Quantitative Methods

Serena DeStefani – Lecture 2 - 7/7/2020

Announcements

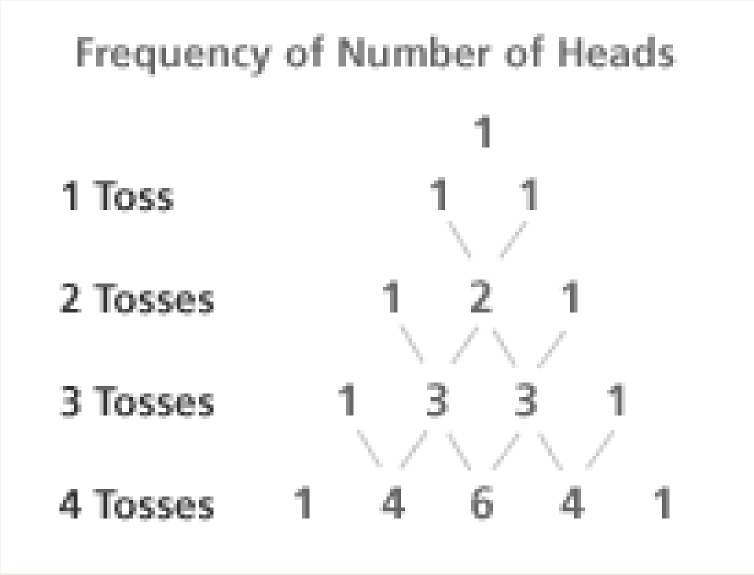
- First HW due tomorrow morning before class:
  - Upload answers on Sakai
  - Email me a screenshot of your work

- Please join Datacamp
- Tutor available

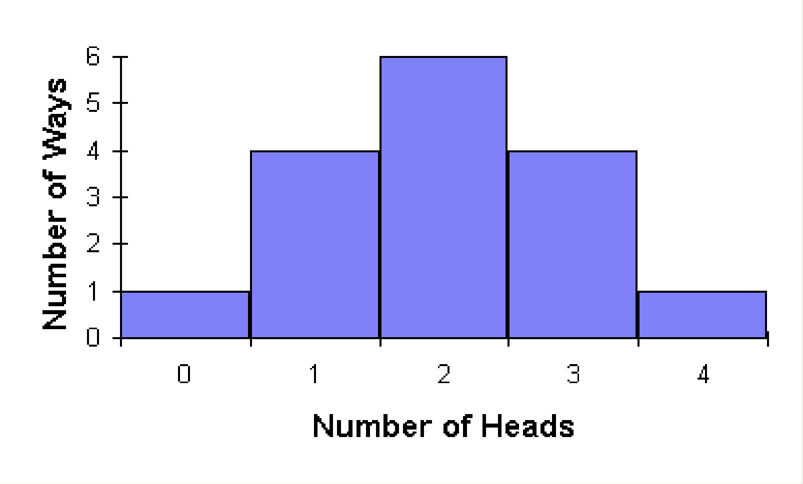
Review

<u>From the binomial expansion to the normal distribution</u>

We can plot the frequency of getting heads on an histogram



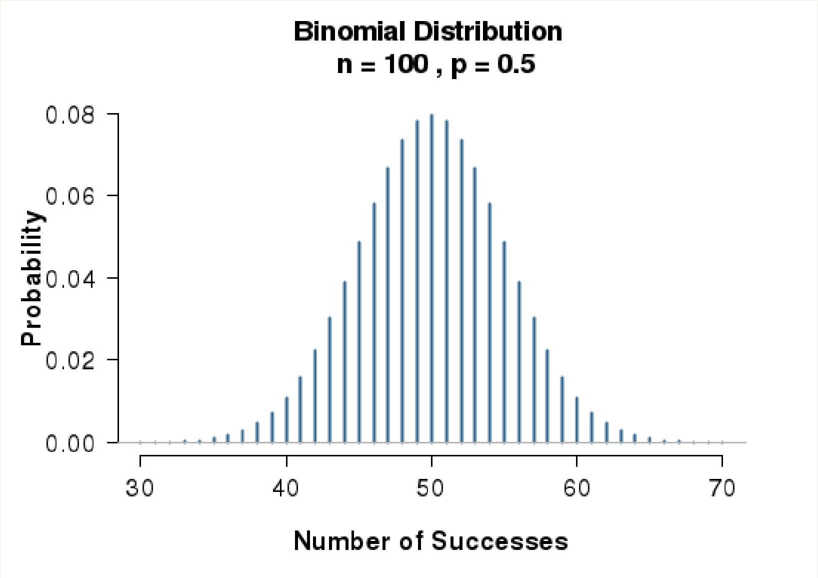
$$(\tfrac{1}{2}+\tfrac{1}{2})^4 = \frac{1}{16}+\frac{4}{16}+\frac{6}{16}+\frac{4}{16}+\frac{1}{16}$$



Review

<u>From the binomial expansion to the normal distribution</u>

The more coin tosses I make, the more this histogram will resemble a curve:



See simulation at:
https://shiny.rit.albany.edu/stat/binomial/

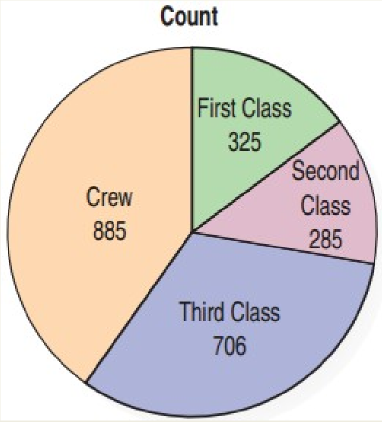
Review

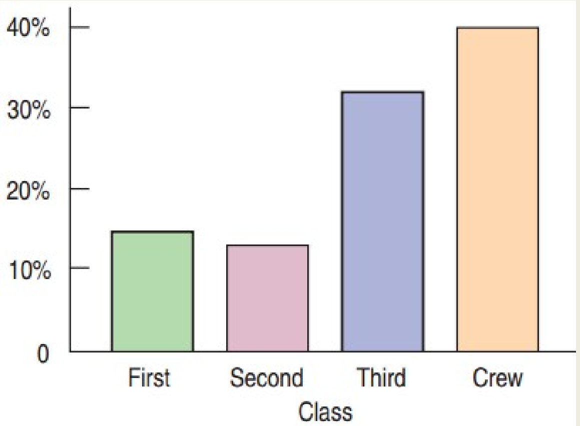
- How to display and describe categorical data

Class	Count
First	325
Second	285
Third	706
Crew	885

Class	%
First	14.77
Second	12.95
Third	32.08
Crew	40.21

- **Frequency tables**
- Bar Charts





- Pie Charts

Review

		Class				
		First	Second	Third	Crew	Total
Survival	Alive	203	118	178	212	**711**
	Dead	122	167	528	673	**1490**
	Total	**325**	**285**	**706**	**885**	**2201**

- How to compare categorical data:
- **Contingency tables**
- Marginal distribution

			Class				
			First	Second	Third	Crew	Total
Survival	Alive	Count	203	118	178	212	**711**
		% of Column	62.5%	41.4%	25.2%	24.0%	**32.3%**
	Dead	Count	122	167	528	673	**1490**
		% of Column	37.5%	58.6%	74.8%	76.0%	**67.7%**
	Total	Count	**325** **100%**	**285** **100%**	**706** **100%**	**885** **100%**	**2201** **100%**

- Conditional distribution:
  - percent of one variable satisfying the conditions of another
  - can be organized by column or by row

Review

- Independence: The distribution of one variable is the same for all categories of another.
- There is no association between the two.

- An association that holds for all of several groups can reverse direction when the data are combined to form a single group. This reversal is called Simpson's paradox.

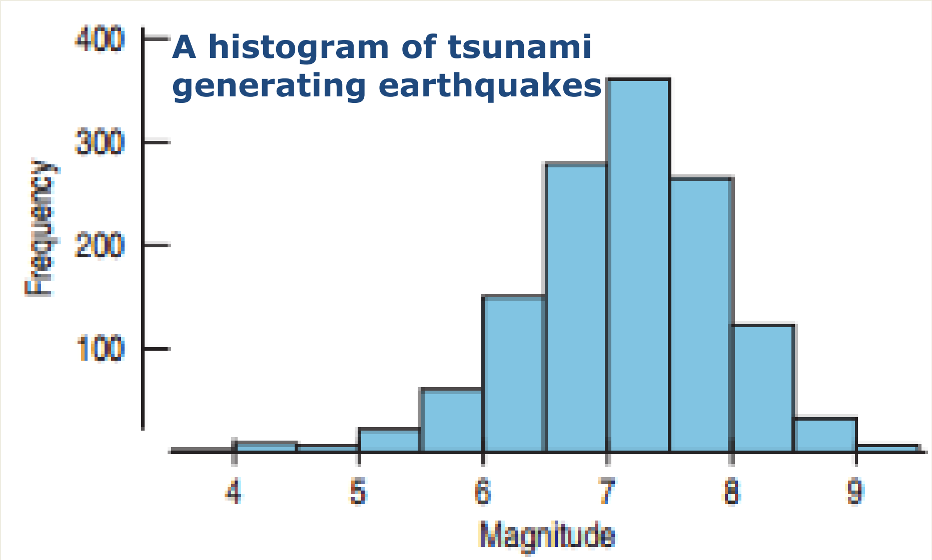
Chapter 3

Displaying and Summarizing **Quantitative** Data

- HISTOGRAMS -

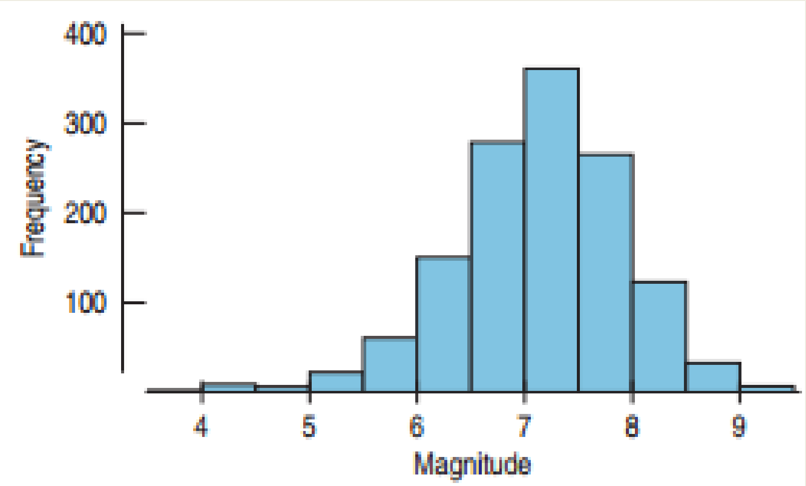
- Histogram: A chart that displays quantitative data

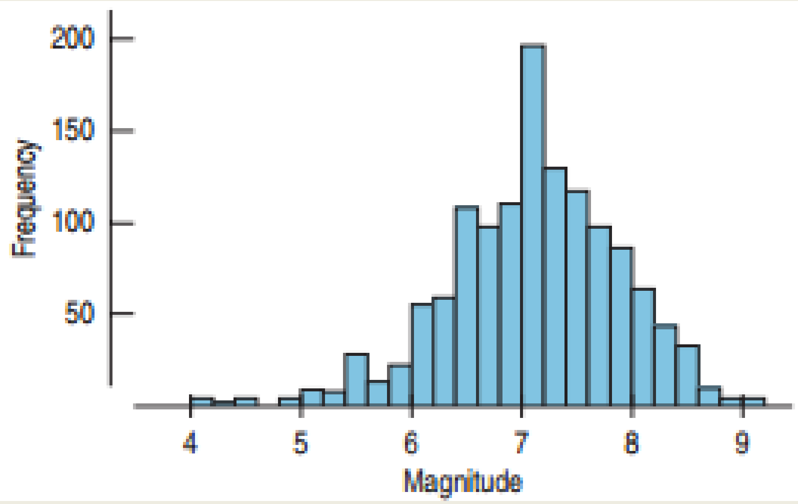
- Great for seeing the distribution of the data



Choosing the Bin Width

- Different bin widths tell different stories.

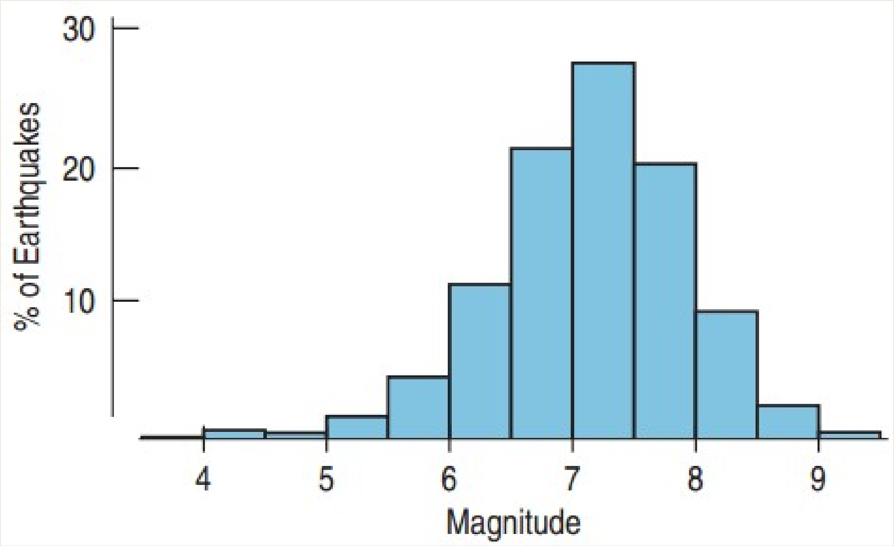




Relative Frequency Histograms

- Relative Frequency Histogram

- The vertical axis represents the relative frequency, the frequency divided by the total.



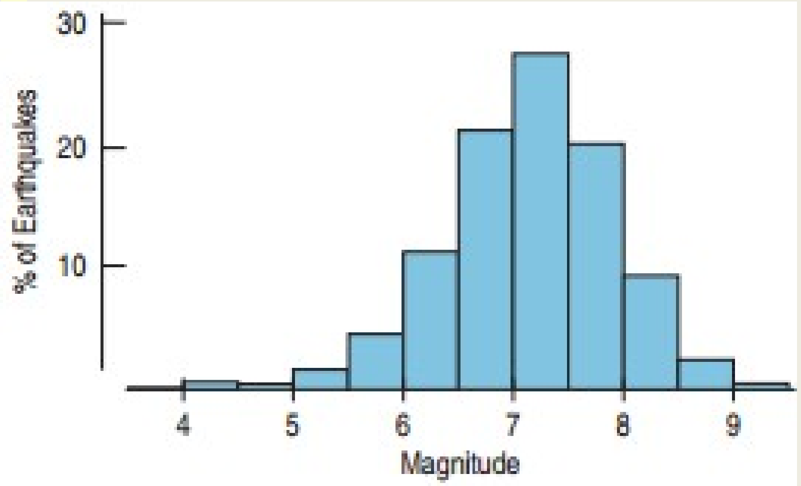
Think Before you Draw

- Is the variable quantitative? Is the **answer** to the survey question or result of the experiment **a number whose units are known**?

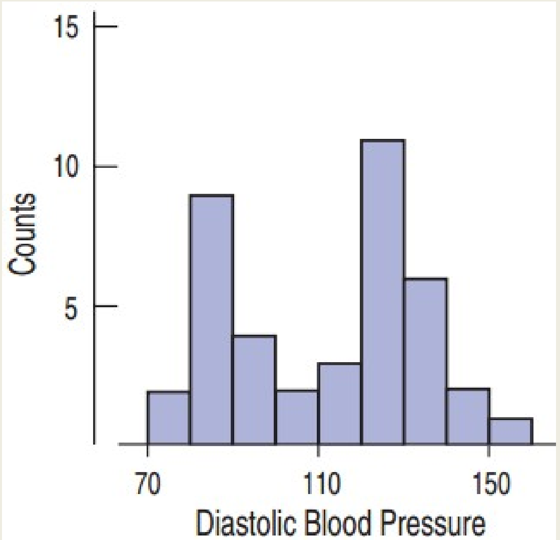
Shape: Modes

- A Mode of a histogram is a hump or high-frequency bin.

