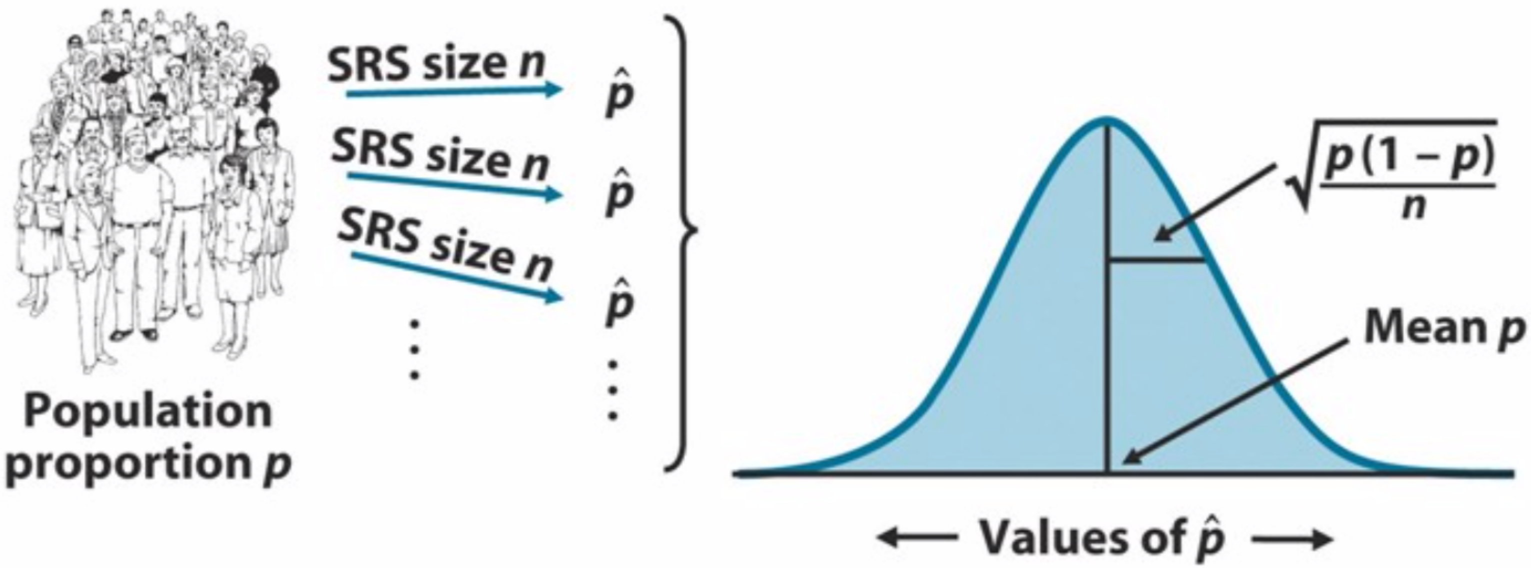
Quantitative Methods

Serena DeStefani – Lecture 14 –7/28/2020

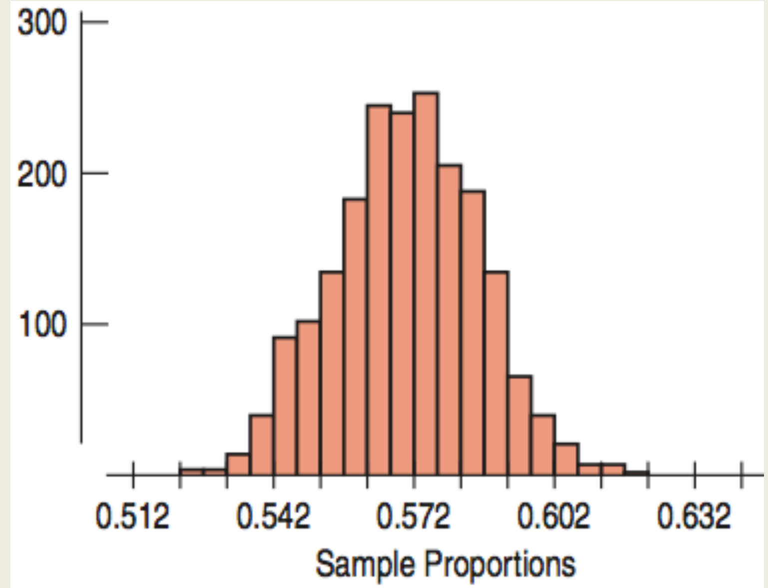
How did we get here?

- Two questions:
  - What's the population parameter?
  - Is my result likely?
- To answer these questions I need a *probability model*
- The rules of probability
- Random variable: the Bernoulli trial
- The geometric and binomial distribution
- The normal approximation to the binomial
- The sampling distribution of a proportion

Visual of How A Model of a Sampling Distribution of Proportions is Formed



Sampling Distributions for Proportions



Sampling Distribution for Proportions

- Symmetric
- Unimodal
- Centered at *p*
- The sampling distribution follows the Normal model.

What does the sampling distribution tell us?

- The sampling distribution allows us to make statements about where we think the corresponding **population parameter** is and how precise these statements are likely to be.

From One Sample to Many Samples

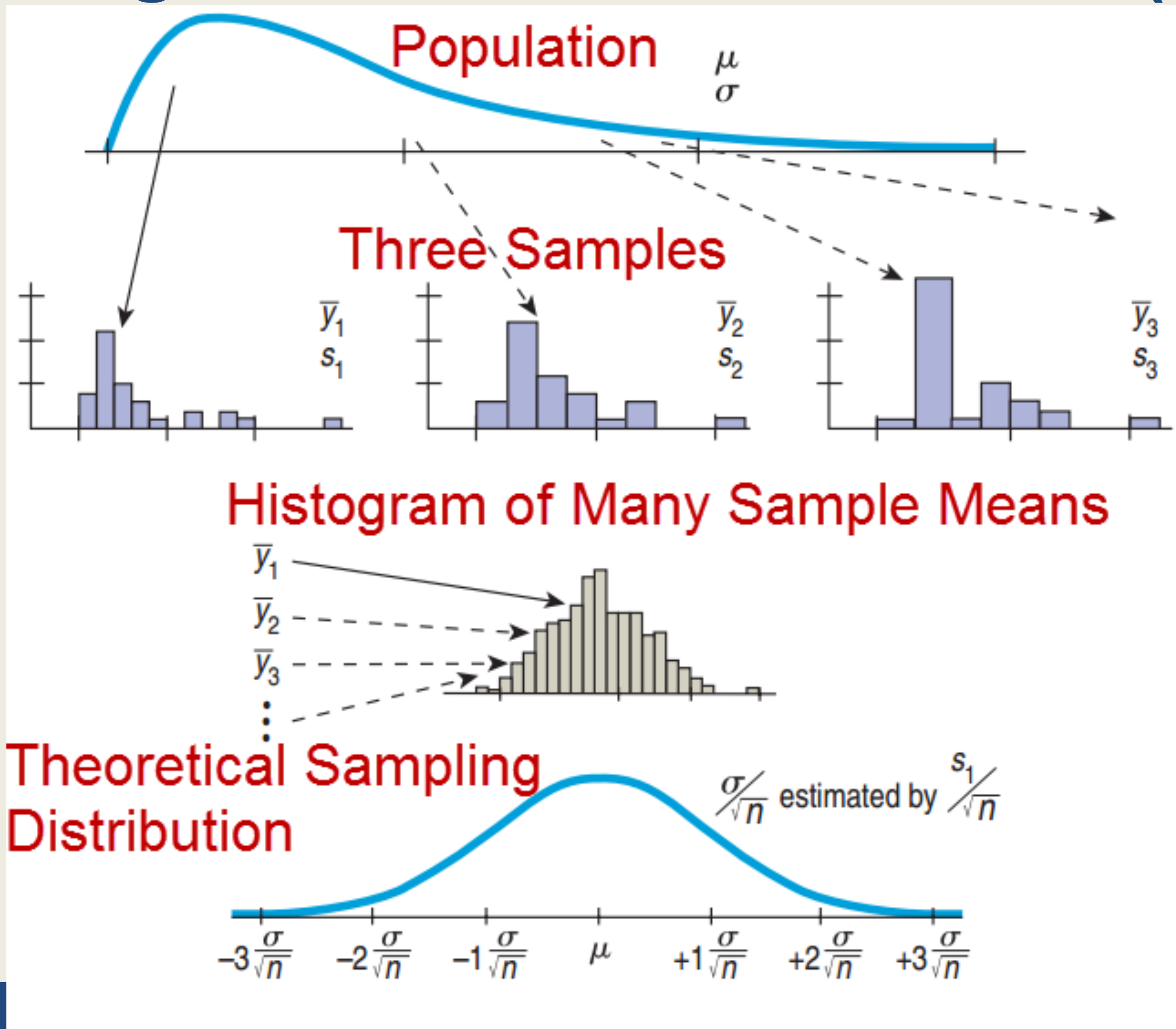
Distribution of One Sample

- **Variable** was the *answer to the survey* question or the *result* of an experiment.
- **Proportion** is a fixed value that comes from the one sample.

Sampling Distribution

- **Variable** is the proportion that comes from the entire sample.
- Many proportions that differ from one to another, each coming from a different sample.

Sampling Distributions for Means (CLT)



Mean and Standard Deviation

Sampling Distribution for <u>Proportions</u>

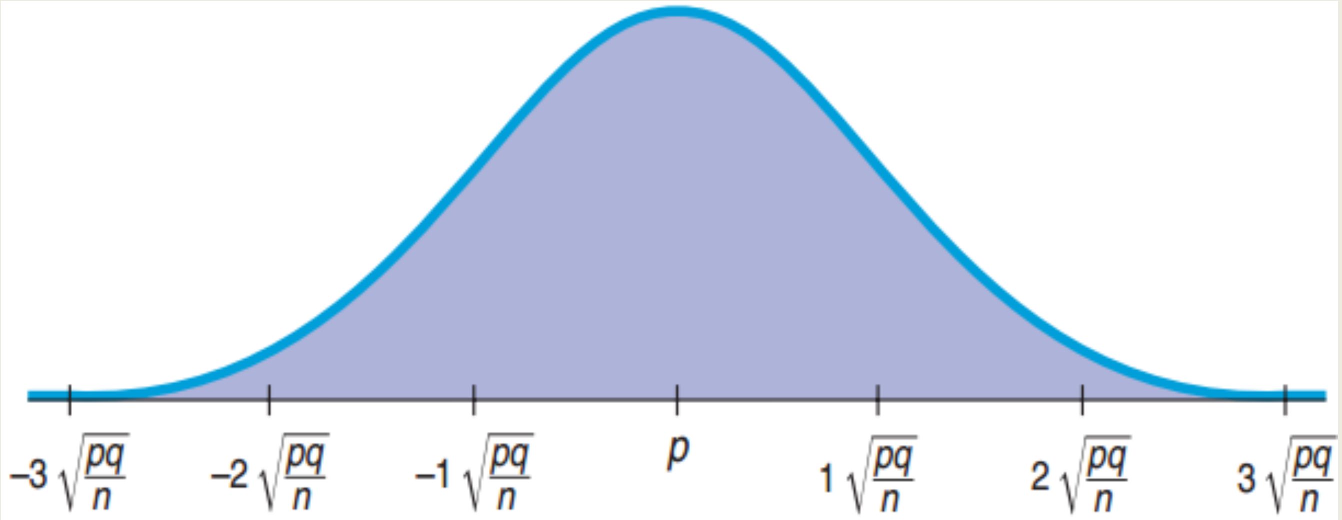
- Mean $= p$

This p is a parameter!

