Quantitative Methods

Serena DeStefani – Lecture 13 –7/27/2020

How did we get here?

- <u>Statistics is about variation</u>, specifically in data
- Types of *variables*
- How to represent variables
  - bar graphs
  - boxplots, histograms
- How to compare apples and oranges
  - by their variation (z scores)
  - are they varying together? correlation and regression

How did we get here?

- Statistics is about variation, specifically in data
- What I am interested in?
- A larger group, or population
- How to get data?
- **Surveys**
- Impossible to survey the population
- → representative sample (SRS)
- But exactly what does the population think?
- What's the population parameter?
- **Experiments**
- Is my results due to the treatment or is it due to chance?

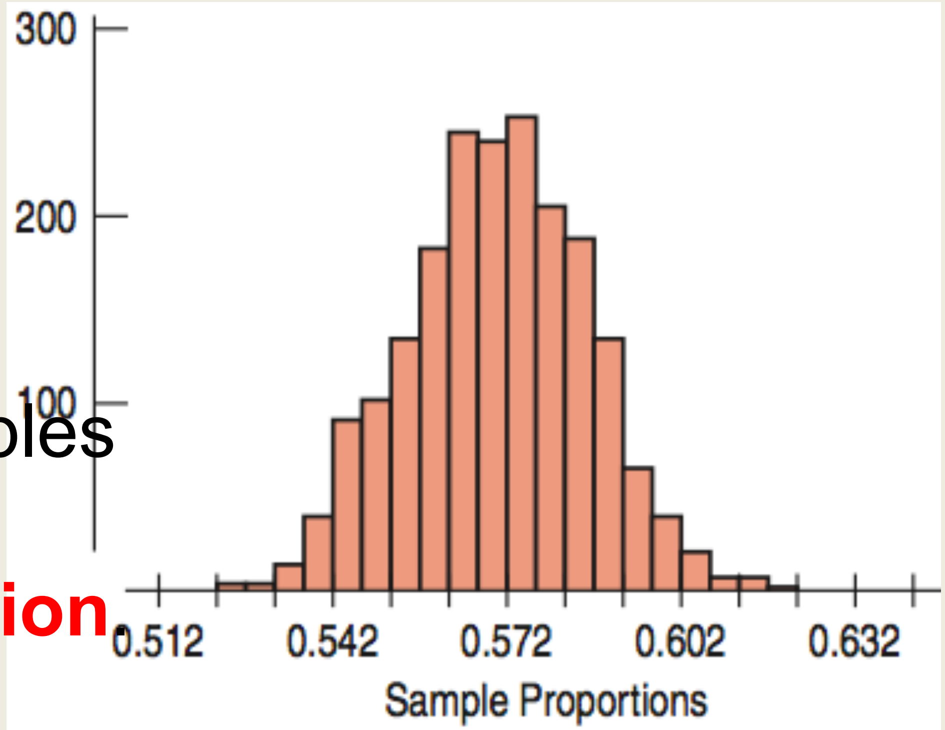
How did we get here?

- Two questions:
  - What’s the population parameter?
  - Is my result likely?
- To answer these questions I need a *probability model*
- The rules of probability
- Random variable: the Bernoulli trial
- The geometric and binomial distribution
- The normal approximation to the binomial
- The sampling distribution
- The Central Limit Theorem

Sampling About Climate Change

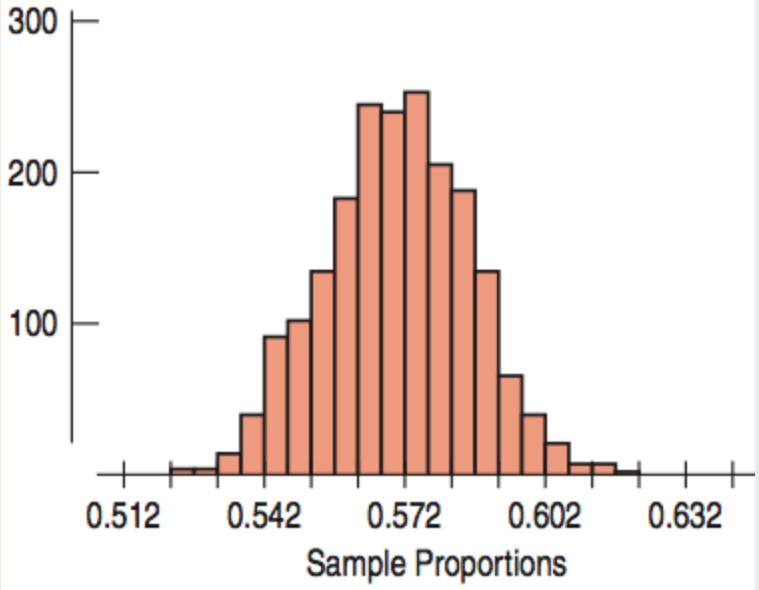
According to a Gallup poll of 1022 Americans, 57% believe that climate change is due to human activity.

- If many surveys were done of 1022 Americans, we could calculate the sample proportion for each.
- The histogram shows the distribution of a simulation of 2000 sample proportions.



- The distribution of all possible sample proportions from samples with the *same sample size* is called the **sampling distribution**.

Sampling Distributions for Proportions



Sampling Distribution for Proportions

- Symmetric
- Unimodal
- Centered at *p*
- The sampling distribution follows the Normal model.

What does the sampling distribution tell us?

- The sampling distribution allows us to make statements about where we think the corresponding **population parameter** is and how precise these statements are likely to be.

From One Sample to Many Samples

Distribution of One Sample

- **Variable** was the *answer to the survey* question or the *result* of an experiment.
- **Proportion** is a fixed value that comes from the one sample.

Sampling Distribution

- **Variable** is the proportion that comes from the entire sample.
- Many proportions that differ from one to another, each coming from a different sample.

Mean and Standard Deviation

Sampling Distribution for Proportions

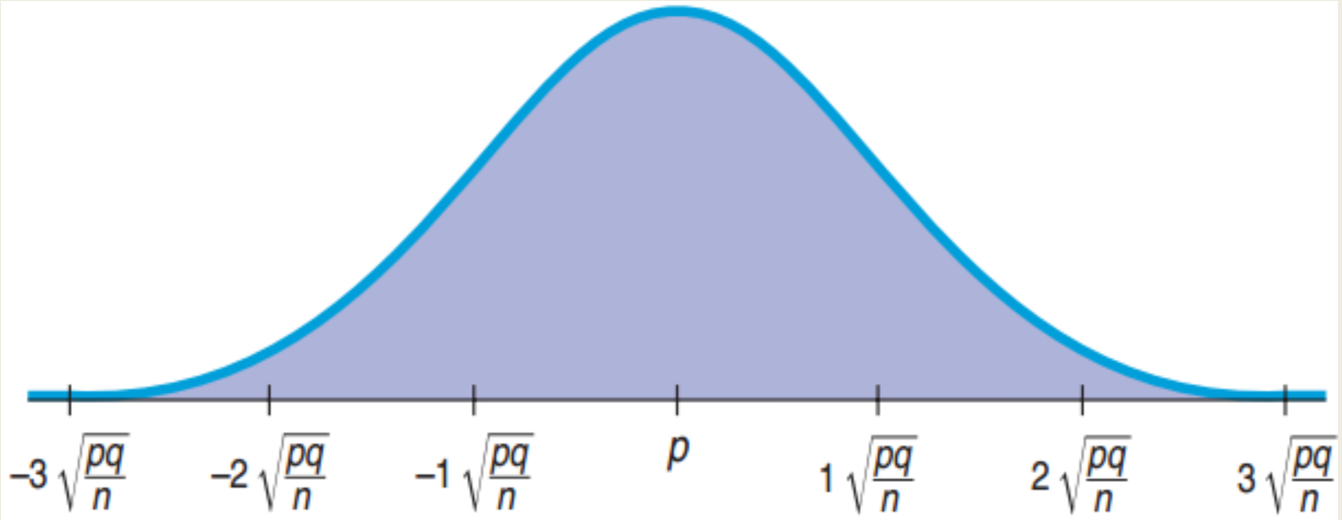
- $\text{Mean} = p$

This *p* is a parameter!