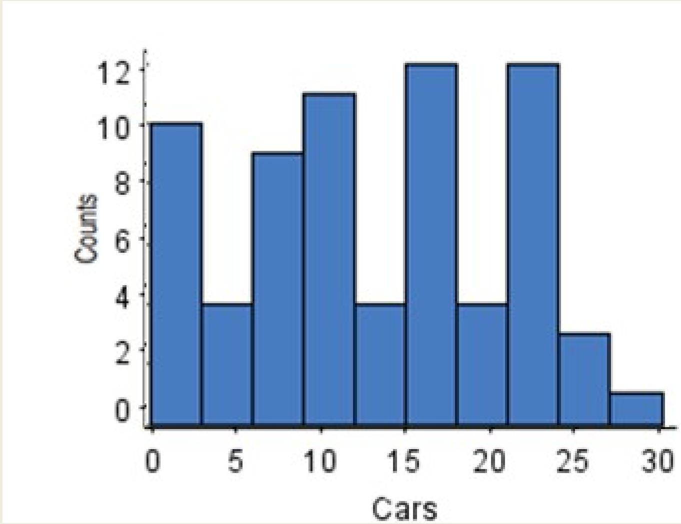
Unimodal



Bimodal

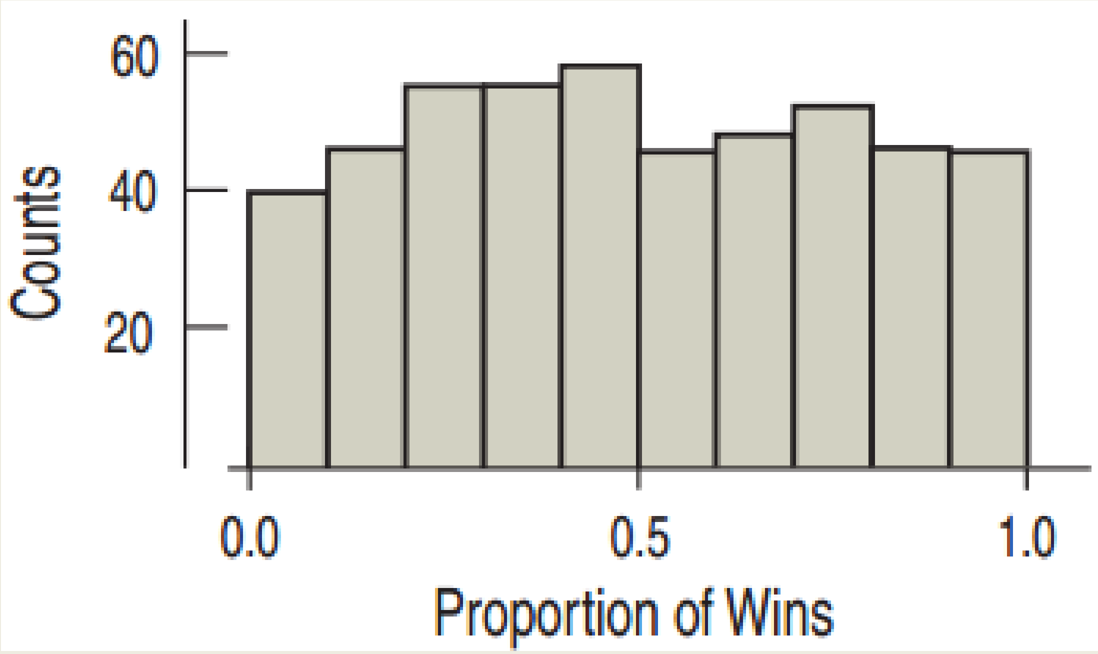


Multimodal



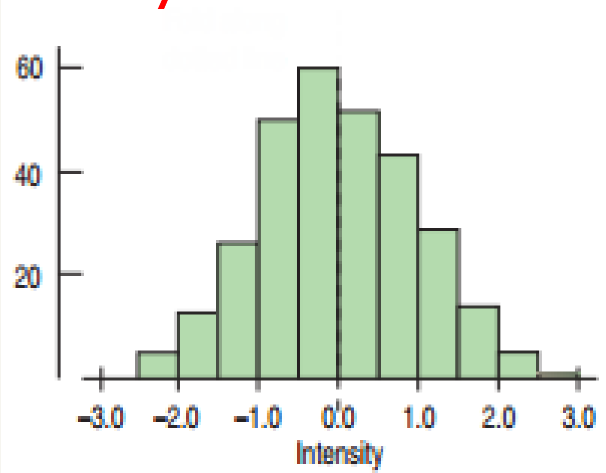
Uniform Distributions

- Uniform Distribution

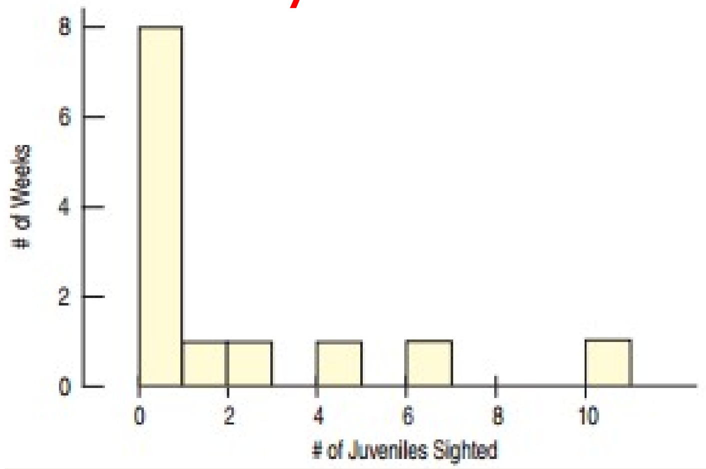


Shape: Symmetry

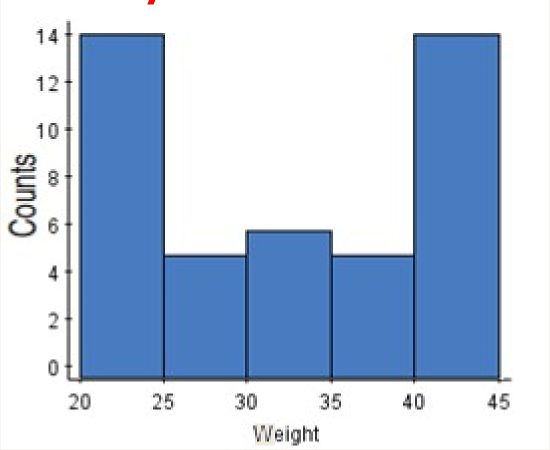
Symmetric



Not Symmetric

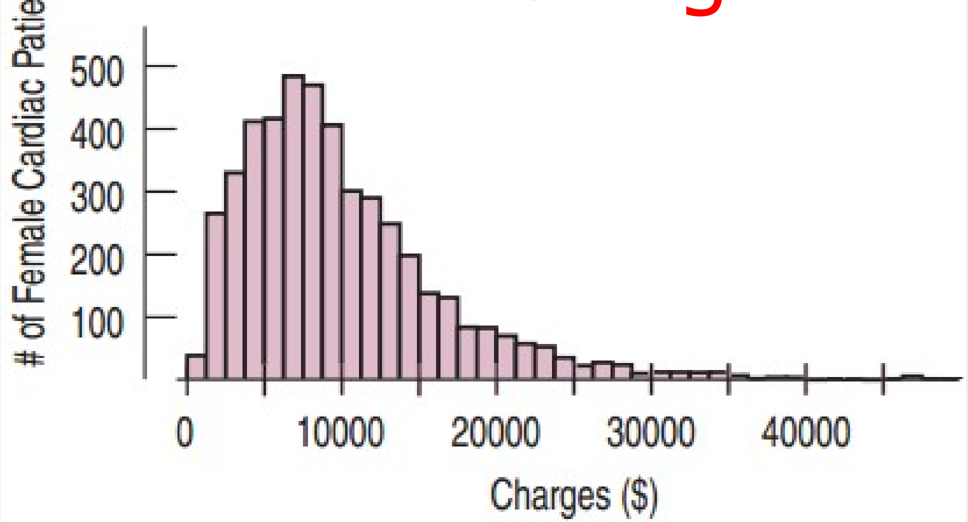


Symmetric

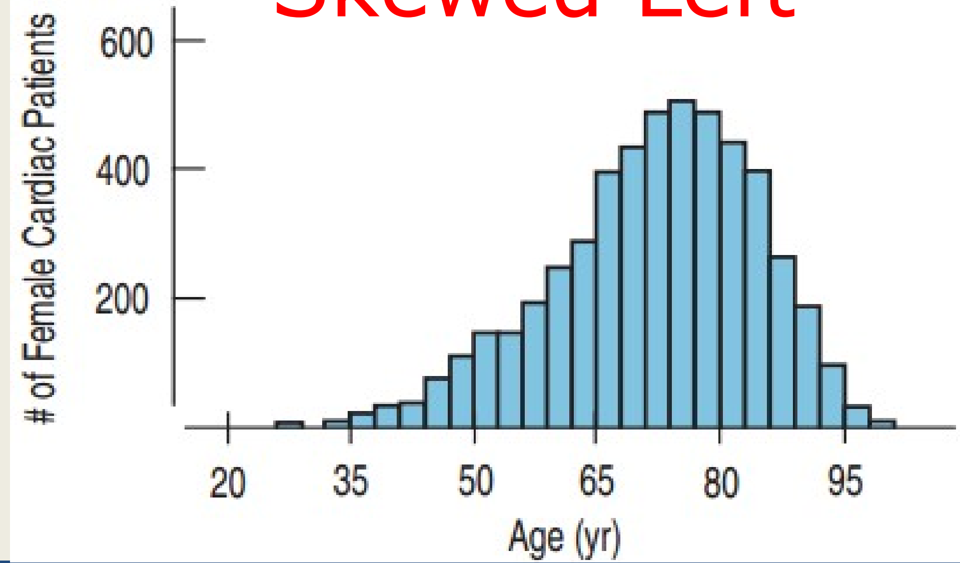


Shape: Skew

Skewed Right

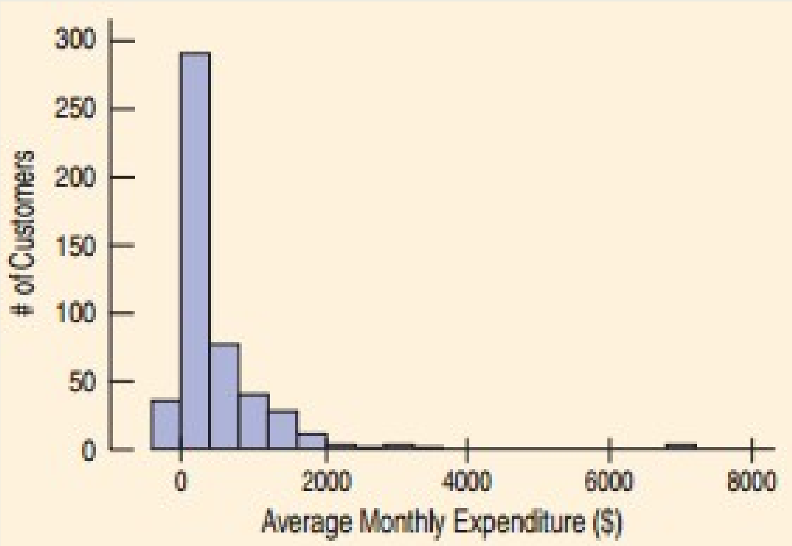


Skewed Left



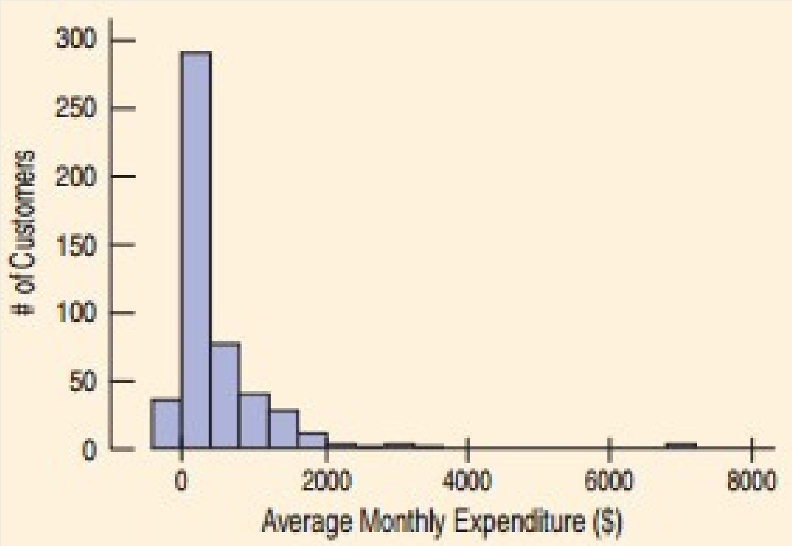
Outliers

•An Outlier is a data value that is far above or far below the rest of the data values.



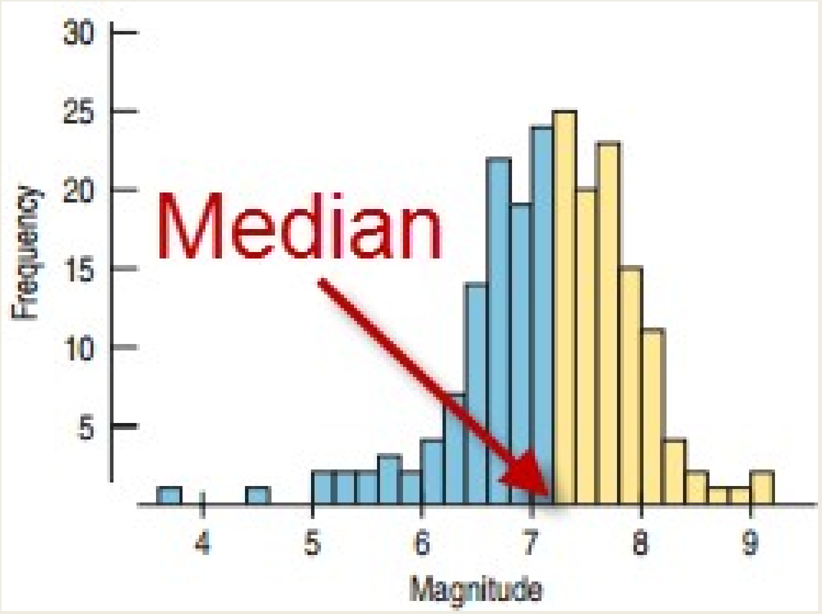
Example

- The histogram shows the amount of money spent by a credit card company's customers. Describe and interpret the distribution.



- CENTER -

•Median: The center of the data values



Calculating the Median: Odd Sample Size

- First order the numbers.
- If there are an odd number of numbers, *n*, the median is at position $\frac{n+1}{2}$.
- Find the median of the numbers: 2, 4, 5, 6, 7, 9, 9.
- $\frac{n+1}{2} = \frac{7+1}{2} = 4$

$$2,4,5,\boxed{6},7,9,9$$

- The median is the fourth number: 6
- Note that there are 3 numbers to the left of 6 and 3 to the right.

Calculating the Median: Even Sample Size

- First order the numbers.

- If there are an even number of numbers, n, the median is the average of the two middle numbers: $\frac{n}{2}, \frac{n}{2}+1$.

- Find the median of the numbers: 2, 2, 4, 6, 7, 8.

- $\frac{n}{2} = \frac{6}{2} = 3$

- The median is the average of the third and the fourth numbers: $\text{Median} = \frac{4+6}{2} = 5$

- SPREAD -

- Locating the center is only part of the story

Range

- The range is the difference between the maximum and minimum values.

$$Range = Maximum - Minimum$$

Percentiles and Quartiles

- Percentiles divide the data in one hundred groups.
- The n^{th} percentile is the data value such that n percent of the data lies below that value.
- Median?
- first quartile (Q1).
- third quartile (Q3).

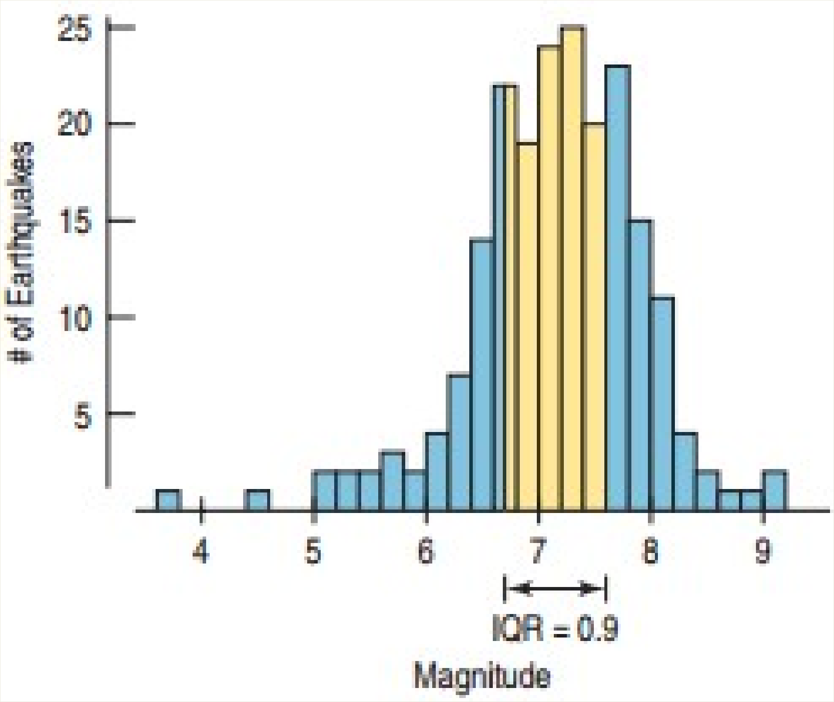
The Interquartile Range

- The Interquartile Range (IQR) is the difference between the upper quartile and the lower quartile

IQR = Q3 – Q1

The Interquartile Range

- The Interquartile Range for earthquake causing tsunamis is 0.9.
- The picture below shows the meaning of the IQR.



Benefits and Drawbacks of the IQR

- Outliers?
- Summary?
- What does it show?
- General audience?

5-Number Summary

- The 5-Number Summary provides a numerical description of the data. It consists of
  - Minimum
  - First Quartile (Q1)
  - Median
  - Third Quartile (Q3)
  - Maximum

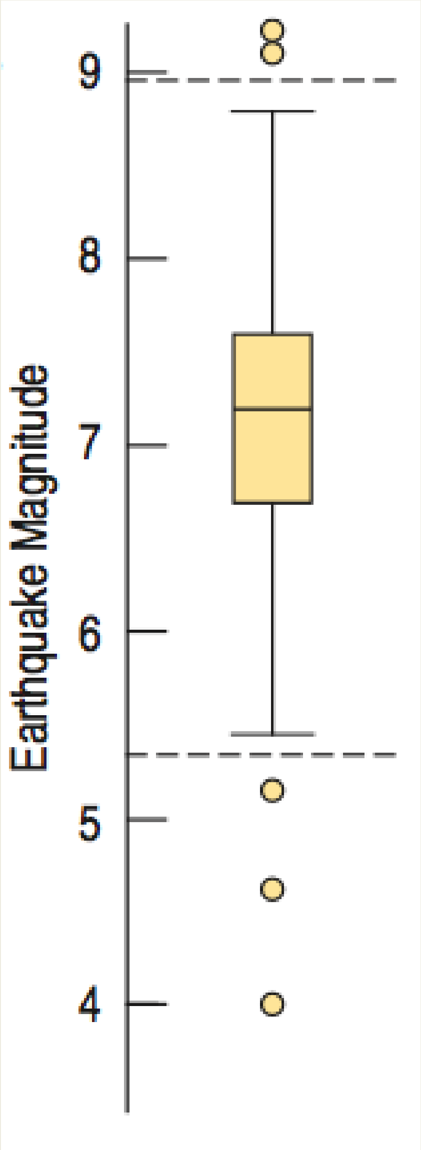
- The list to the right shows the 5-Number Summary for the tsunami data.

Max	9.1
Q3	7.6
Median	7.2
Q1	6.7
Min	4.0

Boxplots

- A Boxplot is a chart that displays the 5-Point Summary and the outliers.
- Box
- Fences
- Whiskers
- Median.

