Queens College, CUNY, Department of Computer Science

Computational Finance

CSCI 365 / 765

Summer 2018

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Midterm 2

Wednesday July 18, 2018

- **NOTE: It is the policy of the Computer Science Department to issue a failing grade to any student who either gives or receives help on any test.**
- **A student caught cheating on any question in an exam, project or quiz will fail the entire course.**
- **Any problem to which you give two or more (different) answers receives the grade of zero automatically.**
- *This is a take home exam. Answers should be typed in a file. See below for instructions.*
- Please submit your solution via email, as a file attachment, to `Sateesh.Mane@qc.cuny.edu`.
- Please submit one zip archive with all your files in it.
  1. The zip archive should have either of the names (CS361 or CS761):

     `StudentId_first_last_CS365_midterm2_July2018.zip`
     `StudentId_first_last_CS765_midterm2_July2018.zip`
  2. The archive should contain one "text file" named "Midterm2.[txt/docx/pdf]" and one cpp file per question named "Q1.cpp" and "Q2.cpp" etc.
  3. Note that text answers may not be required for all questions.
  4. Note that not all questions may require a cpp file.
- **In all questions where you are asked to submit programming code, programs which display any of the following behaviors will receive an automatic F:**
  1. Programs which do not compile successfully (non-fatal compiler warnings are excluded).
  2. Array out of bounds, reading of uninitialized variables (including null pointers).
  3. Operations which yield NAN or infinity, e.g. divide by zero, square root of negative number, etc. *Infinite loops.*
  4. Programs which do NOT implement the public interface stated in the question.
- **In addition, note the following:**
  1. All debugging statements (for your personal testing) should be commented out.
  2. Program performance will be graded solely on the public interface stated in the questions.

General information

- **The statements below are for general information only.**
- Ignore them if they are not relevant for the exam questions below.
- The questions in this exam do not involve problems of overflow or underflow.
- Solutions involving the writing of algorithms will not be judged if they work on a 64–bit instead of a 32–bit computer.
- **Value of π to machine precision on any computer.**

  1. Some compilers support the constant `M_PI` for π, in which case you can write

     ```
     const double pi = M_PI;
     ```

  2. If your compiler does not support `M_PI`, the value of π can be computed via

     ```
     const double pi = 4.0*atan2(1.0,1.0);
     ```

Material to be used in later questions

- Form a set of eight digits $(d_1, \ldots, d_8)$ as follows.
- **Take the 8 digits of your student id and define $(d_1, \ldots, d_8)$ as follows:**

$$
\begin{aligned}
d_1 &= \text{digit 1 of student id}\,,\\
d_2 &= \text{digit 2 of student id}\,,\\
d_3 &= \text{digit 3 of student id}\,,\\
d_4 &= \text{digit 4 of student id}\,,\\
d_5 &= \text{digit 5 of student id}\,,\\
d_6 &= \text{digit 6 of student id}\,,\\
d_7 &= \text{digit 7 of student id}\,,\\
d_8 &= \text{digit 8 of student id}\,.
\end{aligned}
$$

- For example if your student id is 23054617, then

$$
\begin{aligned}
d_1 &= 2\,,\\
d_2 &= 3\,,\\
d_3 &= 0\,,\\
d_4 &= 5\,,\\
d_5 &= 4\,,\\
d_6 &= 6\,,\\
d_7 &= 1\,,\\
d_8 &= 7\,.
\end{aligned}
$$

- *For some student ids, it is possible that some of the digits may be zero. It is also possible that some of the digits may be equal.* Do not worry.
- For the student id 11111111, all the digits are equal.
- For the student id 33330000, four digits are zero and the other four are all equal to 3.

1 Question 1

- The price of a stock is $S_0 = 100.0$ at time $t_0 = 0$.
- The stock pays no dividends.
- The interest rate is $r = 5.0\%$.
- The expiration time of the forward contract is $T = 1.0$ years.
- **Question: For each case below, formulate an arbitrage strategy to take advantage of the forward price.**
  1. The forward price is $F = 105.0$.
  2. The forward price is $F = 106.0$.
- **Question: For each case above, state how much profit your arbitrage strategy yields at the expiration time T.**
- ***Show all the steps in your arbitrage strategies. Do not just state the final profit.***

2 Question 2

- A stock trades today ($t_0 = 0$) at a price of $S_0 = 100.0$.
- A futures contract on the stock trades today at a price $F_0 = 103.25$.
- The futures contract expiration date is 5 days from today.
- Every day for $t_i = i$, $i = 1, 2, 3, 4, 5$, the stock price is S_1, S_2, S_3, S_4, S_5.
- The futures price every day is F_1, F_2, F_3, F_4, F_5.
- On the expiration day, $F_5 = S_5$ (the futures price converges to the stock price).
- **The stock pays a dividend of** 0.1 **on day 3.**
- Here is a list of the stock and futures prices, for $i = 1, 2, 3, 4, 5$.

i	S_i	F_i	dividend
1	100.75	104.25	
2	S_2	F_2	
3	S_3	F_3	0.1
4	S_4	F_4	
5	S_5	F_5	

