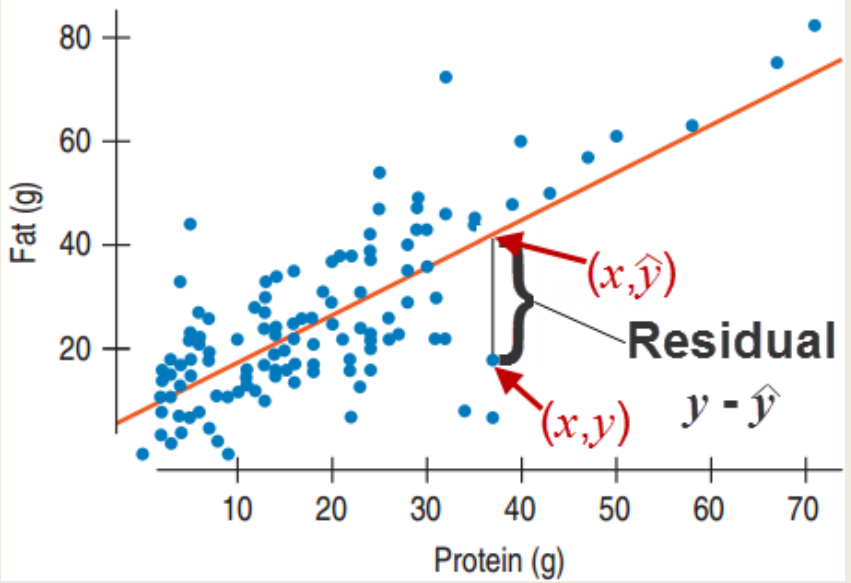
Quantitative Methods

Serena DeStefani – Lecture 6 – 7/14/2020

Review: the Residual



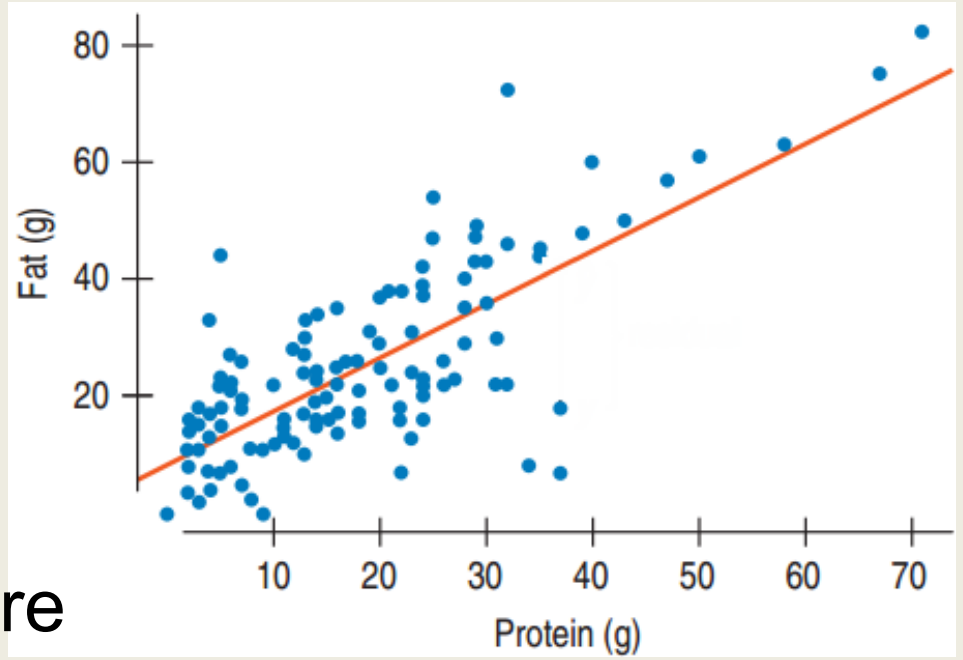
- $\hat{y}$ is the value on the line
- It is called the predicted value.

- For each point (x,y) look at the point $(x,\hat{y})$ on the line with the same x-coordinate.

- The residual is defined by $y - \hat{y}$

- The residual is the difference between the observed value and the predicted value.

The Line of Best Fit



- The best fitting line will have small residuals.
- High negative residuals are just as “bad” as high positive residuals.
- Squaring all residuals makes them all positive.
- The line of best fit is the line for which the sum of the squares of the residuals is the smallest, also called the **least squares line**.

What's the equation of the Line of Best Fit?

Line equation from Algebra

- $y = b + mx$

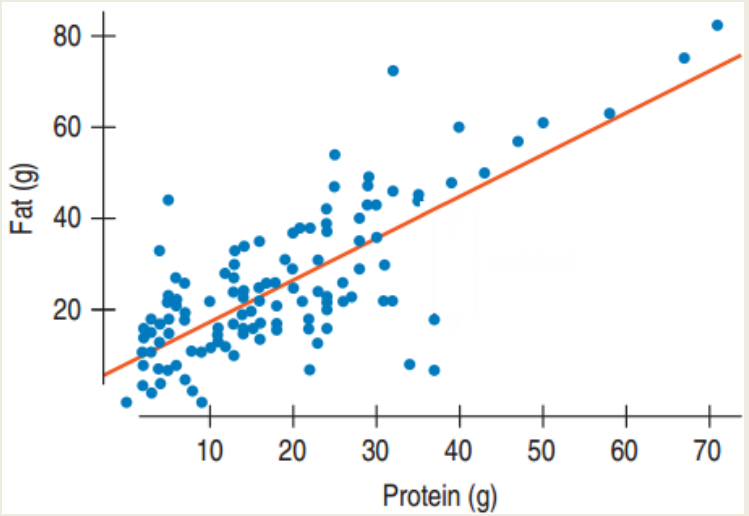
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Line of Best Fit

- $\hat{y} = b_0 + b_1 x$
- b_1 is the slope: how rapidly $\hat{y}$ changes with respect to x.
- b_0 is the y-intercept: The value of $\hat{y}$ when x is 0.

