Quantitative Methods

Serena De Stefani – Lecture 6 – 7/17/2018

Lottery for the Dorms

57 students are in a lottery for the spacious triple dorm room. 20 were from the varsity team and all three winners were from this team.

- How likely is this? Was it rigged?

Conclusions

In the simulation, only 3 out of 100 were “All Varsity.” While 3% is only a small chance, it is not impossible. It looks pretty suspicious.

Is 3% a small enough chance to make a formal accusation?

Idea 1: Examine a Part of the Whole

The Goal

- Learn about the entire group of individuals (called the population)

The Problem

- It is usually impossible to collect data on the entire population.

The Compromise

- Collect data on a smaller group of individuals (called a sample) selected from the population.

Bias

The Challenge

- Obtain a sample that is perfectly representative of the population.

- Avoid bias – over or under emphasizing some characteristic of the population that is pertinent to the study.

Greek for Parameter, Latin for Statistic

Examples of parameters

- μ, σ, p, ρ, β

Examples of Statistics

- $\bar{y}$, s, $\hat{p}$, r, b

- Notice that π would be confusing as a parameter.

Name	Statistic	Parameter
Mean	$\bar{y}$	μ (mu, pronounced "meeoo," not "moo")
Standard Deviation	s	σ (sigma)
Correlation	r	ρ (rho, pronounced like "row")
Regression Coefficient	b	β (beta, pronounced "baytah"[5])
Proportion	$\hat{p}$	p (pronounced "pee"[6])

Idea 2: Randomize

Can we list the characteristics of the population and ensure we represent them all without bias?

- Race, age, ethnicity, income, marital status, work type, family size, …
- The list would go on forever. There are more types of people than the number of people.
- So… what can we do???

Randomizing can lead to a representative sample.

- Randomizing protects us from the influences of all the features of the population.
- On average, the sample will look like the population.

Idea 3: It’s the Sample Size

If you need 100 students to get a random sample at the university, how many Americans would you need to achieve the same level of randomness from the entire U.S.A.?

- Answer: 100
- It is the number of individuals, not the percent of individuals that matters.
- The number of individuals in the sample is called the sample size.

Simple Random Sampling

SRS

- Order the students from 1 to 40.
- Use a computer to randomly select 20 numbers from 1 to 40.
- Select the students with the chosen numbers.

Simple Random Sampling (SRS) is when every combination has an equally likely chance to be selected.

- SRS is the standard which all other sampling techniques are measured.
- Statistical theory is based on SRS.

Sampling Variability

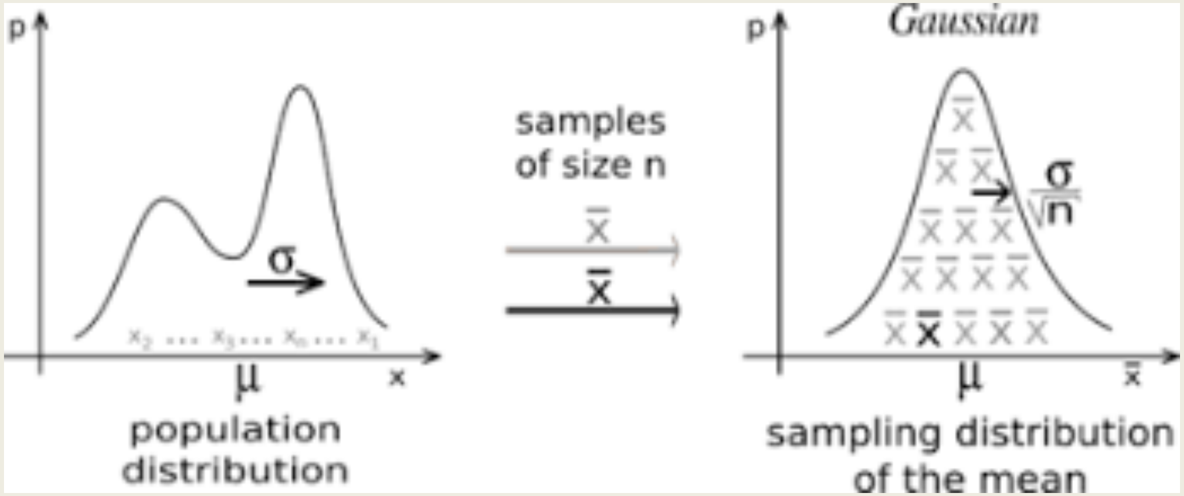
- Samples will vary from one to the next.
  - The first sample of five students' weight might average 131 pounds.
  - The second might average 138 pounds.