- **The values of** $S_2, \ldots, S_5$ **are arbitrary, but satisfy the following inequalities:**

$$100.75 > S_2 > S_3 > S_4 > S_5\,. \tag{2.1}$$

- **The values of** $F_2, \ldots, F_5$ **are arbitrary, but satisfy the following inequalities:**

$$104.25 > F_2 > F_3 > F_4 > F_5 (= S_5)\,. \tag{2.2}$$

- **See next page.**

2.1 Investor A

- Investor A goes long one share of stock on day 0.
- Investor A sells the stock on day 5.
- **Calculate (or state) the money paid/received by A every day, starting from day 0, until A's portfolio is closed out.**
- **Calculate the total profit/loss for A after selling the stock.**
  Ignore interest rate compounding for any borrowed money, to answer this question.
- **State on which day A makes that profit/loss.**

2.2 Investor B

- Investor B goes long one futures contract on day 0.
- Investor B holds the futures contract to expiration.
- **Calculate the money paid/received by B every day, starting from day 0, until B's portfolio is closed out.**
- **Calculate the total money paid by B after closing the futures contract.**
  Assume that money in a mark to market account does not pay interest.
- **State what B receives in exchange for closing the futures contract.**

2.3 Investor C

- Investor C goes long one futures contract on day 0.
- Investor C sells the futures contract on day 1.
- **Calculate the money paid/received by C every day, starting from day 0, until C's portfolio is closed out.**
- **Calculate the total profit/loss for C after selling the futures contract.**
  Assume that money in a mark to market account does not pay interest.
- **State on which day C makes that profit/loss.**

2.4 Investor D

- Investor D goes long a **forward contract** on day 0.
- The forward price is $F_{\text{fwd}} = 103.25$ and the expiration time is 5 days.
- **Explain what (if any) trades D can perform on day 1, to lock in a guaranteed profit.**
- **Calculate the money paid/received (if any) by D every day, starting from day 0, until D's portfolio is closed out.**
- **Calculate the total profit/loss for D (if any).**
  Assume that money in a mark to market account does not pay interest.
- **State on which day D makes that profit/loss (if any).**

3 Question 3

- **Ignore interest rate compounding for all profit/loss calculations in this question.**
- A stock trades today ($t_0 = 0$) at a price of $S_0 = 100.0$.
- A futures contract on the stock trades today at a price $F_0 = 103.25$.
- The futures contract expiration date is 5 days from today.
- Every day for $t_i = i$, $i = 1, 2, 3, 4, 5$, the stock price is S_1, S_2, S_3, S_4, S_5.
- The corresponding futures price every day is F_1, F_2, F_3, F_4, F_5.
- On the expiration day, $F_5 = S_5$ (the futures price converges to the stock price).
- **The stock pays a dividend of** 0.07 **on day 3.**
- Here is a list of the stock and futures prices, for $i = 0, 1, 2, 3, 4, 5$.

i	S_i	F_i	dividend
0	100.0	103.25	
1	98.75	101.55	
2	S_2	F_2	
3	S_3	F_3	0.07
4	S_4	F_4	
5	S_5	F_5	

- **The values of** $S_2, \ldots, S_5$ **are arbitrary, but satisfy the following inequalities:**

$$98.75 < S_2 < S_3 < S_4 < S_5\,. \tag{3.1}$$

- **The values of** $F_2, \ldots, F_5$ **are arbitrary, but satisfy the following inequalities:**

$$101.55 < F_2 < F_3 < F_4 < F_5 (= S_5)\,. \tag{3.2}$$

See next page

3.1 Investor A

- **Investor A goes long one share of stock on day 0.**
- Investor A sells the stock on day 5.
- **Calculate (or state) the money paid/received by A every day, starting from day 0, until A's portfolio is closed out.**
  (Note that in some cases your answer may be a formula not a dollar number.)
- **Calculate the total profit/loss for A after selling the stock.**
- **State on which day A makes that profit/loss.**

3.2 Investor B

- **Investor B goes long one futures contract on day 1.**
- Investor B holds the futures contract to expiration.
- **Calculate the money paid/received every day in B's mark to market account, until B's portfolio is closed out.**
  (Note that in some cases your answer may be a formula not a dollar number.)
- **Calculate the total money paid by B after closing the futures contract.**
- **State what B receives in exchange for closing the futures contract.**

3.3 Investor C

- **Investor C goes short a forward contract on day 0.**
- The forward price is $F_{\text{fwd}} = 103.25$ and the expiration time is 5 days.
- **From the data on day 1, it is possible for C to lock in a guaranteed profit?**
  1. **If yes, state the strategy to lock in a guaranteed profit.**
  2. **If no, explain why not.**
- **State all the trades performed on the day C's portfolio is closed out.**
- **Calculate the total profit/loss for C (if any).**
- **State on which day C makes that profit/loss (if any).**

3.4 Investor D

- **Investor D goes long one futures contract on day 1.**
- Investor D sells the futures contract on day 3.
- **Calculate the money paid/received every day in D's mark to market account, until D's portfolio is closed out.**
  (Note that in some cases your answer may be a formula not a dollar number.)
- **Calculate the total profit/loss for D after selling the futures contract.**
- **State on which day D makes that profit/loss.**
- **State the trades (stock/cash/futures) which happen for D on day 5 (futures expiration).**