- $\sigma(\hat{p}) = \frac{\sqrt{npq}}{n} = \sqrt{\frac{pq}{n}}$

Instead *p_hat* is a statistics!

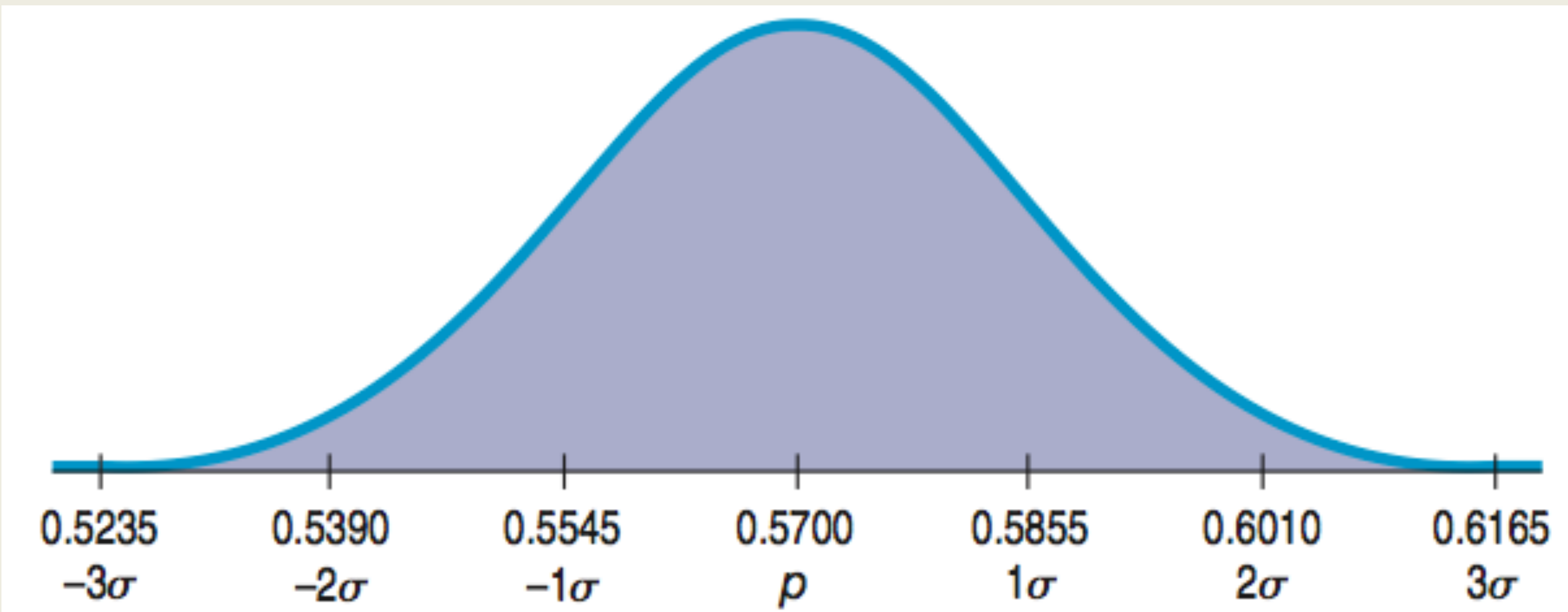
- $N\left(p, \sqrt{\frac{pq}{n}}\right)$



The Normal Model for Climate Change

Population: $p = 0.57$, $n = 1022$. Sampling Distribution:

- Mean = 0.57
- Standard deviation $= SD(\hat{p}) = \sqrt{\dfrac{(0.57)(0.43)}{1022}} \approx 0.0155$



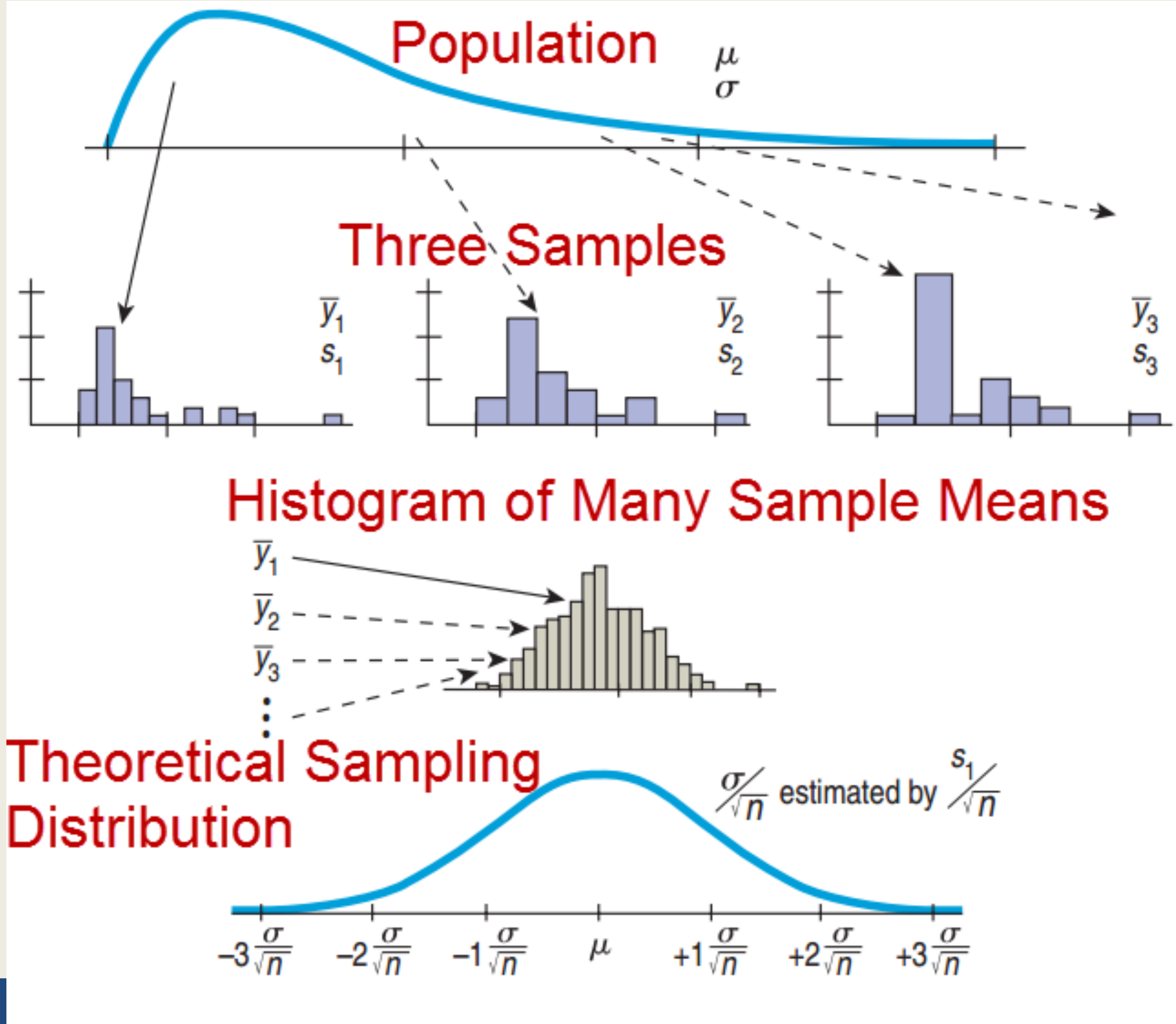
Standard Error

The sample-to-sample standard deviation is called the **standard error** or **sampling variability**.

- The standard error is not a “real” error, since no error has been made.

The problem is, we don’t know the population parameter.

Sampling Distributions for Means



How did we get here?

- What's the population parameter?
  → Confidence Intervals
- Is my result likely?
  → Hypothesis testing
-

Where are we going?

- Confidence intervals
- Hypothesis testing for proportions
- Hypothesis testing for means
- What if I have more than two groups?