John Tukey



Finding the Fences

- The lower fence is defined by

  Lower Fence = Q1 – (1.5 × IQR)

- The upper fence is defined by

  Upper Fence = Q3 + (1.5 × IQR)

- Tsunami Example: Q1 = 6.7, Q3 = 7.6
-

  IQR = 7.6 – 6.7 = 0.9

- Lower Fence = 6.7 – 1.5 × 0.9 = 5.35
- Upper Fence = 7.6 + 1.5 × 0.9 = 8.95

Step-by-Step Example of Shape, Center, Spread: Flight Cancellations

- Question: How often are US flights cancelled?

We have a dataset with flight cancellations by month

- Who? Months
- What? Percentage of Flights Cancelled at U.S. Airports
- When? 1994 – 2013
- Where? United States
- How? Bureau of Transportation Statistics Data

Flight Cancellations

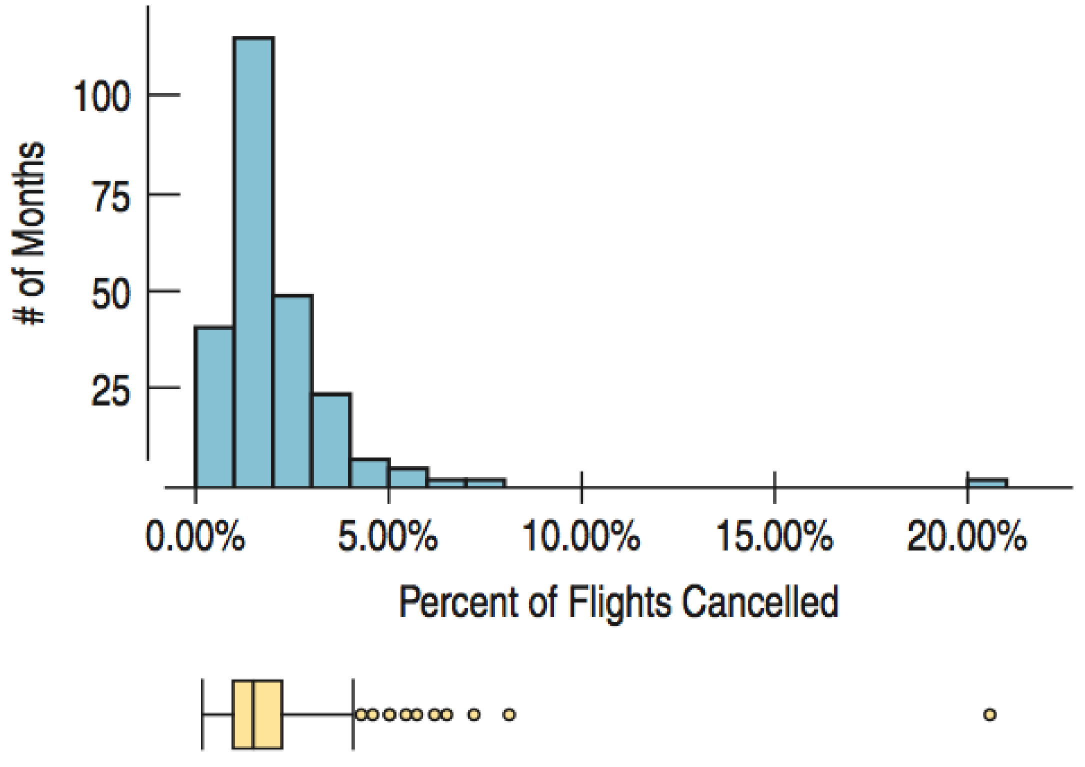
- Identify the Variable
  - Percent of flight cancellations at U.S. airports
  - Quantitative: Units are percentages.

- How will be data be summarized?
  - Histogram
  - Numerical Summary
  - Boxplot

Flight Cancellations

-

Count	238
Max	20.24
Q3	2.39
Median	1.68
Q1	1.16
Min	0.38
IQR	1.23



Flight Cancellations

- How is the data skewed?
- Skewed to the Right: Can’t be a negative percent. Bad weather and other airport troubles can cause extreme cancellations.

- What can we say about the IRQ?
- IQR is small: 1.23%. Consistency among cancellation percents

- Do we have an outlier?
- Extraordinary outlier at 20.2%: September 2001

Symmetric Distributions

- A symmetric distribution is easier to describe
- mean
- standard deviation

The Center of Symmetric Distributions: the Mean

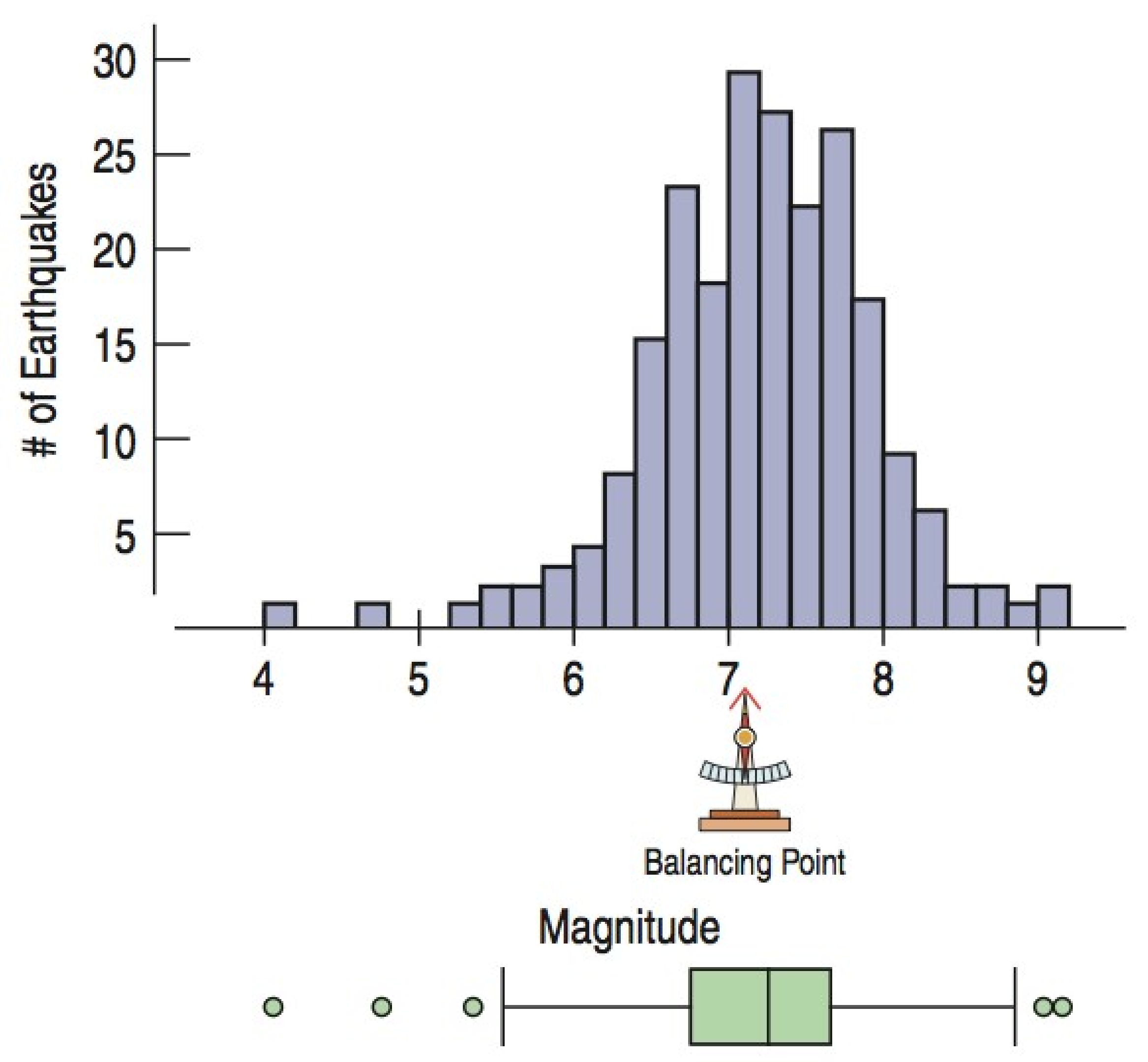
- The Mean is what most people think of as the average.
- Add up all the numbers and divide by the number of numbers.

$$\bar{y} = \frac{\sum y}{n}$$

- Recall that $\sum$ means “Add them all.”

The Mean is the “Balancing Point”

- If you put your finger on the mean, the histogram will balance perfectly.



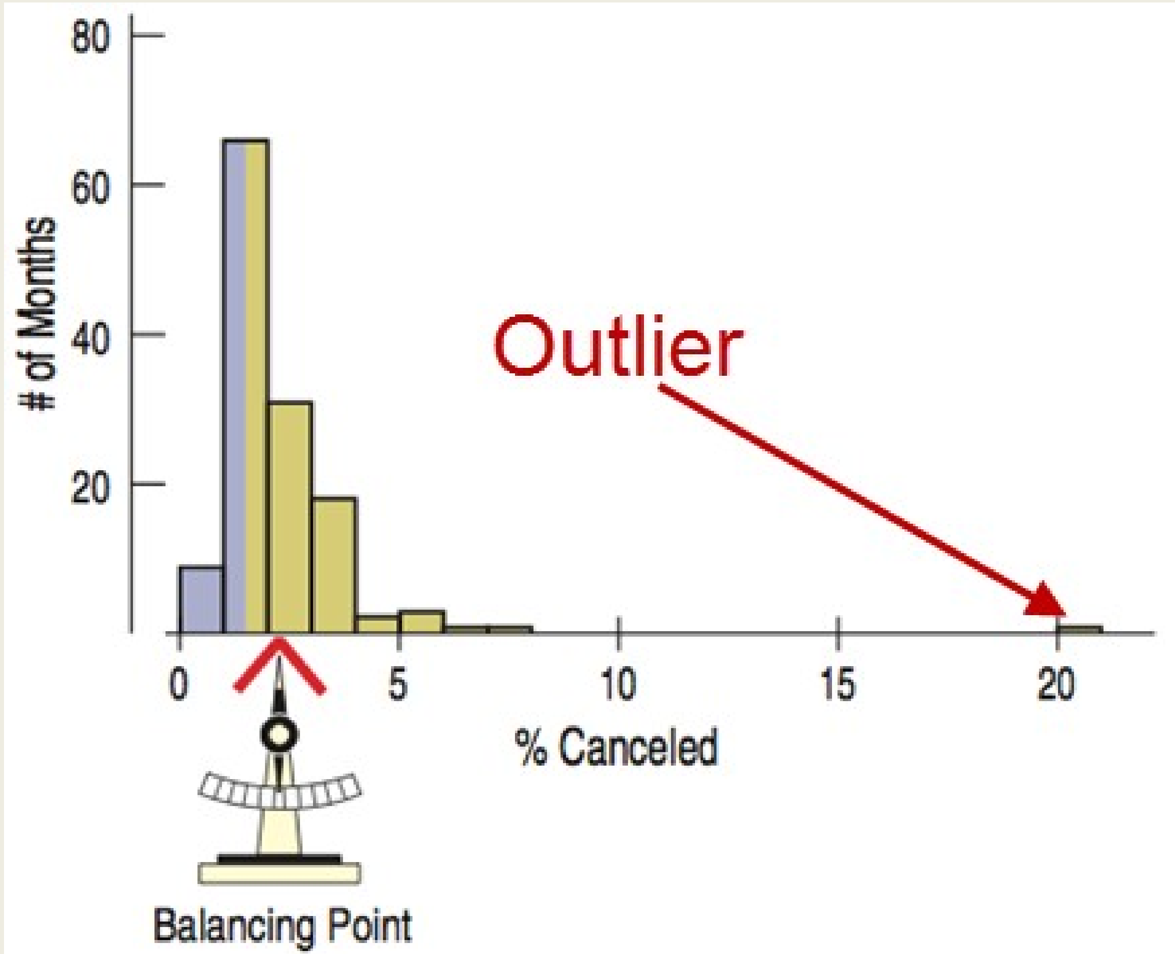
Mean Vs. Median

- For symmetric distributions, the mean and the median are equal.
  - The balancing point is at the center.

- The tail “pulls” the mean towards it more than it does to the median.

- The mean is more sensitive to outliers than the median.

The Mean Is Attracted to the Outlier



Why Use the Mean?

- Which one is easier to work with?
- Which one weights the data better?
- Symmetric data?
- Report both?

The Variance

$$s^2 = \frac{\sum (y - \bar{y})^2}{n - 1}$$

- The variance is a measure of how far the data is spread out from the mean.
- The difference from the mean is: $y - \bar{y}$.
- To make it positive, square it.
- Then find the average of all of these distances, except instead of dividing by n, divide by $n - 1$.
- Use s^2 to represent the variance.
- The variance will mostly be used to find the standard deviation s which is the square root of the variance.

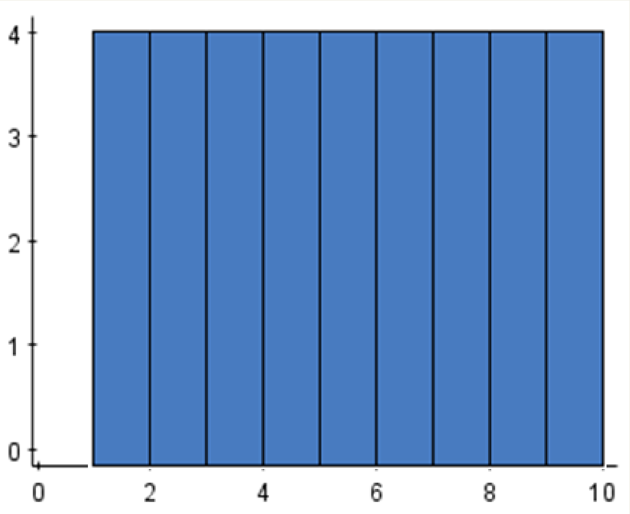
		Class				
		First	Second	Third	Crew	Total
Survival	Alive	203	118	178	212	711
	Dead	122	167	528	673	1490
	Total	325	285	706	885	2201

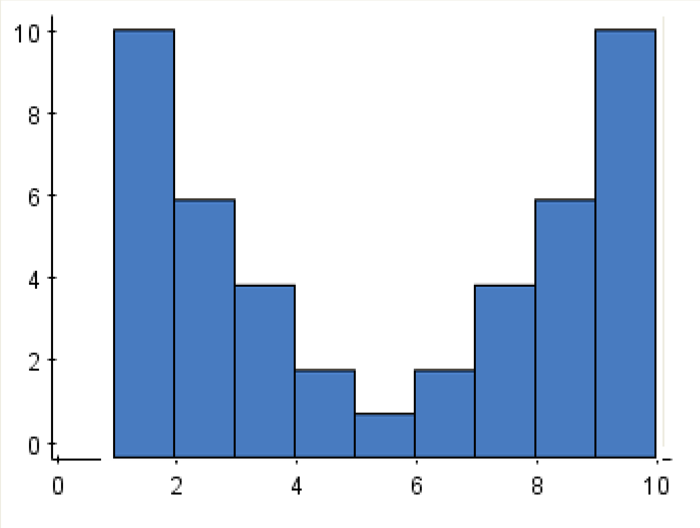
Standard Deviation

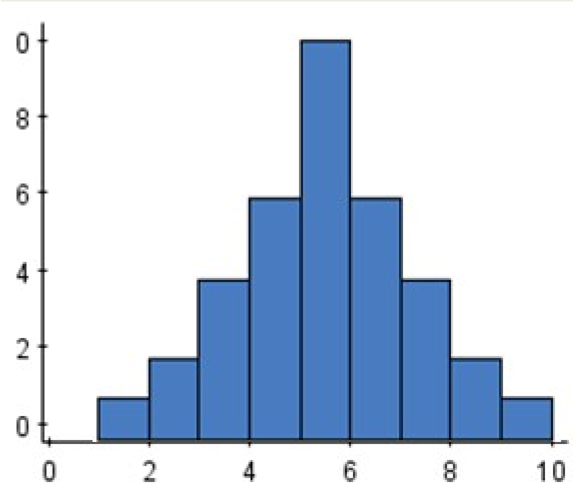
- The variance's units are the square of the original units.
- Taking the square root of the variance gives the standard deviation, which will have the same units as *y*.
- The standard deviation is a number that is close to the average distances that the *y* values are from the mean.
- If data values are close to the mean (less spread out), then the standard deviation will be small.
- If data values are far from the mean (more spread out), then the standard deviation will be large.

The Standard Deviation and Histograms

Order the histograms below from smallest standard deviation to largest standard deviation.







A B C

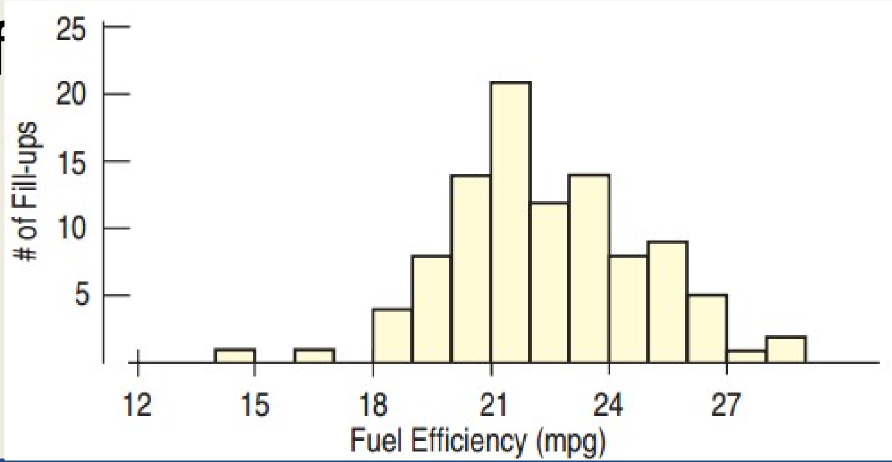
Answer: C, A, B

Summary

- Histogram, Boxplot
  - What to describe?
- Center and Spread
  - if not symmetric?
  - if symmetric?
  - Unimodal symmetric data?
- Unusual Features
  - For multiple modes?
  - Outliers?

