

November 15, 2017

## 4 Lecture 4

### 4.1 Stocks (equities): basic notions

- In the previous lectures we dealt with bonds. It is time now to turn to the other building block of derivatives, i.e. stocks. This will introduce us to notions of probability, i.e. cashflows which *might* be paid at times in the future, depending on what happens in the future. The present value of those cashflows will then depend on their probabilities. That will lead us to the best known of all derivatives, namely options on a stock (or a stock index), which is what many of you would like to see, or learn about.
- Before we take that headlong plunge, however, we need to learn some fundamental concepts. The most important concept is that of **arbitrage**.
- We must also learn about **hedging** (in particular the notion of a “**perfect hedge**”).
- We also need to understand that the theory we shall employ in these lectures is based on various simplifying assumptions. These assumptions are required to enable us to get started, as new students learning the subject, but are not exactly satisfied in real financial markets. The most important of these assumptions is that of **efficient markets**. There is also the concept of **liquidity**.
- It is perhaps simplest to learn about arbitrage and efficient markets, etc. by studying the simplest of all derivatives, which are forward and futures contracts on a stock.
- **Notation:**  
Throughout this lecture (and many others to follow), we shall assume the interest rate is a constant ( $= r$ ). We shall employ continuously compounded interest rates only. The time today will be denoted by  $t_0$ . In many cases, to simplify the appearance of the formulas, we can set  $t_0 = 0$ .

## 4.2 Efficient markets, liquidity, friction, etc.

- We shall assume that the financial markets are **efficient**. This means that all information about the stock market is available to everyone, at the same time. For example, if a major news item occurs which affects the financial markets, it will be known to everyone, at the same time. Hence nobody can take advantage because they have information not available to others. This is obviously not true in real markets, but in many cases it is a good approximation.
- We shall also assume that the financial markets are **liquid**. This means that we can buy or sell any stock (or bond, etc.) at any time we wish. This means that if we wish to buy a stock, there are always other people willing to sell it. Conversely, if we wish to sell a stock, there are always other people willing to buy it. (Not just a stock, but any other financial instrument.)
- We shall actually assume that the financial markets are **infinitely liquid**. This means we can buy or sell *any quantity* of stock (or bonds, etc.) at the price quoted in the market, all at the same time. This is obviously an approximation. If we wanted to buy a million shares of a stock, even in the USA it might be difficult to find enough people willing to sell so many shares all at the same time. It would require time to find enough sellers, and the stock price would change during that time interval.
- A stock which is difficult to buy or sell is called **illiquid**.
- An important corollary of the notion of infinitely liquid markets is that there is nobody (or no organization) who is so powerful that they can control the market. There is nobody who can dictate to others what the price of a financial instrument should be. Nobody can manipulate the prices on the financial market. For an illiquid stock, this assumption may not be true,
- We shall also assume that the financial markets are **frictionless**. This means there is no fee for doing a trade in the financial markets. This is clearly not true. If we wish to buy or sell a stock, we have to pay our stockbroker a fee. In these lectures, we assume there is no fee. This is an assumption that is made in all textbooks on finance. Incorporating friction into financial pricing theory is still a subject of research.
- We shall also assume that the financial markets have no **counterparty risk**. When we perform a trade, the other side to the trade is called the **counterparty**. We assume the counterparty will not cheat or refuse to pay, etc. (for example if the counterparty loses money on the trade). Incorporating counterparty risk into financial pricing theory is still a subject of research.
- We shall also assume that the financial markets allow **unrestricted short selling**. It will be simpler to explain this concept with some examples below, after we have studied what “short selling” means. Unrestricted short selling is actually not true in the financial markets.

## 4.3 Spreads

### 4.3.1 Bid-ask (bid-offer) spread

- In the stock market, two prices are quoted for a stock, called the **bid** and **ask** price.
- The ask price is also known as the **offer** price.
- The bid price is the price buyers are willing to pay to buy a stock.
- The ask price is the price sellers demand, to sell their shares of stock.  
(It is the price at which sellers “offer” to sell their shares of stock.)
- The ask (or offer) price is higher than the bid price.
- For example, for recent information about Microsoft (MSFT) shares, the bid price was \$73.29 and the ask price was \$73.30.
- There are bid-ask or bid-offer spreads for all financial securities.  
Basically, sellers want to sell at a higher price and buyers want to buy at a lower price.
- The difference between the bid and ask prices is called the **bid-ask spread** or **bid-offer spread**.
- **We shall assume the bid-ask spread is zero.**
- This is not true in practice.
- The bid-ask spread is an example of **market friction**. We cannot buy and sell at the same price. The bid-ask spread is caused because the market is not completely liquid. For a highly liquid stock, the bid-ask spread is small. For an **illiquid stock** (= not liquid), the bid-ask spread is large. If it is difficult to buy and sell the stock (maybe because nobody is interested), the bid-ask spread will be large.