Chapter 18

Confidence Intervals for Proportions

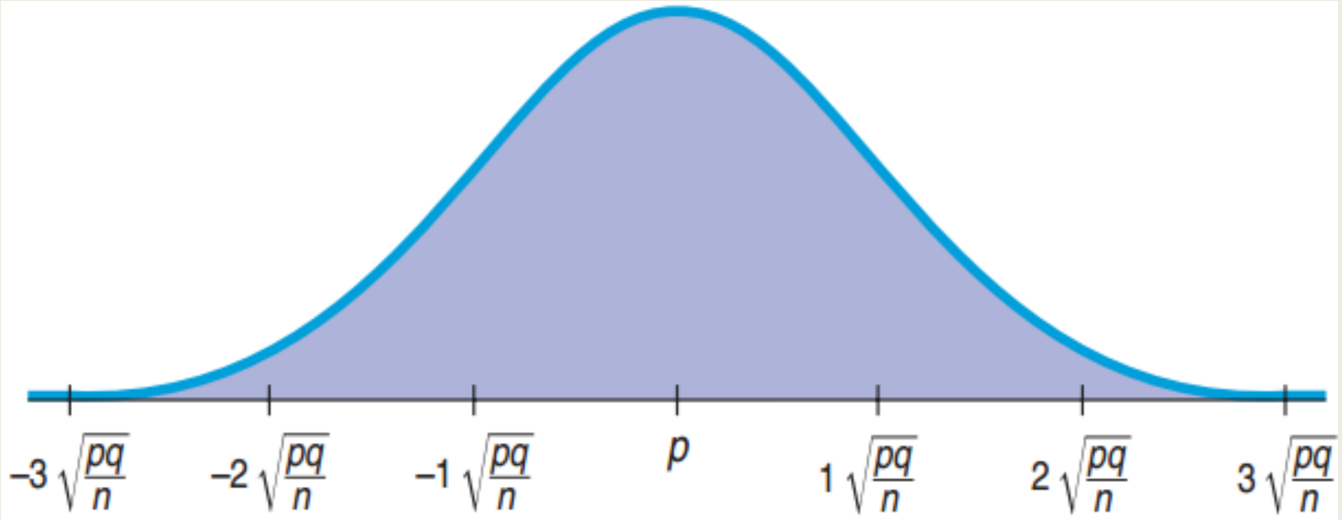
18.1

A Confidence Interval

Mean and Standard Deviation

Sampling Distribution for Proportions

- Mean $= p$
- $\sigma(\hat{p}) = \dfrac{\sqrt{npq}}{n} = \sqrt{\dfrac{pq}{n}}$
- $N\left(p, \sqrt{\dfrac{pq}{n}}\right)$



Standard Deviation for a Proportion?

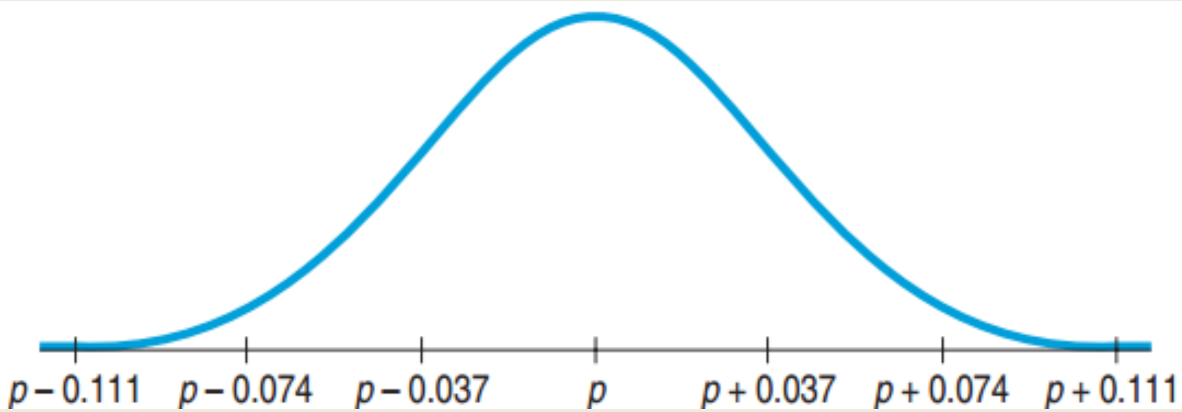
What is the sampling distribution?

- Usually we do not know the population proportion *p*.
- We cannot find the standard deviation of the sampling distribution: $\sqrt{\frac{pq}{n}}$
- After taking a sample, we only know the sample proportion, which we use as an approximation.
- The sample-to-sample standard deviation is called the **standard error** or **sampling variability**
- The standard error is given by $SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}}$

Facebook Daily Status Updates

A recent survey found that 48 of 156 or 30.8% update their Facebook status daily.

- $SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{(0.308)(0.692)}{156}} \approx 0.037$
- The sampling distribution is approximately normal.

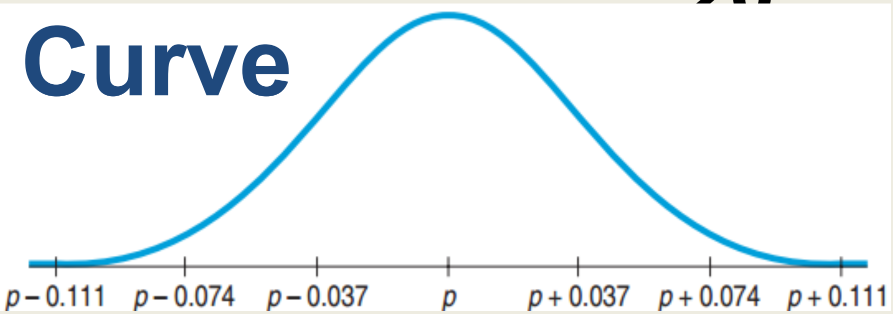


Facebook: true population proportion?

A recent survey found that 48 of 156 or 30.8% update their Facebook status daily.

- This is the sample proportion
- What is the true population proportion?
- To find it, we need to make an inference using the sampling distribution we just found, based on SE

Interpreting this Normal Curve



- By normality, about 95% of all possible samples of 156 young Facebook users will have $\hat{p}$'s within 2 SE (0.037) of p
- If $\hat{p}$ is close to p, then p is close to $\hat{p}$.
- If you stand at $\hat{p}$, then you can be 95% sure that p is within 2SE's from where you are standing.
- → Our confidence interval: (0.234, 0.382)

What You Can Say About p if You Know $\hat{p}$

We don't know exactly what percent of all Facebook users update their status daily, but the interval from 23.4% and 38.2% **probably** contains the **true** proportion.

- Note, we admit we are unsure about both the exact proportion and whether it is in the interval.

We are 95% confident that between 23.4% and 38.2% of all Facebook users update their status daily.

- Notice "*% confident*" and an *interval* rather than an exact value are stated.

What You Cannot Say About p if You Know $\hat{p}$

30.8% of all Facebook users update their status daily.

- We can't make such absolute statements about p.

It is probably true that 30.8% of all Facebook users update their status daily.

- We still cannot commit to a specific value for p, only a range.

We don't know exactly what percent of all Facebook users update their status daily, but we know it is within the interval 30.8% $\pm$ 2 x 3.7%.

- We cannot be *certain* it is in this interval.

Naming the Confidence Interval

This confidence interval is a **one-proportion *z*-interval**.

- "**One**" since there is a single survey question.
- "**Proportion**" since we are interested in the *proportion* of Facebook users who update their status daily.
- "***z*-interval**" since the distribution is approximately normal.