The sample to sample differences are called the **sampling variability** (or sampling error).

- Natural
- Not a problem

Central Limit Theorem

- As the number of samples increases, the distribution of the *means* of the *samples* becomes normal, regardless of the shape of the original population distribution



Classical Statistical Testing

- Why do we talk about:
  - Simulation (the dorm lottery)
  - Sampling

- Classical Statistical Testing is based on the idea of:
  - running a study on a representative sample → results A
  - "theoretically" repeating the study numerous times
  - obtaining a theoretical probability distribution
  - seeing how result A fares compares with this distribution

  - It's like running a simulation, except that we don't do it
  - Because of the Central Limit Theorem, we already know the distribution!

Chapter 13

From Randomness to Probability

13.1

Random Phenomena

Red Light, Green Light

Each day you drive through an intersection and check if the light is red, green, or yellow.

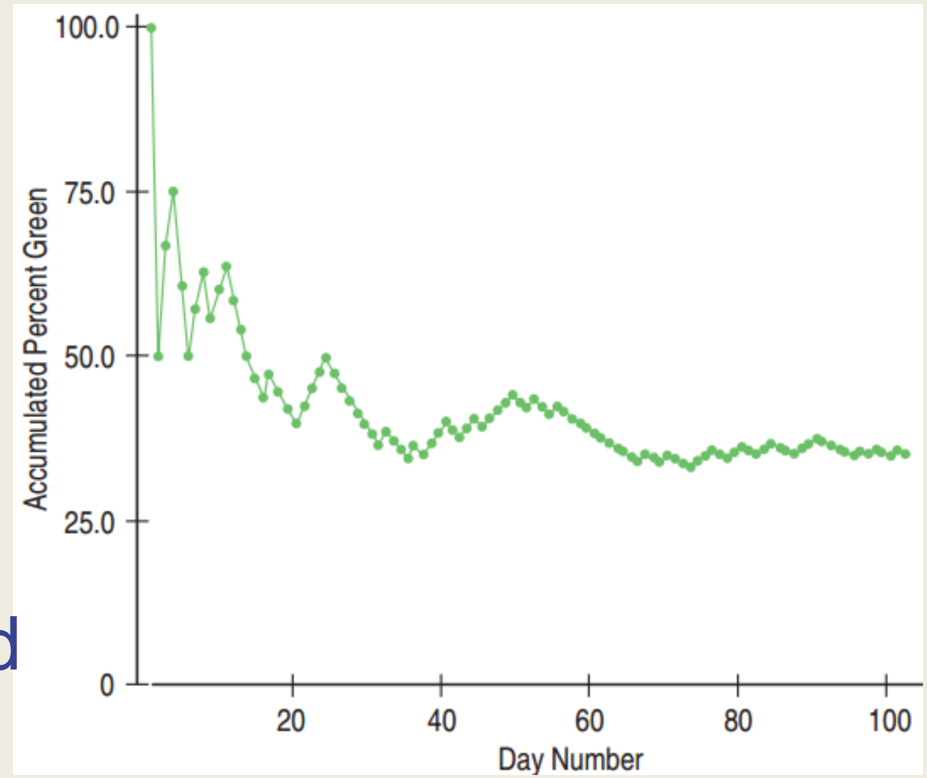
- Day 1: green
- Day 2: red
- Day 3: green

Before you begin, you know:

- An outcome will occur.
- The possible outcomes.

After you finish, you know:

- The outcomes that occurred



Random Phenomena Vocabulary

Trial

- Each occasion which we observe a random phenomena (the "thing" that is happening)

Outcome

- The value of the trial for the random phenomena (what *could* happen in a trial, specifically)

Event

- The combination of the trial's outcomes (what actually happened)

Sample Space

- The collection of all possible outcomes (what could ever possibly happen)

Flipping Two Coins

Trial

- The flipping of the two coins

Outcome

- Heads or tails for each flip

Event

- HT, for example

Sample Space

- **S** = {HH, HT, TH, TT}

The Law of Large Numbers

- If you flip a coin once, you will either get 100% heads or 0% heads.
- If you flip a coin 1000 times…
- …you will probably get close to 50% heads.

