Queens College, CUNY, Department of Computer Science

Computational Finance

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4 Lecture 4

4.1 Stocks (equities): basic notions

- In the previous lectures we dealt with bonds. It is time now to turn to the other building block of derivatives, i.e. stocks. This will introduce us to notions of probability, i.e. cashflows which *might* be paid at times in the future, depending on what happens in the future. The present value of those cashflows will then depend on their probabilities. That will lead us to the best known of all derivatives, namely options on a stock (or a stock index), which is what many of you would like to see, or learn about.

- Before we take that headlong plunge, however, we need to learn some fundamental concepts. The most important concept is that of **arbitrage**.

- We must also learn about **hedging** (in particular the notion of a "**perfect hedge**").

- We also need to understand that the theory we shall employ in these lectures is based on various simplifying assumptions. These assumptions are required to enable us to get started, as new students learning the subject, but are not exactly satisfied in real financial markets. The most important of these assumptions is that of **efficient markets**. There is also the concept of **liquidity**.

- It is perhaps simplest to learn about arbitrage and efficient markets, etc. by studying the simplest of all dervatives, which are forward and futures contracts on a stock.

- **Notation:**
  Throughout this lecture (and many others to follow), we shall assume the interest rate is a constant ($= r$). We shall employ continuously compounded interest rates only. The time today will be denoted by t_0. In many cases, to simplify the appearance of the formulas, we can set $t_0 = 0$.

4.2 Efficient markets, liquidity, friction, etc.

- We shall assume that the financial markets are **efficient**. This means that all information about the stock market is available to everyone, at the same time. For example, if a major news item occurs which affects the financial markets, it will be known to everyone, at the same time. Hence nobody can take advantage because they have information not available to others. This is obviously not true in real markets, but in many cases it is a good approximation.

- We shall also assume that the financial markets are **liquid**. This means that we can buy or sell any stock (or bond, etc.) at any time we wish. This means that if we wish to buy a stock, there are always other people willing to sell it. Conversely, if we wish to sell a stock, there are always other people willing to buy it. (Not just a stock, but any other financial instrument.)

- We shall actually assume that the financial markets are **infinitely liquid**. This means we can buy or sell *any quantity* of stock (or bonds, etc.) at the price quoted in the market, all at the same time. This is obviously an approximation. If we wanted to buy a million shares of a stock, even in the USA it might be difficult to find enough people willing to sell so many shares all at the same time. It would require time to find enough sellers, and the stock price would change during that time interval.

- A stock which is difficult to buy or sell is called **illiquid**.

- An important corollary of the notion of infinitely liquid markets is that there is nobody (or no organization) who is so powerful that they can control the market. There is nobody who can dictate to others what the price of a financial instrument should be. Nobody can manipulate the prices on the financial market. For an illiquid stock, this assumption may not be true,

- We shall also assume that the financial markets are **frictionless**. This means there is no fee for doing a trade in the financial markets. This is clearly not true. If we wish to buy or sell a stock, we have to pay our stockbroker a fee. In these lectures, we assume there is no fee. This is an assumption that is made in all textbooks on finance. Incorporating friction into financial pricing theory is still a subject of research.

- We shall also assume that the financial markets have no **counterparty risk**. When we perform a trade, the other side to the trade is called the **counterparty**. We assume the counterparty will not cheat or refuse to pay, etc. (for example if the counterparty loses money on the trade). Incorporating counterparty risk into financial pricing theory is still a subject of research.

- We shall also assume that the financial markets allow **unrestricted short selling**. It will be simpler to explain this concept with some examples below, after we have studied what "short selling" means. Unrestricted short selling is actually not true in the financial markets.

4.3 Spreads

4.3.1 Bid-ask (bid-offer) spread

- In the stock market, two prices are quoted for a stock, called the **bid** and **ask** price.
- The ask price is also known as the **offer** price.
- The bid price is the price buyers are willing to pay to buy a stock.
- The ask price is the price sellers demand, to sell their shares of stock.
  (It is the price at which sellers "offer" to sell their shares of stock.)
- The ask (or offer) price is higher than the bid price.
- For example, for recent information about Microsoft (MSFT) shares, the bid price was \$73.29 and the ask price was \$73.30.
- There are bid-ask or bid-offer spreads for all financial securities.
  Basically, sellers want to sell at a higher price and buyers want to buy at a lower price.
- The difference between the bid and ask prices is called the **bid-ask spread** or **bid-offer spread.**
- **We shall assume the bid-ask spread is zero.**
- This is not true in practice.
- The bid-ask spread is an example of **market friction**. We cannot buy and sell at the same price. The bid-ask spread is caused because the market is not completely liquid. For a highly liquid stock, the bid-ask spread is small. For an **illiquid stock** (= not liquid), the bid-ask spread is large. If it is difficult to buy and sell the stock (maybe because nobody is interested), the bid-ask spread will be large.

4.3.2 Borrowing and lending interest rates

- We shall also assume that the interest rates at which we can **borrow and lend money are equal.** This is not true in practice. If we deposit money in a bank savings account (effectively lending money to the bank), the interest rate we receive on our savings is lower than the interest rate the bank will charge us if we apply to the same bank for a personal loan. The borrowing rate is higher than the lending rate.
- We shall assume the borrowing and lending rates are equal.
- Unequal borrowing and lending rates are also an example of **market friction**.