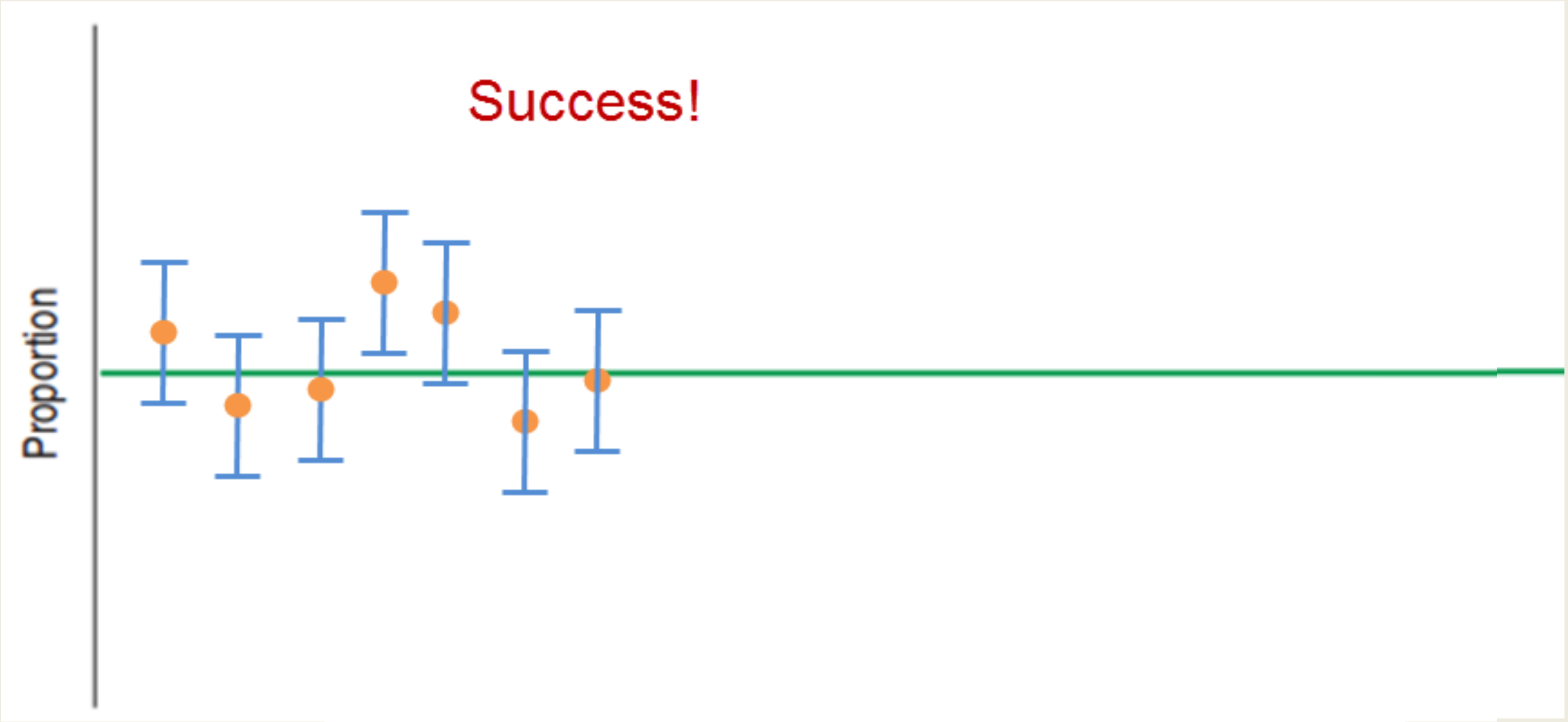
18.2

Interpreting Confidence Intervals:
What Does 95% Confidence Really Mean?

Capturing a Proportion

- The confidence interval may or may not contain the true population proportion.
- Consider repeating the study over an over again, each time with the same sample size.
  - Each time we would get a different $\hat{p}$.
  - From each $\hat{p}$, a different confidence interval could be computed.
  - About 95% of these confidence intervals will capture the true proportion.
  - 5% will not.

Simulating Confidence Intervals



Confidence Intervals

There are a huge number of confidence intervals that could be drawn.

- In theory, all the confidence intervals could be listed.
- 95% will “work” (capture the true proportion).
- 5% will not capture the true proportion.

What about our confidence interval (0.234, 0.382)?

- We will never know whether it captures the population proportion.

“Statistics Means Never Having to Say You Are Certain”

Facebook Status Updates

Technically Correct

- I am 95% confident that the interval from 23.4% to 38.2% captures the true proportion of Facebook users who update daily.

More Casual But Fine

- I am 95% confident that between 23.4% and 38.2% of Facebook users update daily.

18.3

Margin of Error: Certainty vs. Precision

Margin of Error

- Confidence interval for a population proportion (95%):

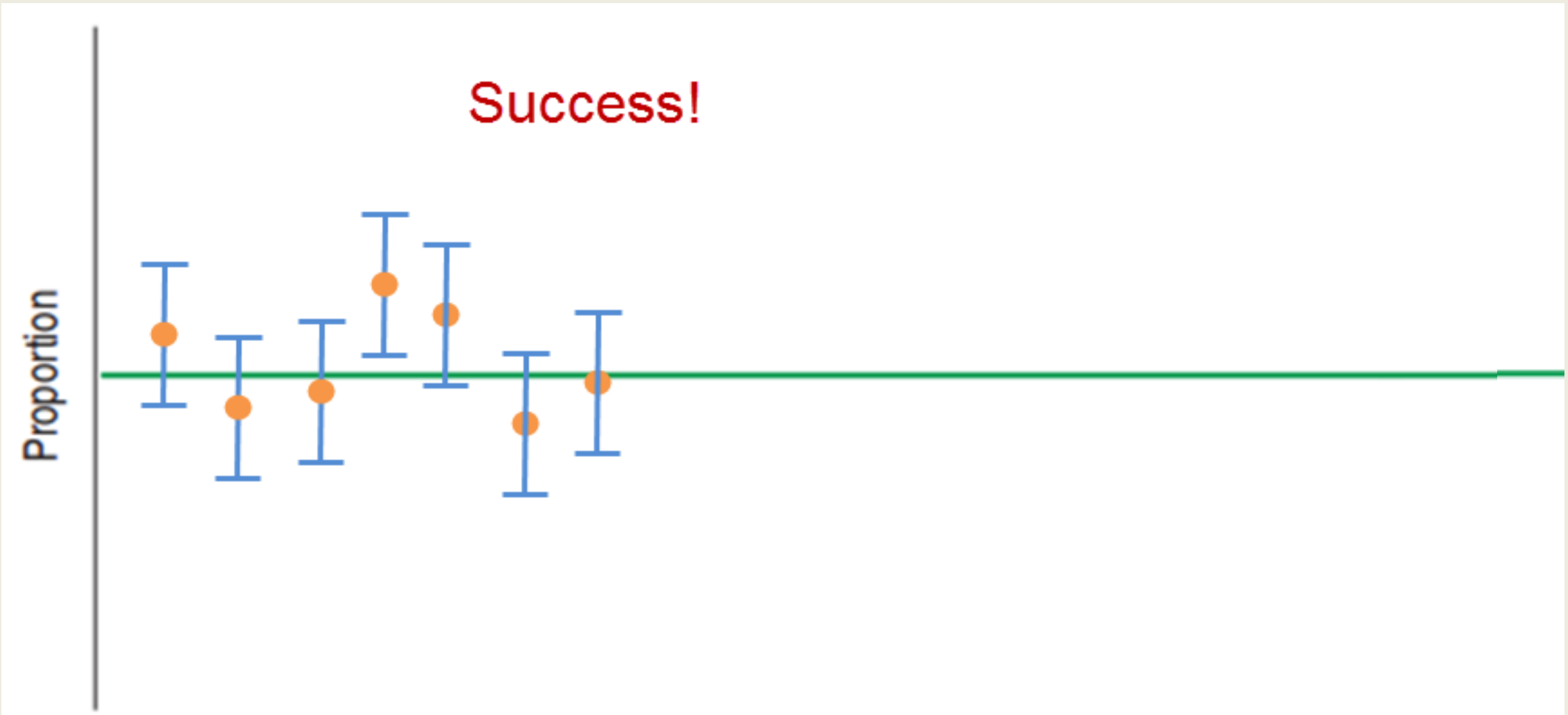
$$\hat{p} \pm 2SE(\hat{p})$$

- The distance, $2SE(\hat{p})$, from $\hat{p}$ is called the margin of error.

- Confidence intervals also work for means, regression slopes, and others. In general, the confidence interval has the form

$$Estimate \pm ME$$

Simulating Confidence Intervals



Certainty vs. Precision

- Instead of a 95% confidence interval, any percent can be used.
- Increasing the confidence (e.g. 99%) increases the margin of error.
- Decreasing the confidence (e.g. 90%) decreases the margin of error.