The **Law of Large Numbers** states that for many trials, the proportion of times an event occurs settles down to one number.

•This number is called the empirical probability.

The Law of Large Numbers Requirements

Identical Probabilities

- The probabilities for each event must remain the same for each trial.

Independence

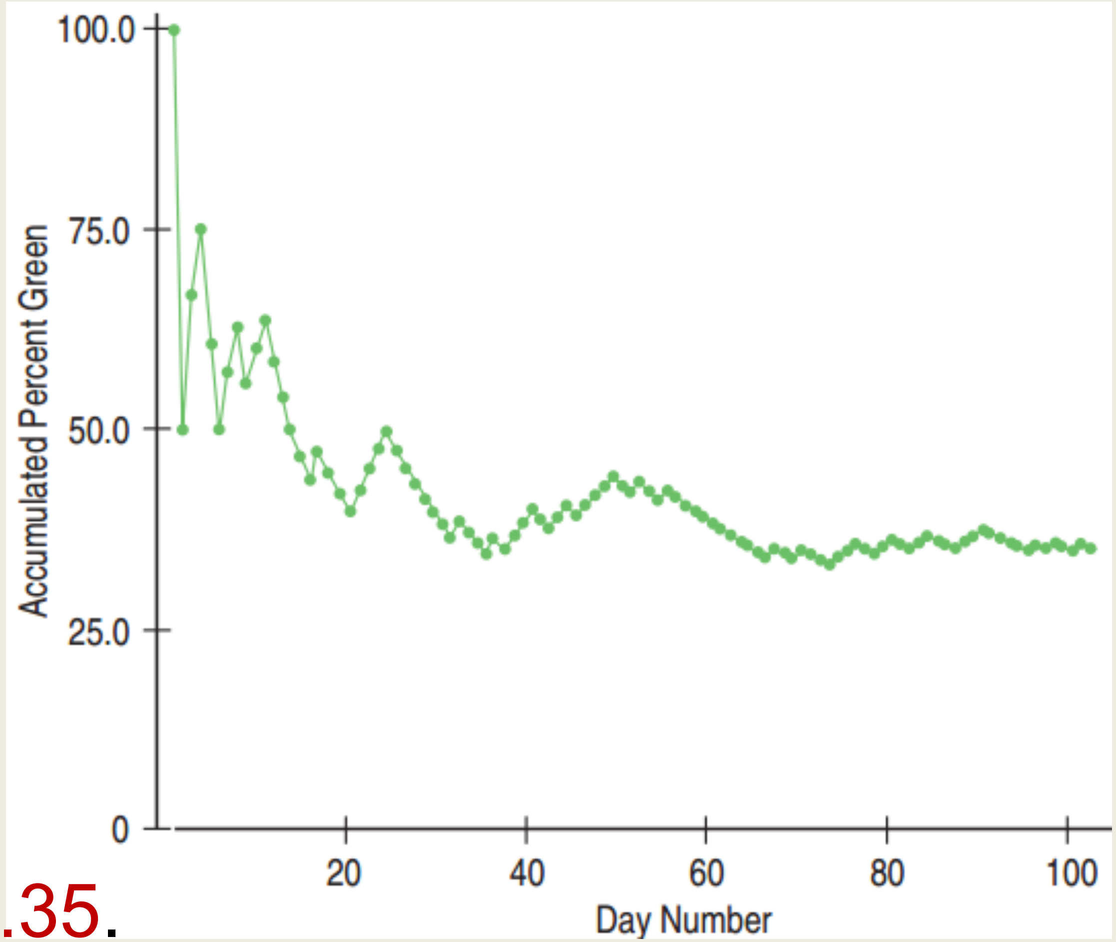
- The outcome of a trial is not influenced by the outcomes of the previous trials.

Empirical probability

- $P(\mathbf{A}) = \dfrac{\#\text{ times } \mathbf{A} \text{ occurs}}{\#\text{ of trials}}$ (in the long run)

Red Light, Green Light: The Law of Large Numbers

- After many days, the proportion of green lights encountered is approximately 0.35.
- *P*(green) = 0.35.
- If we recorded more days, the probability would still be about 0.35.