- $\sigma(\hat{p}) = \dfrac{\sqrt{npq}}{n} = \sqrt{\dfrac{pq}{n}}$

Instead *p_hat* is a statistics!

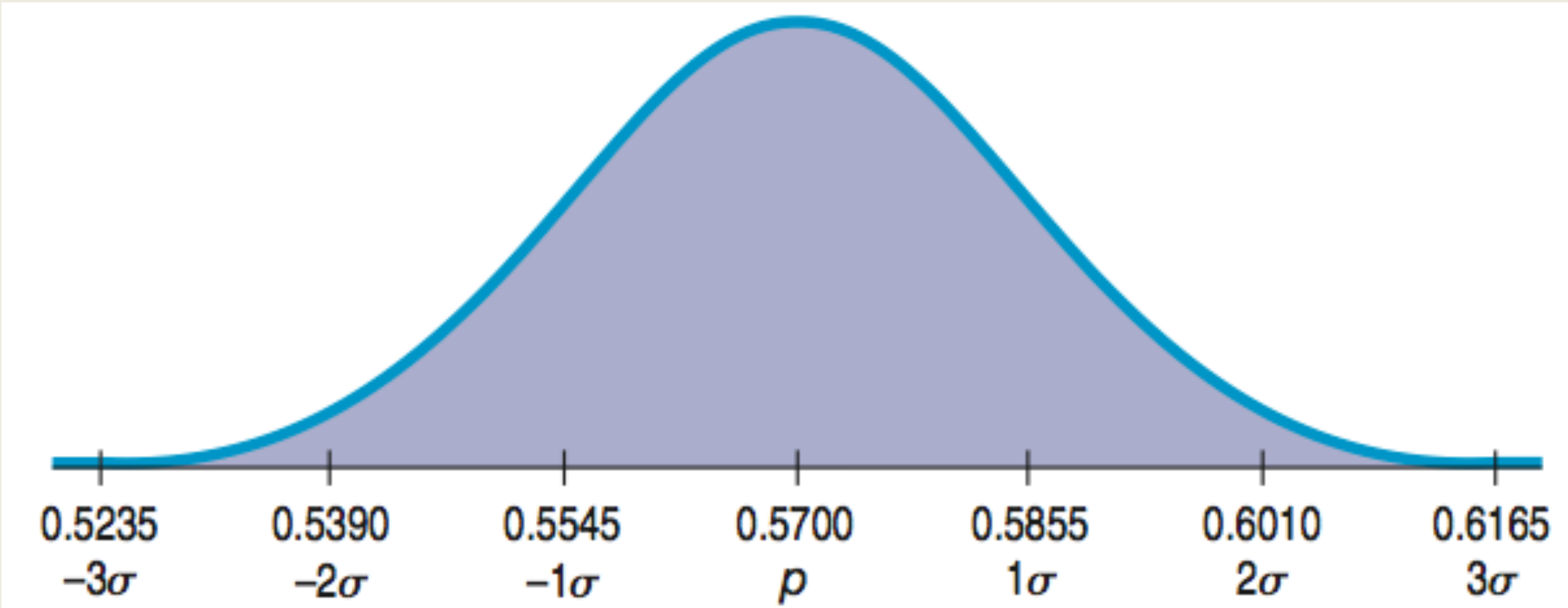
- $N\left(p, \sqrt{\dfrac{pq}{n}}\right)$



The Normal Model for Climate Change

Population: p = 0.57, n = 1022. Sampling Distribution:

- Mean = 0.57
- Standard deviation $= SD(\hat{p}) = \sqrt{\frac{(0.57)(0.43)}{1022}} \approx 0.0155$



Standard Deviation for a Proportion?

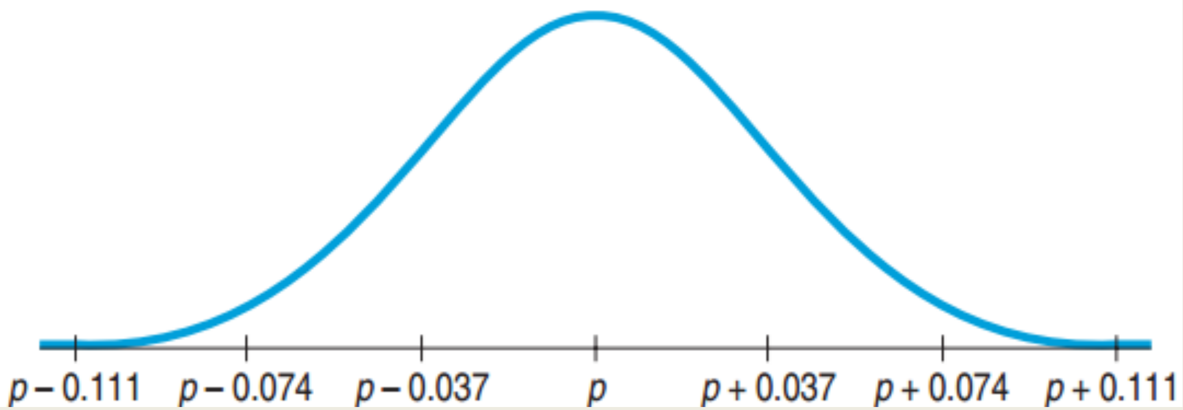
What is the sampling distribution?

- Usually we do not know the population proportion p.
- We cannot find the standard deviation of the sampling distribution: $\sqrt{\frac{pq}{n}}$
- After taking a sample, we only know the sample proportion, which we use as an approximation.
- The sample-to-sample standard deviation is called the **standard error** or **sampling variability**
- The standard error is given by $SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}}$

Facebook Daily Status Updates

A recent survey found that 48 of 156 respondents or 30.8% update Facebook status daily.

- $SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{(0.308)(0.692)}{156}} \approx 0.037$
- The sampling distribution is approximately normal.

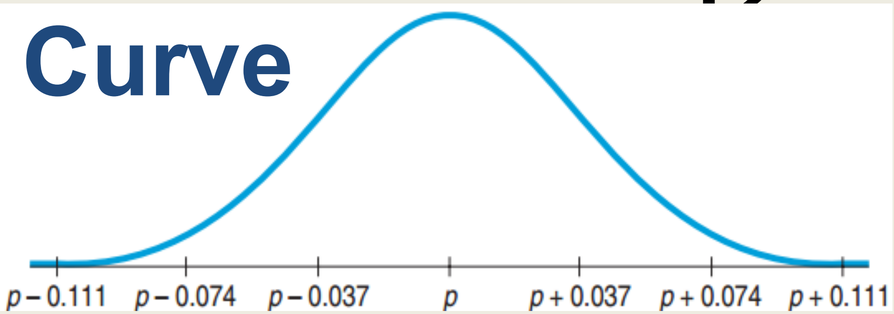


Facebook: true population proportion?

A recent survey found that 48 of 156 or 30.8% update their Facebook status daily.

- This is the <u>sample proportion</u>
- What is the true <u>population proportion?</u>
- To find it, we need to make an inference using the sampling distribution we just found, based on SE

Interpreting this Normal Curve



- By normality, about 95% of all possible samples of 156 young Facebook users will have $\hat{p}$'s within 2 SE (0.037) of p
- If $\hat{p}$ is close to p, then p is close to $\hat{p}$.
- If you stand at $\hat{p}$, then you can be 95% sure that p is within 2SE's from where you are standing.
- → Our confidence interval: (0.234, 0.382)

What <u>You Can Say</u> About *p* if You Know $\hat{p}$

We don't know exactly what percent of all Facebook users update their status daily, but <u>the interval from 23.4% and 38.2% **probably** contains the **true** proportion</u>.

- Note, we admit we are unsure about both the exact proportion and whether it is in the interval.

We are 95% confident that between 23.4% and 38.2% of all Facebook users update their status daily.

- Notice "*% confident*" and an *interval* rather than an exact value are stated.

What You Cannot Say About *p* if You Know $\hat{p}$

30.8% of all Facebook users update their status daily.

- We can't make such absolute statements about *p*.

It is probably true that 30.8% of all Facebook users update their status daily.

- We still cannot commit to a specific value for *p*, only a range.

We don't know exactly what percent of all Facebook users update their status daily, but we know it is within the interval 30.8% $\pm$ 2 x 3.7%.

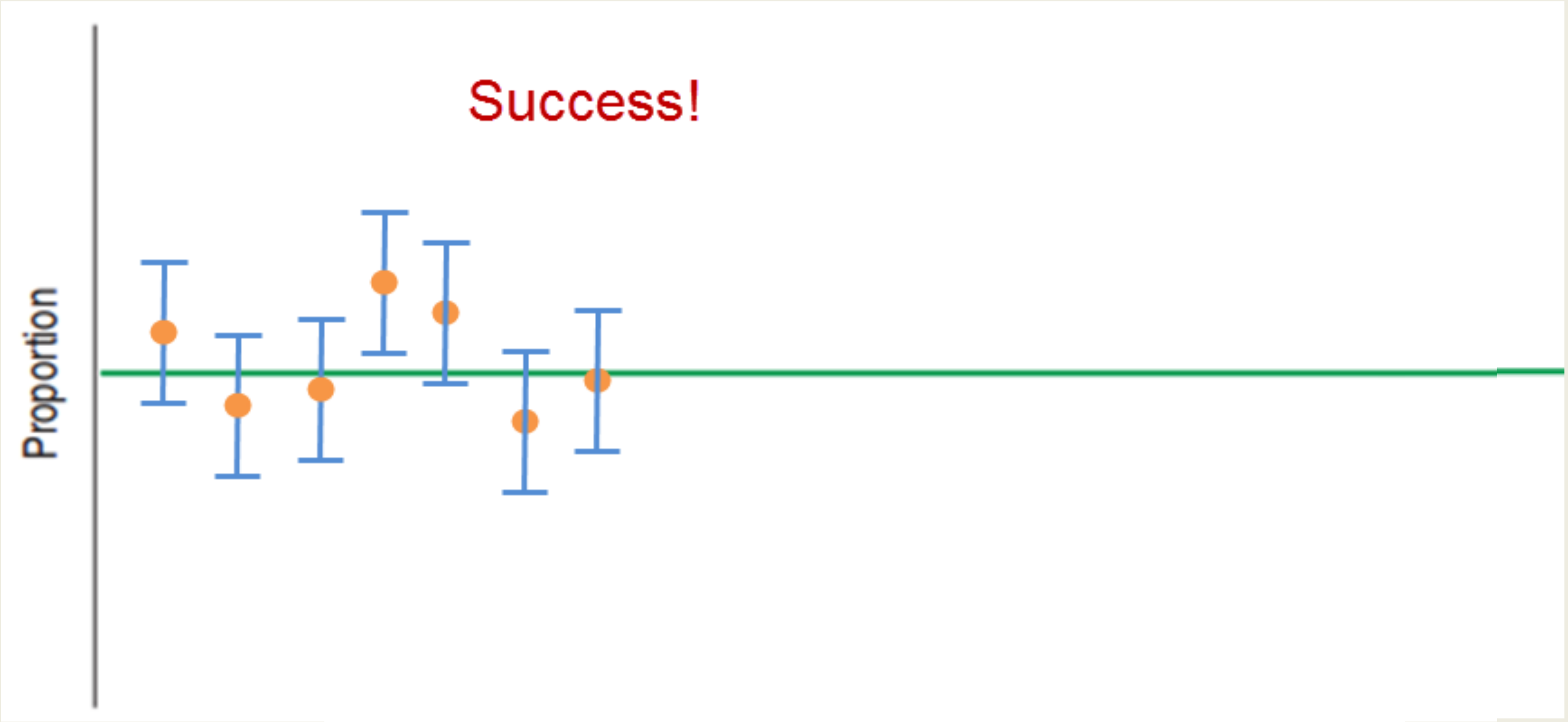
- We cannot be *certain* it is in this interval.