Slope and Correlation

Formula $b_1 = r \frac{s_y}{s_x}$



- Since the standard deviations are always positive, the slope and the correlation always have the same sign.
- The correlation has no units, but the slope has units of *y* over units of *x*.
  - For the Burger King example, the units for the slope are grams of fat per grams of protein.

The *y*-Intercept: how to find it?

The *y*-intercept and the slope are related by

$$\bar{y} = b_0 + b_1 \bar{x}$$

- The point corresponding to the means of *x* and *y*: $(\bar{x}, \bar{y})$ will always lie on the line of best fit.

- Given the mean of *x,* the mean *y*, and the slope, we can find the y-intercept:

$$b_0 = \bar{y} - b_1 \bar{x}$$

Finding the Regression Equation

PROTEIN $\bar{x}$ = 18.0 gr, s_x = 13.5 gr

FAT $\bar{y}$ = 24.8 gr, s_y = 16.2 gr

r = 0.76

$$b_1 = r\frac{s_y}{s_x} \qquad \bar{y} = b_0 + b_1\bar{x} \qquad b_0 = \bar{y} - b_1\bar{x}$$

b_1 = 0.76 X (16.2 gr fat /13.5 gr protein) =0.91 gr fat / gr prot

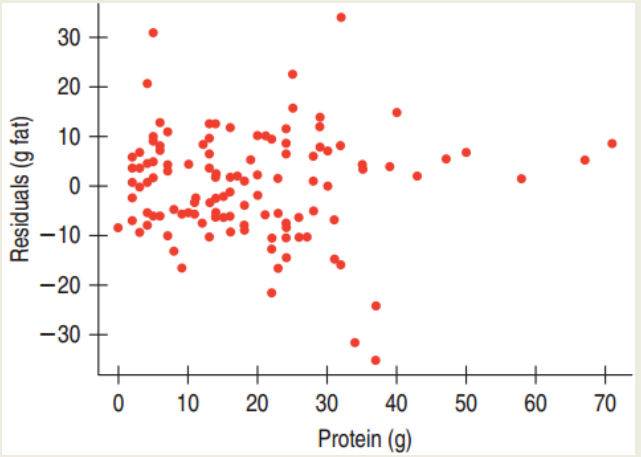
b_0 = 24.8 – 0.91 gr fat / gr prot X 18.0 gr prot = 8.4 gr fat

Fat = 8.4 + 0.91 *Protein*

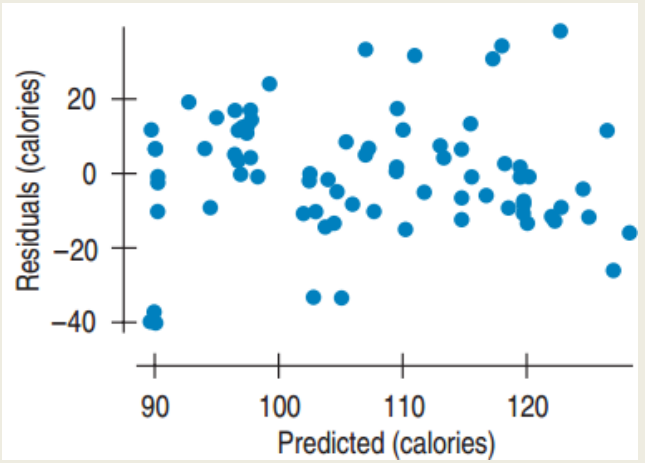
Correlation and Prediction, thinking in z-scores

- What would your guess for height be if you knew the new student's shoe size had $z = 2$?
  - We know that the correlation is positive, but not perfect ($0 < z_{in} < 2$).
  - There is a way to connect correlation and z-scores
  - Let's think about the line of best fit for z-scores
  - Since $b_0 = b_1\bar{x} - \bar{y}$ and the means for *z*-scores are both 0, this gives $b_0 = 0$.
  - Since the standard deviations are both 1, $b_1 = r\frac{s_y}{s_x}$ gives $b_1 = r$. Plugging into $\hat{y} = b_0 + b_1 x$ gives: $\hat{z}_y = rz_x$

Conditions on the Scatterplot of the Residuals

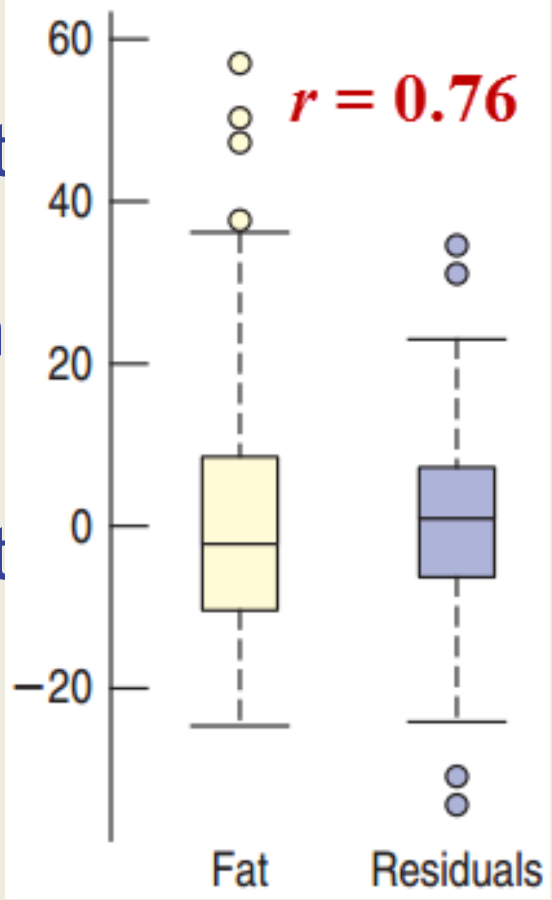


- There should be no bends.
- There should be no outliers.
- There should be no changes in the spread from one part of the plot to another.