Confidence Interval on Global Warming

Yale and George Mason University interviewed 1010 US adults about beliefs and attitudes on global warming. They presented a 95% confidence interval on the proportion who think there is disagreement among scientists.

- Had the polling been done repeatedly, 95% of all random samples would yield confidence intervals that contain the true population proportion of all US adults who believe there is disagreement among scientists.

Yale/George Mason Study Revisited

The poll of 1010 adults reported a margin of error of 3%.
(by convention,95% with "worst case" ME based on p = 0.5)

- How was the 3% computed?

$$SE(\hat{p}) = \sqrt{\frac{(0.5)(0.5)}{1010}} \approx 0.0157$$

- For 95% confidence

$$ME = 2(0.0157) = 0.031$$

- The margin of error is close to 3%.

Critical Values

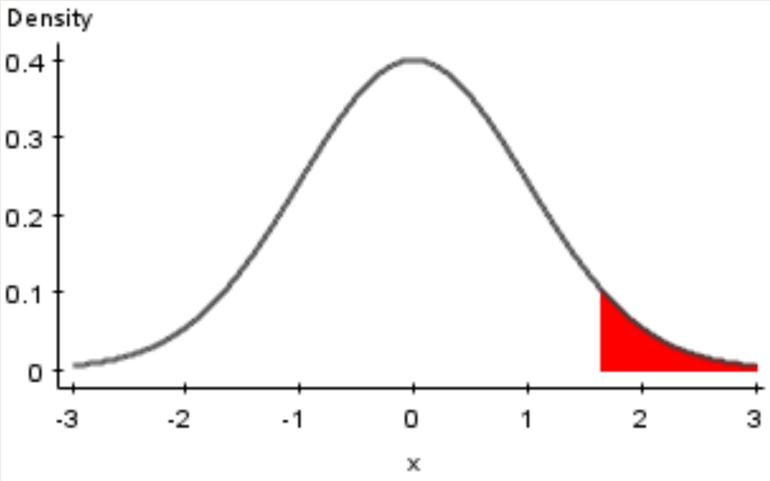
- For a 95% confidence interval, the margin of error was 2*SE*.
  - The 2 comes from the normal curve.
  - 95% of the area is within about 2*SE* from the mean.

- In general the *number* of *SE* is called the critical value. Since we use the normal distribution here we denote it z^*
- To be more precise, z^* for 95%CI is 1.96

Finding the Critical Value

Find the critical value corresponding to 90% confidence.

- 90% inside gives 10% outside.
- 2 tails outside with 10% means 1 tail with 5% or 0.05.



- The critical value is about

  $z^* = 1.645$.

Finding the Margin of Error (Take 2)

Yale/George Mason Poll: 1010 US adults, 40% think scientists disagree about global warming. At 95% confidence *ME* = 3%.

- Find the margin of error at 90% confidence.

Finding the Margin of Error (Take 2)

Yale/George Mason Poll: 1010 US adults, 40% think scientists disagree about global warming. At 95% confidence *ME* = 3%.

- Find the margin of error at 90% confidence.

$$SE(\hat{p}) = \sqrt{\frac{(0.4)(0.6)}{1010}} \approx 0.0154$$

- For 90%, $z^* \approx 1.645$: $ME = (1.645)(0.0154) = 0.025$.
- This gives a smaller margin of error which is *good*.
- **Drawback:** lower level of confidence which is *bad*

18.4

Assumptions and Conditions

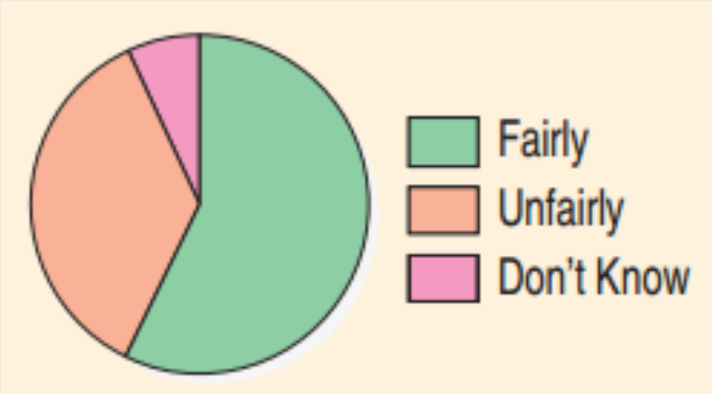
Independence and Sample Size

- Independence Condition
  - If data is collected using SRS or a randomized experiment → Randomization Condition
  - Some data values do not influence others.
  - Check for the 10% Condition: The sample size is less than 10% of the population size.
- Success/Failure Condition
  - There must be at least 10 successes.
  - There must be at least 10 failures.

One-Proportion z-Interval

- First check for randomization, independence, 10%, and conditions on sample size.
- Confidence level C, sample size *n*, proportion $\hat{p}$.
- Confidence interval: $\hat{p} \pm z^* SE\left(\hat{p}\right)$
- $SE\left(\hat{p}\right) = \sqrt{\frac{\left(\hat{p}\right)\left(\hat{q}\right)}{n}}$
- z*: the critical value that specifies the number of *SE*'s needed for C% of random samples to yield confidence intervals that capture the population proportion.

Do You Believe the Death Penalty is Applied Fairly?



- Sample size: 510

- Answers:
  - 58% “Fairly”
  - 36% “Unfairly”
  - 7% “Don’t Know”

- Construct a confidence interval for the population proportion that would reply “Fairly.”

Do You Believe the Death Penalty is Applied Fairly?

- **Plan:** Find a 95% confidence interval for the population proportion.
- **Model:**
  - ✓ Randomization: Randomly selected by Gallup Poll
  - ✓ 10% Condition: Population is all Americans
  - ✓ Success/Failure Condition
    - ✓ $(510)(0.58) = 296 \geq 10$, $(510)(0.42) = 214 \geq 10$
  - Use the Normal Model to find a one-proportion z-interval.

Do You Believe the Death Penalty is Applied Fairly?

- **Mechanics:** $n = 510, \ \hat{p} = 0.58$
-
-
-
-

Do You Believe the Death Penalty is Applied Fairly?

- **Mechanics:** $n = 510$, $\hat{p} = 0.58$
- $SE(\hat{p}) = \sqrt{\dfrac{(0.58)(0.42)}{510}} \approx 0.022$
- $z^* \approx 1.96$
- $ME \approx (1.96)(0.022) \approx 0.043$
- The 95% Confidence Interval is:
  0.58 ± 0.043 or $(0.537, 0.623)$

Do You Believe the Death Penalty is Applied Fairly?

- **Conclusion:** I am 95% confident that between 57.3% and 62.3% of all US adults think that the death penalty is applied fairly.

What Sample Size?

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

- For example, to ensure a $ME < 3\%$:
- For 95%, $z^* = 1.96$
-
-

What Sample Size?

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

- For example, to ensure a *ME* < 3%:
- For 95%, $z^* = 1.96$
- Values that make *ME* largest are $\hat{p} = 0.5,\ \hat{q} = 0.5$
- $$0.03 = 1.96\sqrt{\frac{(0.5)(0.5)}{n}}$$
- Solving for n, gives $n \approx 1067.1$.
- We need to survey at least 1068 to ensure a *ME* less than 0.03 for the 95% confidence interval.

The Yale/George Mason Survey and Sample Size

Poll: 40% believe scientists disagree on global warming.

- For a **follow-up survey**, what sample size is needed to obtain a 95% confidence interval with $ME \leq 2\%$?

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

-

The Yale/George Mason Survey and Sample Size

Poll: 40% believe scientists disagree on global warming.

- For a **follow-up survey**, what sample size is needed to obtain a 95% confidence interval with $ME \leq 2\%$?

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} \qquad 0.02 = 1.96\sqrt{\frac{(0.4)(0.6)}{n}}$$

- $n \approx 2304.96$

- The group will need at least 2305 respondents.