Example: Fuel Efficiency

- The car owner has checked the fuel efficiency each time he filled the tank of his new car. How would you describe the fuel efficiency?
- Plan: Summarize the distribution of the car's fuel efficiency.
- Variable: mpg for the first 100 f
- Mechanics: show a histogram
  - Fairly symmetric
  - Low outlier



Fuel Efficiency Continued

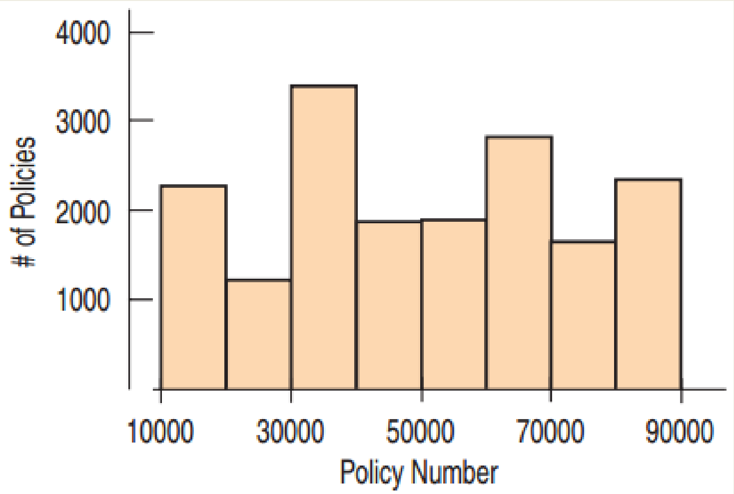
Count	100
Mean	22.4 mpg
StdDev	2.45
Q1	20.8
Median	22.0
Q3	24.0
IQR	3.2

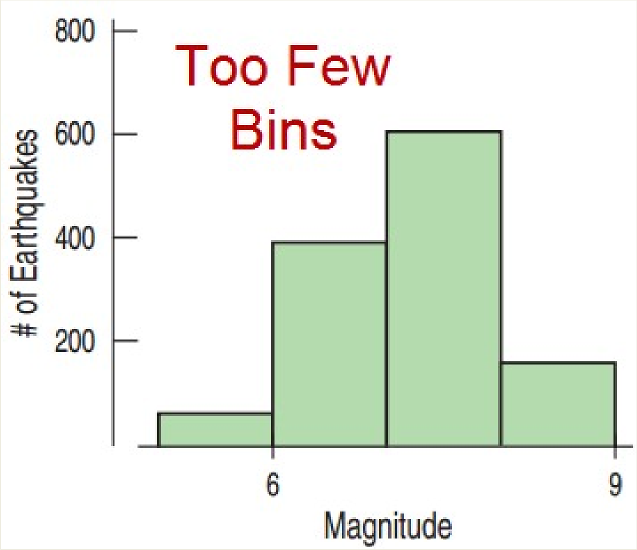
- Which to report?
  - The mean and median are close.
  - Report the mean and standard deviation.

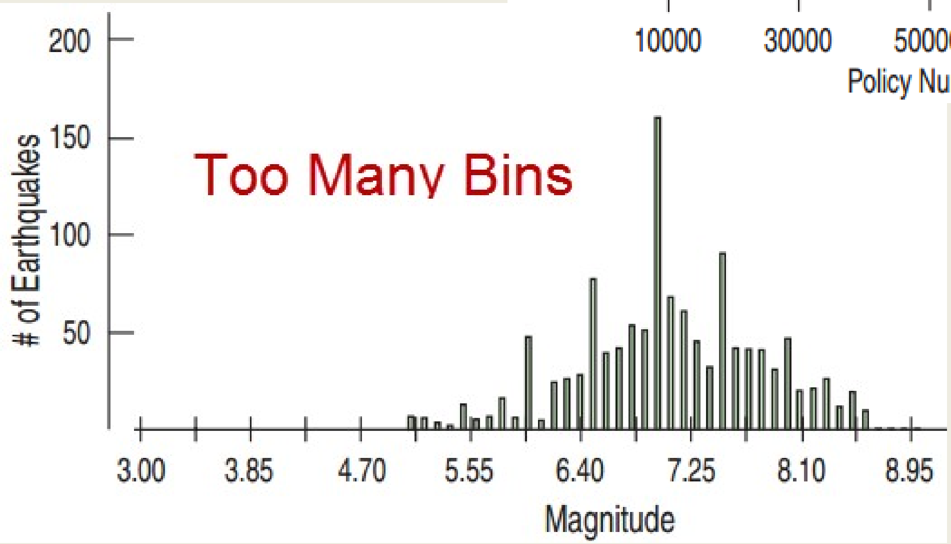
- Conclusion
  - Distribution is unimodal and symmetric.
  - Mean is 22.4 mpg.
  - Low outlier may be investigated, but limited effect on the mean
  - $s = 2.45$; from one filling to the next, fuel efficiency differs from the mean by an average of about 2.45 mpg.

What Can Go Wrong?

- Don’t make a histogram for categorical data.
- Don’t look for shape, center, and spread for a bar chart.
- Choose a bin width appropriate for the data.







What Can Go Wrong? Continued

- Do a reality check
  - Don’t blindly trust your calculator. For example, a mean student age of 193 years old is nonsense.
- Sort before finding the median and percentiles.
  - 315, 8, 2, 49, 97 does not have median of 2.
- Don’t compute numerical summaries for a categorical variable.
  - The mean Social Security number is meaningless.

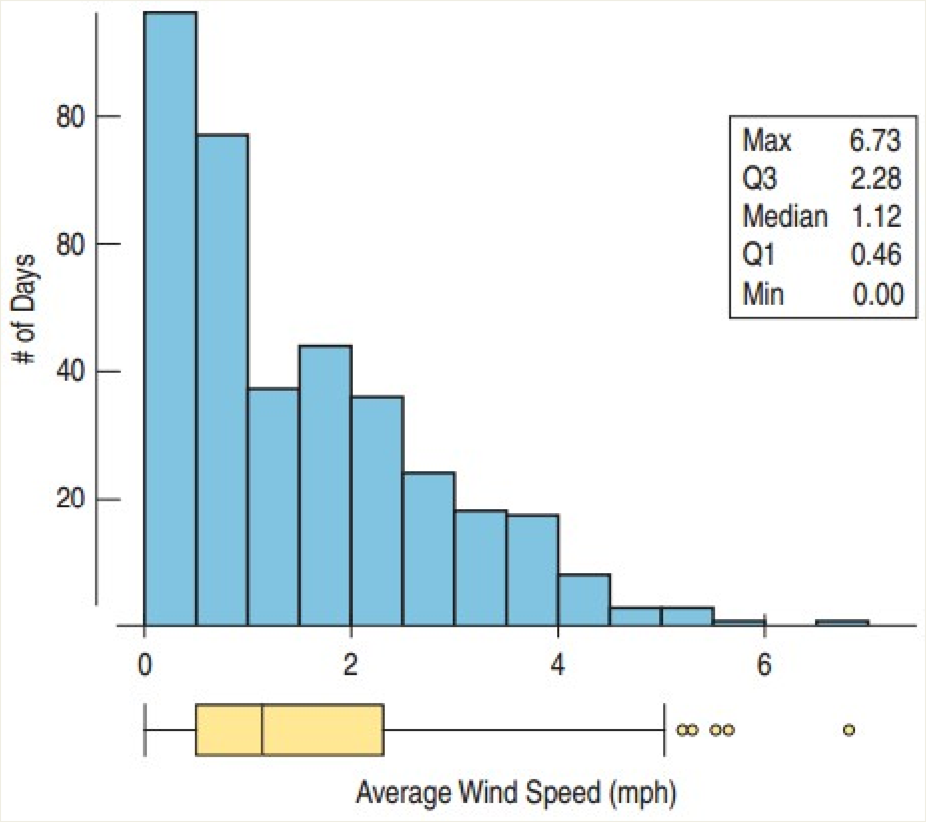
What Can Go Wrong? Continued

- Don't report too many decimal places.
  - Citing the mean fuel efficiency as 22.417822453 is going overboard.
- Don't round in the middle of a calculation.
- For multiple modes, think about separating groups.
  - Heights of people → Separate men and women
- Beware of outliers, the mean and standard deviation are sensitive to outliers.
  - Use a histogram or dotplot to ensure that the mean and standard deviation really do describe the data.

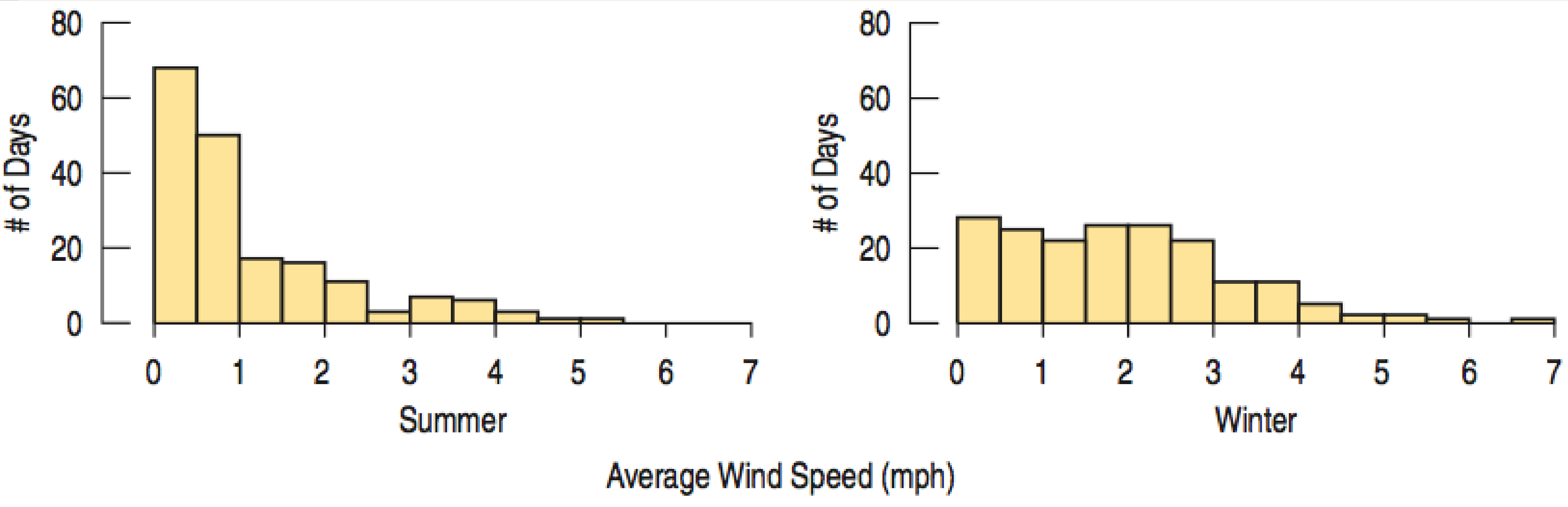
Chapter 4

Understanding and Comparing Distributions

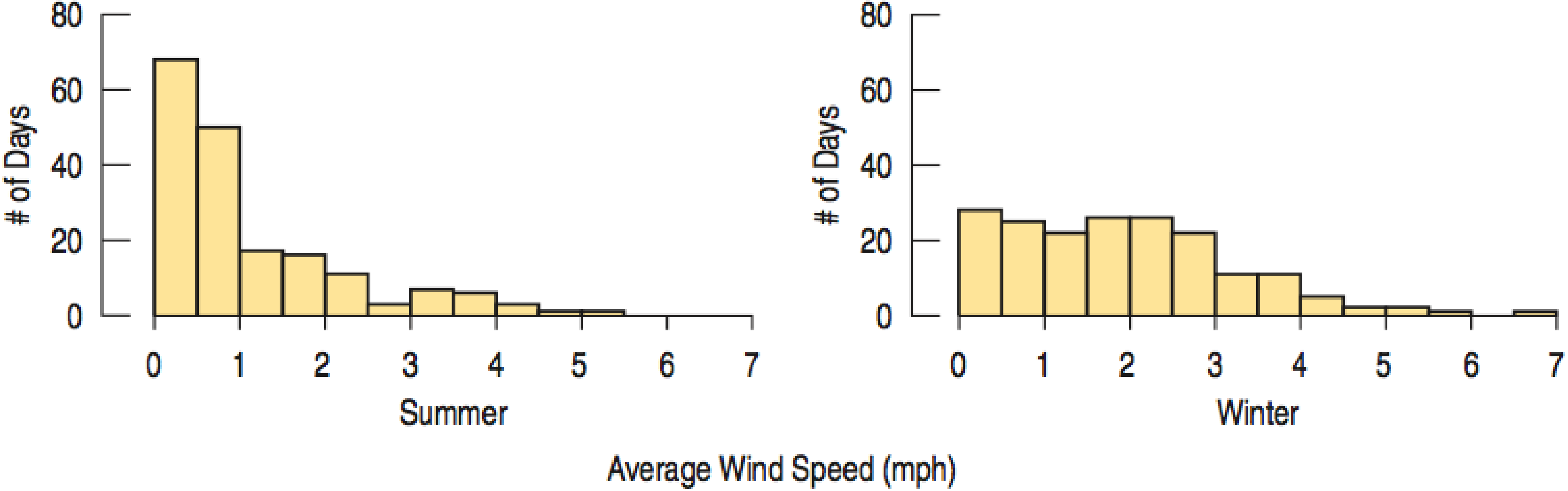
Wind Speeds in the Hopkins Memorial Forest



Comparing Seasons



Comparing Seasons (Continued)



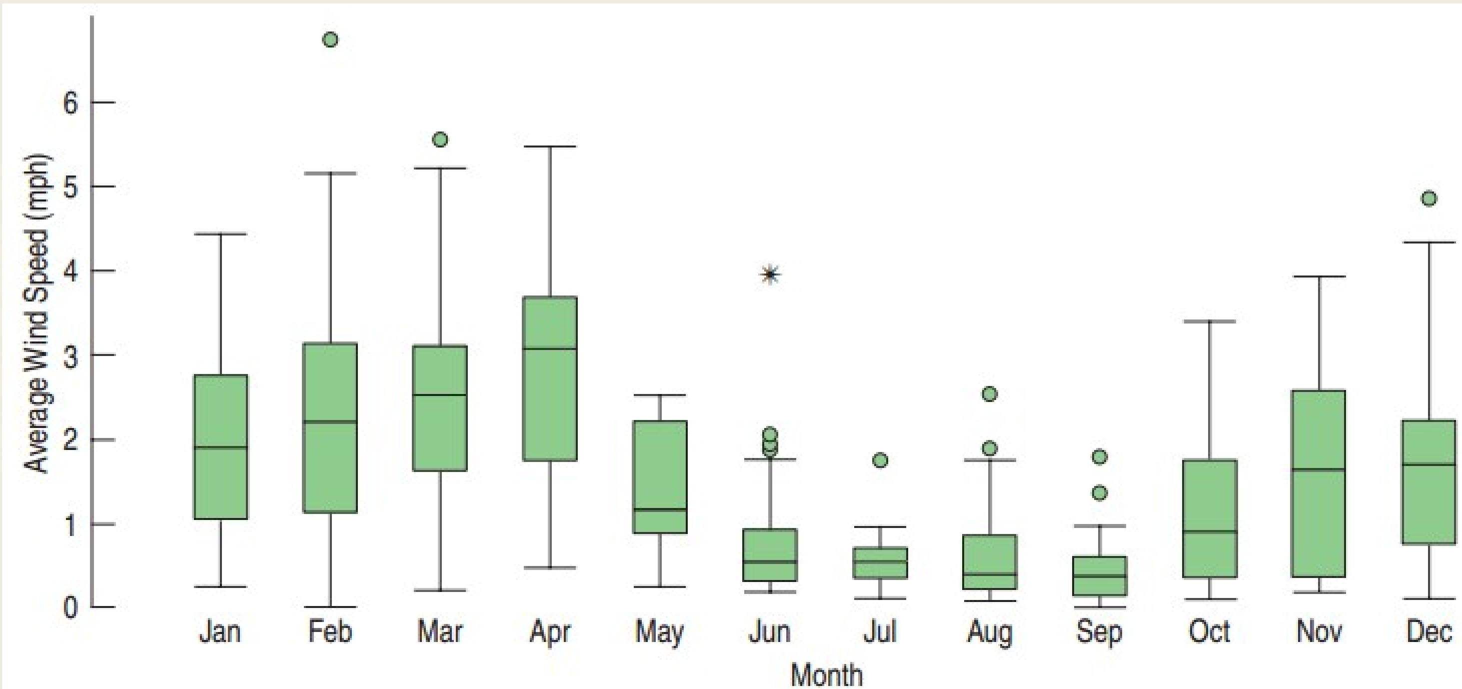
Comparing Seasons (Continued)

Summaries for *Average Wind Speed* by Season

Season	Mean	StdDev	Median	IntQRange
Summer	1.11	1.10	0.71	1.27
Winter	1.90	1.29	1.72	1.82

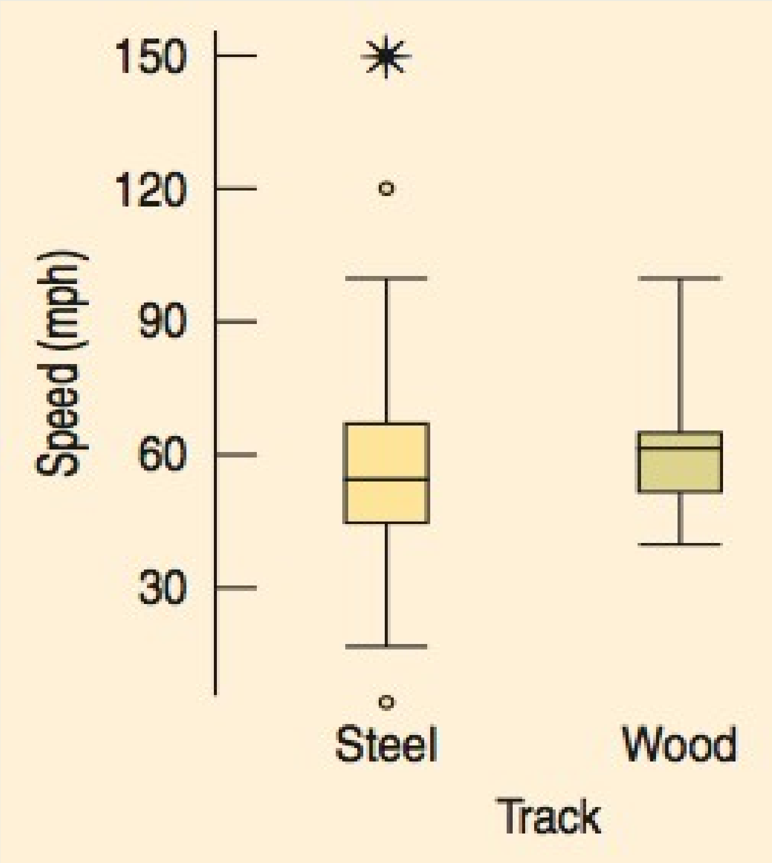
Using Boxplots for Comparisons

- Are some months windier than others?



