Queens College, CUNY, Department of Computer Science

Computational Finance

CSCI 365 / 765

Spring 2018

Instructor: Dr. Sateesh Mane

July 14, 2018

due Tuesday July 17, 2018 at 11.59 pm

4 Homework: Forwards & Futures

Note: Continuous interest rate compounding is used in all questions.

- Please email your solution, as a file attachment, to `Sateesh.Mane@qc.cuny.edu`.
- Please submit one zip archive with all your files in it.
  1. The zip archive should have either of the names (CS365 or CS765):

     `StudentId_first_last_CS365_hw4.zip`
     `StudentId_first_last_CS765_hw4.zip`

  2. The archive should contain one "text file" named "hw4.[txt/docx/pdf]" and one cpp file per question named "Q1.cpp" and "Q2.cpp" etc.
  3. Note that not all homework assignments may require a text file.
  4. Note that not all questions may require a cpp file.

4.1 Forward price

- Suppose the price of a stock is $S_0 = 100.5$ at time $t_0 = 0$. The stock pays no dividends. Suppose also the interest rate is $r = 5.5\%$.
  Question: Calculate the fair value of the forward price at the time $T = 0.75$ years.

$$F = S_0 e^{r(T-t_0)} . \tag{4.1.1}$$

- Suppose the price of a stock is $S_0 = 95.5$ at time $t_0 = 0$ and it pays a continuous dividend yield $q = 1.1\%$. Suppose also the interest rate is $r = 5.1\%$.
  Question: Calculate the fair value of the forward price at the time $T = 0.65$ years.

$$F = S_0 e^{(r-q)(T-t_0)} . \tag{4.1.2}$$

4.2 Forwards: arbitrage

- The price of a stock is $S_0 = 100.0$ at time $t_0 = 0$.
- The stock pays no dividends.
- The interest rate is $r = 5.0\%$.
- The expiration time of the forward contract is $T = 1.0$ years.
- **Question: For each case below, formulate an arbitrage strategy to take advantage of the forward price.**
  1. The forward price is $F = 105.0$.
  2. The forward price is $F = 106.0$.
- **Question: For each case above, state how much profit your arbitrage strategy yields at the expiration time T.**
- ***Show all the steps in your arbitrage strategies. Do not just state the final profit.***

4.3 Futures: mark to market

- Suppose the stock price today is $S_0 = 100.0$. We have a futures contract which will expire in 5 days. The futures price today is $F_0 = 105.5$. Today is $t_0 = 0$ and the expiration date is $t_5 = 5$ (measured in days, not years).
- Every day for $t_i = i$, $i = 1, 2, 3, 4, 5$, the stock price is S_1, S_2, S_3, S_4, S_5. The futures price every day is F_1, F_2, F_3, F_4, F_5. On the expiration day, $F_5 = S_5$ (the futures price converges to the stock price).
- Every day the futures contract is **marked to market.** This means that if the futures price on day i is F_i, (for $i = 1, 2, 3, 4, 5$), the following happens.
  1. If $F_i > F_{i-1}$, the investor receives a cash amount $F_i - F_{i-1}$ in the mark to market account.
  2. If $F_i < F_{i-1}$, the investor pays a cash amount $F_{i-1} - F_i$ into the mark to market account.
  3. If $F_i = F_{i-1}$, nothing happens.
- **Note: the money in the mark to market account does not earn interest.**
- **See next page.**

4.3.1 Random walk 1

- Here is a list of the stock and futures prices, for $i = 1, 2, 3, 4, 5$.

i	S_i	F_i	money received	money paid
1	99.5	103.3		
2	101.3	104.1		
3	101.3	102.1		
4	100.2	101.3		
5	99.3	99.3		

- **Question: For each day, calculate the amount of money paid or received in the mark to market account.**

- On the final day (expiration), the futures price converges to the stock price so $F_5 = S_5$.,
  The investor pays the stock price S_5 and takes delivery of the underlying asset.

- **Question: Add up the money the investor pays on the final day (to take delivery) plus the money in the mark to market account. What is the total amount of money the investor pays?**

4.3.2 Random walk 2

- Here is a **different** list of the stock and futures prices, for $i = 1, 2, 3, 4, 5$.
  (Remember that $S_0 = 100.0$ and $F_0 = 105.5$.)

i	S_i	F_i	money received	money paid
1	100.9	106.3		
2	103.8	108.7		
3	106.1	109.2		
4	107.5	108.3		
5	108.3	108.3		

- **Question: For each day, calculate the amount of money paid or received in the mark to market account.**

- On the final day (expiration), the futures price converges to the stock price so $F_5 = S_5$.
  The investor pays the stock price S_5 and takes delivery of the underlying asset.

- **Question: Add up the money the investor pays on the final day (to take delivery) plus the money in the mark to market account. What is the total amount of money the investor pays?**

4.3.3 Summary

- **Question: Does the random walk of the futures prices affect the total amount paid by the investor?**

- **Question: Does the random walk of the stock prices affect the total amount paid by the investor?**

4.3.4 Addendum (added Oct 11, 2017)

- ***The investor sells the futures contract prior to expiration.***
- When the investor sells the futures contract, one of three things happens:
  1. Money received in account > money paid: the net profit is given to the investor.
  2. Money paid into account > money received: the investor loses the net loss.
  3. Money paid into account = money received: nothing happens (no profit or loss).

4.3.5 Random walk 3

- Here is a list of the stock and futures prices, for $i = 1, 2, 3, 4, 5$.
  (Remember that $S_0 = 100.0$ and $F_0 = 105.5$.)

i	S_i	F_i	money received	money paid
1	99.5	106.3		
2	101.3	105.1		
3	101.3	105.8		
4	100.1	104.2		
5	102.3	102.3		

- **Question: For each case below, calculate the net profit or loss to the investor.**
  1. The investor sells the futures contract on day 2.
  2. The investor sells the futures contract on day 3.

4.3.6 Random walk 4

- In this random walk the stock prices are **different** but the futures prices are the **same** as in random walk #3 above.
  (Remember that $S_0 = 100.0$ and $F_0 = 105.5$.)

i	S_i	F_i	money received	money paid
1	99.7	106.3		
2	101.2	105.1		
3	101.5	105.8		
4	102.8	104.2		
5	102.3	102.3		

- **Question: For each case below, calculate the net profit or loss to the investor.**
  1. The investor sells the futures contract on day 2.
  2. The investor sells the futures contract on day 3.

4.3.7 Summary

- **Question: Does the random walk of the stock prices affect the profit/loss of the investor?**