Thoughts on Sample Size and *ME*

- Obtaining a large sample size can be expensive and/or take a long time.
- For a pilot study, $ME = 10\%$ can be acceptable.
- For full studies, $ME \leq 5\%$ is better.
- Public opinion polls typically use $ME = 3\%$, $n = 1000$.
- If p is expected to be very small such as 0.005, then much smaller ME such as 0.1% is required.

Credit Cards and Sample Size

A pilot study showed that 0.5% of credit card offers in the mail end up with the person signing up.

- To be within 0.1% of the true rate with 95% confidence, how big does the test mailing have to be?

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Credit Cards and Sample Size

A pilot study showed that 0.5% of credit card offers in the mail end up with the person signing up.

- To be within 0.1% of the true rate with 95% confidence, how big does the test mailing have to be?

$$ME = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} \qquad 0.001 = 1.96\sqrt{\frac{(0.005)(0.995)}{n}}$$

- $n \approx 19{,}111.96$
- The test mailing should include at least 19,112 offers.

What Can Go Wrong?

Don't claim other samples will agree with yours.

- **Wrong:** In 95% of samples, between 43% and 51% agree with decriminalization of marijuana.

Don't be certain about the parameter.

- **Wrong:** Between 23% and 38% of Facebook users update daily. Don't forget to include the confidence.

Don't forget that it's about the parameter (not the statistics)

- **Wrong:** I'm 95% confident that $\hat{p}$ is between 23% and 38%. You know for sure exactly what $\hat{p}$ is.

What Can Go Wrong?

Don't claim to know too much.

- **Wrong:** I'm 95% confident that between 23% and 38% of all Facebook users in the world update daily. The survey was just about US residents between 18 and 22.

Do take responsibility.

- Accept that you are only 95% confident, not sure.

Don't suggest that the parameter varies… it does not!

- **Wrong:** There is a 95% chance that the true parameter is between 23% and 38%.

What Can Go Wrong?

Do treat the whole interval equally.

- The middle of the interval is not necessarily more plausible than the edges.

Beware of margins of error that are too large to be useful.

- Between 10% and 90% update daily is not useful. Use a larger sample size to shrink the *ME*.

Watch out for biased sampling.

- Biased samples produce an unreliable CIs.

Think about independence

- Be careful in your sample design to ensure randomization.

Chapter 19

Testing Hypotheses About Proportions

19.1

Hypotheses

Cracking Rate < 20%?

General cracking rate: 20%
After a new engineering process,
the cracking rate of 400 casts fell to 17%!

Is this due to the new engineering or just random chance?

- **Null Hypothesis (H_0)**: Nothing has changed
  - H_0: proportion = hypothesized value
  - H_0: $p = 0.20$
- Alternative Hypothesis:
  - H_A: $p < 0.20$

How Small to Convince Us?

- Had the new cracking rate been 1%, it would clearly indicate a change from 20%.
  - Extremely unlikely that this could happen just by random chance
- Had the new cracking rate been 19.8%, we would be skeptical.
  - Not so unlikely to be just random chance
- How about 17%?
  - **How likely is it that a random sample would have a cracking rate 17% or less?**

Checking Conditions and Finding the Standard Error

Checking Conditions: $n = 400$, $p = 0.20$

✓ $np = (400)(0.20) = 80 \geq 10$
$nq = (400)(0.80) = 320 \geq 10$

✓ Independence plausible

✓ The Normal model applies.

Find the standard deviation of the **model**.

- $$SD(\hat{p}) = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(0.20)(0.80)}{400}} = 0.02$$
- **Note:** Use p and not $\hat{p}$ to find standard deviation.

Using the Normal Model

$p = 0.20,\ \hat{p} = 0.17,\ SD(\hat{p}) = 0.02$

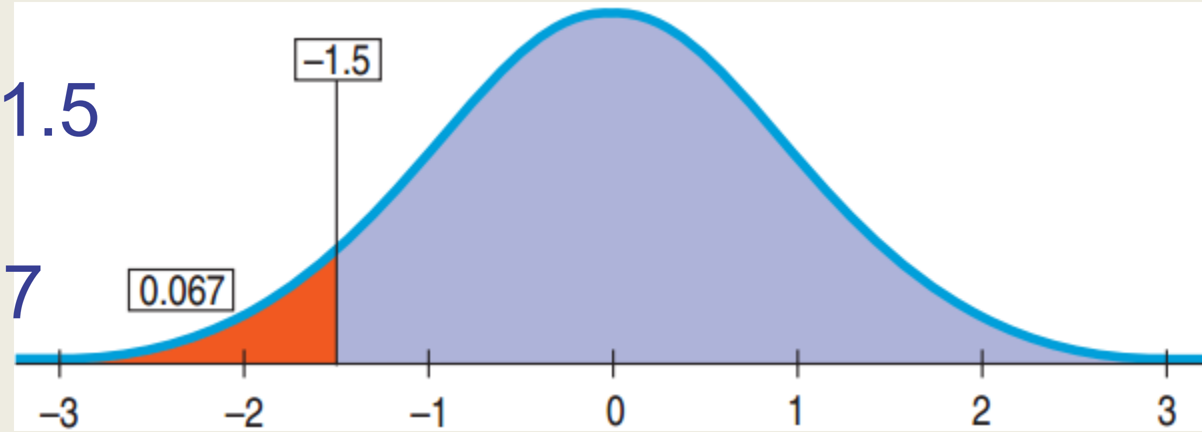
-

-

Using the Normal Model

$p = 0.20,\ \hat{p} = 0.17,\ SD(\hat{p}) = 0.02$

- $z = \dfrac{0.17 - 0.20}{0.02} = -1.5$
- $P(z < -1.5) \approx 0.067$



- If the null hypothesis is true that the cracking rate is still equal to 20%, then the probability of observing a cracking rate of 17% in a random sample of 400 is 6.7%.

Innocent until Proven Guilty

- Begin with the presumption of innocence (H_0).
- Collect evidence.
  - Bank money in house
  - Still wearing mask
  - Getaway car found in his name
- Evidence beyond a reasonable doubt?
  - Is 5% small enough chance?
  - How about 1%?
  - 6.7%?

19.2

P-Values

The P-Value and Surprise

The P-value is the probability of seeing data like these (or even more unlikely-extreme data) given the null hypothesis is true.

- Tells us how **surprised** we would be to get these data given H_0 is true.
- **P-value small:** Either H_0 is not true or something remarkable occurred
- **P-value not small enough:** Not a surprise. Data consistent with the model. Do not reject H_0.

Guilty or Not Enough Evidence?

Defendant is either

- **Guilty:** P-value too small. The evidence is clear.
- **Not Guilty:** P-value not small enough. The evidence is not sufficient. Not the same as innocent. Maybe innocent or maybe guilty, but not enough evidence found.

Two Choices

- **Fail to reject H_0 if P-value large.** Never accept H_0.
- **Reject H_0 if P-value is small.** Accept H_A.

When the P-Value is Not Small

Wrong

- Accept H_0.
- We have proven H_0.

Right

- Fail to reject H_0
- There is insufficient evidence to reject H_0.
- H_0 may or may not be true.

Example: H_0: All swans are white.

- If we sample 100 swans that are all white, there could still be a black swan.

19.3

The Reasoning of Hypothesis Testing

Step 1: State the Hypotheses

H_0:

- H_0 usually states that there's nothing different.
- H_0: parameter = hypothesized value
- Note the parameter describes the population not the sample.
- H_0 is called the null hypothesis.

H_A:

- H_A is a statement that something has changed, gotten bigger or smaller
- H_A is called the alterative hypothesis.

Hypotheses About the DMV

The DMV claims **80% of all drivers** pass the driving test.
In a survey of 90 teens, only 61 passed.

- Is there evidence that **teen pass rates** are below 80%?
  - H_0: $p = 0.80$
  - H_A: $p < 0.80$

Step 2: Model

Decide on the model to test the null hypothesis and parameter.

- Check conditions, e.g. independence, sample size.
- If the conditions are not met, either quit or redesign the study.
- Normal models use z-scores. Other models may not use z-scores.
- Name the model, e.g. 1-proportion z-test.