Wooden Vs. Steel

- Which type of roller coaster is faster: steel or wooden?



Please, No Cold Coffee!

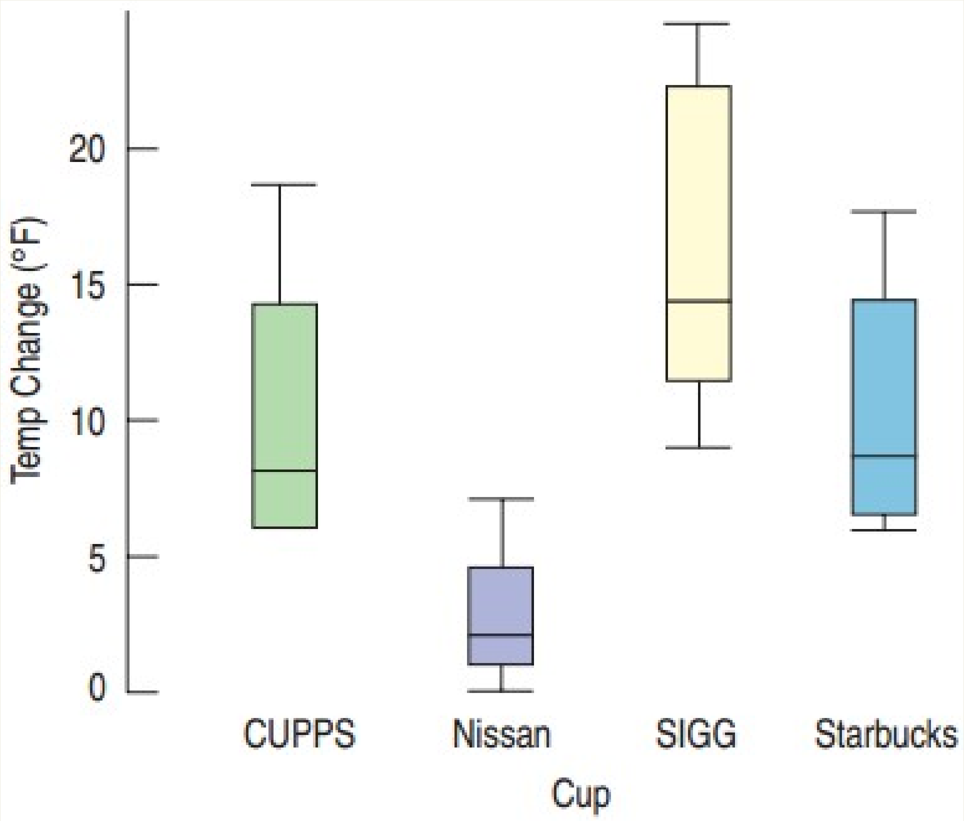
- We want to compare which of 4 different coffee cups keeps the coffee hot.

- Measure the temperature 30 minutes after being poured for each of the four types. Repeat the experiment 8 times.

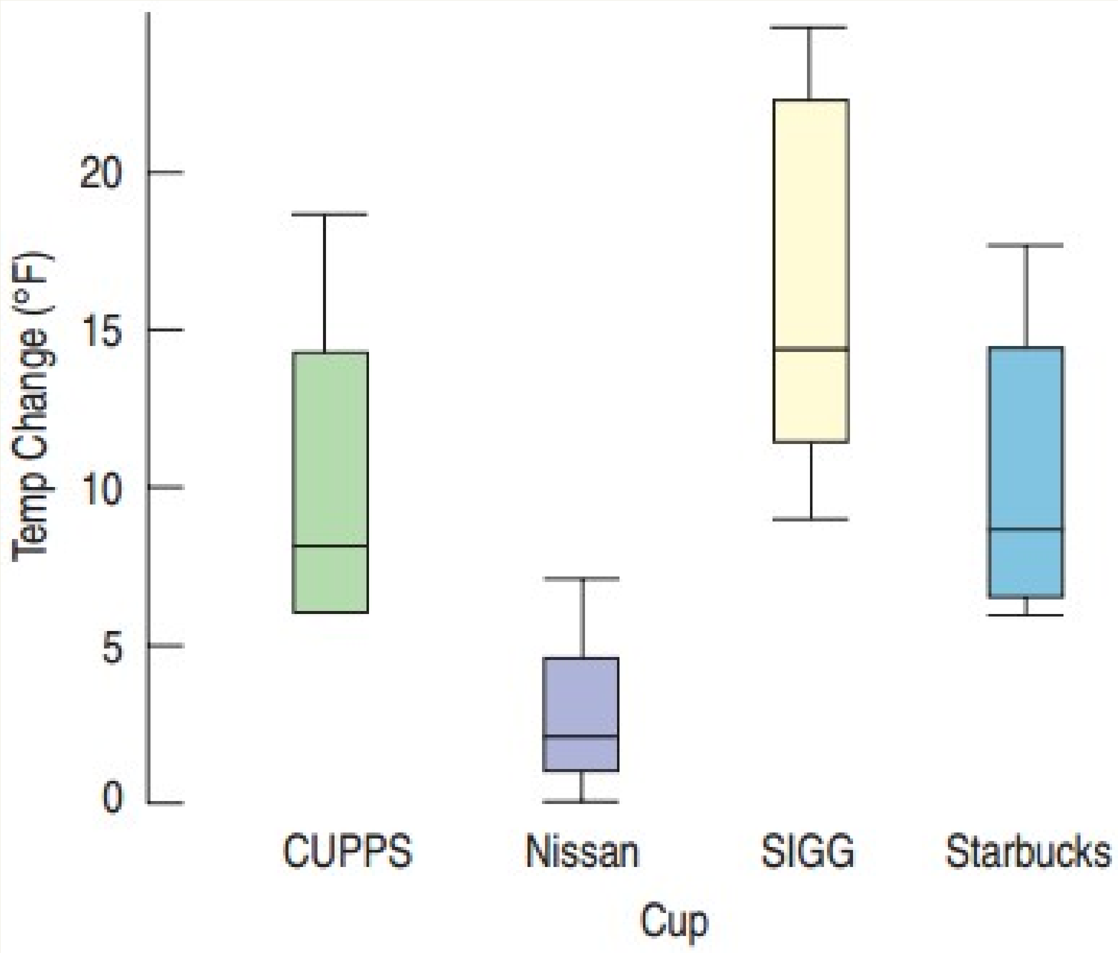
- **Plan:** Compare the data sets for the four types.
- **Variables:** Quantitative – Temperature change of coffee

Mechanics

	Min	Q1	Median	Q3	Max	IQR
CUPPS	6°F	6	8.25	14.25	18.50	8.25
Nissan	0	1	2	4.50	7	3.50
SIGG	9	11.50	14.25	21.75	24.50	10.25
Starbucks	6	6.50	8.50	14.25	17.50	7.75



Conclusion



4.3

Outliers

How to Approach Outliers

- Check to see if there may have been an error in the data collection or data input.
  - If the reported heights of students includes a student that is 170 inches tall (14 feet), maybe that student was measured in centimeters.

- Check to see if there was an extraordinary outcome.
  - The median number of daily customers at the Punxsutawney, PA, gift store may be 42 with an IQR of 12, but on December 23rd, there were 831 customers.

Common Errors Causing an Outlier

- Transposing the digits
- A respondent not understanding the survey question
- Misreading results
- Confusion about units
- Cheating

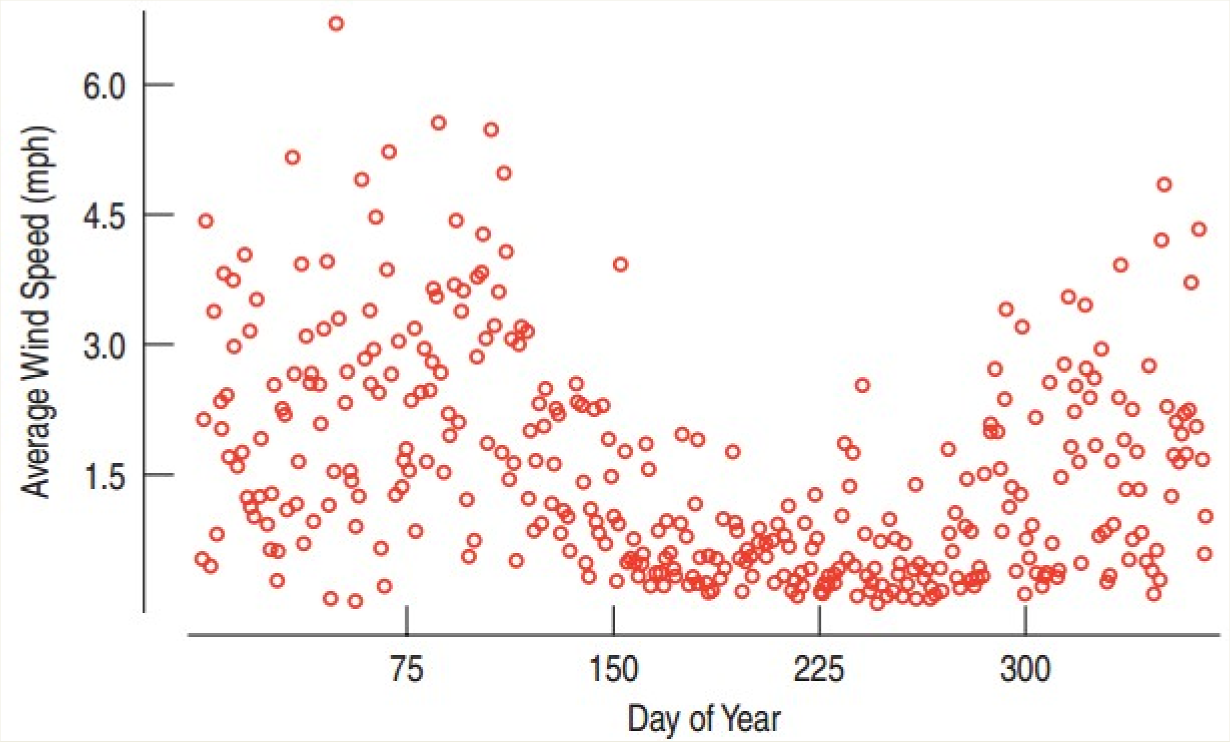
The Outliers Can be the Most Interesting Data Values

- Income Data:
  - The CEO
- Student Height:
  - The basketball team's center
- Snowfall:
  - The great blizzard of '98
- Exam Score:
  - The curve breaker

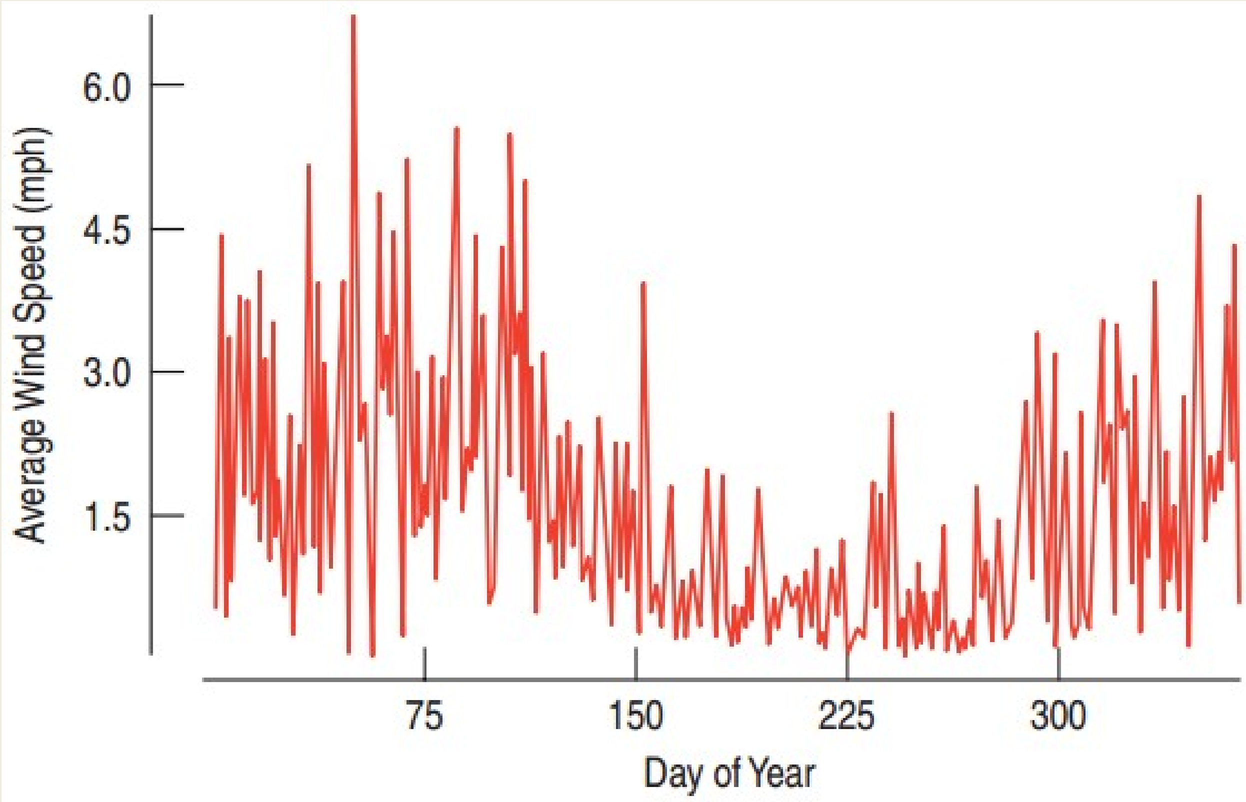
- Always comment on the outliers.

Timeplots

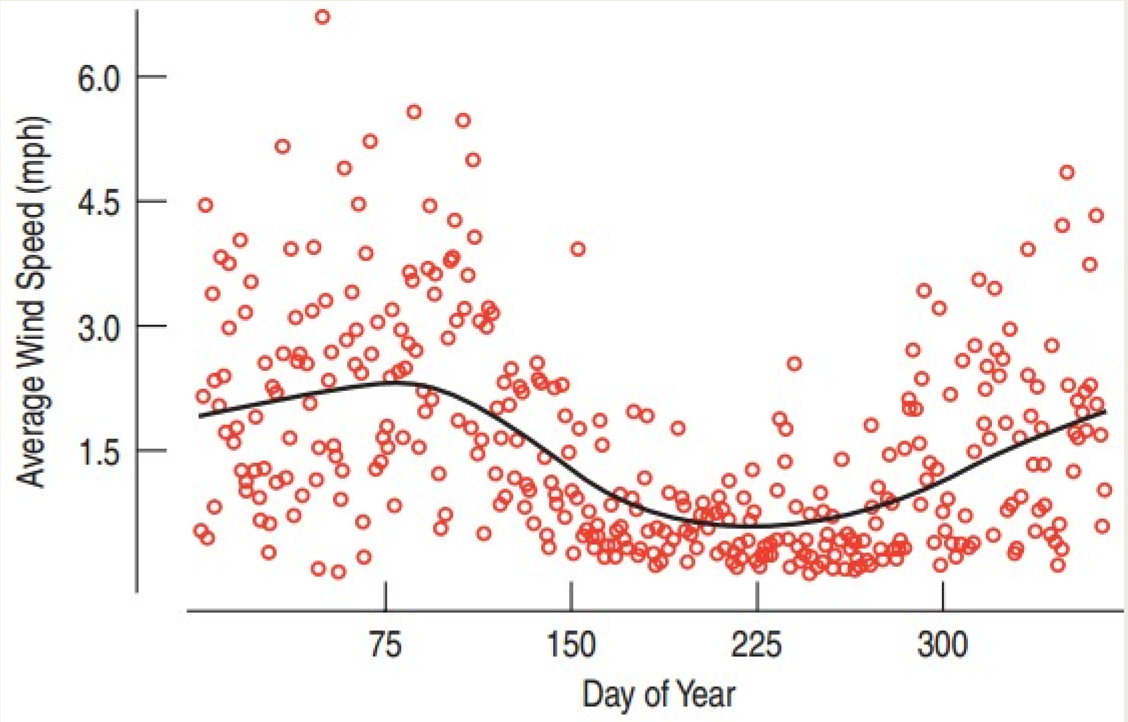
- Timeplots display every data value on a timeline.



