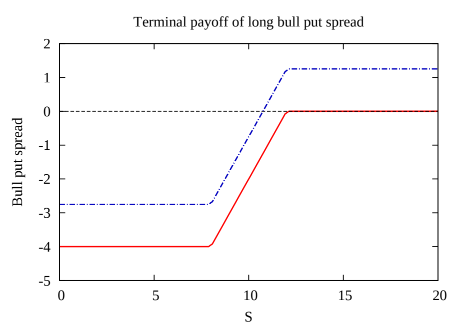

Figure 3: The payoff for a long bull put spread. The red curve shows the terminal payoff from the put options and the dotdash curve displays the profit after taking into account the income from the initial trade of the spread.

8.10 Bear put spread

- Next let us consider a bearish trade, in particular a bear put spread.
- A bear put spread is *short a put option at the low strike* K_1 *and long a put option at the high strike* K_2.
- As already explained, the high strike put at K_2 is worth more than the low strike put at K_1.
- This means that it costs money up front to buy (long position) a bear put spread.
- In a bear option spread, the investor has a bearish view that the stock price at expiration will go down, below the strike K_2. However, the investor also believes that the stock price will not go down below the strike K_1.
- The cost of the bear put spread is lower than the cost of buying a put option struck at K_2.
- However, the profit from the option payoffs is capped at $K_2 - K_1$.
- The profit at expiration is plotted in Fig. 4. The red solid curve plots the terminal payoff of the put options and the blue dotdash curve shows the profit after taking into account the initial cost to buy the bear put spread.
- Clearly, the stock price at expiration must go above a level lower than the high strike K_2, for the overall trade to make a profit.
- Note that if the stock price goes up, the investor may suffer a loss (because of the initial cost to buy the spread).
- Using the notation $p(S, K, t)$ and $P(S, K, t)$ to denote the price of a European or American put with strike K, the no-arbitrage arguments yield the inequalities

$$0 \leq p(S, K_2, t) - p(S, K_1, t) \leq \mathrm{PV}(K_2 - K_1)\,, \tag{8.10.1}$$

$$0 \leq P(S, K_2, t) - P(S, K_1, t) \leq K_2 - K_1\,. \tag{8.10.2}$$

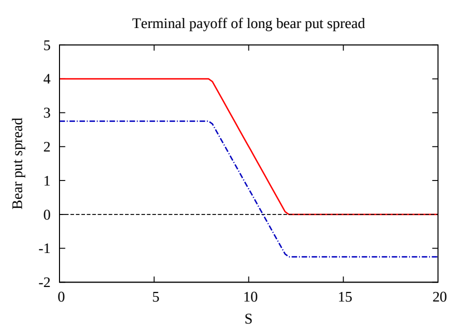

Figure 4: The payoff for a long bear put spread. The red curve shows the terminal payoff from the put options and the dotdash curve displays the profit after taking into account the cost of the initial trade to buy the spread.

8.11 Bear call spread

- Next let us consider a bearish trade, in particular a bear call spread.
- We are *short a call option at the low strike* K_1 *and long a call option at the high strike* K_2.
- As already explained, the low strike call at K_1 is worth more than the high strike call at K_2.
- This means that the buyer (long position) in a bear call spread ***receives money up front.***
- The profit at expiration is plotted in Fig. 3. The red solid curve plots the terminal payoff of the call options and the blue dotdash curve shows the profit after taking into account the income from the initial trade to buy the bear call spread.
- Clearly, the stock price at expiration must go above a level lower than the high strike K_2, for the overall trade to make a profit.
- Note that if the stock price goes up, the investor may suffer a loss (although the amount of loss is capped).
- The profit (if the stock price goes down) comes from the initial income when the bear call spread was purchased.
- The amount of profit using a bear call spread is not as great as the profit from a bear put spread with the same strikes.
- Nevertheless, a bear spread option trade *can* be done using call options.

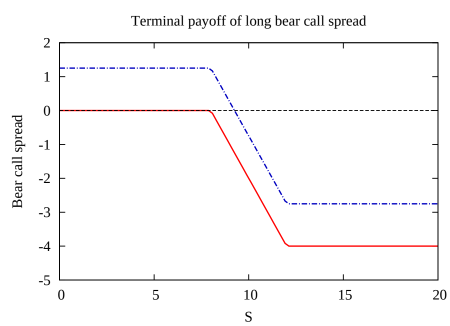

Figure 5: The payoff for a long bear call spread. The red curve shows the terminal payoff from the call options and the dotdash curve displays the profit after taking into account the income from the initial trade of the spread.