1-Proportion z-Test

Conditions

- Same as a 1-Proportion *z*-Interval

Null Hypothesis

- H_0: $p = p_0$

Test Statistics

- $$z = \frac{\hat{p} - p_0}{SD(\hat{p})} \qquad SD(\hat{p}) = \sqrt{\frac{p_0 q_0}{n}}$$

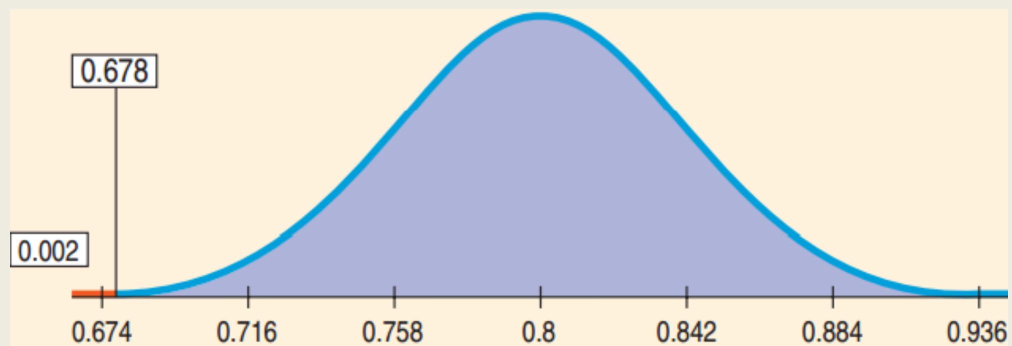
DMV Study: Checking Conditions

- **Randomization Condition:** The 90 teens were a random sample of all teens.
- **10% Condition:** 90 is fewer than10% of the total number of all teens who take the driving test.
- **Success/Failure Condition:**
  - $np_0 = (90)(0.80) = 72 \geq 10$
  - $nq_0 = (90)(0.20) = 12 \geq 10$
- The conditions are satisfied. We can use the Normal model and perform a 1-Proportion *z*-Test.

DMV Study: Mechanics

Claim: 80% pass. 61 of 90 teens tested passed.

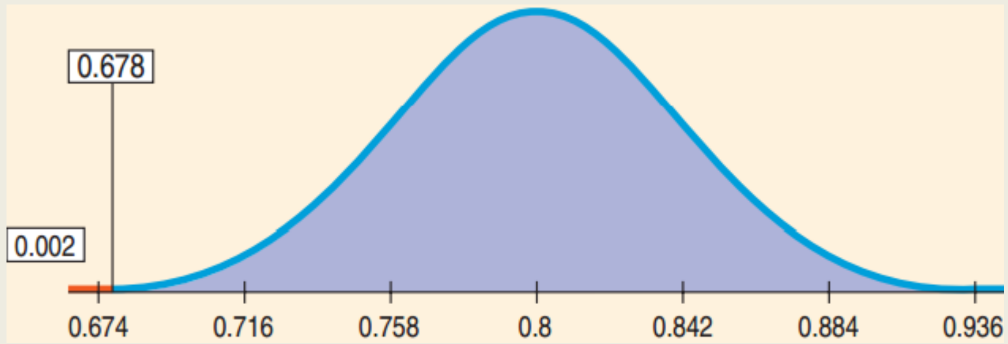
- Find P-value.
    - $n = 90$, $x = 61$, $p_0 = 0.80$, $\hat{p} = \dfrac{61}{90} \approx 0.678$
    -
    -
    -



DMV Study: Mechanics

Claim: 80% pass. 61 of 90 teens tested passed.

- Find P-value.
  - $n = 90$, $x = 61$, $p_0 = 0.80$, $\hat{p} = \dfrac{61}{90} \approx 0.678$
  - $SD(\hat{p}) = \sqrt{\dfrac{p_0 q_0}{n}} \approx \sqrt{\dfrac{(0.80)(0.20)}{90}} \approx 0.042$
  - $z = \dfrac{0.678 - 0.80}{0.042} \approx -2.90$



  - $\text{P-value} = P(z < -2.90) \approx 0.002$

DMV Study: Conclusion

Is the teen pass rate less than 80%? P-value = 0.002

- What can be concluded? What does the P-value mean?
  - P-value = 0.002 is very small → Reject H_0
  - The survey data provide strong evidence that the pass rate for teens is less than 80%.
  - This should not be the end of the conversation.
  - The next step would be to see if the pass rate is low enough to take further action.

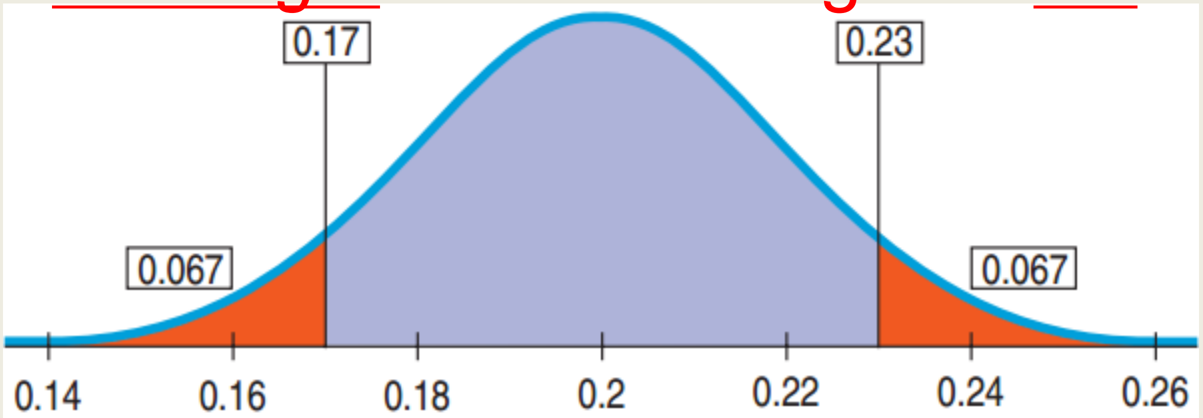
19.4

Alternative Alternatives

Two-Sided Alternative

For the new process the engineer may be interested in whether there has been a *change* in the cracking rate not just a decrease.

- H_0: $p = 0.20$
- H_A: $p \neq 0.20$

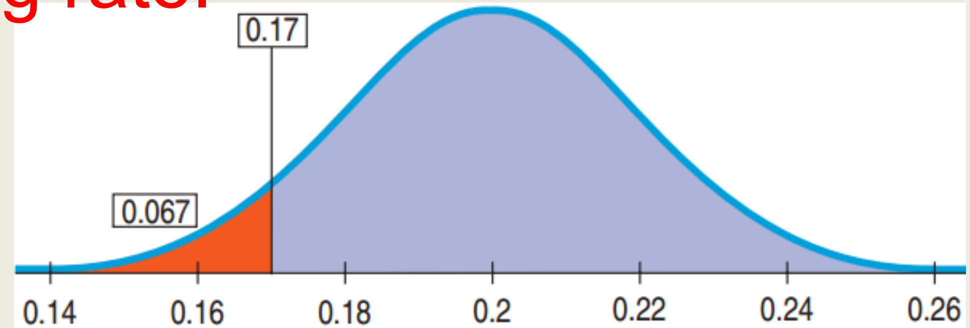


- An alternative hypothesis where we are interested in deviation on either side is called a two-sided alternative.
- The P-value is the probability of deviating from *either* direction from the null hypothesis.

One-Sided Alternative

The engineer may be interested in whether there has been a decrease in the cracking rate.

- H_0: $p = 0.20$
- H_A: $p < 0.20$



- An alternative hypothesis where we are interested in deviation on only one side is called a one-sided alternative.
- The P-value for a one-sided alternative is always half the P-value for the two-sided alternative.

Not the Right Proportion of Male Babies?

Under natural conditions, 51.7% of births are male. In Punjab India's hospital 56.9% of the 550 births were male.

- **Question:** Is there evidence that the proportion of male births is <u>different</u> for this hospital?
- Plan: We will have a <u>two-tailed alternative</u>. The parameter of interest is *p*.
  - H_0: $p = 0.517$
  - H_A: $p \neq 0.517$

Not the Right Proportion of Male Babies?