Connecting the Dots



Smoothing the Data



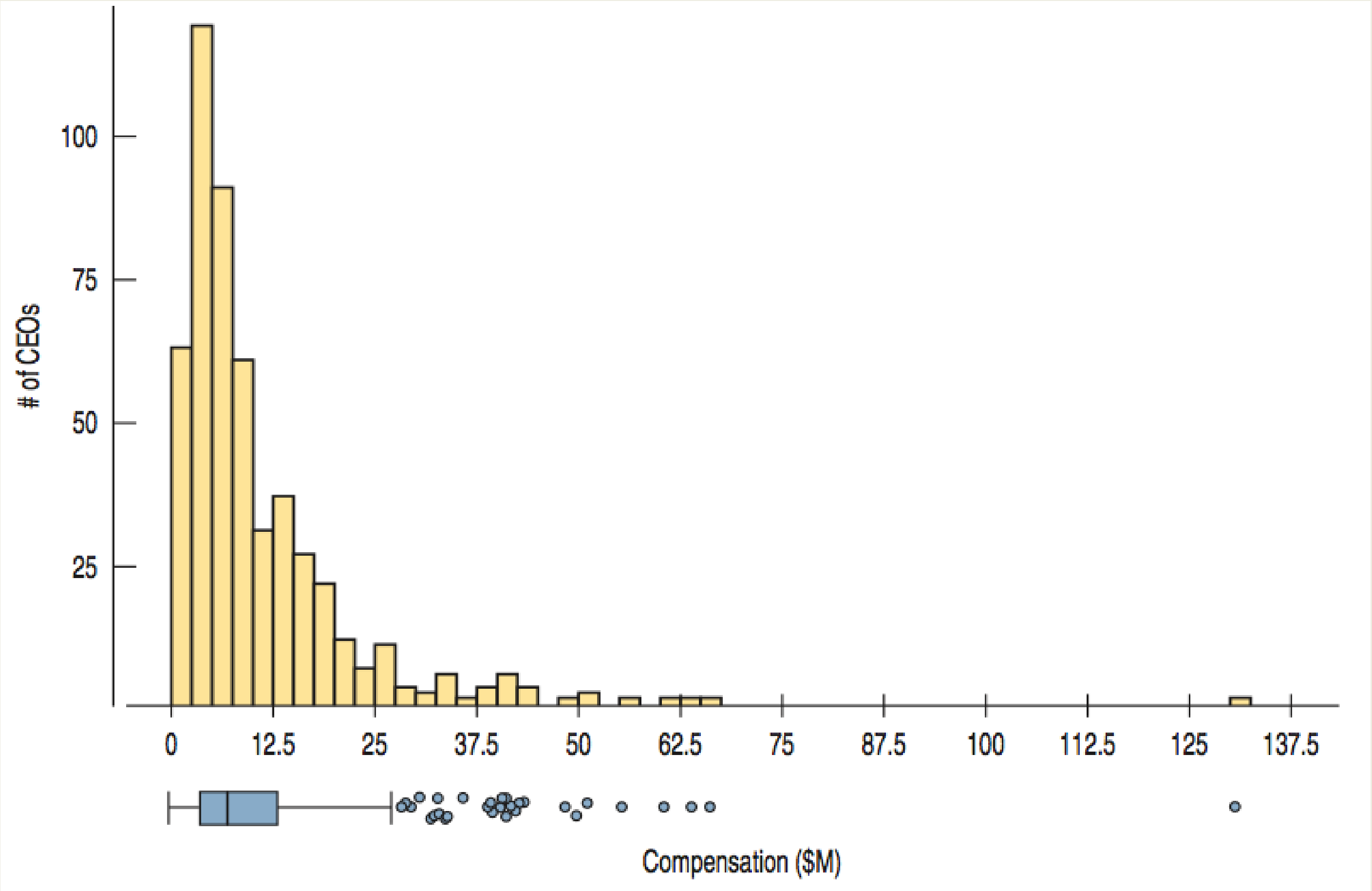
- Lowess curve

Looking into the Future

- Time plots can sometimes be used to predict future trends.
  - Knowing that last summer was calmer than last winter can be used to make predictions about next summer and next winter.

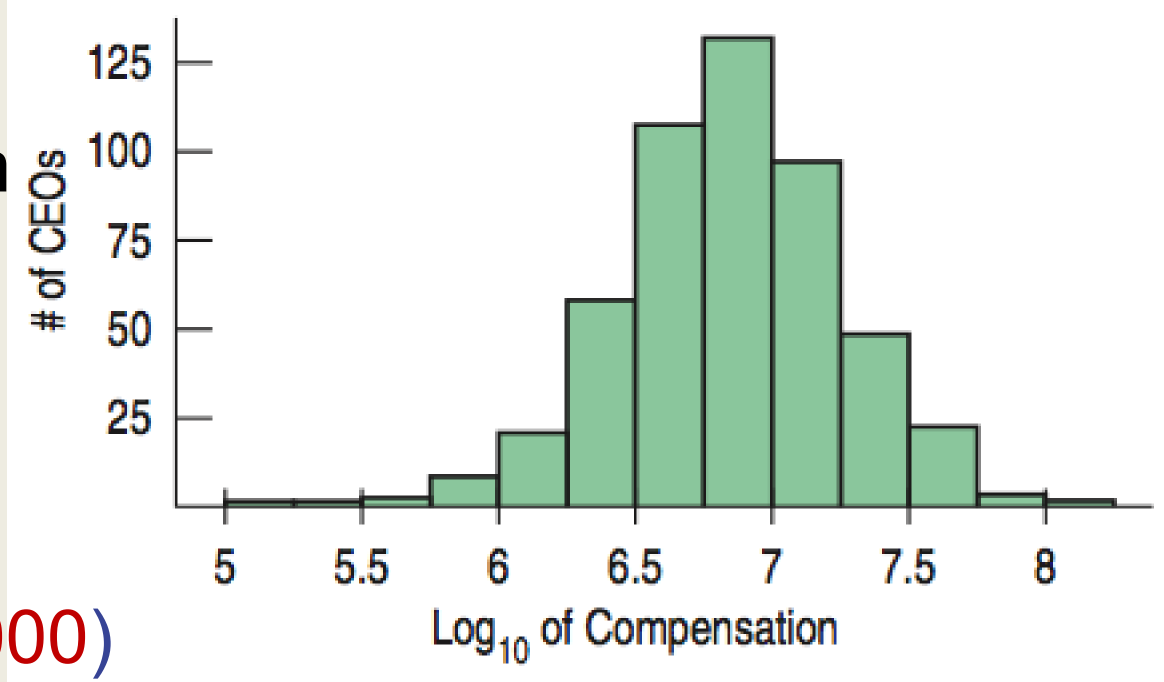
- Predicting the future with a time plot does not always work.
  - Last year's hurricane outlier will not tell you about a hurricane for this year.
  - Stock prices cannot be predicted with a time plot.
  - Roller coaster speeds will not increase forever.

Trouble with Too Many Outliers



Transformation of the Data

- Taking logarithm of the salaries makes histogram much easier to interpret.
- Symmetric
- Typical log salary: between 5 and 7.5 (\$100,000 and \$31,600,000)



  - Median log salary: 6.67 or \$4,786,301
  - Mean log salary: 6.68 or \$4,677,351
  - Three high log salaries are still outliers!

Common Transformations

Skewed Right:

- Use log, ln, or $\frac{1}{x}$

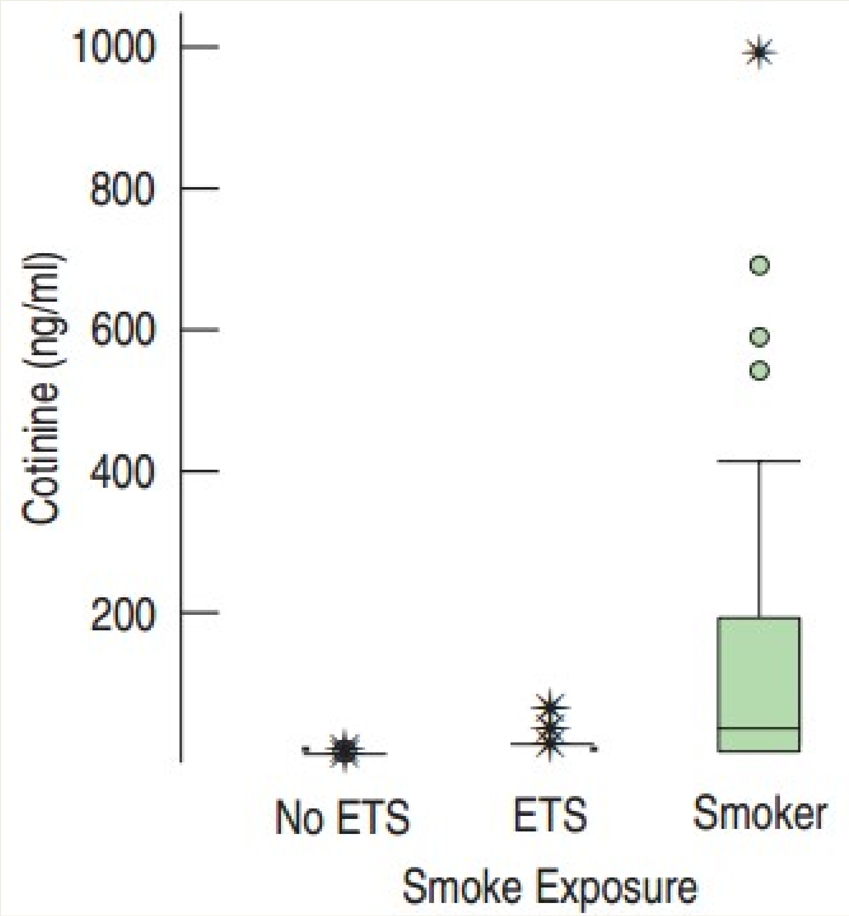
Skewed Left:

- Use x^2

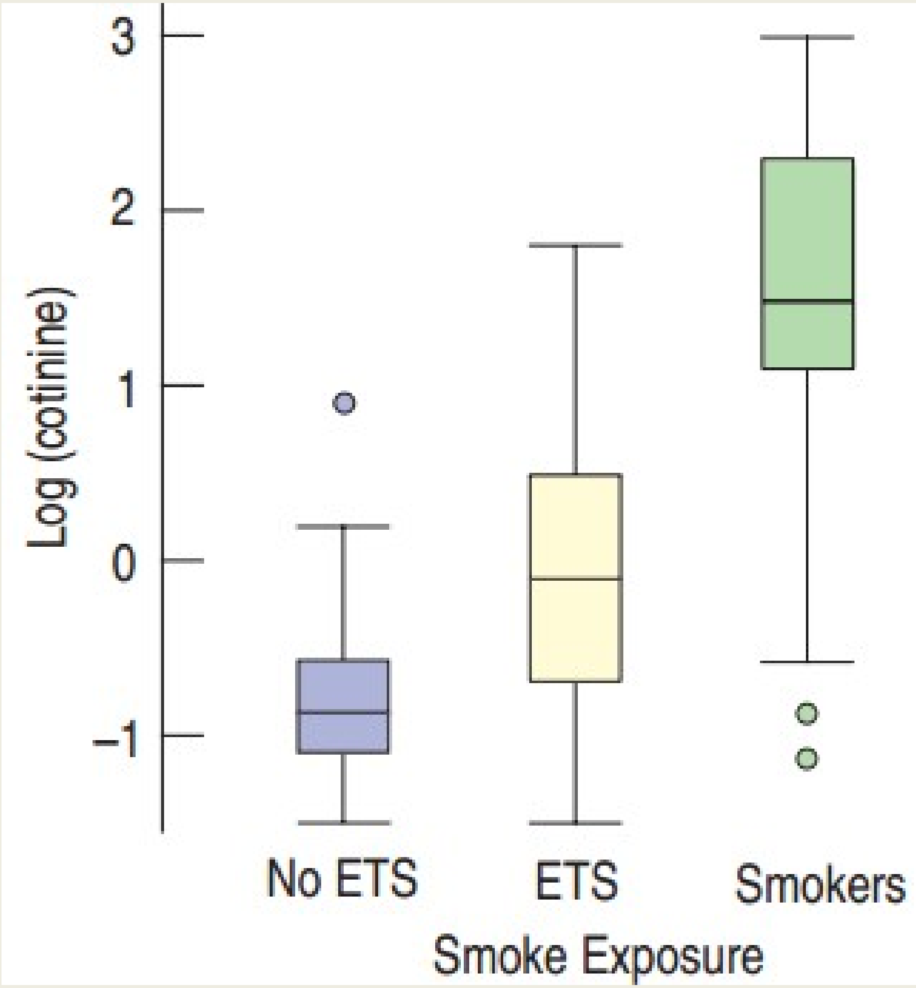
In General:

- Get creative using a computer.

Transforming Boxplots



Log Transformation



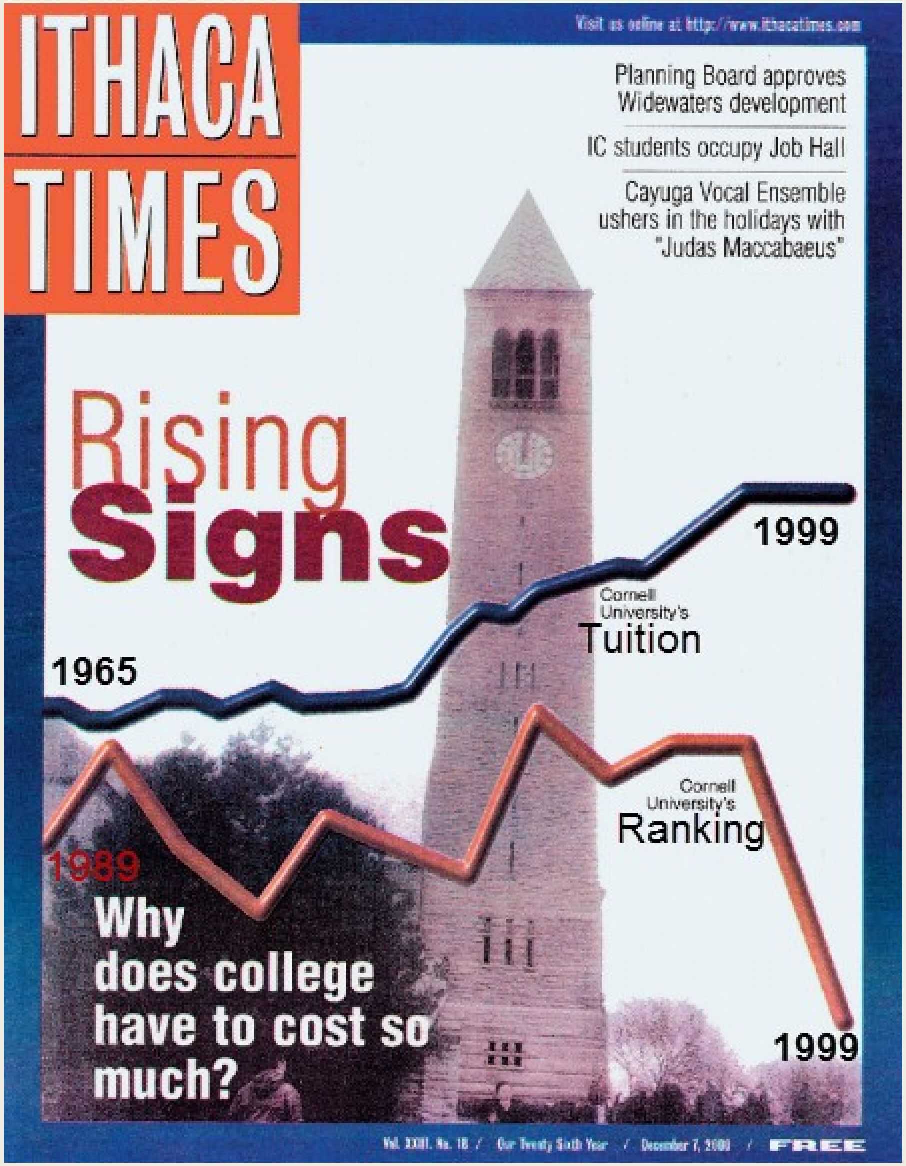
What Can Go Wrong?

- Avoid inconsistent scales.
  - Don’t try to compare one thing measured in feet to another measured in meters.

- Label Clearly.
  - Variables should be identified, and axes labeled.

- Beware of Outliers!
  - If the outliers are errors, remove them.
  - Otherwise, considering presenting with and without the outliers.

What's Wrong With This?



Things to remember

- Choose the right tool.
  - Use histograms to compare two or three groups.
  - Use boxplots to compare many groups.
- Treat outliers with attention and care.
  - Investigate if the outliers are errors or remarkable.
- Use a timeplot to track trends over time.
- Re-express or transform data for better understanding.
  - Can transform skewed distributions to symmetric ones
  - Can help to compare spreads of different groups

Chapter 5

The Standard Deviation as a Ruler and the Normal Model

5.1

Standardizing with *z*-Scores

Comparing Athletes

- Chernova took the gold in the Olympics with a long jump of 6.545 m
- about 0.5 m farther than the mean distance.

- Jessica Ennis won the 200 m run with a time of 22.83 s
- more than 2 s faster than average.

- Whose performance was more impressive?

How Many Standard Deviations Above?

- The standard deviation helps us compare.
- Chernova's long jump was more than 1 standard deviation better than the mean.

	Long Jump	200 m
Mean (all contestants)	5.91 m	24.48 s
SD	0.56 m	0.80 s
n	35	36
Chernova	6.54 m	23.67 s
Ennis	6.48 m	22.83 s

- Ennis's winning time in the 200 m was more than 2 standard deviations faster than than the mean.

Is there an even more precise way to calculate these?

The *z*-Score

- In general, to find the distance between the value and the mean in standard deviations:

1. Subtract the mean from the value.
2. Divide by the standard deviation.

$$z = \frac{y - \bar{y}}{s}$$

- This is called the *z*-score.

The *z*-score

- The *z*-score measures the distance of the value from the mean in standard deviations.

- Positive z-score?
- Negative z-score?
- Small z-score?
- Large z-score?

How Many Standard Deviations from Mean?

	Long Jump	200 m
Mean (all contestants)	5.91 m	24.48 s
SD	0.56 m	0.80 s
n	35	36
Chernova	6.54 m	23.67 s
Ennis	6.48 m	22.83 s

How Many Standard Deviations from Mean?

- Chernova's long jump

$$z = \frac{6.54 - 5.91}{0.56} \approx 1.1$$

- Ennis's 200 m run

$$z = \frac{22.83 - 24.48}{0.80} \approx -2.1$$

	Long Jump	200 m
Mean (all contestants)	5.91 m	24.48 s
SD	0.56 m	0.80 s
n	35	36
Chernova	6.54 m	23.67 s
Ennis	6.48 m	22.83 s

- Ennis's winning time is a little more impressive.
- Judges could assign points based on standard deviations from mean and this system would have a correlation of 0.99 with the one currently used!

How Many Standard Deviations from Mean?

- $-1 < z < 1$: Not uncommon
- $z = \pm 3$: Rare
- $z = 6$: Shouts out for attention!

