Queens College, CUNY, Department of Computer Science

Numerical Methods

CSCI 361 / 761

Spring 2018

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due Friday, Aug. 3, 2018, 11.59 pm

9 Homework lecture 9

9.1 Linear algebra: triangular systems

- **Question: The equations below are in upper triangular form. Solve the equations for the values of x_1, x_2 and x_3.**

$$x_1 - 2x_2 + 3x_3 = 2\,, \tag{9.1.1a}$$
$$3x_2 + x_3 = 1\,, \tag{9.1.1b}$$
$$x_3 = 4\,. \tag{9.1.1c}$$

- **Question: The equations below are in lower triangular form. Solve the equations for the values of x_1, x_2 and x_3.**

$$x_1 = 2\,, \tag{9.1.2a}$$
$$x_1 - 2x_2 = 4\,, \tag{9.1.2b}$$
$$x_1 - x_2 - 3x_3 = 8\,. \tag{9.1.2c}$$

- **As experience has demonstrated, if you do not understand the above expressions/questions, THEN ASK.**
- **If you do not understand the words/sentences in Lecture 6, etc. THEN ASK.**
- **Send me an email, explain what you do not understand.**
- **Do not just keep quiet and produce nonsense in exams.**

9.2 Linear algebra: inconsistency and linear dependence

- You are given the following equations

$$x_1 + 2x_2 + 3x_3 = 1\,, \tag{9.2.1a}$$
$$2x_1 + 2x_2 + x_3 = 1\,, \tag{9.2.1b}$$
$$3x_1 + 2x_2 - x_3 = 1\,. \tag{9.2.1c}$$

- **Question: Add or subtract multiples of eq.** (9.2.1a) **from eq.** (9.2.1b) **to eliminate** x_1 **from eq.** (9.2.1b)**. Next add or subtract multiples of eq.** (9.2.1a) **from eq.** (9.2.1c) **to eliminate** x_1 **from eq.** (9.2.1c)**.**

- Write the resulting equations in the following form.
(You have to calculate the values of a, b, c, d, r_2 and r_3.)

$$x_1 + 2x_2 + 3x_3 = 1\,, \tag{9.2.2a}$$
$$ax_2 + bx_3 = r_2\,, \tag{9.2.2b}$$
$$cx_2 + dx_3 = r_3\,. \tag{9.2.2c}$$

- **Question: Are eqs.** (9.2.2a) – (9.2.2c) **inconsistent?**

- **Question: Are eqs.** (9.2.2a) – (9.2.2c) **not linearly independent?**

- **STOP HERE: You do not have to solve eqs.** (9.2.2a) – (9.2.2c)**.**

9.3 Linear algebra: Gaussian elimination without pivoting

- You are given the following equations

$$x_1 + 2x_2 + x_3 = 1\,, \tag{9.3.1a}$$
$$2x_1 + 2x_2 - x_3 = 1\,, \tag{9.3.1b}$$
$$-3x_1 + 2x_2 + 3x_3 = 7\,. \tag{9.3.1c}$$

- **Question: Add or subtract multiples of eq. (9.3.1a) from eq. (9.3.1b) to eliminate x_1 from eq. (9.3.1b). Next add or subtract multiples of eq. (9.3.1a) from eq. (9.3.1c) to eliminate x_1 from eq. (9.3.1c).**

- Write the resulting equations in the following form.
(You have to calculate the values of a, b, c, d, r_2 and r_3.)

$$x_1 + 2x_2 + x_3 = 1\,, \tag{9.3.2a}$$
$$ax_2 + bx_3 = r_2\,, \tag{9.3.2b}$$
$$cx_2 + dx_3 = r_3\,. \tag{9.3.2c}$$

- **Question: Add or subtract multiples of eq. (9.3.2b) from eq. (9.3.2c) to eliminate x_2 from eq. (9.3.2c).**

- Write the resulting equations in the following form.
(You have to calculate the values of d', and r'_3.)

$$x_1 + 2x_2 + x_3 = 1\,, \tag{9.3.3a}$$
$$ax_2 + bx_3 = r_2\,, \tag{9.3.3b}$$
$$d'x_3 = r'_3\,. \tag{9.3.3c}$$

- **Question: Now eqs. (9.3.3a) – (9.3.3c) are in upper triangular form. Solve eqs. (9.3.3a) – (9.3.3c) by backsubstitution and calculate the values of x_1, x_2 and x_3.**

9.4 Linear algebra: Gaussian elimination & zero pivot

- You are given the following equations

$$x_1 + 2x_2 + x_3 = 1\,, \tag{9.4.1a}$$
$$2x_1 + 4x_2 - x_3 = 1\,, \tag{9.4.1b}$$
$$-3x_1 + 2x_2 + 3x_3 = 7\,. \tag{9.4.1c}$$

- **Question: Add or subtract multiples of eq.** (9.4.1a) **from eq.** (9.4.1b) **to eliminate** x_1 **from eq.** (9.4.1b)**. Next add or subtract multiples of eq.** (9.4.1a) **from eq.** (9.4.1c) **to eliminate** x_1 **from eq.** (9.4.1c)**.**

- Write the resulting equations in the following form.
(You have to calculate the values of a, b, c, d, r_2 and r_3.)

$$x_1 + 2x_2 + x_3 = 1\,, \tag{9.4.2a}$$
$$ax_2 + bx_3 = r_2\,, \tag{9.4.2b}$$
$$cx_2 + dx_3 = r_3\,. \tag{9.4.2c}$$

- **If you have done your work correctly, you should obtain** $a = 0$ **(zero pivot).**

- **NOTE: You do not have to solve eqs.** (9.4.2a) – (9.4.2c)**.**

9.5 Linear algebra: Gaussian elimination with partial pivoting

- **We employ the equations from the previous section eqs.** (9.4.1a) – (9.4.1c)**.**
- **Question: Calculate the value of** $\hat{a}_i$ **for** $i = 1, 2, 3$ **in eqs.** (9.4.1a) – (9.4.1c)**.**
- **Question: Calculate the value of** $|a_{i1}|/\hat{a}_i$ **for** $i = 1, 2, 3$ **in eqs.** (9.4.1a) – (9.4.1c)**.**
- **Question: Which pair of equations in eqs.** (9.4.1a) – (9.4.1c) **will be swapped?**
- Swap the relevant pair of equations and write the resulting equations in the following form. (You have to calculate the values of a_{ij}, and b_i, for $i, j = 1, 2, 3$.)

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1\,, \tag{9.5.1a}$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2\,, \tag{9.5.1b}$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3\,. \tag{9.5.1c}$$

- **Question: Add or subtract multiples of eq.** (9.5.1a) **from eq.** (9.5.1b) **to eliminate** x_1 **from eq.** (9.5.1b)**. Next add or subtract multiples of eq.** (9.5.1a) **from eq.** (9.5.1c) **to eliminate** x_1 **from eq.** (9.5.1c)**.**
- Write the resulting equations in the following form. (You have to calculate the values of a'_{ij}, and b'_j for $i, j = 2, 3$.)

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1\,, \tag{9.5.2a}$$
$$a'_{22}x_2 + a'_{23}x_3 = b'_2\,, \tag{9.5.2b}$$
$$a'_{32}x_2 + a'_{33}x_3 = b'_3\,. \tag{9.5.2c}$$

- **If you have done your work correctly, you should obtain** $a'_{22} \neq 0$ **(nonzero pivot).**
- **NOTE: You do not have to solve eqs.** (9.5.2a) – (9.5.2c)**.**

9.6 Linear algebra: (optional)

- **Solve eqs.** (9.4.1a) – (9.4.1c)**.**