Naming the Confidence Interval

This confidence interval is a **one-proportion *z*-interval**.

- “**One**” since there is a single survey question.
- “**Proportion**” since we are interested in the *proportion* of Facebook users who update their status daily.
- “***z*-interval**” since the distribution is approximately normal.

Simulating Confidence Intervals



Margin of Error

- Confidence interval for a population proportion (95%):

$$\hat{p} \pm 2SE\left(\hat{p}\right)$$

- The distance, $2SE\left(\hat{p}\right)$, from $\hat{p}$ is called the margin of error.

- Confidence intervals also work for means, regression slopes, and others. In general, the confidence interval has the form

$$Estimate \pm ME$$

Certainty vs. Precision

- Instead of a 95% confidence interval, any percent can be used.
- Increasing the confidence (e.g. 99%) increases the margin of error.
- Decreasing the confidence (e.g. 90%) decreases the margin of error.

Critical Values

- For a 95% confidence interval, the margin of error was 2*SE*.
  - The 2 comes from the normal curve.
  - 95% of the area is within about 2*SE* from the mean.

- In general the *number* of *SE* is called the critical value. Since we use the normal distribution here we denote it z^*
- To be more precise, z^* for 95%CI is 1.96

Independence and Sample Size

- Independence Condition
  - If data is collected using SRS or a randomized experiment → Randomization Condition
  - Some data values do not influence others.
  - Check for the 10% Condition: The sample size is less than 10% of the population size.
- Success/Failure Condition
  - There must be at least 10 successes.
  - There must be at least 10 failures.

One-Proportion z-Interval

- First check for randomization, independence, 10%, and conditions on sample size.
- Confidence level C, sample size n, proportion $\hat{p}$.
- Confidence interval: $\hat{p} \pm z^* SE(\hat{p})$
- $SE(\hat{p}) = \sqrt{\frac{(\hat{p})(\hat{q})}{n}}$
- z*: the critical value that specifies the number of *SE*'s needed for C% of random samples to yield confidence intervals that capture the population proportion.

Do You Believe the Death Penalty is Applied Fairly?

- **Mechanics:** $n = 510, \ \hat{p} = 0.58$
-
-
-
-

Do You Believe the Death Penalty is Applied Fairly?

- **Mechanics:** $n = 510$, $\hat{p} = 0.58$
- $SE(\hat{p}) = \sqrt{\dfrac{(0.58)(0.42)}{510}} \approx 0.022$
- $z^* \approx 1.96$
- $ME \approx (1.96)(0.022) \approx 0.043$
- The 95% Confidence Interval is:
  0.58 ± 0.043 or $(0.537, 0.623)$

Do You Believe the Death Penalty is Applied Fairly?

- **Conclusion:** I am 95% confident that between 57.3% and 62.3% of all US adults think that the death penalty is applied fairly.

What Sample Size?

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

- For example, to ensure a $ME < 3\%$:
- For 95%, $z^* = 1.96$
-
-

What Sample Size?

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

- For example, to ensure a *ME* < 3%:
- For 95%, $z^* = 1.96$
- Values that make *ME* largest are $\hat{p} = 0.5,\ \hat{q} = 0.5$
- $$0.03 = 1.96\sqrt{\frac{(0.5)(0.5)}{n}}$$
- Solving for n, gives $n \approx 1067.1$.
- We need to survey at least 1068 to ensure a *ME* less than 0.03 for the 95% confidence interval.

The Yale/George Mason Survey and Sample Size

Poll: 40% believe scientists disagree on global warming.

- For a **follow-up survey**, what sample size is needed to obtain a 95% confidence interval with *ME* $\leq$ 2%?

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

-

The Yale/George Mason Survey and Sample Size

Poll: 40% believe scientists disagree on global warming.

- For a **follow-up survey**, what sample size is needed to obtain a 95% confidence interval with $ME \leq 2\%$?

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} \qquad 0.02 = 1.96\sqrt{\frac{(0.4)(0.6)}{n}}$$

- $n \approx 2304.96$

- The group will need at least 2305 respondents.

Thoughts on Sample Size and *ME*

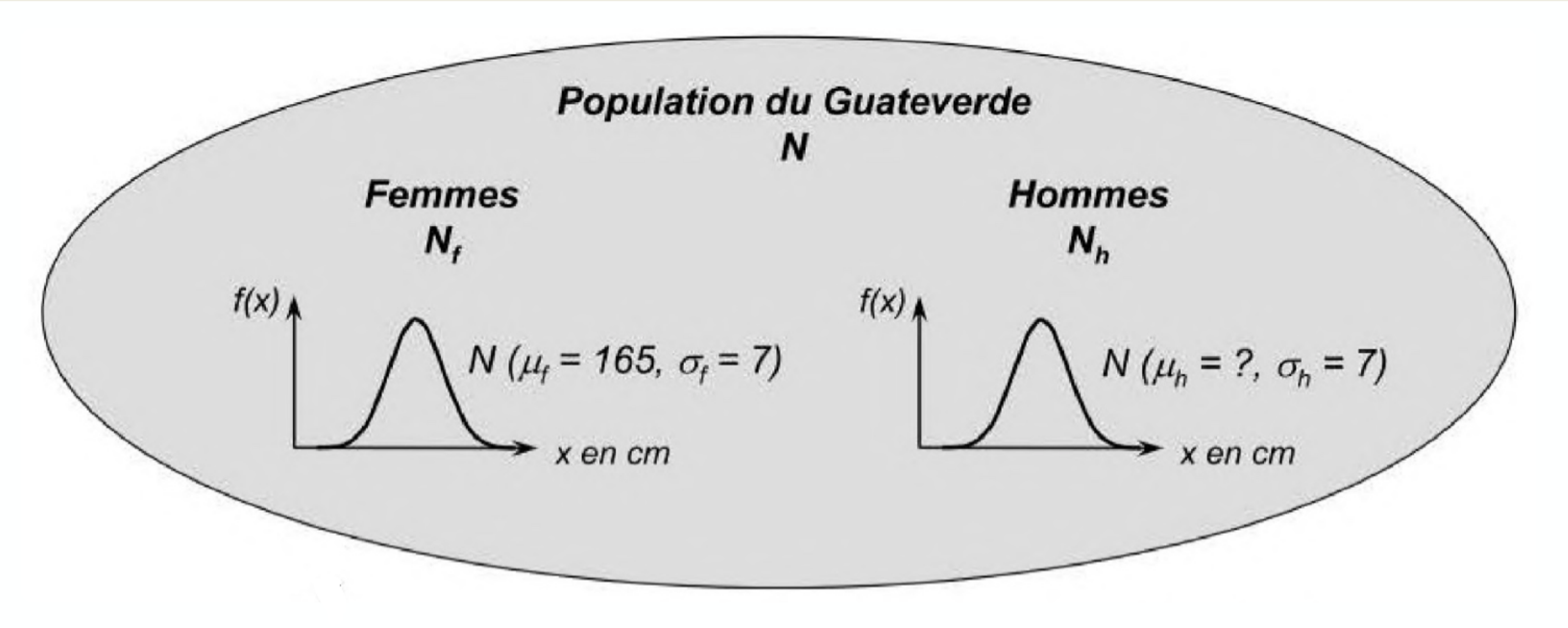
- Obtaining a large sample size can be expensive and/or take a long time.
- For a pilot study, *ME* = 10% can be acceptable.
- For full studies, *ME* $\leq$ 5% is better.
- Public opinion polls typically use *ME* = 3%, $n = 1000$.
- If p is expected to be very small such as 0.005, then much smaller *ME* such as 0.1% is required.