5.2

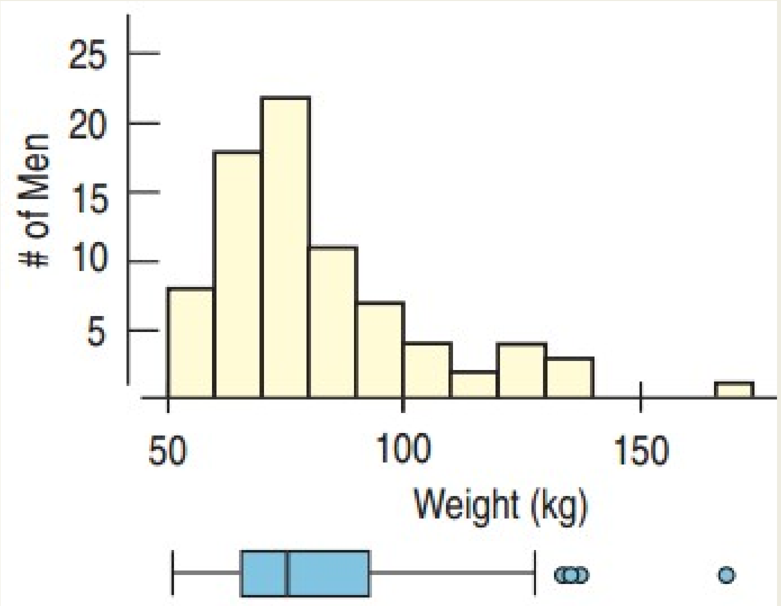
Shifting and scaling

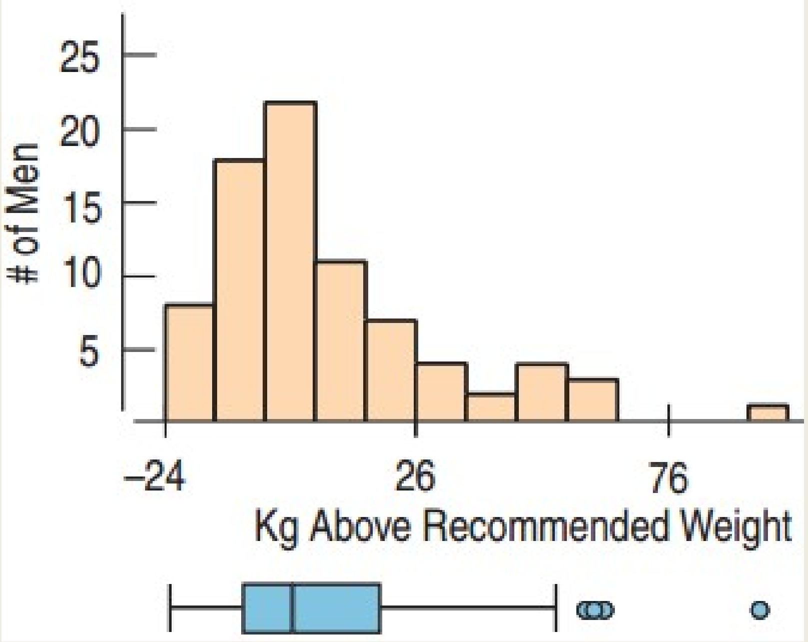
National Health and Examination Survey

- Who? — 80 male participants between 19 and 24 who measured between 68 and 70 inches tall
- What? — Their weights in kilograms
- When? — 2001 – 2002
- Where? — United States
- Why? — To study nutrition and health issues and trends
- How? — National survey

Shifting Weights

- Mean: 82.36 kg
- Maximum Healthy Weight: 74 kg
- How are shape, center, and spread affected when 74 is ***subtracted*** from all values?
  - Shape and spread are unaffected.
  - Center is shifted by 74.



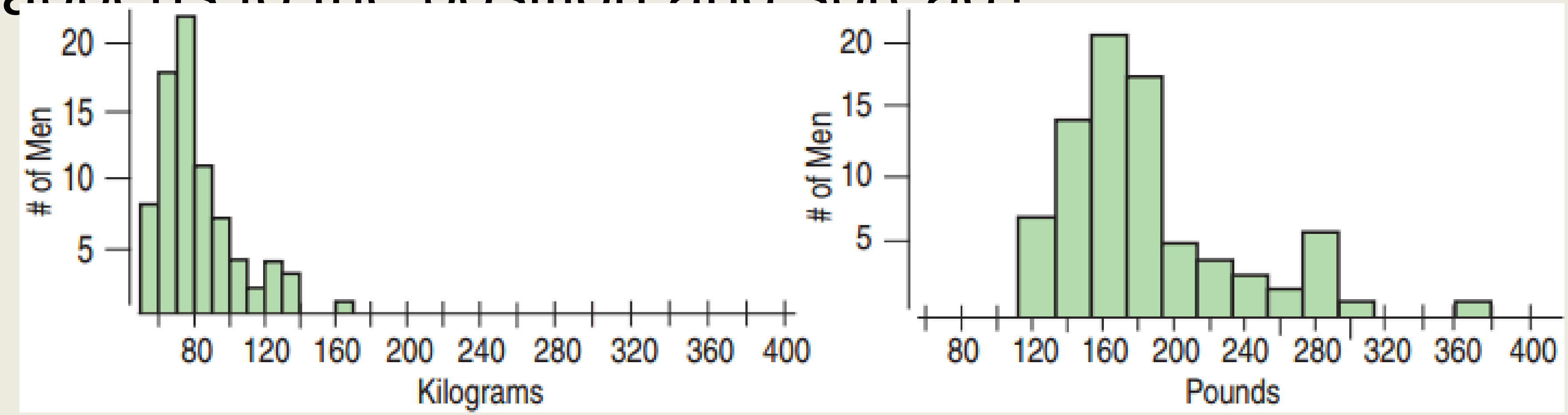


Rules for Shifting

- If the same number is subtracted or added to all data values, then:
  - The measures of the spread – standard deviation, range, and IQR – are all unaffected.
  - The measures of position – mean, median, and mode – are all changed by that number.

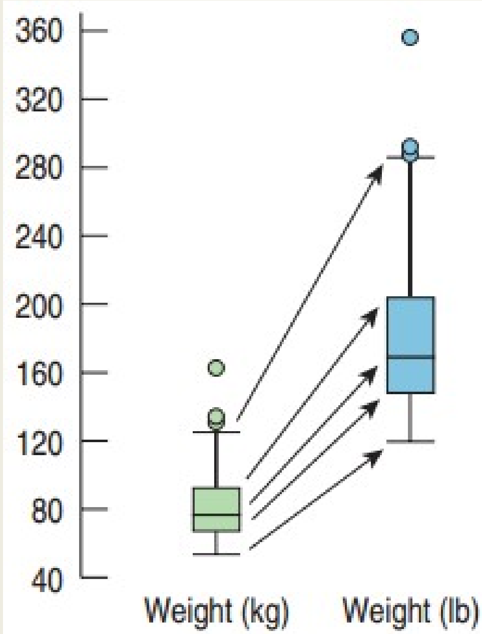
Rescaling

- If we multiply all data values by the same number, what happens to the position and spread?



- To go from kg to lbs, multiply by 2.2.
- The mean and spread are also multiplied by 2.2.

How Rescaling Affects the Center and Spread



	Weight (kg)	Weight (lb)
Min	54.3	119.46
Q1	67.3	148.06
Median	76.85	169.07
Q3	92.3	203.06
Max	161.5	355.30
IQR	25	55
SD	22.27	48.99

- When we multiply (or divide) all the data values by a constant, all measures of position and all measures of spread are multiplied (or divided) by that same constant.

Example: Rescaling Combined Times in the Olympics

- The mean and standard deviation in the men's combined event at the Olympics were 168.93 seconds and 2.90 seconds, respectively.

- If the times are measured in minutes, what will be the new mean and standard deviation?
  - Mean: 168.93 / 60 = 2.816 minutes
  - Standard Deviation: 2.90 / 60 = 0.048 minute

Shifting, Scaling, and *z*-Scores

$$z = \frac{y - \bar{y}}{s}$$

- Converting to *z*-scores:
  - Subtract the mean $\bar{y} - \bar{y} = 0$
  - Divide by the standard deviation $s/s = 1$
  - Shape ?
  - Center ?
  - Spread?

Example: SAT and ACT Scores

- How high does a college-bound senior need to score on

  the **ACT** in order to make it into the top quarter of equivalent of SAT scores for a college with middle 50% between 1530 and 1850?

- SAT: Mean = 1500, Standard Deviation = 250

- ACT: Mean = 20.8, Standard Deviation = 4.8

- **Plan:** Want ACT score for upper quarter. Have $\bar{y}$ and s
- **Variables:** Both are quantitative. Units are points.

Show →Mechanics: Standardize the Variable

- It is known that the middle 50% of SAT scores are between 1530 and 1850, $\bar{y} = 1500$, $s = 250$

- The top quarter starts at 1850.

- Find the *z*-score: $z = \dfrac{1850 - 1500}{250} = 1.40$

- For the ACT, 1.40 standard deviations above the mean:

$$20.8 + 1.40(4.8) = 27.52$$

Conclusion

- To be in the top quarter of applicants for that specific college in terms of combined SAT scores, a college-bound senior would need to have an ACT score of at least 27.52.

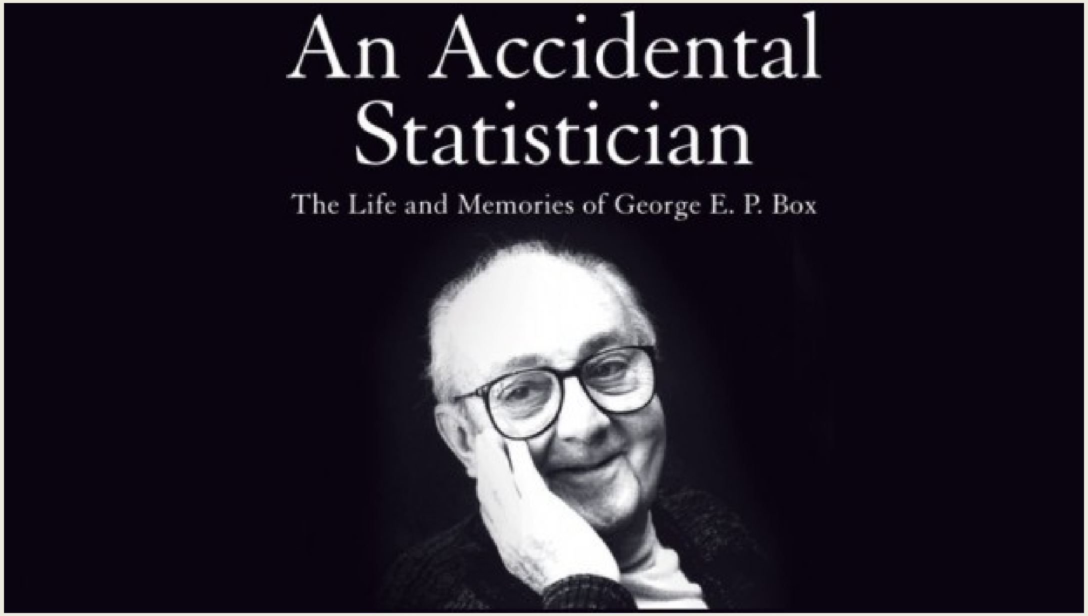
5.3

Normal Models

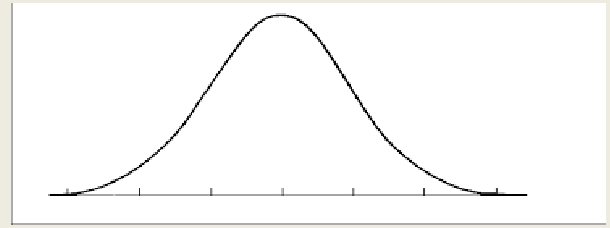
Models

- “All models are wrong, but some are useful.”

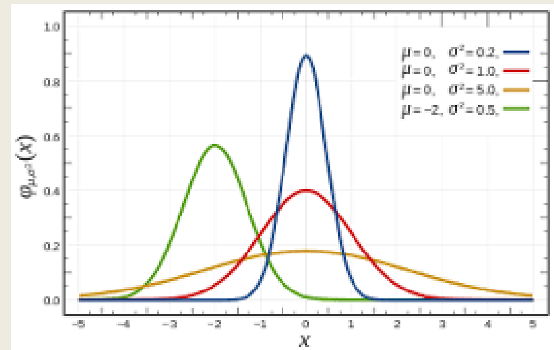
  George Box, statistician



The Normal Model



- Bell Shaped: unimodal, symmetric



- A Normal model for every mean
- and standard deviation.

- μ (read “mew”) represents the population mean.
- σ (read “sigma”) represents the population standard deviation.
- $N(\mu, \sigma)$ represents a Normal model with mean μ and standard deviation σ.

Parameters and Statistics

- Parameters: Numbers that help specify the model
  - μ, σ

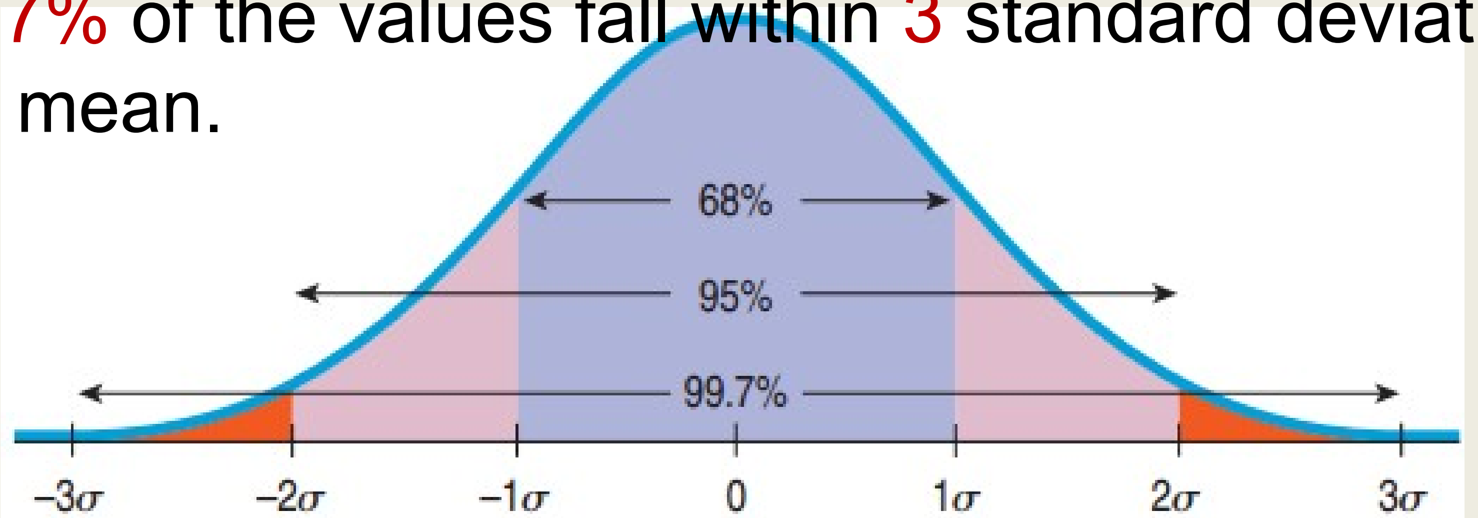
- Statistics: Numbers that summarize the data
  $\bar{y}$, s, median, mode

- $N(0, 1)$ is called the standard Normal model, or the standard Normal distribution.

- The Normal model should only be used if the data is approximately symmetric and unimodal.

The 68-95-99.7 Rule

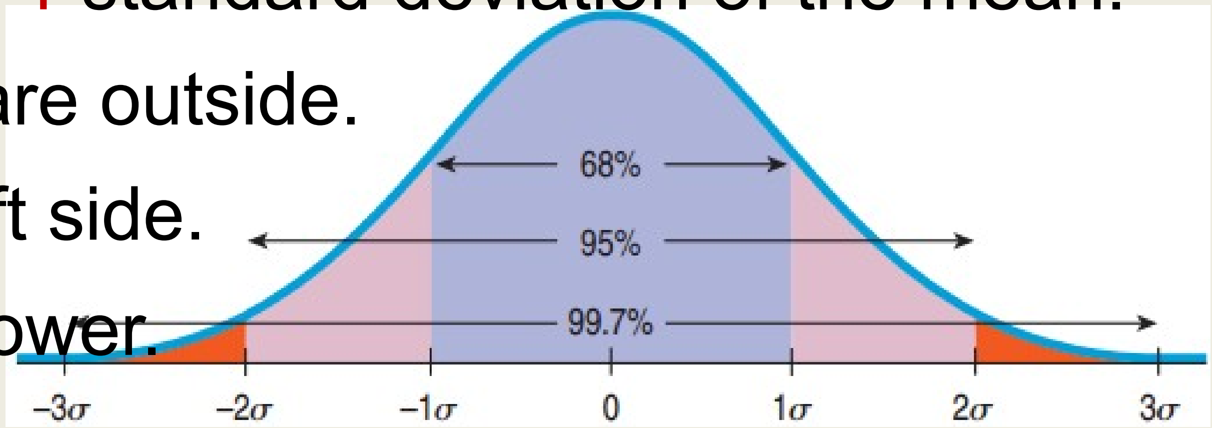
- 68% of the values fall within 1 standard deviation of the mean.
- 95% of the values fall within 2 standard deviations of the mean.
- 99.7% of the values fall within 3 standard deviations of the mean.



Example of the 68-95-99.7 Rule

- In the 2010 winter Olympics men's slalom, Li Lei's time was 120.86 sec, about 1 standard deviation slower than

  the mean. Given the Normal model, how many of the 48 skiers were slower?
- About 68% are within 1 standard deviation of the mean.
- 100% – 68% = 32% are outside.
- "Slower" is just the left side.
- 32% / 2 = 16% are slower.
- 16% of 48 is 7.7.
- About 7 are slower than Li Lei.



Three Rules For Using the Normal Model

- When data is provided, first make a histogram to make sure that the distribution is symmetric and unimodal.

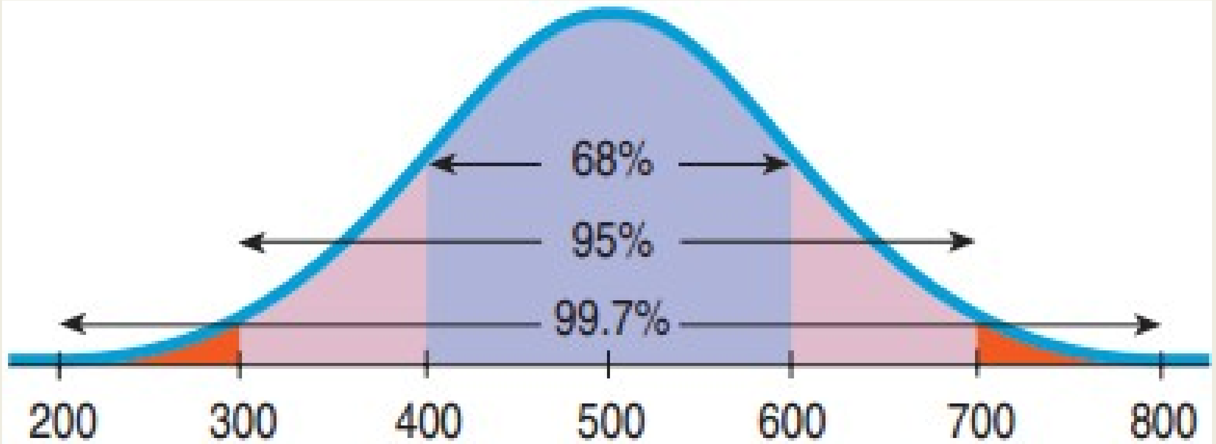
- Then sketch the Normal model.