The Nonexistent Law of Averages

Wrong

- If you flip a coin 6 times and get 6 heads, then you are due for a tail on the next flip.
- You put 10 quarters in the slot machine and lose each time. You are just a bad luck person, so you have a smaller chance of winning on the 11^{th} try.

- There is no such thing as the Law of Averages for **short** runs.

13.2

Modeling Probability

Theoretical Probability

American Roulette

- 18 Red, 18 Black, 2 Green
- If you bet on Red, what is the probability of winning?

Theoretical Probability

- $$P(\mathbf{A}) = \frac{\text{\# of outcomes in } \mathbf{A}}{\text{\# of possible outcomes}}$$
- $$P(\text{red}) = \frac{18}{38}$$

Heads or Tails

Flip 2 coins. Find *P*(HH)

- List the sample space:
  - **S** = {HH, HT, TH, TT}
- *P*(HH) = ¼

Flip 100 coins. Find the probability of all heads.

- The sample space would involve 1,267,650,600,228,229,401,496,703,205,376 different outcomes.
- Later, we will see a better way.

Equally Likely?

What's wrong with this logic?

- Randomly pick two people.
- Find the probability that both are left-handed.
- Sample Space

  S = {LL, LR, RL, RR}
- $P(\text{LL}) = 1/4$

Since left-handed and right-handed are not equally likely, this method does not work.

Personal Probability

What's your chance of getting an A in a new course you are taking?

- You cannot base this on your long-run experience.
- There is no sample space of events with equal probabilities to list.
- You can only base your answer on subjective experience and guesswork.
- Probabilities based on personal experience rather than long-run relative frequencies or equally likely events are called personal probabilities.

13.3

Formal Probability

Rules 1 and 2

Rule 1: $0 \le P(\mathbf{A}) \le 1$

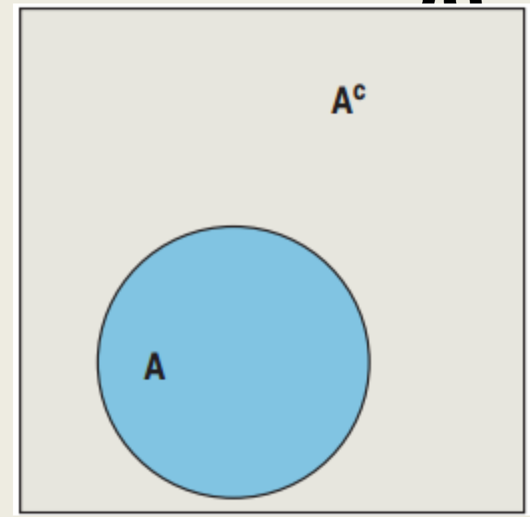
- You can't have a −25% chance of winning.
- A 120% chance also makes no sense.

- **Note:** Probabilities are written in decimals.
  - 45% chance → $P(\mathbf{A}) = 0.45$

Rule 2: $P(\mathbf{S}) = 1$

- The set of all possible outcomes has probability 1.
  - There is a 100% chance that you will get a head or a tail.

Rule 3: The Complement Rule



Complements

- Define $\mathbf{A^C}$ as the complement of **A**.

- $\mathbf{A^C}$ is the event of "A not happening".
  - If **A** is the event of rolling a 5 on a six sided die, then $\mathbf{A^C}$ is the event of "not rolling a 5": {1, 2, 3, 4, 6}
  - $P(\mathbf{A}) = 1/6$. $P(\mathbf{A^C}) = ?$
  - $P(\mathbf{A^C}) = 5/6 = 1 – 1/6$

The Rule of Complements: $P(\mathbf{A^C}) = 1 – P(\mathbf{A})$

Red Light Green Light and Complements

We know that P(green) = 0.35. Find P(not green)

- **Not green** is the complement of **green**.

- Use the rule of complements:
  - $P(\text{not green}) = P(\text{green}^C)$
    $= 1 – P(\text{green})$
    $= 1 – 0.35$
    $= 0.65$

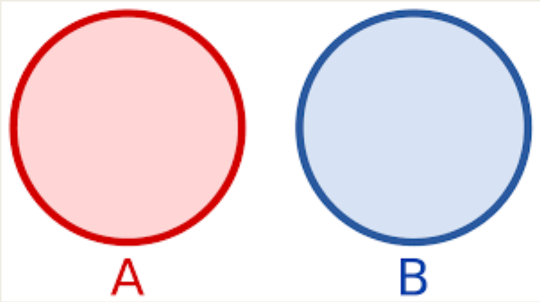
- The probability of the light not being green is 0.65.