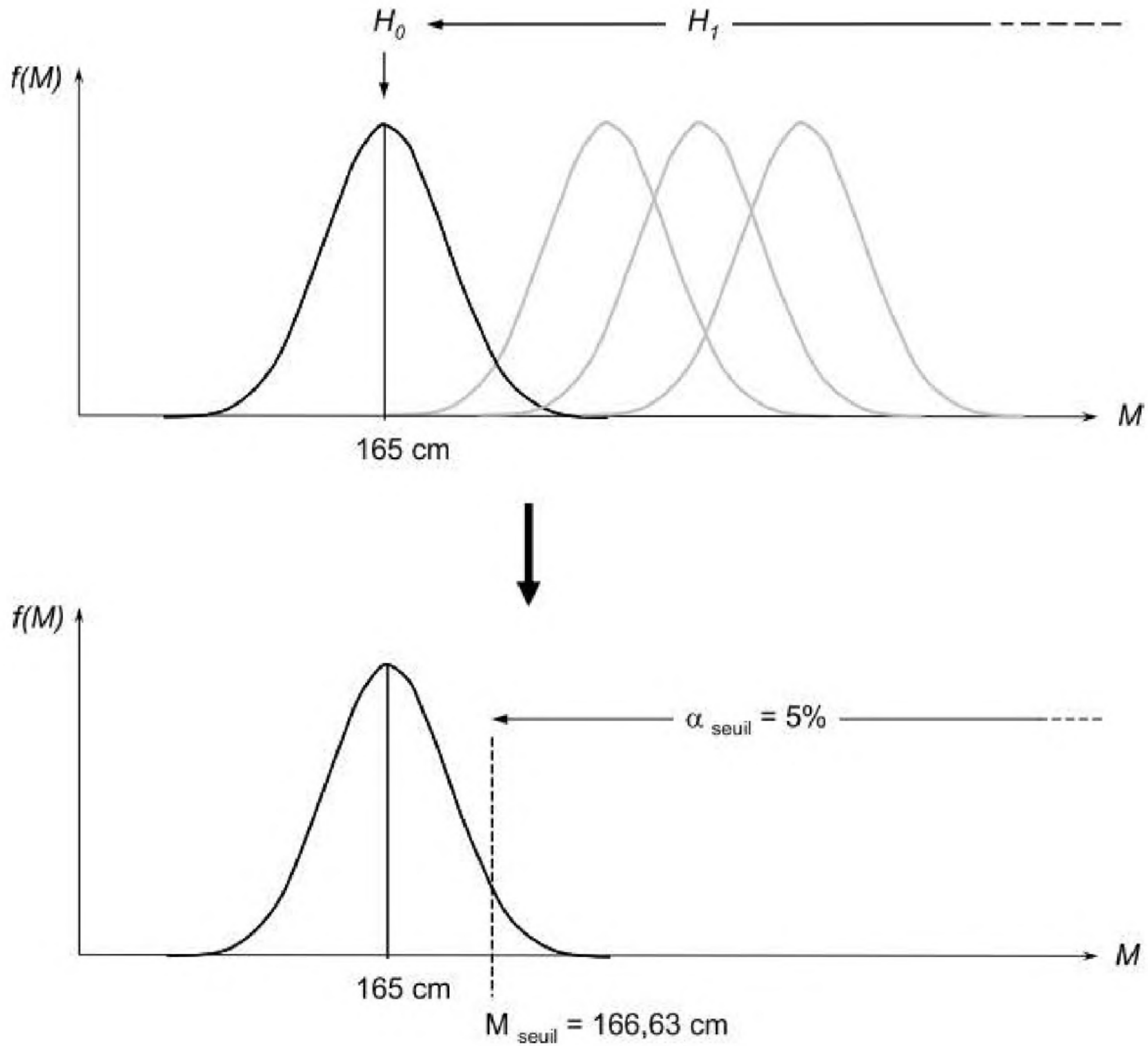
Chapter 19

Testing Hypotheses About Proportions

19.1

Hypotheses


Population du Guateverde
N
Femmes
N_f
f(x)
$N\ (\mu_f = 165,\ \sigma_f = 7)$
x en cm
Hommes
N_h
f(x)
$N\ (\mu_h = ?,\ \sigma_h = 7)$
x en cm



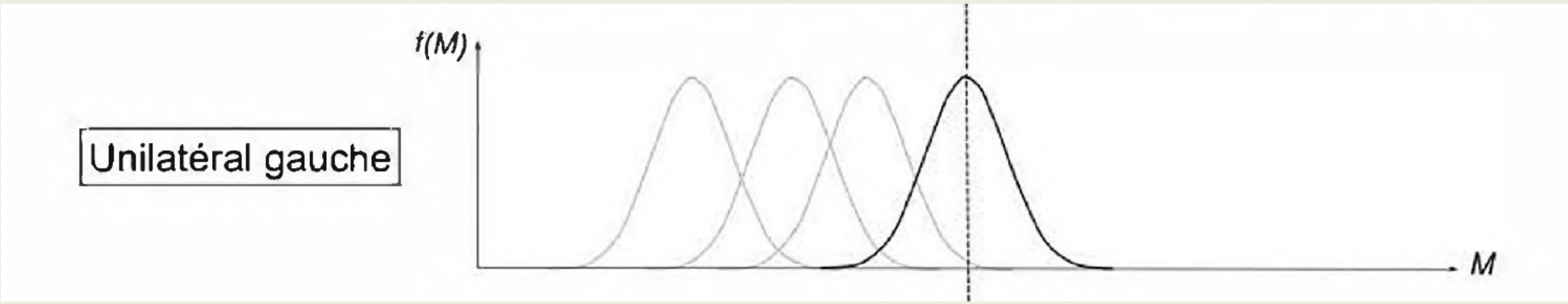
H0
H1
f(M)
M
165 cm
f(M)
α seuil = 5%
M
165 cm
M seuil = 166,63 cm

Cracking Rate < 20%?

General cracking rate: 20%
After a new engineering process,
the cracking rate of 400 casts fell to 17%!

Is this due to the new engineering or just random chance?

- **Null Hypothesis (H_0):** Nothing has changed
  - H_0: hypothesized value = parameter
  - H_0: $p = 0.20$
- Alternative Hypothesis:
  - H_A: $p < 0.20$


Unilatéral gauche
f(M)
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Step 1: State the Hypotheses

H_0:

- H_0 usually states that there's nothing different.
- H_0: hypothesized value = parameter value
- Note the parameter describes the population not the sample.
- H_0 is called the null hypothesis.

H_A:

- H_A is a statement that something has changed, gotten bigger or smaller
- H_A is called the alterative hypothesis.

1-Proportion z-Test

Conditions

- Same as a 1-Proportion *z*-Interval

Null Hypothesis

- H_0: $p = p_0$

Test Statistics

- $z = \dfrac{\hat{p} - p_0}{SD(\hat{p})}$ $\quad SD(\hat{p}) = \sqrt{\dfrac{p_0 q_0}{n}}$

Checking Conditions and Finding the Standard Error

Checking Conditions: $n = 400$, $p = 0.20$

- ✓ $np = (400)(0.20) = 80 \geq 10$
  $nq = (400)(0.80) = 320 \geq 10$
- ✓ Independence plausible
- ✓ The Normal model applies.

Find the standard deviation of the **model**.

- $$SD(\hat{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.20)(0.80)}{400}} = 0.02$$
- **Note:** Use p and not $\hat{p}$ to find standard deviation.

Using the Normal Model

$$z = \frac{\hat{p} - p_0}{SD\left(\hat{p}\right)}$$

$p = 0.20,\ \hat{p} = 0.17,\ SD(\hat{p}) = 0.02$

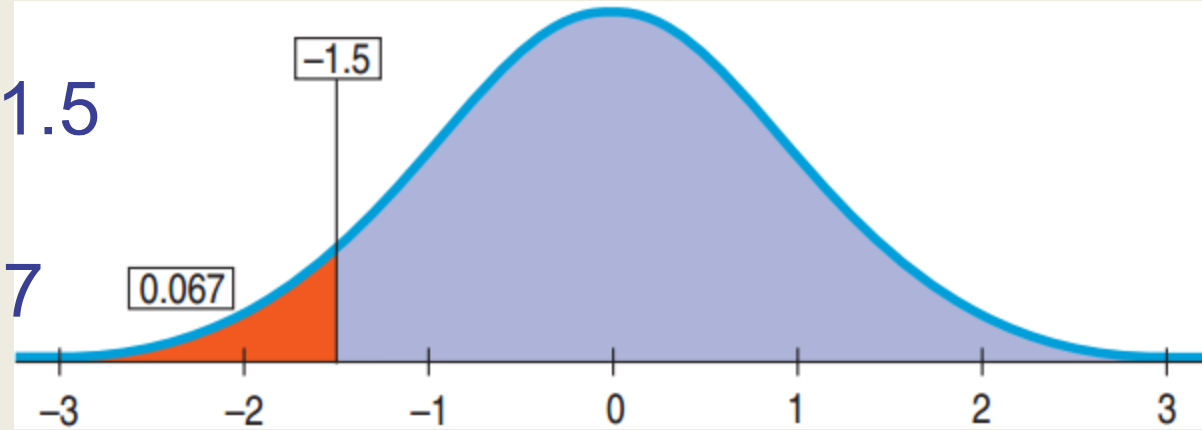
-

-

Using the Normal Model

$p = 0.20,\ \hat{p} = 0.17,\ SD(\hat{p}) = 0.02$

- $z = \dfrac{0.17 - 0.20}{0.02} = -1.5$
- $P(z < -1.5) \approx 0.067$



- If the null hypothesis is true that the cracking rate is still equal to 20%, then the probability of observing a cracking rate of 17% in a random sample of 400 is 6.7%.

The P-Value and Surprise

The P-value is the probability of seeing data like these (or even more unlikely-extreme data) given the null hypothesis is true.

- Tells us how **surprised** we would be to get these data given H_0 is true.
- **P-value small:** Either H_0 is not true or something remarkable occurred
- **P-value not small enough:** Not a surprise. Data consistent with the model. Do not reject H_0.

Guilty or Not Enough Evidence?

Defendant is either

- **Guilty:** P-value too small. The evidence is clear.
- **Not Guilty:** P-value not small enough. The evidence is not sufficient. Not the same as innocent. Maybe innocent or maybe guilty, but not enough evidence found.

Two Choices

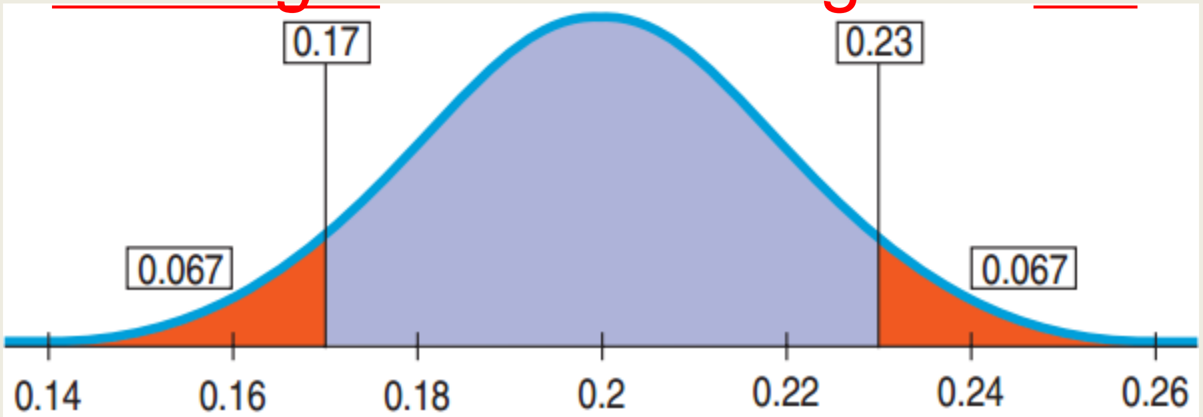
- **Fail to reject H_0 if P-value large.** Never accept H_0.
- **Reject H_0 if P-value is small.** Accept H_A.

19.4

Alternative Alternatives

Two-Sided Alternative

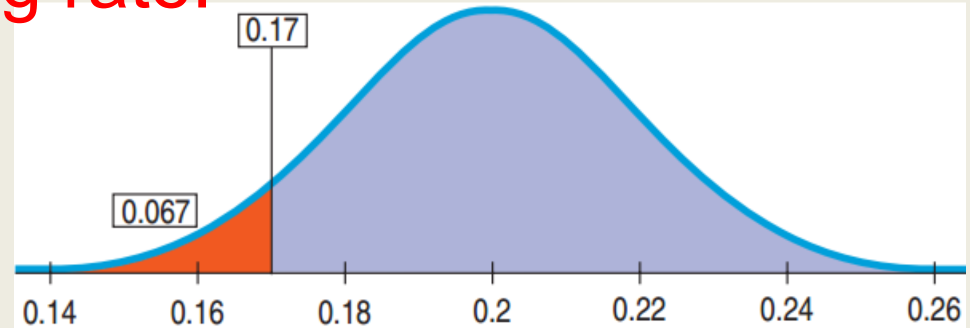
For the new process the engineer may be interested in whether there has been a *change* in the cracking rate not just a decrease.



- H_0: $p = 0.20$
- H_A: $p \neq 0.20$
- An alternative hypothesis where we are interested in deviation on either side is called a two-sided alternative.
- The P-value is the probability of deviating from *either* direction from the null hypothesis.

One-Sided Alternative

The engineer may be interested in whether there has been a decrease in the cracking rate.