Working With the 68-95-99.7 Rule

- Each part of the SAT has a mean of 500 and a standard deviation of 100. Assume the data is symmetric and unimodal. If you earned a 600 on one part of the SAT how do you stand among all others who took the SAT?

- **Plan:** The variable is quantitative and the distribution is symmetric and unimodal. Use the Normal model $N(500, 100)$.

Mechanics



- **Mechanics:**
  - Make a picture.
  - 600 is 1 standard deviations above the mean.
- **Conclusion:**
  - 68% lies within 1 standard deviation of the mean.
  - 100% - 68% = 32% are outside of 1 standard deviation of the mean.
  - Above 1 standard deviations is half of that.
    - 32% / 2 = 16%
  - Your score is higher than 84% of all scores on this test.

5.4

Finding Normal Percentiles

What if z is not −3, −2, −1, 0, 1, 2, or 3?

- If the data value we are trying to find using the Normal model does not have such a nice z-score, we will use a table.

- **Example:** Where do you stand if your SAT math score was 680? $\mu = 500, \sigma = 100$

- Note that the z-score is not an integer:

$$z = \frac{680 - 500}{100} = 1.8$$

The Z table

Look for the z-score on the table: 1.8

Look for the second decimal place.

Result: 0.9641

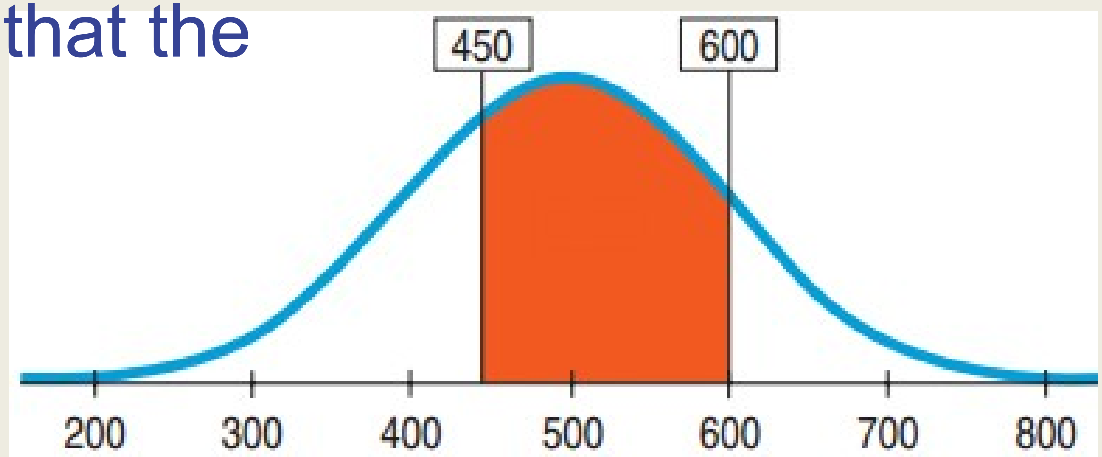
96.4% of SAT scores are below 680.

A Probability Involving "Between"

- What is the proportion of SAT scores that fall between 450 and 600? $\mu = 500, \sigma = 100$

  - **Plan:** Probability that *x* is between 450 and 600 = Probability that *x* < 600 – Probability that x < 450

  - **Variable:** We are told that the Normal model works. *N*(500, 100)



A Probability Involving "Between"

- What is the proportion of SAT scores that fall between 450 and 600? $\mu = 500, \sigma = 100$
- z = (600-500)/100 = 1   z=(450-500)/100= -0.5
- Probability that x is between 450 and 600

  = Probability that x < 600 – Probability that x < 450

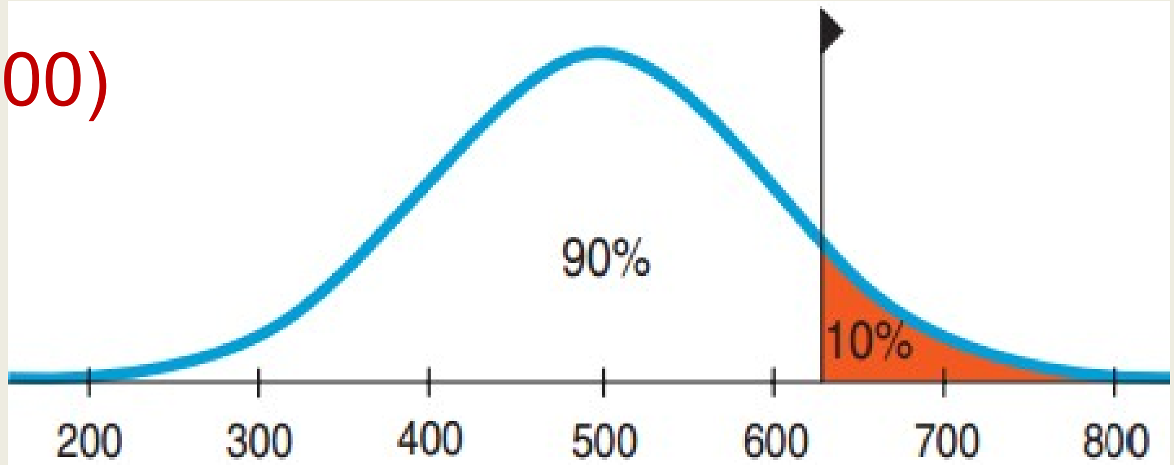
  = 0.8413 – 0.3085 = 0.5328
- Conclusion: The Normal model estimates that about 53.28% of SAT scores fall between 450 and 600.

From Percentiles to Scores: *z* in Reverse

- Suppose a college admits only people with SAT scores in the top 10%. How high a score does it take to be eligible? $\mu = 500$, $\sigma = 100$

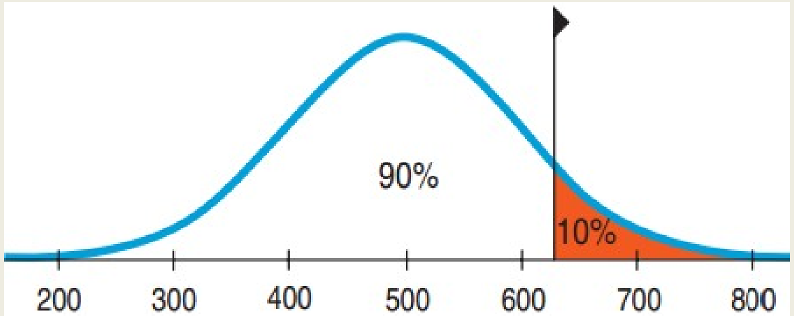
  - **Plan:** We are given the probability and want to go backwards to find *x*.

  - **Variable:** *N*(500, 100)



From Percentiles to Scores: *z* in Reverse

- Suppose a college admits only people with SAT scores in the top 10%. How high a score does it take to be eligible? $\mu = 500$, $\sigma = 100$



- z = 1.29
- (x-500)/100 = 1.29
- x= 1.29*100 +500 = 629
- Conclusion: Because the school wants the SAT Verbal scores in the top 10%, the cutoff is 629.

Underweight Cereal Boxes

- Based on experience, a manufacturer makes cereal boxes that fit the Normal model with mean 16.3 ounces and standard deviation 0.2 ounces, but the label reads 16.0 ounces. What fraction will be underweight?

- **Plan:** Find Probability that $x < 16.0$
- **Variable:** $N(16.3, 0.2)$

Underweight Cereal Boxes

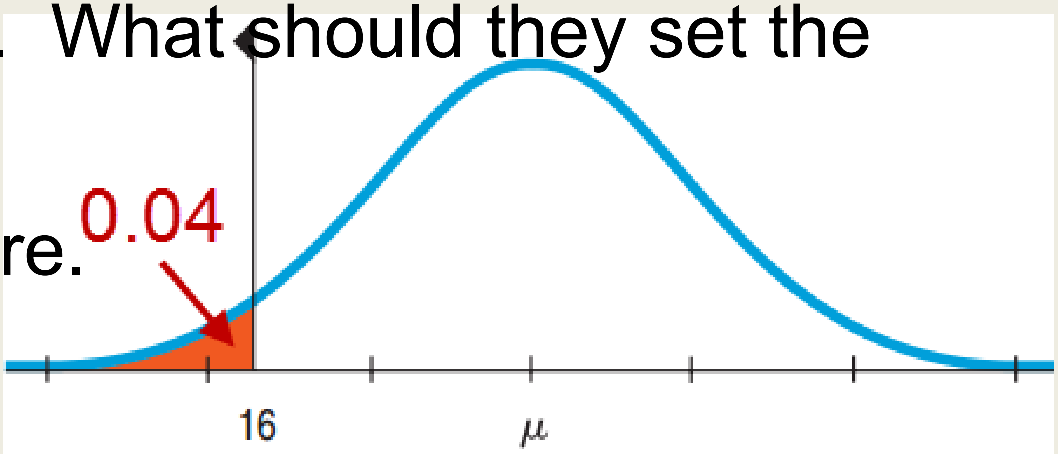
- What fraction of the cereal boxes will be underweight (less than 16.0)?
  $\mu = 16.3$, $\sigma = 0.2$

- $z = (16.0-16.3)/0.2 = -1.5$
- Probability $x < 16.0 = 0.0668$

- Conclusion: I estimate that approximately 6.7% of the boxes will contain less than 16.0 ounces of cereal.

Underweight Cereal Boxes Part II

- Lawyers say that 6.7% is too high and recommend that
  at most 4% be underweight. What should they set the mean at? $\sigma = 0.2$



- Mechanics: Sketch a picture.

- $z = -1.75$
- Find $16 + 1.75(0.02)$
  $= 16.35$ ounces

- Conclusion: The company must set the machine to average 16.35 ounces per box.

Underweight Cereal Boxes Part III

- The CEO vetoes that plan and sticks with a mean of 16.2 ounces and 4% weighing under 16.0 ounces. She demands a machine with a lower standard deviation. What standard deviation must the machine achieve?

- **Plan:** Find σ such that Probability x < 16.0 = 0.04.
- **Variable:** $N(16.2, ?)$

Underweight Cereal Boxes Part III

- What standard deviation must the machine achieve? $N(60.2, ?)$

- From before, $z = -1.75$

$$-1.75 = \frac{16.0 - 16.2}{\sigma}$$

- $1.75\sigma = 0.2, \quad \sigma = 0.114$

- Conclusion: The company must get the machine to box cereal with a standard deviation of no more than 0.114 ounces. The machine must be more consistent.

5.5

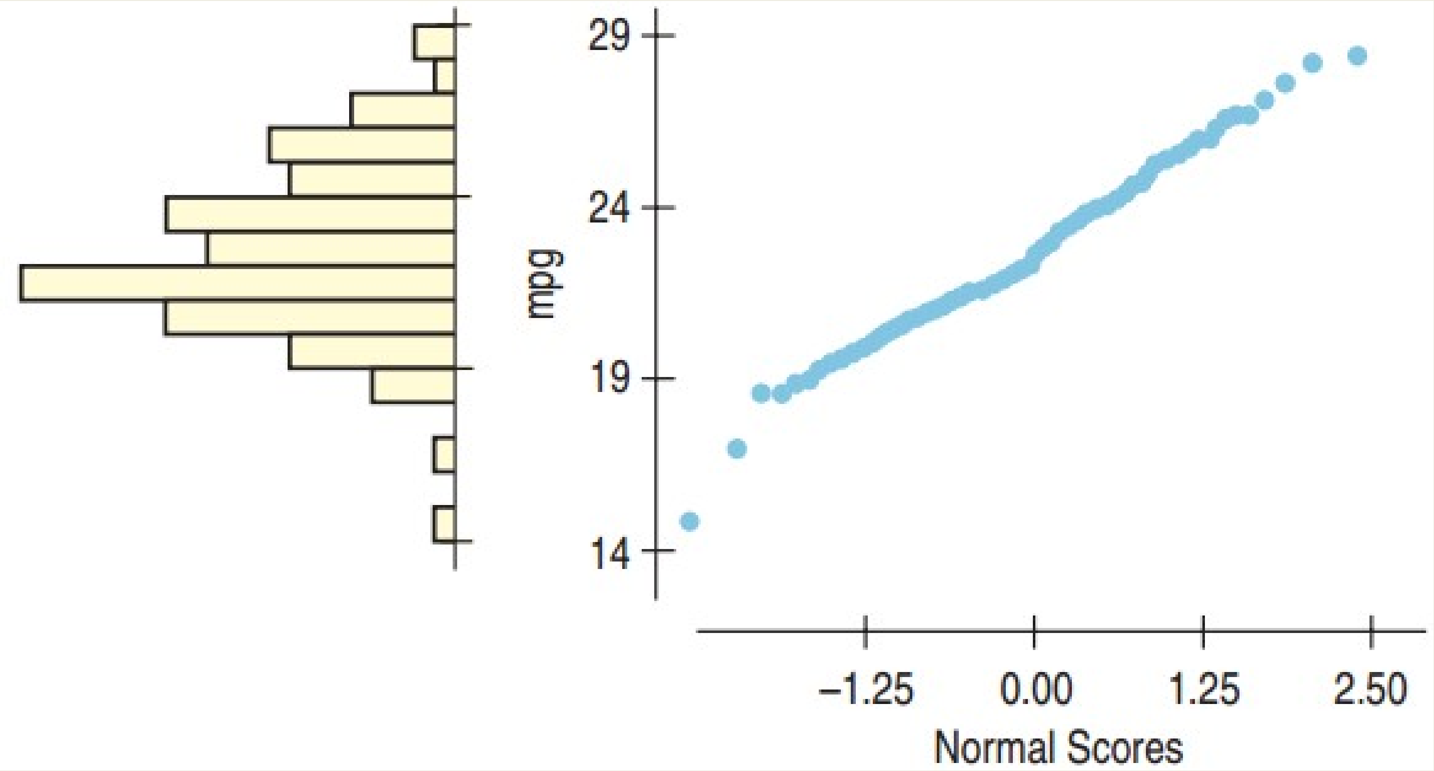
Normal Probability Plots

Checking if the Normal Model Applies

- A histogram will work, but there is an alternative method.

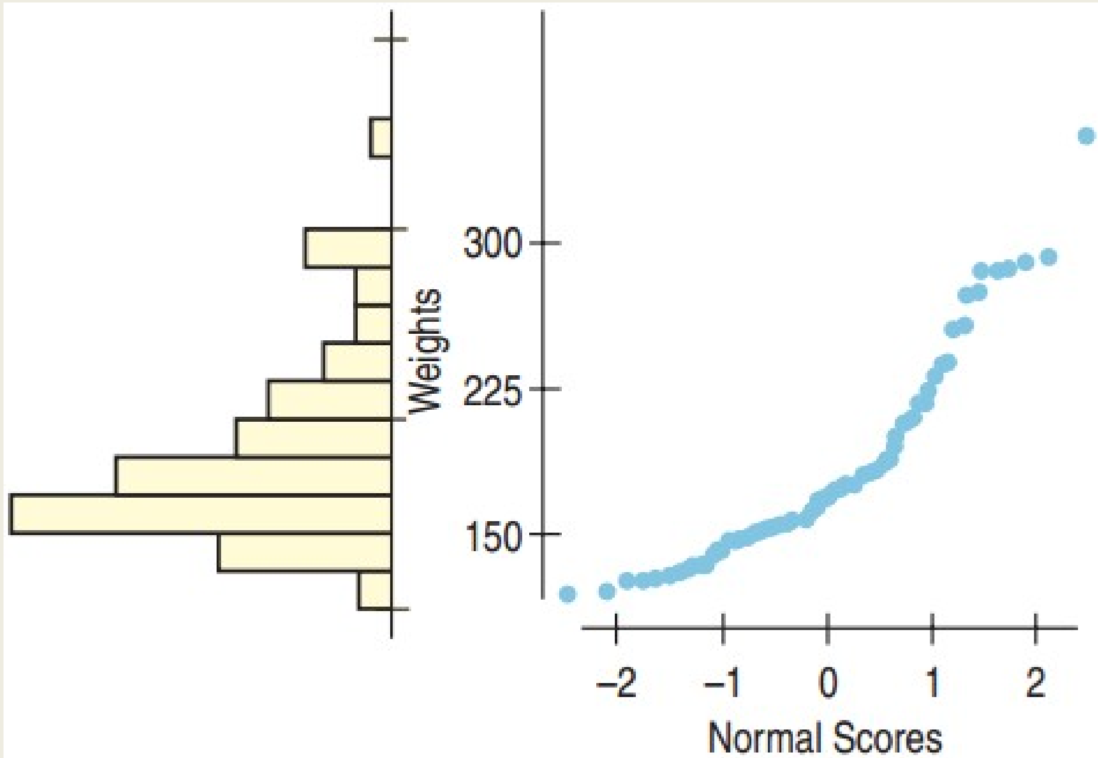
- Instead use a Normal Probability Plot.
  - Plots each value against the *z*-score that would be expected had the distribution been perfectly normal.
  - If the plot shows a line or is nearly straight, then the Normal model works.
  - If the plot strays from being a line, then the Normal model is not a good model.

The Normal Model Applies



- The Normal probability plot is nearly straight, so the Normal model applies. Note that the histogram is unimodal and somewhat symmetric.

The Normal Model Does Not Apply



- The Normal probability plot is not straight, so the Normal model does not apply applies. Note that the histogram is skewed right.

What Can Go Wrong

- Don't use the Normal model when the distribution is not unimodal and symmetric.
  - Always look at the picture first.
- Don't use the mean and standard deviation when outliers are present.
  - Check by making a picture.
- Don't round your results in the middle of the calculation.
  - Always wait until the end to round.
- Don't worry about minor differences in results.
  - Different rounding can produce slightly different results.