Rule 4: The Addition Rule

Suppose

P(sophomore) = 0.2 and P(junior) = 0.3

- Find P(sophomore OR junior)
- Solution: 0.2 + 0.3 = 0.5
- This works because sophomore and junior are **disjoint events**. They have no outcomes in common.



The Addition Rule

- If **A** and **B** are **disjoint** events, then

P(**A** OR **B**) = P(**A**) + P(**B**)

Red Light, Green Light, Yellow Light

Given that $P(\text{green}) = 0.35$ and $P(\text{yellow}) = 0.04$

- Find $P(\text{red})$.
- **Solution:** Use the Rule of Complements and the Addition Rule.
- $$\begin{aligned} P(\text{red}) &= 1 - P(\text{red}^C) \\ &= 1 - P(\text{green OR yellow}) \\ &= 1 - [P(\text{green}) + P(\text{yellow})] \\ &= 1 - [0.35 + 0.04] \\ &= 1 - 0.39 \\ &= 0.61 \end{aligned}$$

The Sum of Probabilities

The sum of all the probabilities of every disjoint event must equal 1.

- What's wrong with the following statement?
  - Probabilities for freshmen, sophomore, junior, senior are: 0.25, 0.23, 0.22, 0.20.
  - 0.25 + 0.23 + 0.22 + 0.20 = 0.90
  - Since they do not add to 1, something is wrong.
- How about the following?
  - *P*(owning a smartphone) = 0.5 and *P*(owning a computer) = 0.9
- This is fine, since they are **not disjoint**.

Rule 5: The Multiplication Rule

The probability that an Atlanta to Houston flight is on time is 0.85.

- If you have to fly every Monday, find the probability that your first two Monday flights will be on time.

Multiplication Rule: For **independent** events **A** and **B**:

$$P(\mathbf{A} \text{ AND } \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B})$$

- $P(1^{st} \text{ on time AND } 2^{nd} \text{ on time})$

$$= P(\text{1st on time}) \times P(\text{2nd on time})$$
$$= 0.85 \times 0.85$$
$$= 0.7225$$

Red Light AND Green Light AND Yellow Light

Find the probability that the light will be red on Monday, green on Tuesday, and yellow on Wednesday.

- The multiplication rule works for more than 2 events.

- P(red Mon. AND green Tues. AND yellow Wed.)

  $= P(\text{red Mon.}) \times P(\text{green Tues.}) \times P(\text{yellow Wed.})$

  $= 0.61 \times 0.35 \times 0.04$

  $= 0.00854$

At Least One Red Light

Find the probability that the light will be red at least one time during the week.

- Use the Complement Rule.
- P(at least 1 red)

$= 1 – P(\text{no reds})$

$= 1 – (0.39 \times 0.39 \times 0.39 \times 0.39 \times 0.39 \times 0.39 \times 0.39)$

≈ 0.9986

Japanese M&M Colors Desires: 38% Pink, 36% Teal, 16% Purple

- Find the probability that one **Japanese survey respondent** will want either **pink or teal**.
- Find the probability that two **Japanese survey respondents** will want **purple**.
- Find the probability that at least one of three **Japanese survey respondents** will want **purple**.

M&Ms: 38% Pink, 36% Teal, 16% Purple

- First notice that

  $0.38 + 0.36 + 0.16 = 0.9 \neq 1$
- There must be other colors.

Question: ***P*(pink OR teal)**

- **Plan:** Pink and teal are disjoint. Apply the Addition Rule.

M&Ms: 38% Pink, 36% Teal, 16% Purple

Question: *P*(pink OR teal)

- **Mechanics:** $P(\text{pink OR teal}) = P(\text{pink}) + P(\text{teal})$
  $= 0.38 + 0.36$
  $= 0.74$
- **Conclusion:** The probability that the respondent chose pink or teal is 0.74.

M&Ms: 38% Pink, 36% Teal, 16% Purple

Question: P(1st purple AND 2nd purple)

- **Plan:** The choice made by the first respondent does not affect the choice of the other. The events are independent.
- **Mechanics:** P(1st purple AND 2nd purple)
  = P(1st purple) × P(2nd purple)
  = 0.16 × 0.16 = 0.0256
- **Conclusion:** The probability that both choose purple is 0.0256.