8.12 Butterfly spread

- There is no restriction to construct a spread using only two options.
- A **butterfly spread** is composed of three options, with strikes K_1, K_2 and K_3.
- For simplicity, we consider only the case where K_2 is halfway between K_1 and K_3, i.e. $K_2 = (K_1 + K_3)/2$.
- A long butterfly spread using call options consists of long one call struck at K_1, short two calls struck at K_2, long one call struck at K_3.
- A long butterfly spread using put options consists of long one put struck at K_1, short two puts struck at K_2, long one put struck at K_3.
- For both puts and calls, there is a positive cost to buy a long position in a butterfly spread. However, to prove this we need to prove that the fair value of an option (put or call) is a *convex function of the strike price.* This is beyond the level of mathematics of tihs course.
- The terminal payoff of the two ways of creating a long position in a butterfly spread (using puts or calls) are identical, and is shown in Fig. 6. An adjustment for the cost of buying the butterfly spread has been subtracted.
- The butterfly spread yields a profit only if the stock price at expiration is close to the central strike K_2.
  1. In many cases, the value of K_2 is close to the current price of the stock.
  2. An investor who buys a butterfly spread has a view that the stock price will not move much (up or down) away from K_2 during the life of the options.
  3. Therefore a butterfly spread is **neither bullish nor bearish.**
  4. A butterfly spread expresses more of a (non-directional) view on the amplitude of the stock price movement during the life of the options.
  5. A butterfly spread is therefore more of a **view on the volatility of the stock price.**

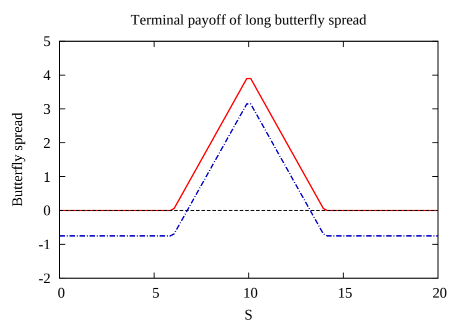

Figure 6: Graph of the terminal payoff of a long position in a butterfly spread. The strikes of the long options are $K_1 = 6$ and $K_3 = 14$ and the strike of the short options is $K_2 = 10$. The dotdash curve includes an adjustment for the initial cost of buying the butterfly spread.

8.13 Calendar spread and diagonal spread

- A **calendar spread** is composed of two options (of the same type), with the same strike but different expiration ddates.
- A calendar spread is constructed by buying an option with a long time to expiration and selling an option at the same strike but with a shorter time to expiration. Both options are puts or both options are calls.
- In a **reverse calendar spread**, we sell an option with a long time to expiration and buy an option at the same strike but with a shorter time to expiration. Both options are puts or both options are calls.
- The option with the longer-term expiration costs more than the shorter-expiration option. Hence there is a positive initial cost to buy a calendar spread.
- Note that if the option with the shorter expiration is exercised, the entire calendar spread terminates at that time.
- However, if the option with the shorter expiration is not exercised, the calendar spread survives to the longer expiration date.
- A calendar spread can be considered as a bearish view in the short term (to the expiration of the near-term option) and a bullish view in the longer term (to the expiration of the longer-term option).
- The payoff diagram for a calendar is not so simple to display because there is no single time at which we can say with certainty what the payoffs of all the options are. Suppose the option expiration times are T_1 and $T_2 > T_1$, We know the terminal payoff at the later time T_2, *but this requires an assumption that the shorter dated option was not exercised at T_1.* Conversely, at the time T_1, we do not know the value of the longer dated option. We require a mathematical formula for the longer dated option, for example, but that requires a mathematical model, with assumptions that may not be valid in practice. Fig. 7 displays the terminal payoff of the shorter dated option ($= c(T_1)$) at its expiration time T_1 (solid curve) and a *sketch* of the value of the longer dated option ($= c(T_2)$). Fig. 8 displays the value of the calendar spread ($= c(T_2) - c(T_1)$) using the options in Fig. 7. The dotdash curve in Fig. 8 includes an adjustment for the initial cost of buying the calendar spread. The calendar spread yields a profit for stock price values $S(T_1)$ near the peak and a loss for stock prices values $S(T_1)$ far from the peak (either too high or too low). **Note: It is an artifact of the inputs used to plot Figs. 7 and 8 that the peak in Fig. 8 occurs at or close to the option strike. This is not guaranteed in general.**
- It is possible to construct calendar spreads with three options (compare butterfly spread for vertical spreads), or more.
- A **diagonal spread** is basically a calendar spread where the option strikes are not equal. Hence it is composed of two options (of the same type) with different strikes and expirations.
- The initial cost of a diagonal spread could therefore be zero, or positive, or the investor could receive an up front payment when the spread is initiated.