Variation of *y* and the Variation of the Residuals (Continued)

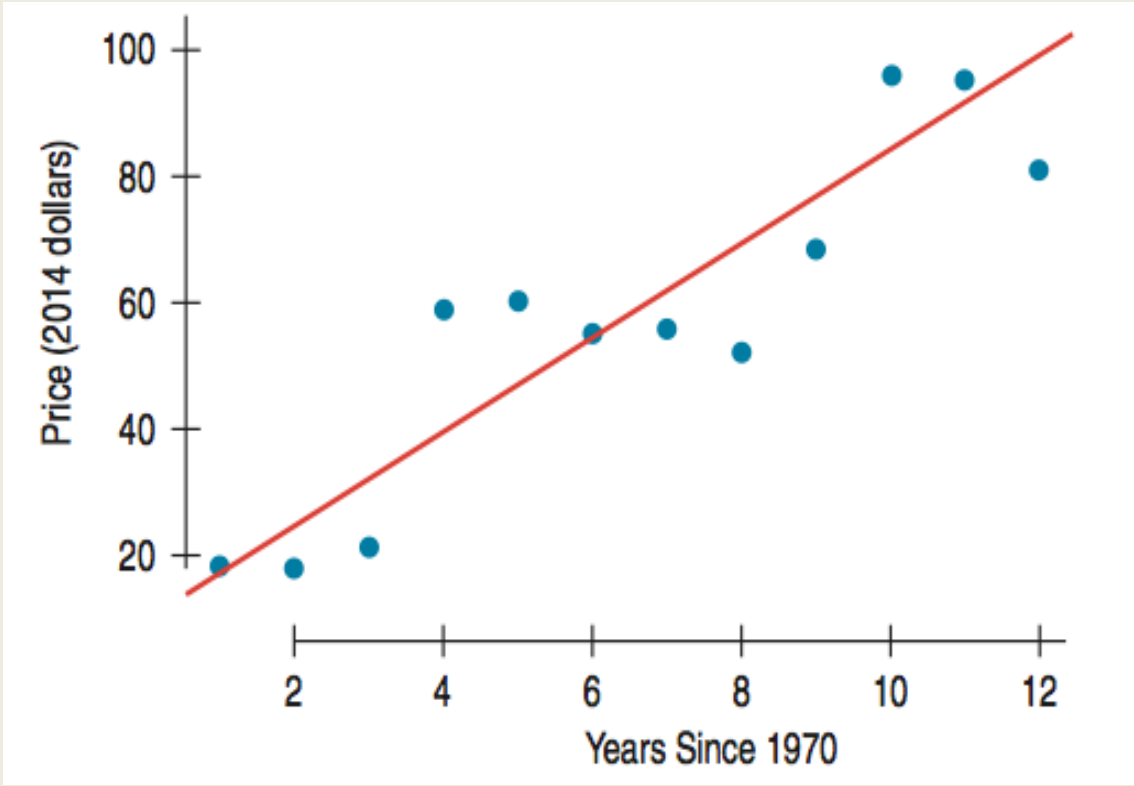
- $R^2 = 0.76^2 = 0.58$
  - 58% of the variability in fat content in Burger King's menu items is accounted for by the variation in protein content.
  - 42% of the variability in fat content is left in the residuals.
    - Other factors such as how the food is prepared account for this remaining variability.



Predicting Gas Prices

$$\widehat{\text{Price}} = 12.79 + 6.75 \text{ Years Since 1970}$$

- The data clearly follows a linear model.
- Can we predict the price of oil for the year 25 (1995)?
- 12.79 + 6.75(25) ≈ $182
- How good is this prediction?



Leverage and Influential Points

- A data point whose *x*-value is far from the mean of the rest of the *x*-values is said to have high leverage.
- Leverage points have the potential to pull strongly on the regression line.
- A point is influential if omitting it from the analysis changes the model enough to make a meaningful difference.
- Influence is determined by
  1. The residual
  2. The leverage

Chapter 10

Understanding Randomness

10.1

What is Randomness?

1 2 3 4

Pick a number from 1 to 4

- Is this random?
- 75% of all people pick 3.
- How would you generate a random number?
- We can use a computer or do it by hand.
- In R there are different ways:
- **runif(1)** generates one random # between 0 and 1
- **floor(runif(3, min=0, max=101))** generates 3 random integers bt 0 and 100

Picking a Card

Randomness by Hand

- Picking a card is another way to generate randomness.
- Must shuffle at least 7 times to achieve randomness.
- Other ways:
  - Rolling a die
  - Flipping a coin
  - Numbers out of a hat

10.2

Simulating by Hand

Lottery for the Dorms

57 students are in a lottery for the spacious triple dorm room. 20 were from the varsity team and all three winners were from this team.

- How likely is this? Was it rigged?

Steps for Simulation

Specify how to model a component outcome using equally likely random digits:

1. Identify the component to be repeated.

2. Explain how you will model the experiment's outcome.

Steps for Simulation

Specify how to simulate trials:

3. Explain how you will combine the components to model the trial.

4. State clearly what the response variable is.

Put it all together to run the simulation:

5. Run several (many) trials

Steps for Simulation

Analyze the response variable:

6. Collect and summarize the results of the trials.
   - As you have learned, look for shape, center, spread, outliers, etc.

7. State your conclusion

Lottery for the Dorms

57 students are in a lottery for the spacious triple dorm room. 20 were from the varsity team and all three winners were from this team.

- How likely is this? Was it rigged?

Plan → Simulation

- **Components to be repeated:** Selection of the students.
- **Outcomes:** Generate numbers from 1 to 57. 1-20 will represent the team members.
- **Trial:** Pick the first three distinct numbers.
- **Response Variable:** Yes if all three are 1-20

Show

Mechanics

We can run the simulation in R:

```
replicate(
        100,
        { floor(runif(3, min=0, max=58))
        }
        )
```

Analyze

- Only 3 out of the 100 trials resulted in “All Varsity.”

Conclusions

In the simulation, only 3 out of 100 were “All Varsity.” While 3% is only a small chance, it is not impossible. It looks pretty suspicious.

Is 3% a small enough chance to make a formal accusation?

What Can Go Wrong?

Don't Overstate Your Case

- Simulation is not reality, it only indicates probability.