M&Ms: 38% Pink, 36% Teal, 16% Purple

Question: P(at least 1 of 3 chose purple)

- **Plan:** Use the Rule of Complements and the Multiplication Rule. Note independence.

- $P(\text{at least 1 of 3}) = 1 - P([\text{at least 1 of 3}]^C)$
  $= 1 - P(\text{none of 3 chose purple})$
  $= 1 - [P(\text{purple}^C)]^3$

- $P(\text{purple}^C) = 1 - P(\text{purple})$

M&Ms: 38% Pink, 36% Teal, 16% Purple

Question: *P*(at least 1 of 3 chose purple)

- **Mechanics:**
- $P(\text{at least 1 of 3 purple}) = 1 - [P(\text{purple}^C)]^3$

$$= 1 - [1 - 0.16]^3$$
$$= 1 - (0.84)^3$$
$$= 0.4073$$

- **Conclusion:** There's about a 40.7% chance that at least one of the respondents chose purple.

What Can Go Wrong?

- Beware of probabilities that don't add up to 1.
  - If they add to <u>less than 1</u>, look for another category.
  - If they add to <u>more than 1</u>, maybe they are not disjoint.
- Don't add probabilities of events if they are not disjoint.
  - Events must be disjoint to use the Addition Rule.

What Can Go Wrong?

- Don't multiply probabilities of events if they are not independent.
  - *P*(over 6' *and* on basketball team) is not equal to *P*(over 6') × *P*(on basketball team)
- Don't confuse disjoint and independent
  - Disjoint events are never independent.

Chapter 14

Probability Part 2

14.1

The General Addition Rule

Or But Not Disjoint

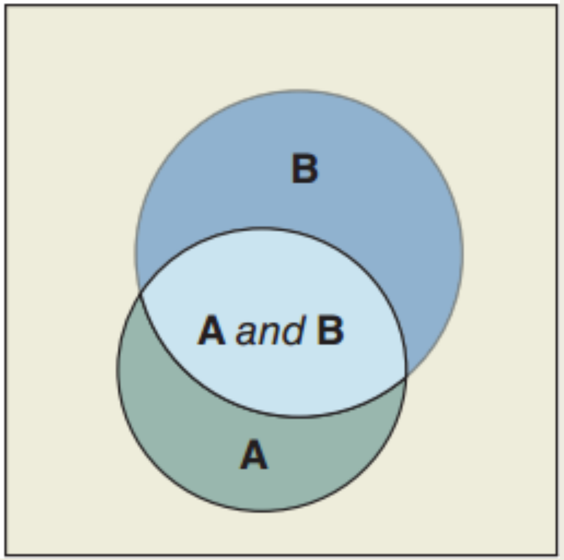
Your Wallet

- **S** = {$1, $2, $5, $10, $20, $50, $100}
- **A** = {odd numbered value} = {$1, $5}
- **B** = {bill with a building} = {$5, $10, $20, $50, $100}

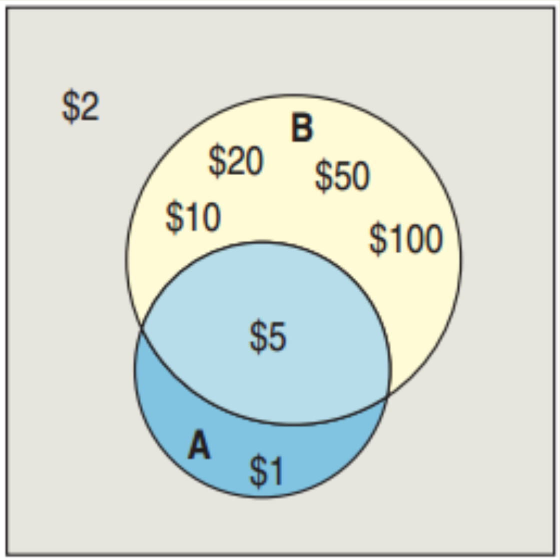
- Why is *P*(**A** *or* **B**) ≠ *P*(**A**) + *P*(**B**)?
- Answer: **A** and **B** are **not disjoint**.
- The intersection **A** *and* **B** = {$5} is double counted.
- To find *P*(**A** *or* **B**), subtract *P*(**A** *and* **B**).