- H_0: $p = 0.20$
- H_A: $p < 0.20$

- An alternative hypothesis where we are interested in deviation on only one side is called a one-sided alternative.
- The P-value for a one-sided alternative is always half the P-value for the two-sided alternative.

Not the Right Proportion of Male Babies?

Under natural conditions, 51.7% of births are male. In Punjab India's hospital 56.9% of the 550 births were male.

- **Question:** Is there evidence that the proportion of male births is different for this hospital?
- What is the probability of getting a results *as different as* 56.9% or *more* different?
- Plan: We will have a two-tailed alternative. The parameter of interest is p.
  - H_0: $p = 0.517$
  - H_A: $p \neq 0.517$

Not the Right Proportion of Male Babies?

- **Model:** Check the conditions
  - ✓ **Independence Assumption:** The sex of one baby should not affect the sex of others.
  - ✓ **Randomization Conditions:** The births were not random, so be cautious of the results.
  - ✓ **10% Condition:** 550 births is certainly less than the total number of all births.

Not the Right Proportion of Male Babies?

- **Model:** Check the conditions (Continued).
  ✓ **Success/Fail Condition:**
  $(550)(0.517) \geq 10$, $(550)(0.483) \geq 10$

- The Normal model does apply.

- We can use a one-proportion z-test.

Not the Right Proportion of Male Babies?

- **Mechanics:**

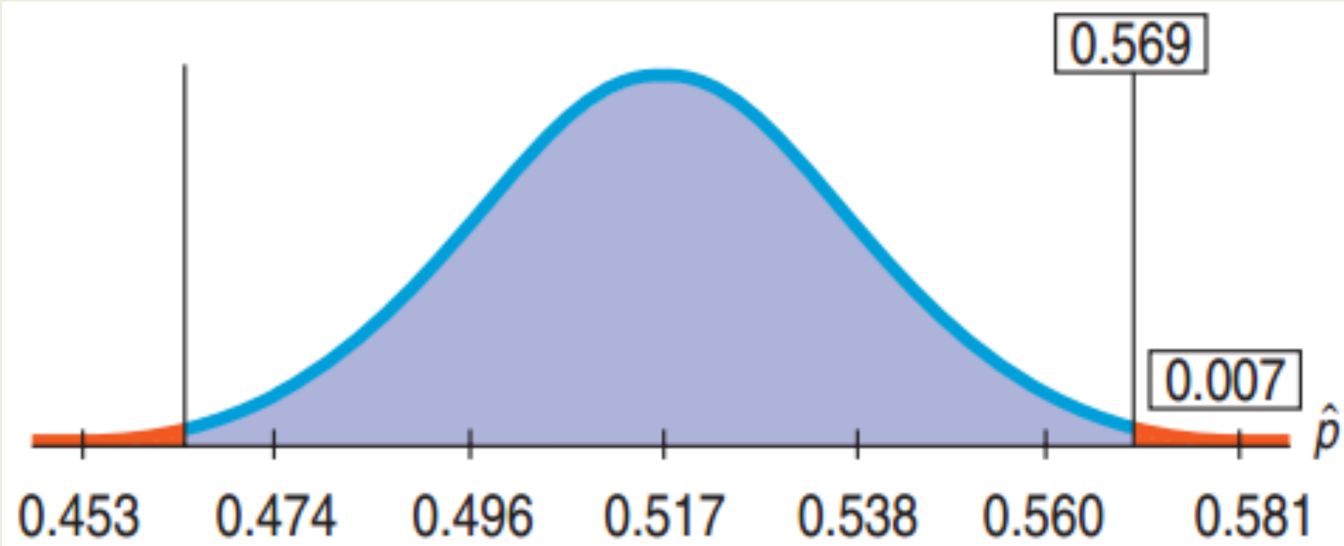
$$SD(\hat{p}) = \sqrt{\frac{p_0 q_0}{n}} \approx \sqrt{\frac{(0.517)(1-0.517)}{550}} \approx 0.0213$$

$$z = \frac{\hat{p} - p_0}{SD(\hat{p})} = \frac{0.569 - 0.517}{0.0213} \approx 2.44$$

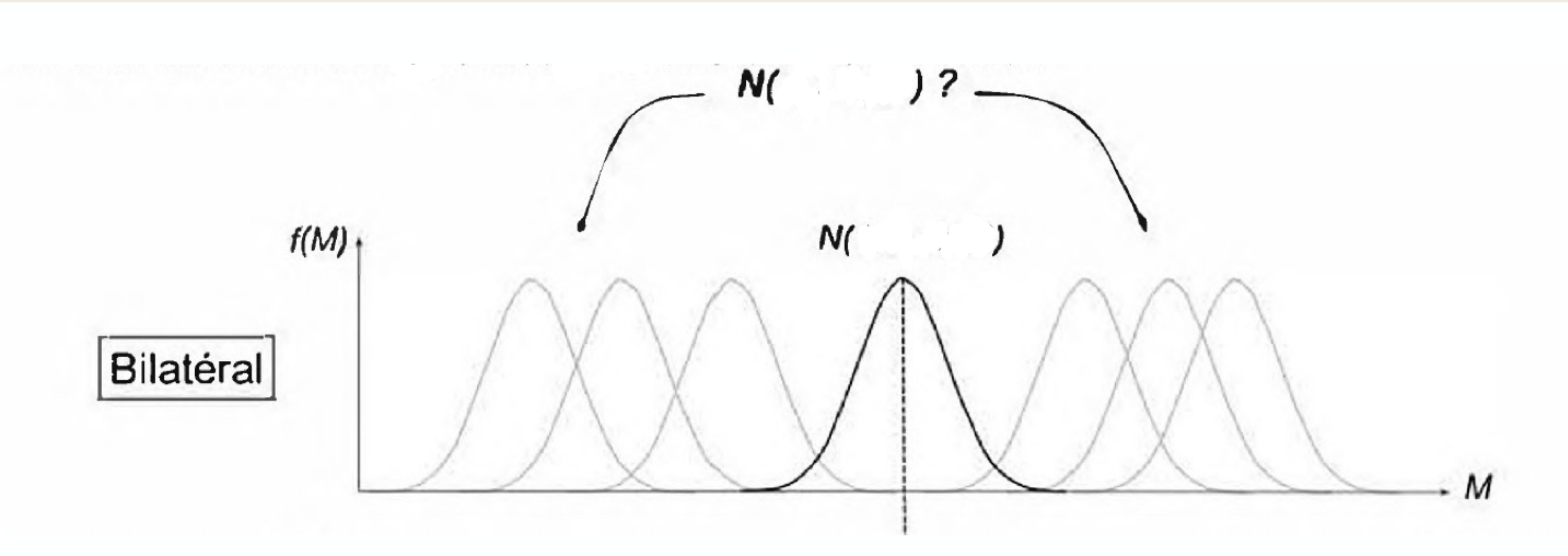
Not the Right Proportion of Male Babies?

Show →

- **Mechanics:**



P-value = $2P(z > 2.44) = 2(0.0073) = 0.0146$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Not the Right Proportion of Male Babies?

- **Conclusion:**
  - Interpreting the P-value = 0.00146
    - If the proportion of male babies were still 51.7%, then an observed proportion as different as 56.9% male babies would occur at random about 15 times in 1000.
    - This is so small a chance that I reject H_0. There is strong evidence that the proportion of boys is not equal to the baseline for the region. It appears larger.

19.5

P-Values and Decisions: What to Tell About a Hypothesis Test

How Small a P-Value is Small Enough?

How small is small enough is context specific.

- Test to see if a renowned musicologist can distinguish between Mozart and Hayden.
  - P-value of 0.1 may be good enough. Just reaffirming known talent.
- A friend claims psychic abilities and can predict heads or tails.
  - Very small P-value such as 0.01 needed. Breaking scientific theory.

Acceptable P-Value Depends on Result's Importance

Proportion of students in school with full time jobs has increased.

- Social issue. P-value = 0.05 will work.

Testing the proportion of faulty rivets that hold together the wings of a commercial aircraft is now below the danger threshold.

- Life and death decision. Need a very small P-value

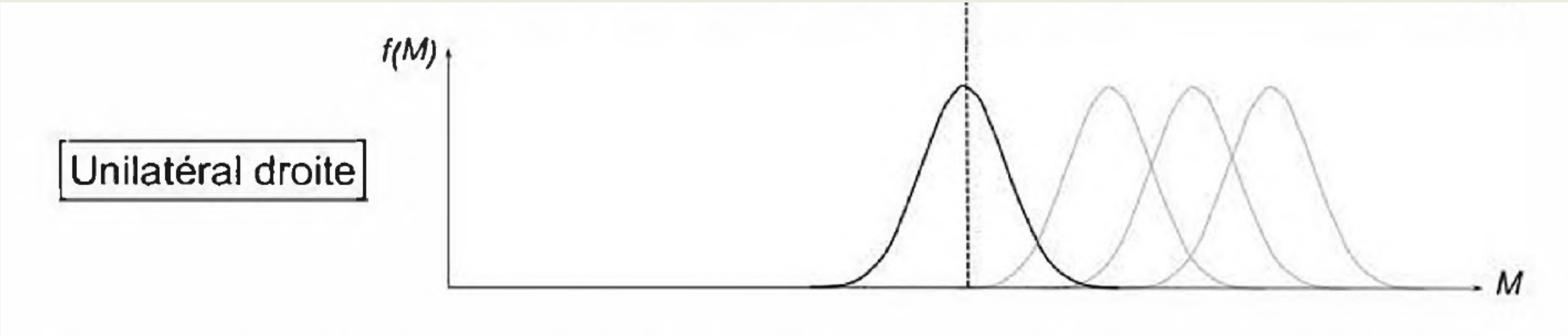
- Whether rejecting or failing to reject, always cite the P-value.
- An accompanying confidence interval helps also.

Home Field Advantage?

Is there a home field advantage in baseball?

- The home team won 1308 of the 2431 (53.81%) games played in the season.
- Is there evidence to suggest that the home team wins more than 50%?

- Plan: p = proportion of home team wins
- Hypotheses
  - H_0: $p = 0.50$
  - H_A: $p > 0.50$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Home Field Advantage?

Model →

- ✓ **Independence Assumption:** Questionable, but the 2013 season may be representative of all games past and future.
- ✓ **10% Condition:** The 2013 season is less than 10% of all games played past and future.
- ✓ **Success/Failure Condition:**
  - $np = (2431)(0.5) \geq 10$
  - $nq = (2431)(0.5) \geq 10$
- **The Normal Model Applies:** Conduct a one-proportion z-test.

Home Field Advantage?

- **Mechanics:**

$$SD(\hat{p}) = \sqrt{\frac{p_0 q_0}{n}} \approx \sqrt{\frac{(0.5)(1-0.5)}{2429}} \approx 0.010141$$

$$z = \frac{\hat{p} - p_0}{SD(\hat{p})} = \frac{0.53805 - 0.5}{0.010141} \approx 3.75$$

- The P-value is about 0.0001.

Home Field Advantage?

- **Conclusion:** The P-value of 0.0001 says that if the true proportion of home teams wins is 0.5 then an observed value of 0.53805 would occur less than 1 time in 1000.
- This is so unlikely, so reject H_0.
- There is reasonable evidence that the true proportion of home team wins is greater than 50%.
- There appears to be a home field advantage.