Model Outcome Chances Accurately

- What would be wrong with generating random numbers 0, 1, 2, 3 to indicate the number of team members that room together?
- There is not a 25% chance of each. They are not equally likely.

Run Enough Trials

- Don't just do a few trials. Err on the side of a large number of trials.

Chapter 11

Sample Surveys (11.1, 11.2, 11.3, 11.7)

11.1

The Three Big Ideas of Sampling

Idea 1: Examine a Part of the Whole

The Goal

- Learn about the entire group of individuals (called the population)

The Problem

- It is usually impossible to collect data on the entire population.

The Compromise

- Collect data on a smaller group of individuals (called a sample) selected from the population.

Examples of Samples

People

- Telephone surveys
- Internet surveys
- Data from a select group of customers
- Student surveys handed out in class
- Medical studies

Experimental units

- Biological research
- Crash dummy tests
- Weather studies

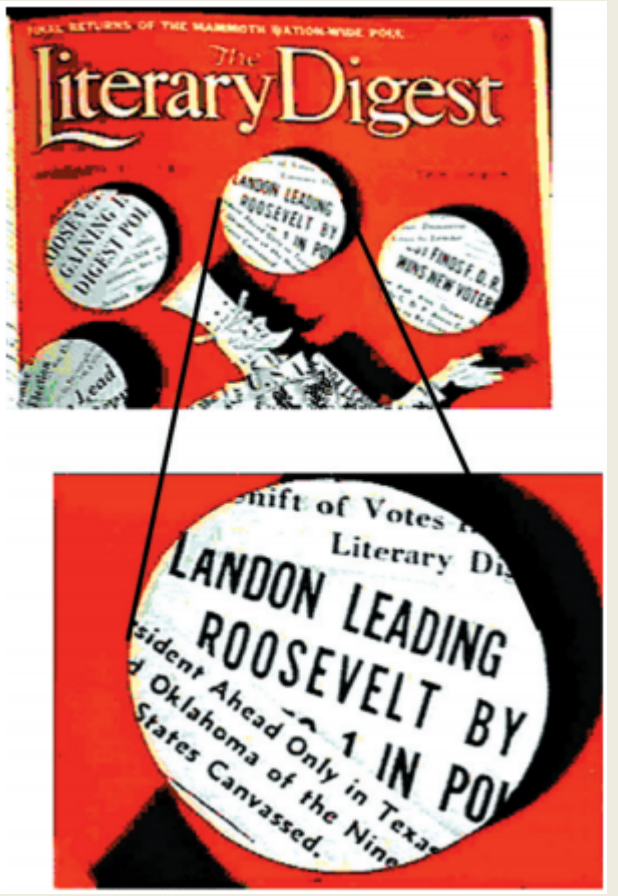
Bias

The Challenge

- Obtain a sample that is perfectly representative of the population.

- Avoid bias – over or under emphasizing some characteristic of the population that is pertinent to the study.

1936: Landon Beats Roosevelt??



The Survey

- 1936, Literary Digest received 2.4 million "mail in" ballots.
- Used names from the phone book

The Results

- Landon leads 57% to 43%
- The Problem
- Only high income earners could
- afford a phone.
- This was a very biased survey.

Literary Digest soon went out of business.

1948: Dewey defeats Truman??

2016: Clinton defeats Trump??

ROYAL STATISTICAL SOCIETY
DATA | EVIDENCE | DECISIONS

Search

SIGNIFICANCE

ROYAL STATISTICAL SOCIETY

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"Clinton defeats Trump" 2016 polls and the shadow of 1948

Dominic Lusinchi

First published: 02 August 2017 | https://doi.org/10.1111/j.1740-9713.2017.01049.x | Citations: 1

Idea 2: Randomize

Can we list the characteristics of the population and ensure we represent them all without bias?

- Race, age, ethnicity, income, marital status, work type, family size, …
- The list would go on forever. There are more types of people than the number of people.
- So… what can we do???

Randomizing can lead to a representative sample.

- Randomizing protects us from the influences of all the features of the population.
- On average, the sample will look like the population.

Idea 3: It's the Sample Size

If you need 100 students to get a random sample at the university, how many Americans would you need to achieve the same level of randomness from the entire U.S.A.?

- Answer: 100
- It is the number of individuals, not the percent of individuals that matters.
- The number of individuals in the sample is called the sample size.

Census

Why not just include everyone?

- Surveying everyone is called a census.
- That would be best **if** we could.

Problems with a census

- Very expensive
- Takes too long
- Usually impossible to find everyone
- Not everyone is willing to participate.
- The population is always changing – births and deaths occur every day.

Issues with a Census

Double Counting

- College students are often counted both at their colleges and where their parents live.

Under Counting: Who is likely to be missed?

- The poor
- Undocumented immigrants
- Homeless

Should we use sampling instead of a census?

- Statisticians think so!
- Politicians don’t agree.

11.2

Populations and Parameters

Parameter and Statistic

Based on a study a report stated that 21.7% of all U.S. teens do not wear seatbelts.

- Do they really know about all U.S. teens?
- They used the sample proportion to make inferences about the population proportion.
- A parameter is a number used in a model of the **population**.
- A statistic is a number that is calculated from the **sample** data.

Greek for Parameter, Latin for Statistic

Examples of parameters

- μ, σ, p, ρ, β

Examples of Statistics

- $\bar{y}$, s, $\hat{p}$, r, b

- Notice that π would be confusing as a parameter.

Name	Statistic	Parameter
Mean	$\bar{y}$	μ (mu, pronounced "meeoo," not "moo")
Standard Deviation	s	σ (sigma)
Correlation	r	ρ (rho, pronounced like "row")
Regression Coefficient	b	β (beta, pronounced "baytah"[5])
Proportion	$\hat{p}$	p (pronounced "pee"[6])

Representative Sampling

Since we can't take a true census, we want to compute statistics that reflect the parameters.

- A sample that does the above is called a representative sample.

- Biased samples tend to not be representative.
  - The statistic tends to be much higher or much lower than the parameter.