The General Addition Rule

$P(\mathbf{A} \text{ or } \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A} \text{ and } \mathbf{B})$



- **The General Addition Rule in words:** Add the probabilities of the two events and then subtract the probability of their intersection.



P(odd amount *or* bill with a building)

$= P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A} \text{ and } \mathbf{B}\}$

$= P(\{\$1, \$5\}) + P(\{\$5, \$10, \$20, \$50, \$100\}) - P(\{\$5\})$

General Addition Rule Example

Survey

- Are you currently in a relationship?
- Are you involved in sports?

Results

- 33% are in a relationship.
- 25% are involved in sports.
- 11% answered “yes” to both.

Problem

- Find the probability that a randomly selected student is in a relationship *or* is involved in sports.

33% Relationship, 25% Sport, 11% Both

Events

- **R** = {in a relationship}
- **S** = {involved in sports}

Calculations

- $P(\mathbf{R} \text{ or } \mathbf{S}) = P(\mathbf{R}) + P(\mathbf{S}) - P(\mathbf{R} \text{ and } \mathbf{S})$

  $= 0.33 + 0.25 - 0.11$

  $= 0.47$

Conclusion

- There is a 47% chance that a randomly selected student is in a relationship or is involved sports.

Using Venn Diagrams

P(not in relationship *and* no sports)

- *P*(**R**^{C} *and* **S**^{C})
- This is the part outside of both circles: 0.53.

P(in a relationship *but* no sports)

- *P*(**R** *and* **S**^{C})
- This is the part in the circle **R** that is outside **S**: 0.22.

P(in a relationship *or* involved in sports *but* not both)

- *P*((**R** *and* **S**^{C}) *or* (**R**^{C} *and* **S**))
- This is the combination of the circles minus the intersection: 0.22 + 0.14 = 0.36

Facebook or Twitter?

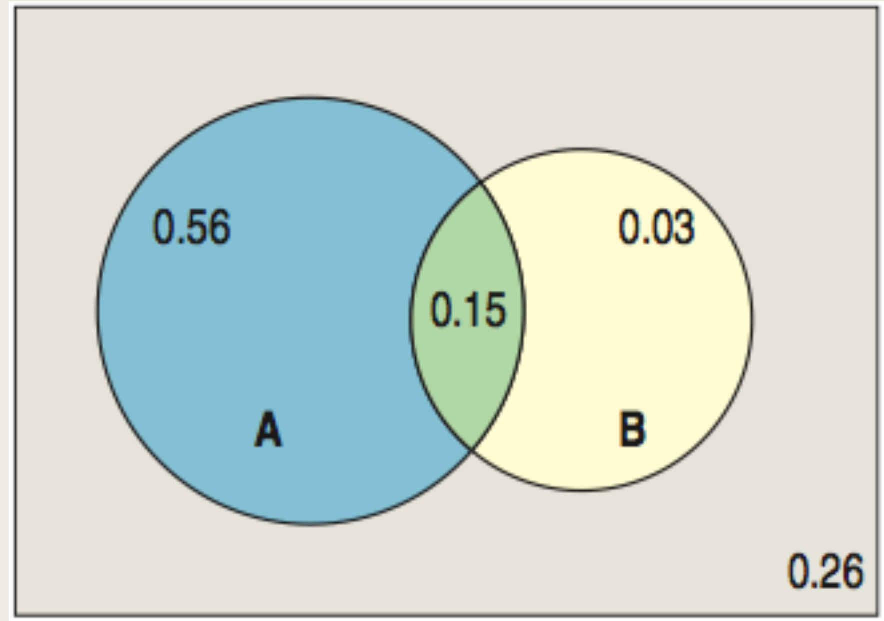
71% use Facebook, 18% Twitter, 15% both

- What is the probability that a randomly selected person:

1. Uses either Facebook or Twitter?
2. Uses either Facebook or Twitter, but not both?
3. Doesn't use Facebook or Twitter?