How Big a Home Field Advantage?

- **Model:**
  - ✓ Success Failure Condition (notice difference!)
    - Home team wins: $1308 \geq 10$
    - Home team losses: $1123 \geq 10$
- Sampling distribution follows the Normal model.
- Find the one-proportion *z*-interval.

How Big a Home Field Advantage?

- **Mechanics (notice difference!):**

$$SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}} \approx \sqrt{\frac{(0.53805)(1-0.53805)}{2431}} \approx 0.01011$$

- For 95% confidence, z^* = 1.96.
- $ME = z^* \times SE(\hat{p}) = 1.96 \times 0.01011 = 0.0198$
- A 95% confidence interval is:
  0.53805 ± 0.0198 or (0.5182, 0.5579).

How Big a Home Field Advantage?

Tell →

- **Conclusion:** I am 95% confident that, in professional baseball, **home teams** win between 51.82% and 55.79% of the games.

- For a 162-game season, the low end gives the home team about 1/2 of an extra victory and the high end, about 4 extra wins.

What Can Go Wrong?

Don’t base your H_0 on what you see in the data.

- Changing the null hypothesis after looking at the data is just wrong.

Don’t base your H_A on the data.

- Both the null and alternative hypotheses must be stated before peeking at the data.

Don’t make H_0 what you want to show to be true.

- H_0 represents the status quo. You can never accept the null hypothesis.

What Can Go Wrong?

Don’t forget to check the conditions.

- Randomization, independence, and sample size

Don’t accept the null hypothesis.

- You can only say you don’t have evidence to reject H_0 (you can only say you “fail to reject”).

If you fail to reject H_0 don’t expect a larger sample would reject H_0.

- Check the confidence interval. If its values would not matter to you, then a larger sample will unlikely be worthwhile.

Chapter 20

Inferences About Means

20.1

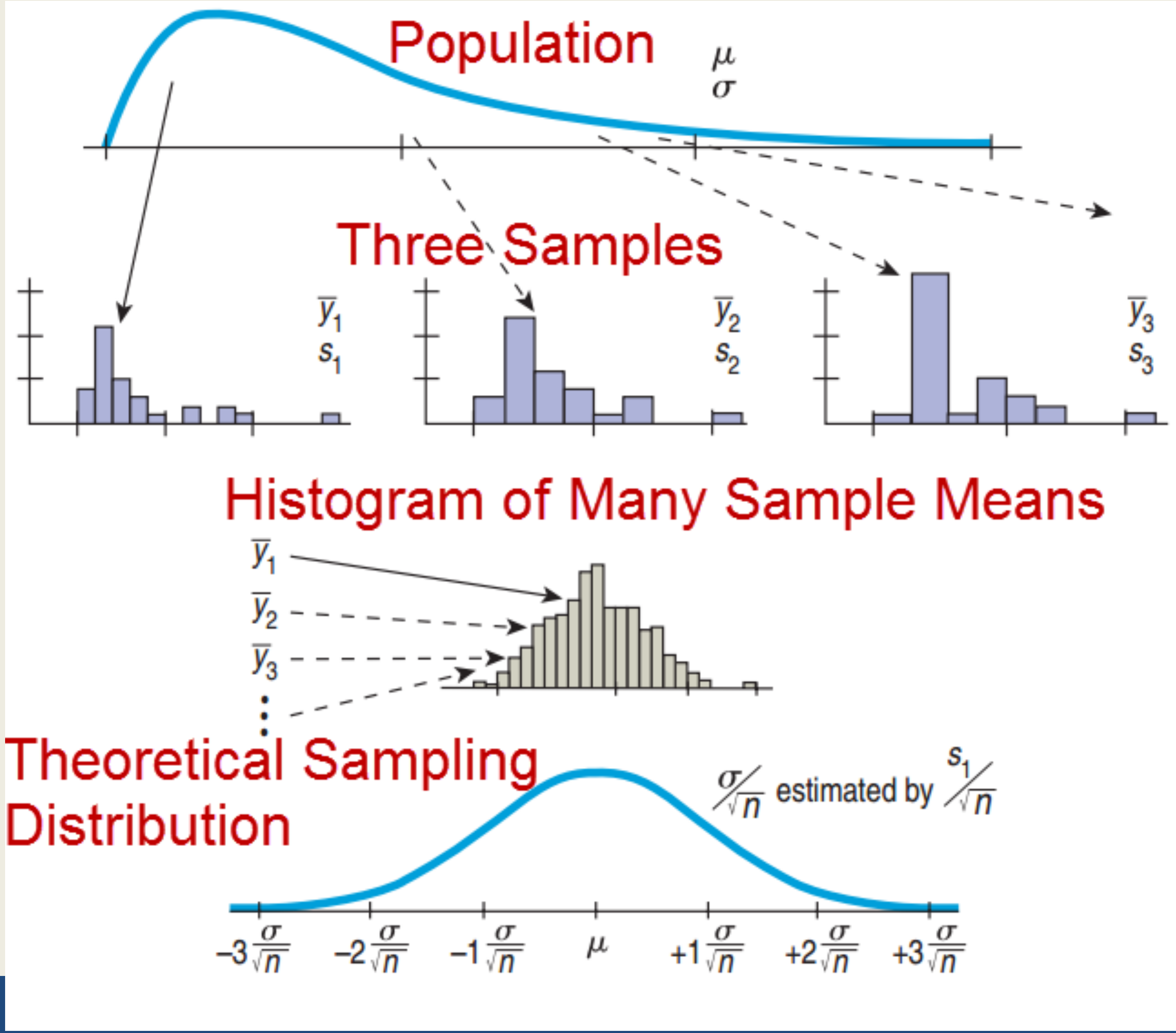
Getting Started: The Central Limit Theorem (Again)

The Central Limit Theorem (CLT)

When random samples are drawn from a population with mean μ and standard deviation σ,
the **sampling distribution** has:

- Mean: μ
- Standard deviation: $\dfrac{\sigma}{\sqrt{n}}$
- Approximately Normal distribution as long as the sample size is large.
- The larger the distribution, the closer to Normal.

Sampling Distributions for Means (CLT)



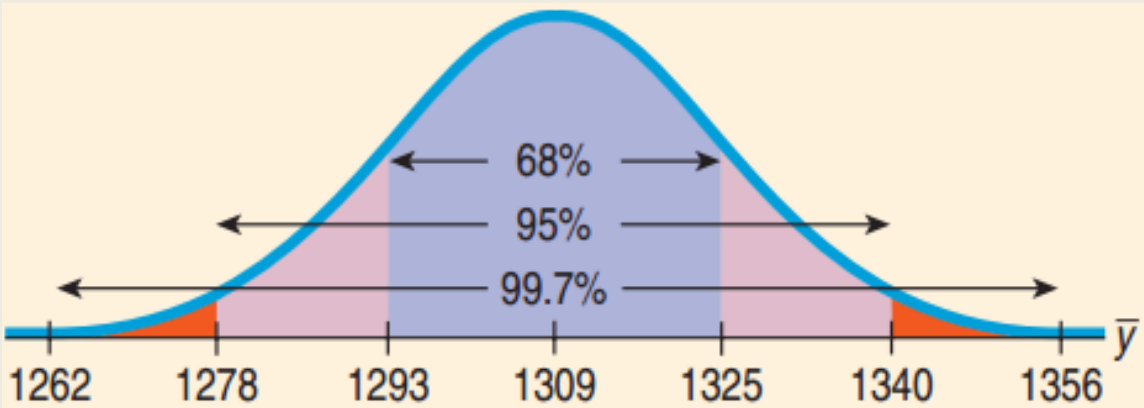
Weight of Angus Cows

The weight of Angus cows is Normally distributed with μ = 1309 pounds and σ = 157 lbs. What does the CLT say about the mean weight of a random sample of 100 Angus cows?

- Sample means $\bar{y}$ will average 1309 pounds.
- $SD(\bar{y}) = \dfrac{\sigma}{\sqrt{n}} = \dfrac{157}{\sqrt{100}} = 15.7$ pounds

Weight of Angus Cows (Continued)

- The CLT says the sampling distribution will be approximately Normal: *N*(1309, 15.7).



For the means of all random samples,

- 68% will be between 1293.3 and 1324.7 lb.
- 95% will be between 1277.6 and 1340.4 lb.
- 99.7% will be between 1261.9 and 1356.1 lb.

The Challenge of the CLT

- CLT tells us $SD(\bar{y}) = \frac{\sigma}{\sqrt{n}}$
- We would like to use this for Confidence Intervals and Hypothesis Testing.
- Unfortunately, we almost never know σ.
- Using s almost works: $SE(\bar{y}) = \frac{s}{\sqrt{n}}$, but not quite.
- When using s, the Normal model has some error.
- **William Gosset** came up with new models, one for each n that works better.

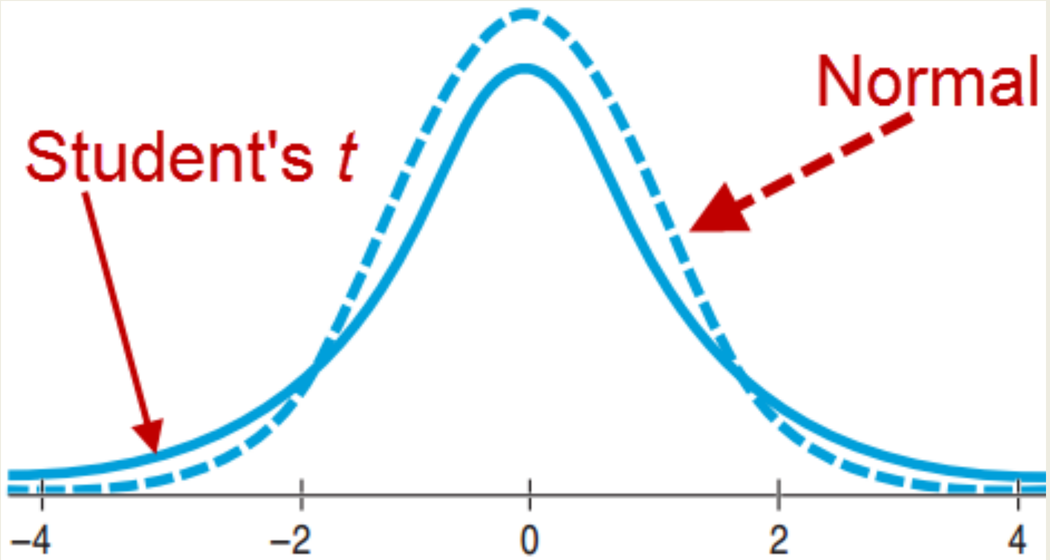
20.2

Gosset's *t*

Gosset at Guinness!

At Guinness, Gosset experimented with beer.

- The Normal Model was not right, especially for small samples.
- Still bell shaped, but details differed, depending on *n*
- Came up with the "Student's *t* Distribution" as the correct model



Degrees of Freedom

- For every sample size n there is a different Student's t distribution.
- Degrees of freedom: $df = n - 1$.
- Similar to the "$n - 1$" in the formula for sample standard deviation
- It is the number of independent quantities left after we've estimated the parameters.