What's Wrong with Each of the Following?

- It is always better to take a census than draw a sample.
- Stopping students outside of the cafeteria is a good way to find out about its quality of food.
- To get the same level of precision that 100 students sampled from a university with 3000 students will have, you need to sample 1000 students from a university with 30,000 students.

What's Wrong with Each of the Following?

- The majority of the 12,357 students who answer a website poll clicked that they enjoyed doing statistics homework. Since the sample size is large, we can conclude that the majority of all students enjoy doing statistics homework.

- The true percentage of all Statistics students who enjoy the homework is called a "population statistic."

11.3

Simple Random Samples

Random But Not Representative

Random

- Suppose there are 100 men and 100 women in a class. Flip a coin.
  - **Heads:** Choose the 100 men.
  - **Tails:** Choose the 100 women.

- Every student has an equally likely chance of being chosen. Randomness was achieved.

- This will **not** produce a **representative sample**.

Simple Random Sampling

SRS

- Order the students from 1 to 40.
- Use a computer to randomly select 20 numbers from 1 to 40.
- Select the students with the chosen numbers.

Simple Random Sampling (SRS) is when every combination has an equally likely chance to be selected.

- SRS is the standard which all other sampling techniques are measured.
- Statistical theory is based on SRS.

Sampling Variability

- Samples will vary from one to the next.
  - The first sample of five students' weight might average 131 pounds.
  - The second might average 138 pounds.

The sample to sample differences are called the **sampling variability** (or sampling error).

- Natural
- Not a problem

11.7

Common Sampling Mistakes, or How to Sample Badly

Mistake 1: Sample Volunteers

Voluntary Response Sample

- Open the survey up to many, only a few respond.
  - Internet polls
  - Letters to Congress
  - “How are we doing?” cards
- Sampling frame does not correspond to population.
- Prone to bias
  - Only most opinionated respond.

Mistake 2: Sample Conveniently

In Convenience Sampling we sample only those who are convenient for us to sample.

- Asking all of your Facebook friends.
- Surveying at shopping malls to find how much people like shopping.
- Asking people in a restaurant how often they eat out.

Convenience sampling is always biased.

- More likely to get people like you.
- “Safe-looking” people.

Mistake 2b: How psychological science works… sometimes

Psychological Science is based on Convenience Sampling : we often sample those who are convenient for us to sample (for surveys)

- Students that see your announcement
- Other graduate students
- Labmates

Convenience sampling is always biased…

But in Psychology, results are extrapolated not only to the respective populations, but to all humanity…!

The WEIRD problem (Western, educated, and from industrialized, rich, and democratic countries)

Mistake 3: Use a Bad Sampling Frame

Complete sampling frames are difficult. In a phone survey you may miss:

- People in prison
- Homeless people
- Students
- Long-term travelers
- Cell-phone users who like their privacy

Mistake 4: Undercoverage

Bias can result when a subpopulation is left out or underrepresented.

- Evening telephone surveys
- Door-to-door surveys
- Surveys given in English

Nonresponse Bias

Nonresponse bias is a concern for most surveys.

- It is better to put forth effort to ensure a smaller group responds than to put the survey out to a large number and only receive a small number of responses.
- Consider whether nonrespondents are likely to think differently from responders.
- ?

Response Bias

Response bias is anything in the survey design that influences the responses.

- People want to please the interviewer.
- People don’t want to admit they are flawed.
- People hide personal facts: income, age, etc.
- Wording of the question can steer the response.

How to Think about Bias

- Always look for bias.
  - If there is bias in a survey that you have already conducted, you must start over. A larger sample size won't help.
- Spend your time and resources reducing bias.
- Who was excluded from the study?
- Pilot-test the survey.
  - Look for misunderstandings, confusion, etc.
  - Refine your survey based on the pilot.
- Always report the sampling method in detail.

Preview

- Why do we talk about:
  - Simulation (the dorm lottery)
  - Sampling

- Classical Statistical Testing is based on the idea of:
  - running a study on a representative sample → results A
  - "theoretically" repeating the study numerous times
  - obtaining a theoretical probability distribution
  - seeing how result A fares compares with the distribution

  - It's like running a simulation

Chapter 12

Experiments and Observational Studies

12.1

Observational Studies

Music and Good Grades

The Study:

- Compared GPAs of music students and non-music students at Mission Viejo High School.

The Results

- Music Students: 3.59
- Non-Music Students: 2.91

Conclusions

- Should we make all students play an instrument?

Issues

- Could there be something else resulting in both?

Observational Studies

Observational Studies

- Researchers don't assign choices.
- Passively observe participants
- Good for discovering relationships related to rare outcomes
- Bad for establishing cause-and-effect relationships
- Tough to handle lurking variables

  - Do musicians have more supportive parents that help GPA?

Retrospective Studies

Retrospective Studies

- Collect data on something that has already occurred
- Similar pros and cons as observational studies
- Additional issues can include:
  - Unreliable memories
  - Incomplete historical records
  - Often limited to a small part of the population

Prospective Studies

A **prospective study** is a study where we identify subjects in advance and collect data as events unfold.

- Pros:
  - Possible to isolate the variables.
  - With care, can establish cause and effect.
  - Can design the study to your specifications.
- Cons:
  - Can be expensive.
  - Rare occurrences require very large samples.
  - Can take too long:
    - Do breast-fed babies live longer than bottle-fed?

Which kind of study?

Early 2007, many dogs and cats died of kidney failure. Should you conduct a retrospective or prospective study to find out why?

- Retrospective Study
  - The event happened in the past.
  - It may have been a rare event.
  - The retrospective study may provide clues on the cause.
- Once possible causes are identified, try a prospective study to verify the causes - if it doesn't kill any pets.

12.2

Randomized, Comparative Experiments

Experiments

Is it possible to establish a cause and effect relationship?

- Take 100 young children. Randomly select 50 to be in a music program. The other 50 will not be allowed to play an instrument.

- An experiment requires random assignment of subjects to treatments.

- Only experiments can establish cause and effect.