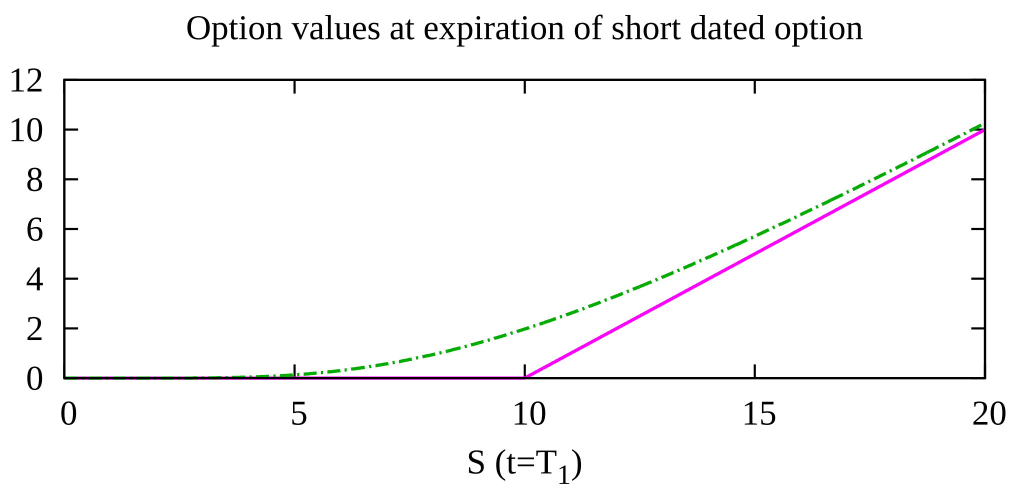

Figure 7: Graph of the values of a long call option with expiration T_1 and a long call option with expiration $T_2 > 1$, plotted at the expiration time T_1 of the shorter dated option. The solid curve shows the terminal payoff of the shorter dated option. The dotdash curve is a *sketch* of the value (*not terminal payoff*) of the longer dated option. Note that the value of the longer dated option is not a definite number, but an estimate using a mathematical model.

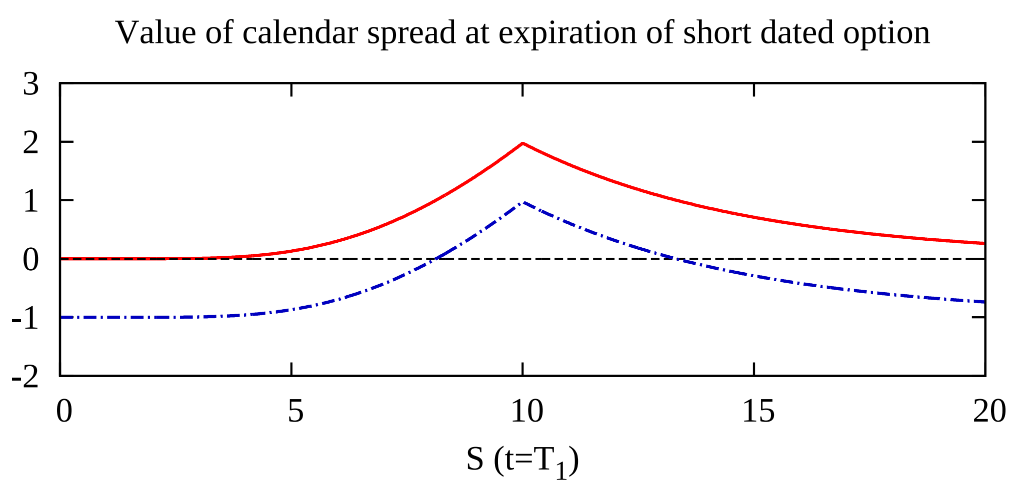

Figure 8: Graph of the value of a long position in a calendar spread, using the options displayed in Fig. 7, i.e. long the longer dated option ($= c(T_2)$) and short the shorter dated option ($= c(T_1)$). The solid curve shows difference in option values $= c(T_2) - c(T_1)$ of the two options. The dotdash curve includes an adjustment for the initial cost of buying the calendar spread. **Note: It is an artifact of the inputs that the peak occurs at or close to the option strike. This is not guaranteed in general. This is not a payoff diagram in the same sense as the vertical option spreads, because the options do not all expire at the same time.** The above plot shows the value of the calendar spread if the long dated option were sold at the same time that the short dated option expires. **However, it is not necessary to sell or exercise the long dated option at the same time that the short dated option expires.**