4.4 Arbitrage

- An **arbitrage trade** is a financial transaction which

  (i) incurs ***no loss in all future scenarios***, and

  (ii) has a ***positive probability of profit in some of those scenarios.***

  Hence by executing an arbitrage trade, we shall never lose money, and will make a positive profit in some cases. Essentially if arbitrage exists, there is a riskless way to make a profit.

- The words "all" and "positive probability" are important in the definition of arbitrage. It is not enough that a trade makes no loss in some scenarios but not all. That is *not* arbitrage: such a trade might make a loss. An arbitrage trade must make no loss in *all* scenarios. It must also make a profit in some scenarios (with a probability greater than zero).

- An arbitrage strategy always begins with a position of zero. We start with zero shares of stock, and zero money in our bank account. We buy or sell any shares of stock to perform the required trades. We borrow any money we need to perform the required trades (and pay interest on the borrowing). If we have cash in hand (for example if we sold stock), we invest the money in a bank and earn interest on that money. We assume that the interest rates at which we borrow or lend money are equal. (This is an important assumption.)

- What if a trade makes no loss in all future scenarios and no profit either? Then the total cost of the trade today must be zero. That is a central concept in financial pricing theory. It leads to the concept of a **perfect hedge**. It yields a **fair value** for a financial security. We shall see many examples in the lectures.

4.5 Stock price: efficient and liquid markets, arbitrage

- Suppose there is a stock S, whose price today is S_0.
- In an efficient market, *all market participants must agree that the stock price is S_0.*
- Consider the following four people $X_1 - X_4$:
  1. X_1 wishes to sell the stock at a price $S_1 < S_0$.
  2. X_2 wishes to buy the stock at a price $S_2 > S_0$.
  3. X_3 wishes to sell the stock at a price $S_3 > S_0$.
  4. X_4 wishes to buy the stock at a price $S_4 < S_0$.
- These are our responses to $X_1 - X_4$:
  1. We buy the stock from X_1 at price S_1 and immediately sell it on the market at price S_0. We make a profit of $S_0 - S_1$.
  2. We buy the stock on the market at price S_0 and immediately sell it to X_2 at price S_2. We make a profit of $S_2 - S_0$.
  3. *We simply don't do business with X_3.* There is no loss to us.
  4. We also don't do business with X_4. There is also no loss to us.
- Note the following important details in formulating the trading strategies in the above cases:
  1. The market must be efficient: we need to know that the market price is S_0. If we did not know the market price is S_0, we could not formulate our trading strategies as above.
  2. The market must be liquid: to execute the trades above, we must be able to buy and sell the stock quickly, before the market price of the stock changes away from S_0.
  3. Our profit was $S_0 - S_1$ (first case) or $S_2 - S_0$ (second case) and our loss was zero in all cases. This is how an arbitrage strategy functions: there must never be a loss in any scenario, and a positive profit (with probability > 0) in some scenarios.
  4. It is also important to note that an arbitrage strategy begins with a position of *zero* in our account. We have no stock initially. We buy and sell what we need, and end up with a profit. Suppose we already owned shares of stock and sold them to X_2 at the price S_2. This is *not* arbitrage. We bought the stock at an earlier time, at a price different from S_0, and selling it now at a price S_2 may or may not make a profit. We might even make a loss, if we paid more than S_2 when we purchased the stock initially.
  5. The market must also be frictionless. Suppose we had to pay transaction fees to perform the above trades. Consider the case of X_1, who was willing to sell the stock at a price $S_1 < S_0$. We buy the stock from X_1 at price S_1 (and pay a fee to our stockbroker) and immediately sell it on the market at price S_0 (and pay a second fee to our stockbroker). If the total fee we pay exceeds $S_0 - S_1$, then there is no profit for us, and in fact a loss. Then we would not perform the above trades.
- Some people express the above concept as follows:

  In an efficient market, one price must rule.

4.6 Price takers

- Most of us, most people who trade in the financial markets, are **price takers.**
- That means we accept the value of the price of something that is quoted in the market. (We "take" the quoted price.) We do not have the power or ability to change that price.
- In the idealized mathematical theory that we use in this class (and in all the textbooks), everyone is a price taker. There is an infinite quantity of shares of any stock, and we can buy or sell unlimited quantities (including short selling) and our actions do not change the stock price.
- That is not exactly true in the real financial markets. It is a good approximation for actively traded highly liquid stocks, where there is a large quantity of shares available to buy/sell.
- However, the following scenario does happen, and I have seen it several times in my career. A company X (maybe a startup) wishes to go public and offers shares for sale to the public. The company X asks for \$50 a share (for example). Another company Y says to X: we will buy *all* of your offering, but we will only pay a maximum of \$40 per share. I have seen this happen. There are companies which are large enough and powerful enough that they can set the price they want to pay. So company X has to make a decision. X can say no, we want \$50 per share. But then X has to go and find customers who will be willing to pay \$50 per share, and that may be difficult to do. There may not be enough interest from investors. On the other hand, there is a customer ($= Y$) who will agree to buy all the shares X is offering for sale. But in exchange for agreeing to buy all the shares, Y will pay a maximum of \$40 per share. So X has to decide what to do.
- I have also seen situations where Y says: we will buy 75% of your offering (at some price, say \$40 per share) and you can find customers to buy the remaining 25%. But then the news will spread that Y is paying only \$40 per share, so other investors may not be willing to pay more that \$40 per share.
- This sort of thing does happen. It is perfectly legal.
- In our idealized mathematical theory, the number of shares is infinite, so no one can control the stock price. If the stock price is S_0 at time t_0, it means there are infinitely many investors who will trade at the price S_0. Everyone is a price taker.