How Experiments Work

- Identify the explanatory variable(s), called the factor(s).
- Identify the response variable.
- Select subjects or participants (if human) or experimental units (if not human).
- Decide on the levels to choose for each factor.
  - Music program or no music program
  - Sleep hours: 4, 6, or 8
- The combination of specific levels from all factors that a subject receives is called its treatment.

Assigning Participants to Treatments

- Don’t let them choose.
- Don’t assign based on what’s best for each.
- **Randomly** assign participants into groups. Each group receives a different treatment.
- Only through random assignment can a cause-and-effect relationship be established.
- What ethical dilemmas might this introduce?

12.3

The Four Principles of Experimental Design

Control and Randomization

1. Control
   - Make all other conditions as similar as possible for all treatment groups.
   - Control allows us to isolate the one thing that is being studied. Helps avoid lurking variables
2. Randomize
   - Equalizes the effects of variation that we cannot control
   - Distributes the uncontrollable factors equally

Control what you can, randomize the rest.

Replicate and Block

Replicate

- Apply each treatment to a number of subjects.
- Repeat the entire experiment on an entirely different population of experimental units.

Block

- Group similar individuals together and randomize within each of these blocks.
- Blocking helps account for the variability due to the difference between blocks.

Is the Pet Food Safe?

Control

- Food and water portions, housing, exercise, sleep. Stick to one breed.

Randomize

- Assign dogs to two feed treatments randomly.

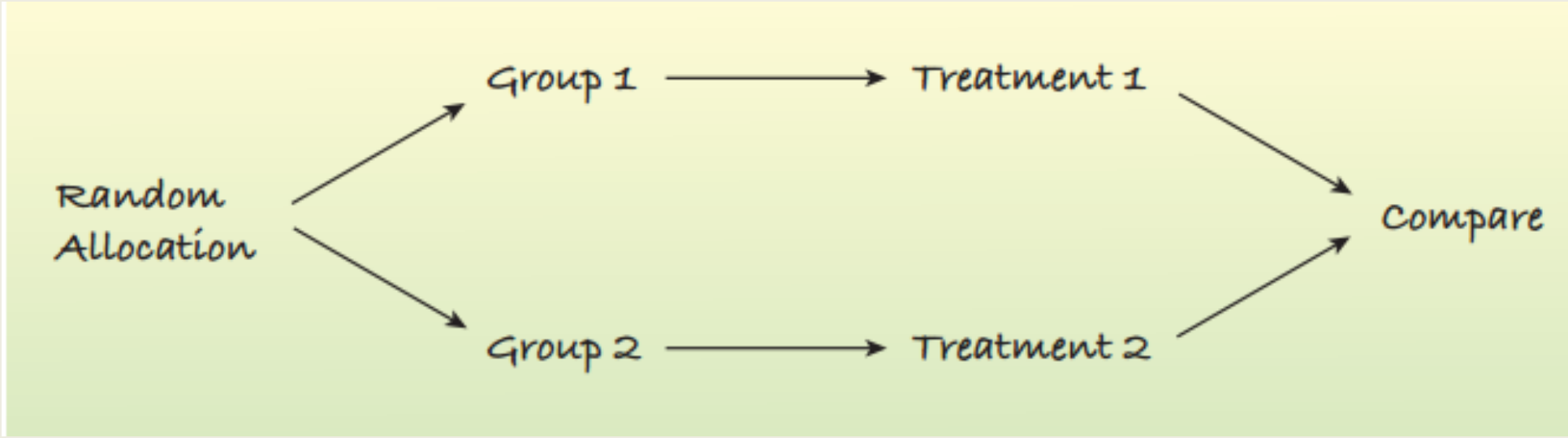
Replicate

- Redo the entire experiment with a different breed.

Diagrams

A diagram is a useful organizational tool.

- Flow charts help to understand the process
- Side-by-side boxplots help compare the treatment groups.

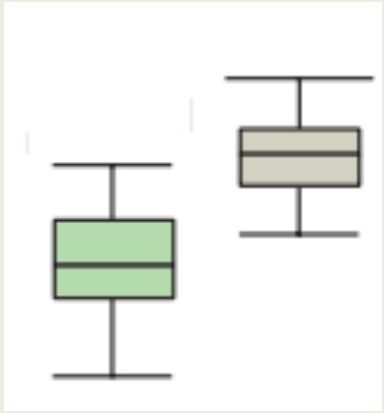


Statistical Significance

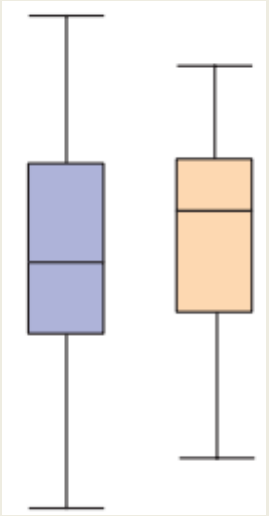
A difference is called **statistically significant** if the difference is greater than what we would expect from random chance, if we'd repeat the test multiple times.

- Flip a coin 100 times:
  - 54 heads is not statistically significant since it would not be surprising to observe this outcome.
  - 94 heads is statistically significant since it would be surprising to observe this outcome.

Statistical Significance



Statistically significant since the medians of each are outside the typical values of the other.



Not statistically significant since the medians of each are within the typical values of the other.

Random Samples and Random Treatments

- **Surveys** use a random group of participants.
- **Experiments** find a homogeneous group, separate them into random subgroups for treatment.
- Experiments do not use a random sample from the population.
- Beware of stating that the participants from the experiment represent the larger population.

12.4

Control Treatments

Control Groups and Control Treatments

Control

- Does eating ten carrots a day help you lose weight?
- Find 200 participants and randomly select 100 of them to eat ten carrots a day.
- The other 100 are the control group.
- **Not** eating ten carrots a day is the control treatment.

Blinding

What brand of cola is the best?

- If you give participants cans of cola and ask how much they like it, the label can be an influence.
- Instead give each an unlabeled cup of soda.
- Single-blinding involves the participants not knowing whether they are in the control or treatment group.
- If the person handing out the cups hands out her favorite soda she may bias the results.
- Double-blinding means neither the participant nor the person handing out the soda knows the label.