8.14 Combinations: straddles and strangles

- A **straddle** consists of long one call and one put, both with the same strike and expiration.
- A **strangle** is the same as a straddle but the call has a higher strike than the put.
- Clearly, there is an initial cost to buy either a straddle or a strangle.
- A graph of the terminal payoff of a straddle is shown in Fig. 9. The dotdash curve displays the profit after taking into account the cost of the initial trade to buy the straddle.
- A graph of the terminal payoff of a strangle is shown in Fig. 10. The dotdash curve displays the profit after taking into account the cost of the initial trade to buy the strangle.
- Both a straddle and a strangle are **non-directional trades.** A straddle yields a profit if the stock price at expiration is far from the common strike of the put and the call. A strangle yields a profit if the stock price at expiration is above the strike of the call or below the strike of the put (with adjustment for the initial cost to buy the strangle).
- Hence both a straddle and a strangle express a view on the **volatility of the stock.**

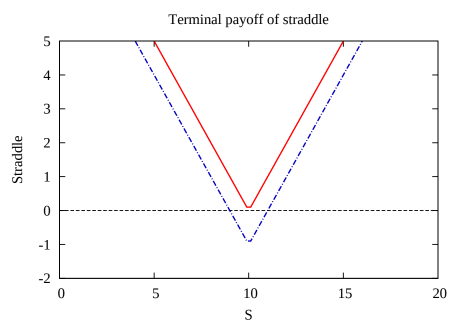

Figure 9: The terminal payoff for a straddle. Both the put and the call are struck at $K = 10$. The red curve shows the terminal payoff from the straddle and the dotdash curve displays the profit after taking into account the cost of the initial trade to buy the straddle.

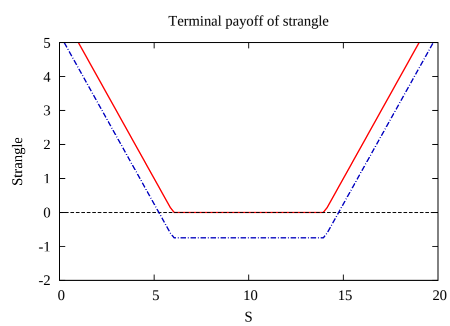

Figure 10: The terminal payoff for a strangle. The put is struck at $K_{\text{put}} = 6$ and the call is struck at $K_{\text{call}} = 14$. The red curve shows the terminal payoff from the strangle and the dotdash curve displays the profit after taking into account the cost of the initial trade to buy the strangle.

8.15 Combinations: collars

- A **collar** consists of long one put struck at K_1 and short a call struck at K_2, where $K_2 > K_1$. Both options have the same expiration.
- For a **zero cost collar** (also known as a **costless collar**), the price of the call equals the price of the put, hence the total cost of the collar is **zero.**
- A graph of the terminal payoff of a collar is shown in Fig. 11.
- Another trading strategy is to buy one share of stock and a collar. (This is equivalent to long a put and short a covered call.) A graph of the terminal payoff of a collar plus stock is shown in Fig. 12. Basically, this strategy protects against downside risk if the stock price drops too low, but it also caps the upside gain if the stock price increases too high.
- ***Basically, many trading strategies are possible.***

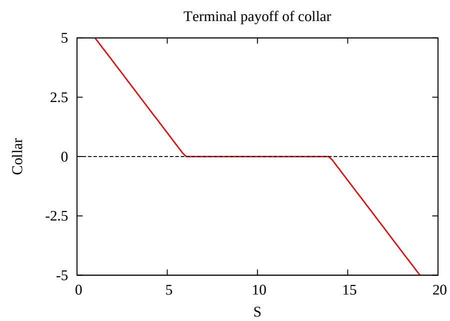

Figure 11: The terminal payoff for a collar. The put is struck at $K_{\text{put}} = 6$ and the call is struck at $K_{\text{call}} = 14$.

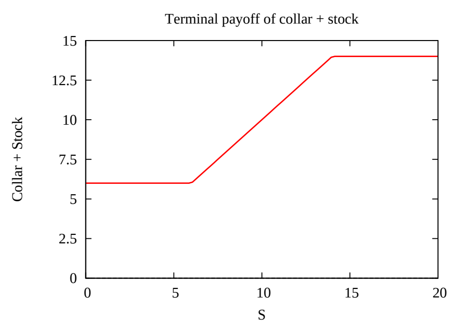

Figure 12: The terminal payoff for a collar plus stock. The put is struck at $K_{\text{put}} = 6$ and the call is struck at $K_{\text{call}} = 14$.