- **Model:** Check the conditions
  - ✓ **Independence Assumption:** The sex of one baby should not affect the sex of others.
  - ✓ **Randomization Conditions:** The births were not random, so be cautious of the results.
  - ✓ **10% Condition:** 550 births is certainly less than the total number of all births.

Not the Right Proportion of Male Babies?

- **Model:** Check the conditions (Continued).
  - ✓ **Success/Fail Condition:** $(550)(0.517) \geq 10$, $(550)(0.483) \geq 10$
- The Normal model does apply.
- We can use a one-proportion z-test.

Not the Right Proportion of Male Babies?

- **Mechanics:**

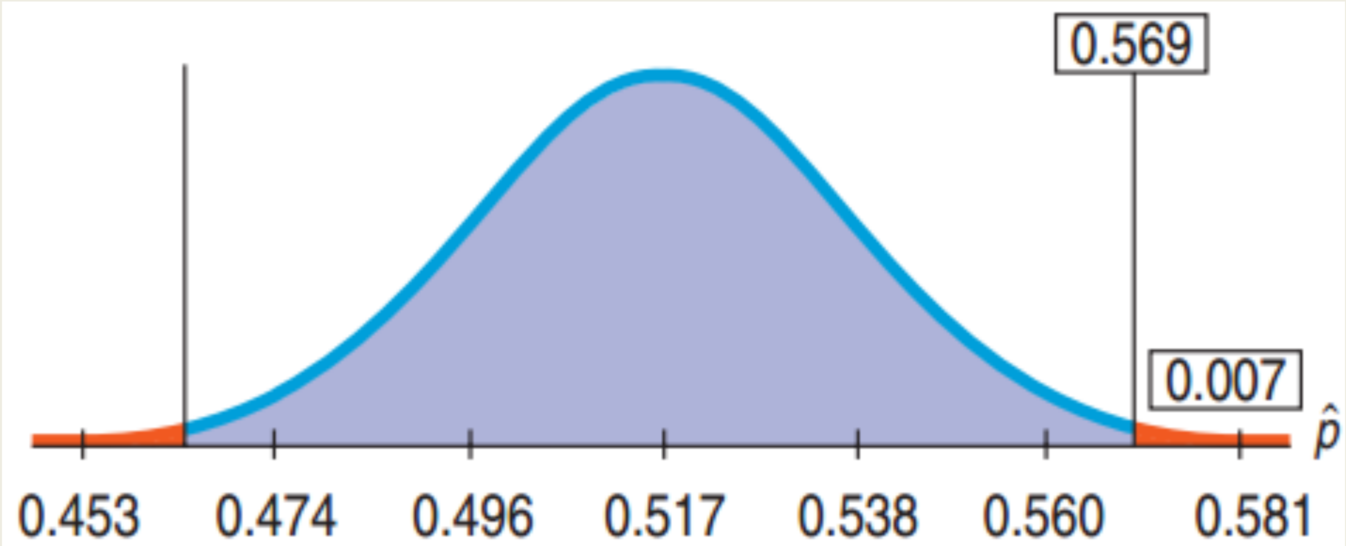
$$SD(\hat{p}) = \sqrt{\frac{p_0 q_0}{n}} \approx \sqrt{\frac{(0.517)(1-0.517)}{550}} \approx 0.0213$$

$$z = \frac{\hat{p} - p_0}{SD(\hat{p})} = \frac{0.569 - 0.517}{0.0213} \approx 2.44$$

Not the Right Proportion of Male Babies?

Show →

- **Mechanics:**



P-value = $2P(z > 2.44) = 2(0.0073) = 0.0146$

Not the Right Proportion of Male Babies?

- **Conclusion:**
  - Interpreting the P-value = 0.00146
    - If the proportion of male babies were still 51.7%, then an observed proportion as different as 56.9% male babies would occur <u>at random</u> about 15 times in 1000.
    - This is so small a chance that I reject H_0. There is strong evidence that the proportion of boys is not equal to the baseline for the region. It appears larger.

19.5

P-Values and Decisions: What to Tell About a Hypothesis Test

How Small a P-Value is Small Enough?

How small is small enough is context specific.

- Test to see if a renowned musicologist can distinguish between Mozart and Hayden.
  - P-value of 0.1 may be good enough. Just reaffirming known talent.

- A friend claims psychic abilities and can predict heads or tails.
  - Very small P-value such as 0.01 needed. Breaking scientific theory.

Acceptable P-Value Depends on Result's Importance

Proportion of students in school with full time jobs has increased.

- Not that important. P-value = 0.05 will work.

Testing the proportion of faulty rivets that hold together the wings of a commercial aircraft is now below the danger threshold.

- Life and death decision. Need a very small P-value

- Whether rejecting or failing to reject, always cite the P-value.
- An accompanying confidence interval helps also.

Home Field Advantage?

Is there a home field advantage in baseball?

- The home team won 1308 of the 2431 (53.81%) games played in the season.
- Is there evidence to suggest that the home team wins more than 50%?

- Plan: p = proportion of home team wins
- Hypotheses
  - H_0: $p = 0.50$
  - H_A: $p > 0.50$

Home Field Advantage?

Model →

- ✓ **Independence Assumption:** Questionable, but the 2013 season may be representative of all games past and future.
- ✓ **10% Condition:** The 2013 season is less than 10% of all games played past and future.
- ✓ **Success/Failure Condition:**
  - $np = (2431)(0.5) \geq 10$
  - $nq = (2431)(0.5) \geq 10$
- **The Normal Model Applies:** Conduct a one-proportion z-test.

Home Field Advantage?

- **Mechanics:**

$$SD(\hat{p}) = \sqrt{\frac{p_0 q_0}{n}} \approx \sqrt{\frac{(0.5)(1-0.5)}{2429}} \approx 0.010141$$

$$z = \frac{\hat{p} - p_0}{SD(\hat{p})} = \frac{0.53805 - 0.5}{0.010141} \approx 3.75$$

- The P-value is about 0.0001.

Home Field Advantage?

- **Conclusion:** The P-value of 0.0001 says that if the true proportion of home teams wins is 0.5 then an observed value of 0.53805 would occur less than 1 time in 1000.
- This is so unlikely, so reject H_0.
- There is reasonable evidence that the true proportion of home team wins is greater than 50%.
- There appears to be a home field advantage.

How Big a Home Field Advantage?

- **Model:**

  ✓ Success Failure Condition (notice difference!)
  - Home team wins: 1308 $\geq$ 10
  - Home team losses: 1123 $\geq$ 10

- Sampling distribution follows the Normal model.

- Find the one-proportion *z*-interval.

How Big a Home Field Advantage?

- **Mechanics (notice difference!):**

$$SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}} \approx \sqrt{\frac{(0.53805)(1-0.53805)}{2431}} \approx 0.01011$$

- For 95% confidence, z^* = 1.96.
- $ME = z^* \times SE(\hat{p}) = 1.96 \times 0.01011 = 0.0198$

- A 95% confidence interval is:
  0.53805 ± 0.0198 or (0.5182, 0.5579).

How Big a Home Field Advantage?

Tell →

- **Conclusion:** I am 95% confident that, in professional baseball, home teams win between 51.82% and 55.79% of the games.

- For a 162-game season, the low end gives the home team about 1/2 of an extra victory and the high end, about 4 extra wins.

What Can Go Wrong?

Don't base your H_0 on what you see in the data.

- Changing the null hypothesis after looking at the data is just wrong.

Don't base your H_A on the data.

- Both the null and alternative hypotheses must be stated before peeking at the data.

Don't make H_0 what you want to show to be true.

- H_0 represents the status quo. You can never accept the null hypothesis.

What Can Go Wrong?

Don't forget to check the conditions.

- Randomization, independence, and sample size

Don't accept the null hypothesis.

- You can only say you don't have evidence to reject H_0 (you can only say you "fail to reject").

If you fail to reject H_0 don't expect a larger sample would reject H_0.

- Check the confidence interval. If its values would not matter to you, then a larger sample will unlikely be worthwhile.