Who Can Affect the Experiment

There are two main classes of individuals who can affect the experiment.

- Those who can influence the results.
  - Subjects
  - Researchers
  - Treatment administrators
  - Technicians

- Those who evaluate the results.
  - Judges
  - Treating Physicians

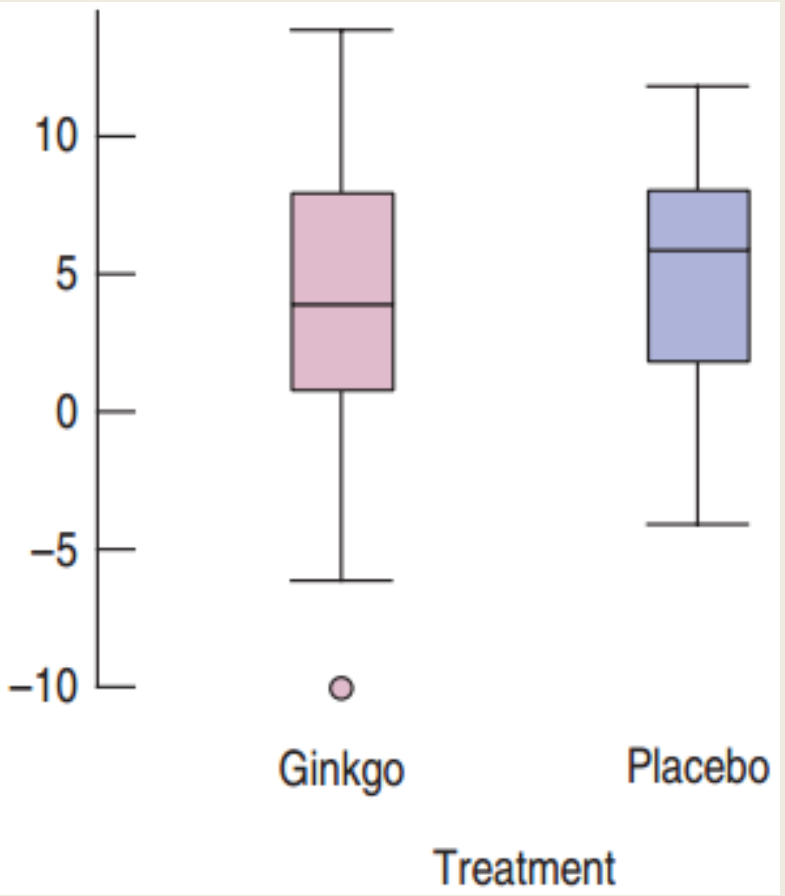
Placebos

- A placebo is a “fake” treatment that looks like the treatment being tested.
- Just telling a patient that they are being treated can aid recovery.
- This is called the placebo effect. The placebo is fake but the placebo effect is real!
- Use a placebo for effective blinding.

Placebo Effect Really is True

The Study

- One group received Ginkgo the other received a placebo.
- 13 memory tests were given.
- Ginkgo better for 6 tests.
- Placebo better for 7 tests.



Summary of Experimental Techniques

- Randomized
- Comparative
- Double-blind
- Placebo-controlled

12.5

Blocking

Comparing Nail Polish with Blocking

- I want to compare two types of nail polish, red and nude, to see how much they are likely to get chipped
- The right hand and the left hand might yield different results

Nail Polish

Comparing Nail Polish on Real Hands

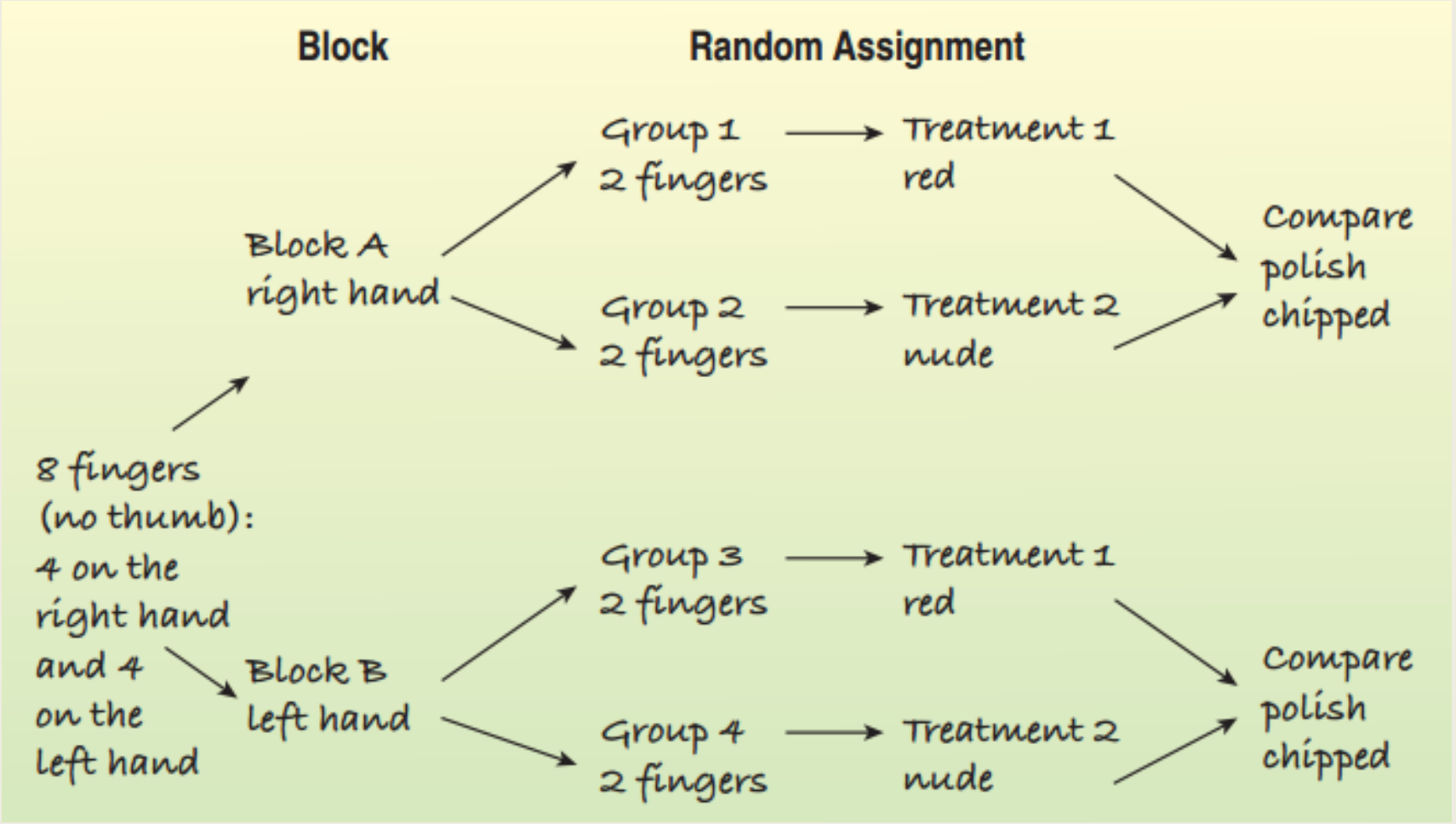
- To decrease variation, randomly assign half the right hand fingers red and other half nude.
- Do the same for the left hand side.
- This ensures that a color is not overrepresented by the right hand.