Confidence Interval for Means

Sampling Distribution Model for Means

- With certain conditions (seen later), the standardized sample mean follows the Student's *t* model with $n-1$ degrees of freedom.

$$t = \frac{\bar{y} - \mu}{SE(\bar{y})}$$

- We estimate the standard deviation with

$$SE(\bar{y}) = \frac{s}{\sqrt{n}}$$

One Sample *t*-Interval for the Mean

- When the assumptions are met (seen later), the confidence interval for the mean is

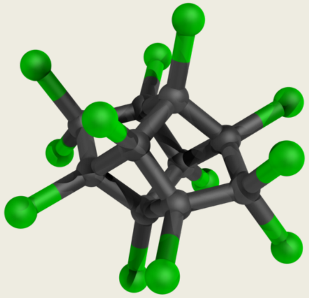
$$\bar{y} \pm t^*_{n-1} \times SE(\bar{y})$$

- The critical value t^*_{n-1} depends on the confidence level, C, and the degrees of freedom $n - 1$.

Contaminated Salmon

$$SE(\bar{y}) = \frac{s}{\sqrt{n}}$$

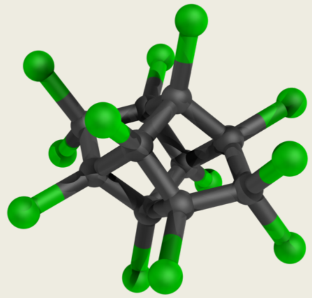
A study of **mirex** concentrations in salmon found



- n = 150, $\bar{y}$ = 0.0913 ppm, s = 0.0495 ppm
- Find a **95% confidence interval** for mirex concentrations in salmon.
- Find df, SE and get t*

Contaminated Salmon

A study of mirex concentrations in salmon found



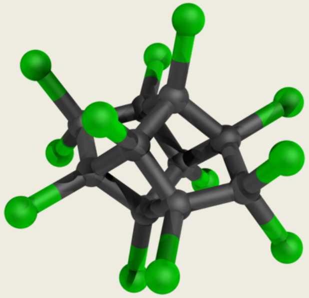
- n = 150, $\bar{y}$ = 0.0913 ppm, s = 0.0495 ppm
- Find a 95% confidence interval for mirex concentrations in salmon.
- df = 150 – 1 = 149

- $SE(\bar{y}) = \dfrac{0.0495}{\sqrt{150}} \approx 0.0040$
- $t^*_{149} = 1.976$
- (from technology, or 1.977 using table with df=140)

Contaminated Salmon

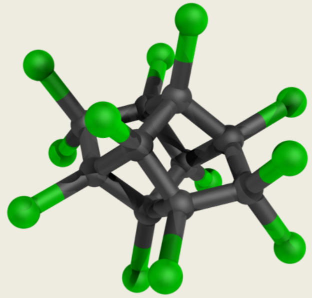
$$\bar{y} \pm t^{*}_{n-1} \times SE(\bar{y})$$

Confidence Interval for μ



Contaminated Salmon

Confidence Interval for μ



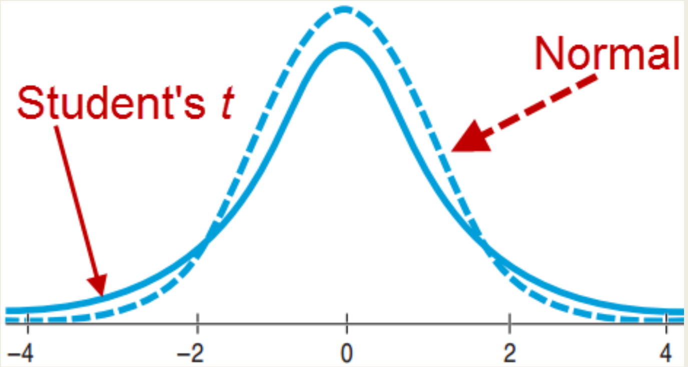
- $\bar{y} \pm t^*_{149} \times SE(\bar{y}) = 0.0913 \pm 1.976(0.0040)$

  $= 0.0913 \pm 0.0079$

  $= (0.0834, 0.0992)$

- I'm 95% confident that the mean level of mirex concentration in farm-raised salmon is between 0.0834 and 0.0992 parts per million.

Thoughts about *z* and *t*



- The Student's *t* distribution:
  - Is unimodal.
  - Is symmetric about its mean.
  - Has higher tails than Normal.
  - Is very close to Normal for large *df*.
  - Is needed because we are using *s* as an estimate for σ.

- *If you happen to know σ, which almost never happens, use the Normal model and not Student's t.*

Assumptions and Conditions

Independence Condition

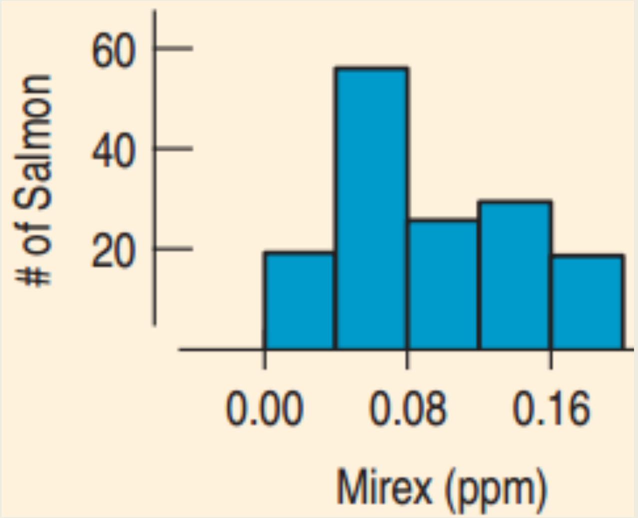
- Randomization Condition: The data should arise from a suitably randomized experiment.
- Sample size < 10% of the population size.

Original Distribution Nearly Normal

- For large sample sizes ($n > 40$), not severely skewed.
- ($15 \leq n \leq 40$): Need unimodal and symmetric.
- ($n < 15$): Need almost perfectly normal.
- Check with a histogram.

Assumptions for Salmon Contamination Study

Researchers tested 150 salmon from 51 farms in eight regions in six countries. Are the assumptions and conditions for inference satisfied?

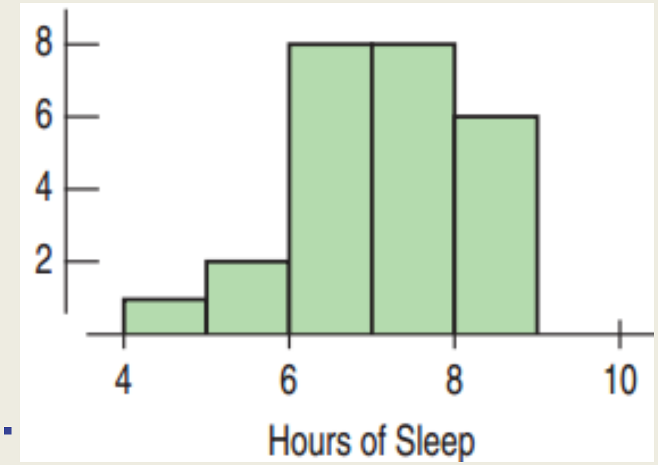


- ✓ **Independence Assumption:** Not a random sample, but likely independently selected.

- ✓ **Nearly Normal Condition:** The histogram is unimodal and not highly skewed. OK since sample size is 51.

How Much Sleep do College Students Get?

Build a 90% Confidence Interval for the Mean.



- **Plan:** Data on 25 Students

Model →

- **Randomization Condition**
  The data are from a random survey.
- **Nearly Normal Condition**
  Unimodal and slightly skewed, so OK
- Use Student's *t*-Model with *df* = 25 – 1 = 24.
- One-sample *t*-interval for the mean

How Much Sleep?

- Mechanics: $n = 25$, $\bar{y} = 6.64$, $s = 1.075$

How Much Sleep?

- Mechanics: $n = 25$, $\bar{y} = 6.64$, $s = 1.075$

$$SE(\bar{y}) = \frac{s}{\sqrt{n}} = \frac{1.075}{\sqrt{25}} = 0.215 \text{ hours}$$

$$t^*_{24} = 1.711$$ (from table, 24 df, CI → 2 tails)

$$ME = t^*_{24} \times SE(\bar{y})$$
$$= 1.711 \times 0.215$$
$$= 0.368 \text{ hours}$$

$$90\% \text{ CI} = 6.64 \pm 0.368 = (6.272, 7.008)$$

How Much Sleep do College Students Get?

- **Conclusion:** I'm 90 percent confident that the interval from 6.272 and 7.008 hours contains the true population mean number of hours that college students sleep.
- Be careful about convenience sampling!

Make a Picture

Always Test the Normality Assumption

- Create a histogram to check for near normality.
  - Good for seeing the nature of the deviations: outliers, not symmetric, skewed
- Also create a normal probability plot to see that it is reasonably straight.
  - Good for checking for normality
- With technology at hand, there is no excuse not to make these two plots.

20.3

Interpreting Confidence Intervals

What Not to Say

Don't Say

- "90% of all students sleep between 6.272 and 7.008 hours each night."
  - The CI is for the *mean* sleep, not *individual* students.

- "We are 90% confident that a randomly selected student will sleep between 6.272 and 7.008 hours per night."
  - We are 90% confident about the mean sleep, not an individual's sleep.

What Not to Say (Continued)

Don’t Say

- “The mean amount of sleep is 6.64 hours 90% of the time.”
  - The population mean never changes. Only sample means vary from sample to sample.

- “90% of all samples will have a mean sleep between 6.272 and 7.008 hours per night.”
  - This interval does not set the standard for all other intervals. This interval is no more likely to be correct than any other.

What You Should to Say

Do Say

- "90% of all possible samples will produce intervals that actually do contain the true mean sleep."
  - This is the essence of the CI, but it may be too formal for general readers.

- "I am 90% confident that the true mean amount that students sleep is between 6.272 and 7.008 hours per night."
  - This is more personal and less technical.

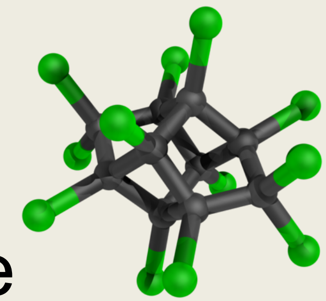
20.4

A Hypothesis Test for the Mean

One-Sample *t*-Test for the Mean

- Assumptions are the same.
- H_0: $\mu = \mu_0$
- $t_{n-1} = \dfrac{\bar{y} - \mu_0}{SE(\bar{y})}$
- Standard Error $SE(\bar{y}) = \dfrac{s}{\sqrt{n}}$
- When the conditions are met and H_0 is true, the statistic follows the Student's *t* Model.
- Use this model to find the P-value.

Are the Salmon Unsafe?