### 4.3.2 Borrowing and lending interest rates

- We shall also assume that the interest rates at which we can **borrow and lend money are equal**. This is not true in practice. If we deposit money in a bank savings account (effectively lending money to the bank), the interest rate we receive on our savings is lower than the interest rate the bank will charge us if we apply to the same bank for a personal loan. The borrowing rate is higher than the lending rate.
- We shall assume the borrowing and lending rates are equal.
- Unequal borrowing and lending rates are also an example of **market friction**.

## 4.4 Arbitrage

- An **arbitrage trade** is a financial transaction which
  - (i) incurs *no loss in all future scenarios*, and
  - (ii) has a *positive probability of profit in some of those scenarios*.

Hence by executing an arbitrage trade, we shall never lose money, and will make a positive profit in some cases. Essentially if arbitrage exists, there is a riskless way to make a profit.

- The words “all” and “positive probability” are important in the definition of arbitrage. It is not enough that a trade makes no loss in some scenarios but not all. That is *not* arbitrage: such a trade might make a loss. An arbitrage trade must make no loss in *all* scenarios. It must also make a profit in some scenarios (with a probability greater than zero).
- An arbitrage strategy always begins with a position of zero. We start with zero shares of stock, and zero money in our bank account. We buy or sell any shares of stock to perform the required trades. We borrow any money we need to perform the required trades (and pay interest on the borrowing). If we have cash in hand (for example if we sold stock), we invest the money in a bank and earn interest on that money. We assume that the interest rates at which we borrow or lend money are equal. (This is an important assumption.)
- What if a trade makes no loss in all future scenarios and no profit either? Then the total cost of the trade today must be zero. That is a central concept in financial pricing theory. It leads to the concept of a **perfect hedge**. It yields a **fair value** for a financial security. We shall see many examples in the lectures.

## 4.5 Stock price: efficient and liquid markets, arbitrage

- Suppose there is a stock  $S$ , whose price today is  $S_0$ .
- In an efficient market, *all market participants must agree that the stock price is  $S_0$* .
- Consider the following four people  $X_1 - X_4$ :
  1.  $X_1$  wishes to sell the stock at a price  $S_1 < S_0$ .
  2.  $X_2$  wishes to buy the stock at a price  $S_2 > S_0$ .
  3.  $X_3$  wishes to sell the stock at a price  $S_3 > S_0$ .
  4.  $X_4$  wishes to buy the stock at a price  $S_4 < S_0$ .
- These are our responses to  $X_1 - X_4$ :
  1. We buy the stock from  $X_1$  at price  $S_1$  and immediately sell it on the market at price  $S_0$ . We make a profit of  $S_0 - S_1$ .
  2. We buy the stock on the market at price  $S_0$  and immediately sell it to  $X_2$  at price  $S_2$ . We make a profit of  $S_2 - S_0$ .
  3. *We simply don't do business with  $X_3$* . There is no loss to us.
  4. We also don't do business with  $X_4$ . There is also no loss to us.
- Note the following important details in formulating the trading strategies in the above cases:
  1. The market must be efficient: we need to know that the market price is  $S_0$ . If we did not know the market price is  $S_0$ , we could not formulate our trading strategies as above.
  2. The market must be liquid: to execute the trades above, we must be able to buy and sell the stock quickly, before the market price of the stock changes away from  $S_0$ .
  3. Our profit was  $S_0 - S_1$  (first case) or  $S_2 - S_0$  (second case) and our loss was zero in all cases. This is how an arbitrage strategy functions: there must never be a loss in any scenario, and a positive profit (with probability  $> 0$ ) in some scenarios.
  4. It is also important to note that an arbitrage strategy begins with a position of *zero* in our account. We have no stock initially. We buy and sell what we need, and end up with a profit. Suppose we already owned shares of stock and sold them to  $X_2$  at the price  $S_2$ . This is *not* arbitrage. We bought the stock at an earlier time, at a price different from  $S_0$ , and selling it now at a price  $S_2$  may or may not make a profit. We might even make a loss, if we paid more than  $S_2$  when we purchased the stock initially.
  5. The market must also be frictionless. Suppose we had to pay transaction fees to perform the above trades. Consider the case of  $X_1$ , who was willing to sell the stock at a price  $S_1 < S_0$ . We buy the stock from  $X_1$  at price  $S_1$  (and pay a fee to our stockbroker) and immediately sell it on the market at price  $S_0$  (and pay a second fee to our stockbroker). If the total fee we pay exceeds  $S_0 - S_1$ , then there is no profit for us, and in fact a loss. Then we would not perform the above trades.
- Some people express the above concept as follows:

**In an efficient market, one price must rule.**