- Each hand serves as a separate experiment.

Blocking

- Experimental units can be separated into groups that are not the treatment, we call these groups blocks.
- Blocking involves randomly assigning the treatments within each block.
- Blocking helps isolate the variability due to the differences between blocks.
- Blocking helps clarify the difference between the treatments.
- The design is called a randomized block design.

Chart of a Randomized Block Design



12.6

Confounding

Animated Teaching vs. Subdued Teaching

Professor Ceci taught the same course in the fall and the spring.

- Fall: No slides, everything else the same
- Spring: Slides

Results: How much did you learn? (1-5)

- Fall: 2.93
- Spring: 4.05

Conclusions

- Slides better than no slides???
- Weather: Fall ends gloomy, spring ends pleasant.

Confounding Factors

Two factors are **confounded** if the levels of one are associated with the levels of the other.

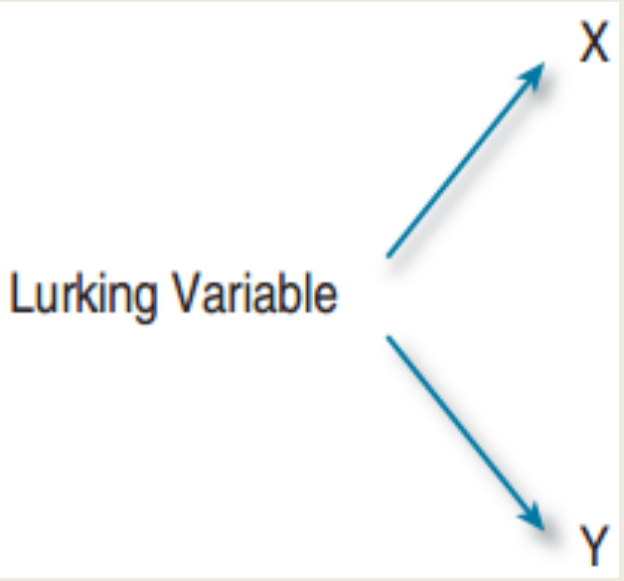
- Weather and Professor Cecil's style were confounded.
- Try to avoid confounding factors, but it is difficult and sometimes impossible.
- Avoiding confounding factors can introduce new ones.
  - Compare morning and afternoon fall courses.

Two Factors

A bank sent out 50,000 low-rate-no-fee offers and 50,000 high rate with fee offers.

- Many more responses for low rate, no fee
- Was it the low rate or no fee that customers liked?
- Should have sent out four types of offers:
  - Low rate, no fee
  - Low rate, with fee
  - High rate, no fee
  - High rate, with fee

Lurking and Confounding

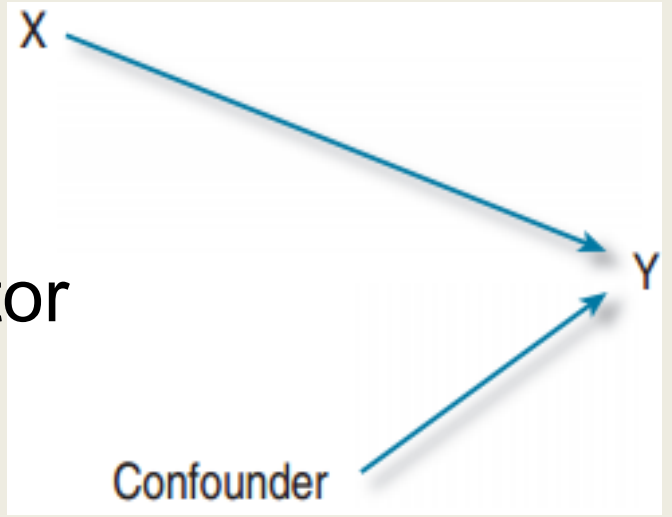


Lurking Variable

- Associated with both *x* and *y*
- Makes it appear that *x* causes *y*

Confounding Variable

- Associated in a noncausal way with a factor
- Affects the response
- Can't tell if the cause was the factor or confounding variable



What Can Go Wrong?

- Don't give up just because you cannot run the experiment.
  - Sometimes we have to resort to an observational study. Do airbags lower the risk of dying?

- Beware of Confounding
  - If possible, use randomization.
  - If not possible, report out likely confounding factors.

What Can Go Wrong? (Continued)

- Bad things can happen even to good experiments.
  - Record additional information that may help in the analysis.

- Don’t spend your entire budget on the first run.
  - Try a small pilot run first.
  - Then refine the factor levels and fix other problems.
  - Finally, perform the full-scale study.