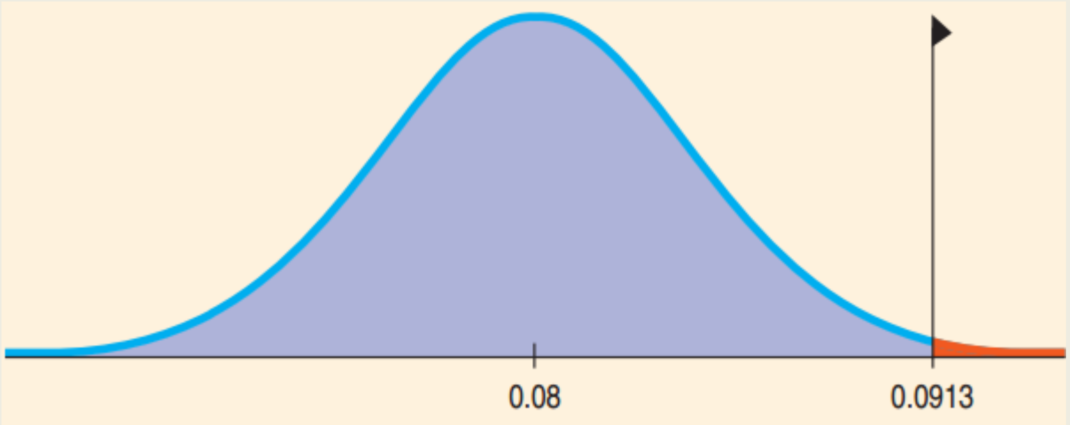


EPA recommended mirex screening is 0.08 ppm.

- Are farmed salmon contaminated beyond the permitted EPA level?

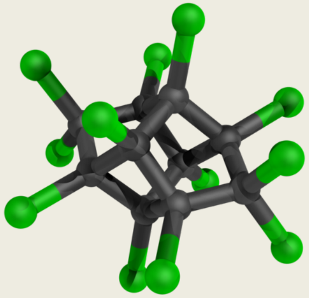
- Recap: Sampled 150 salmon. Mean 0.0913 ppm, Standard Deviation 0.0495 ppm.

- H_0: $\mu = 0.08$
- H_A: $\mu > 0.08$



Are the Salmon Unsafe?

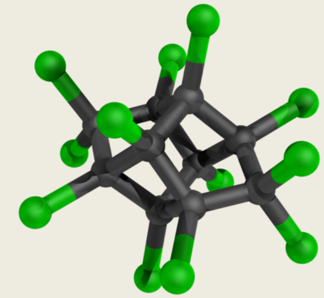
One-Sample *t*-Test for the Mean



- $n = 150$, $df = 149$, $\bar{y} = 0.0913$, $s = 0.0495$ in this case we can calculate t!

$$SE(\bar{y}) = \frac{s}{\sqrt{n}} \qquad t = \frac{\bar{y} - \mu}{SE(\bar{y})}$$

Are the Salmon Unsafe?



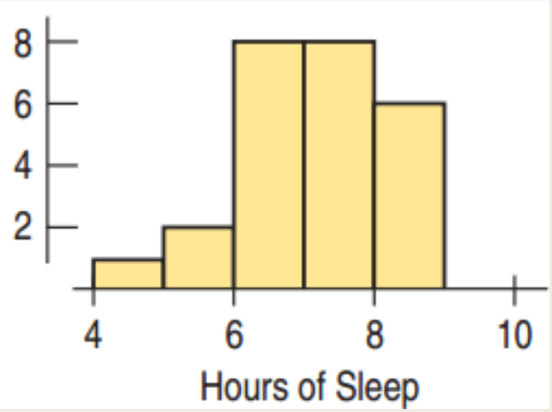
One-Sample *t*-Test for the Mean

- $n = 150$, $df = 149$, $\bar{y} = 0.0913$, $s = 0.0495$

- $SE(\bar{y}) = \dfrac{0.0495}{\sqrt{150}} \approx 0.0040$, $t_{149} = \dfrac{0.0913 - 0.08}{0.0040} = 2.825$
- $P(t_{149} > 2.825) = 0.0027$
  or $P < 0.005$ using the t-table
- Since the P-value is so low, reject H_0 and conclude that the population mean mirex level does exceed the EPA screening value.

Do Students Get at Least Seven Hours of Sleep? 25 Surveyed.

- **Plan:** Is the mean amount of sleep less than 7 hours?
- **Hypotheses:** H_0: $\mu = 7$ H_A: $\mu < 7$
- **Model**



  ✓ **Randomization Condition:** The students were randomly and independently selected .

  ✓ **Nearly Normal Condition:** Unimodal and symmetric

  - Use the Student's *t*-model, *df* = 24
  - One-sample *t*-test for the mean

Do Students Get at Least Seven Hours of Sleep? 25 Surveyed.

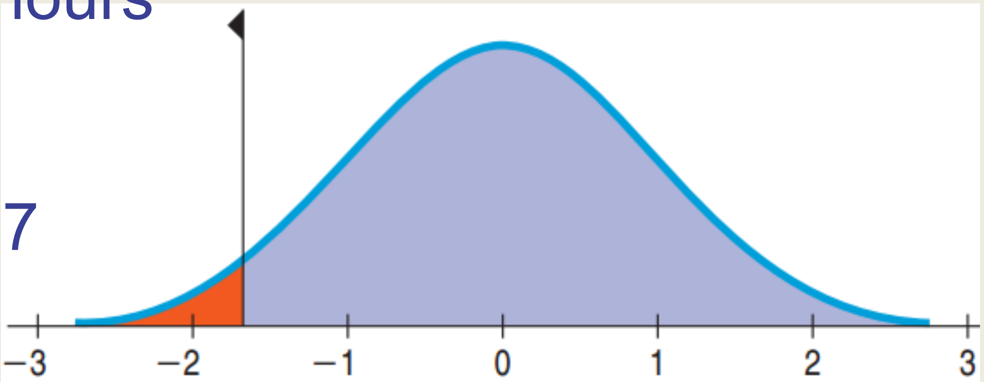
- **Mechanics:** $n = 25, \bar{y} = 6.64, s = 1.075$

Do Students Get at Least Seven Hours of Sleep? 25 Surveyed.

- **Mechanics:** $n = 25, \bar{y} = 6.64, s = 1.075$

$$SE(\bar{y}) = \frac{s}{\sqrt{n}} = \frac{1.075}{\sqrt{25}} = 0.215 \text{ hours}$$

$$t = \frac{\bar{y} - \mu_0}{SE(\bar{y})} = \frac{6.64 - 7.0}{0.215} \approx -1.67$$



$$\text{P-value} = P(t_{25} < -1.67) \approx 0.054$$

Or between 0.05 and 0.10 using the t-table

Do Students Get at Least Seven Hours of Sleep? 25 Surveyed.

- **Conclusion:** P-value = 0.054 says that if students do sleep an average of 7 hrs., samples of 25 students can be expected to have an observed mean of 6.64 hrs. or less about 54 times in 1000.
- With 0.05 cutoff, there is not quite enough evidence to conclude that the mean sleep is less than 7.
- The 90% CI: (6.272, 7.008) contains 7.
- Collecting a larger sample would reduce the *ME* and give us a better handle on the true mean hours of sleep.

Relationship between Intervals and Tests

Confidence Intervals

- Start with data and find plausible values for the parameter.
- Always 2-sided

Hypothesis Tests

- Start with a proposed parameter value and then use the data to see if that value is or is not still plausible.
- **2-sided test:** Within the confidence interval means fail to reject H_0. P-value = 1 – *C* is the cutoff.
- **1-sided test:** P-value = (1 – *C*)/2 is the cutoff.

Sleep, Confidence Intervals and Hypothesis Tests

90% Confidence interval mean hours slept : (6.272, 7.008)

- For a 2-tailed test with a 10% cutoff, any $6.272 \le \mu_0 \le 7.008$ would result in failing to reject H_0.

- For a 1-tailed test ("<") with a 5% cutoff, any $\mu_0 \le 7.008$ would result in failing to reject H_0.

- For a 1-tailed test (">") with a 5% cutoff, any $6.272 \le \mu_0$ would result in failing to reject H_0.

The Special Case with Proportions

Confidence Intervals

- Use $\hat{p}$ to calculate $SE(\hat{p})$.

Hypothesis Tests

- Use p to calculate $SD(p)$.

If $SE(\hat{p})$ and $SD(p)$ are far from each other, the relationship between the confidence interval and the hypothesis test breaks down.

20.5

Choosing the Sample Size

The Challenge of Finding the Sample Size

$$ME = t^*_{n-1} \times \frac{s}{\sqrt{n}}$$

- To find the necessary sample size in order to have a small enough margin of error:
  - Decide on acceptable *ME*.
  - Determine *s*: Use a pilot to estimate *s*.
  - Determine t^*_{n-1}: Use *z** as an estimate. By the 68-95-99.7 Rule, use 2 for 95% confidence.

$$ME = t^*_{n-1} \times \frac{s}{\sqrt{n}}$$

Finding the Sample Size

Should we buy a movie download accelerator? With a free trial we can test several download times in order to obtain a 95% CI to estimate the mean download time with a *ME* < 8 minutes.

Assume σ = 10 minutes.

- With z^* = 2 ,

Finding the Sample Size

Should we buy a movie download accelerator? With a free trial we can test several download times in order to obtain a 95% CI to estimate the mean download time with a *ME* < 8 minutes.

Assume σ = 10 minutes.

- With z^* = 2, $8 = 2 \times \dfrac{10}{\sqrt{n}}, \quad \sqrt{n} = \dfrac{20}{8}, \quad n = 6.25$

Finding the Sample Size

Should we buy a movie download accelerator? With a free trial we can test several download times in order to obtain a 95% CI to estimate the mean download time with a ME < 8 minutes.

Assume σ = 10 minutes.

- With z^* = 2, $8 = 2 \times \frac{10}{\sqrt{n}}, \quad \sqrt{n} = \frac{20}{8}, \quad n = 6.25$
- With the small sample size, z^* and t^* differ.

$$t_5^* = 2.571$$

Finding the Sample Size

- Estimate n again: $8 = 2.571 \times \dfrac{10}{\sqrt{n}}$

$$\sqrt{n} = \frac{2.571 \times 10}{8} \approx 3.214, \quad n \approx 10.33$$

- To make sure *ME* is no larger than 8 minutes, round up.
- We'll find the downloading times for 11 movies.

What Can Go Wrong?

Don’t confuse proportions and means.

- When counting successes with a proportion, use the Normal model.
- With quantitative data (means), use Student’s *t*.

Beware of multimodality.

- If the histogram is not unimodal, consider separating into groups and analyzing each group separately.

Beware of skewed data.

- Look at the normal probability plot and histogram.
- Consider re-expressing if the data is skewed.

What Can Go Wrong?

Set outliers aside.

- Outliers violate the Nearly Normal Condition.
- If removing a data value substantially changes the conclusions, then you have an outlier.
- Analyze without the outliers, but be sure to include a separate discussion about the outliers.

Watch out for bias.

- With bias, even a large sample size will not save you.

What Can Go Wrong?

Make sure cases are independent.

- Look for violations of the independence assumption.

Make sure the data are from an appropriately randomized sample.

- Without randomization, both the confidence interval and the p-value are suspect.

Interpret your confidence interval correctly.

- The CI is about the mean of the population, not the mean of the sample, individuals in samples, or individuals in the population.