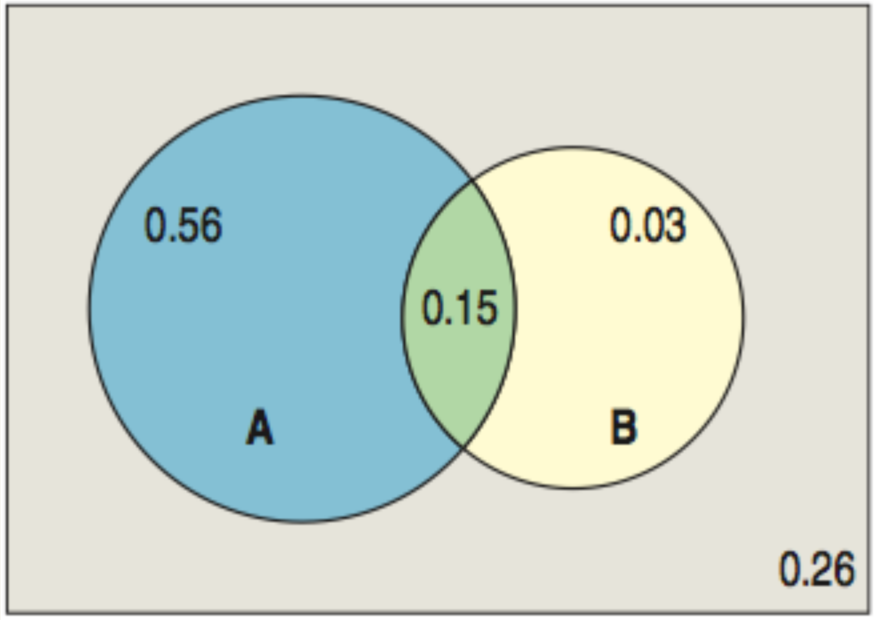
- Plan:
  - **A** = {uses Facebook}
  - **B** = {uses Twitter}

71% Facebook, 18% Twitter, 15% both



- Plot:
  - $P(\mathbf{A}) = 0.71$
  - $P(\mathbf{B}) = 0.18$
  - $P(\mathbf{A} \textit{ and } \mathbf{B}) = 0.15$
  - $P(\mathbf{A} \textit{ and } \mathbf{B}^C) = 0.71 – 0.15 = 0.56$
  - $P(\mathbf{A}^C \textit{ and } \mathbf{B}) = 0.18 – 0.15 = 0.03$
  - $P(\mathbf{A}^C \textit{ and } \mathbf{B}^C) = 1 – (0.56 + 015 + 0.03) = 0.26$

71% Facebook, 18% Twitter, 15% both



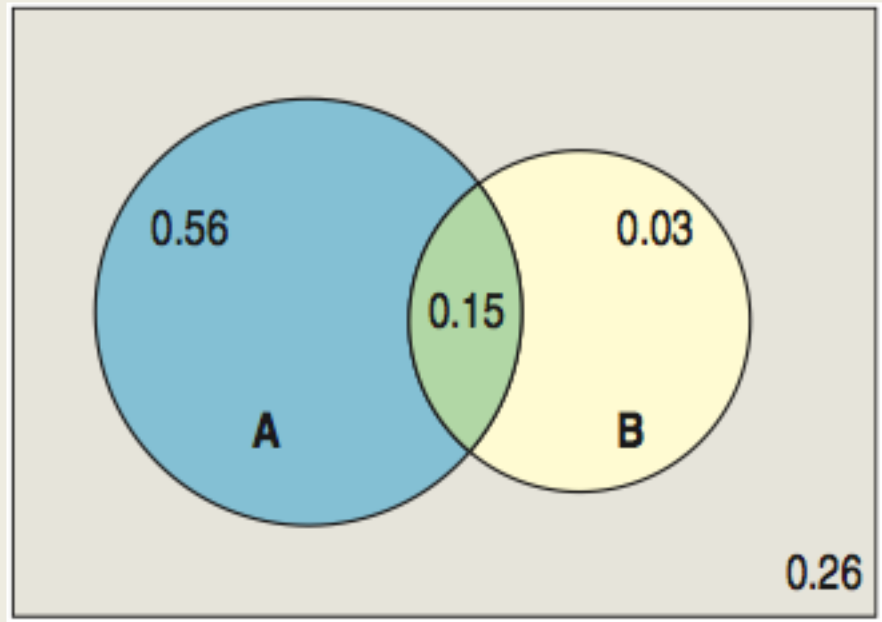
1. *P*(Facebook or Twitter)

- **Mechanics**
  - $P(\mathbf{A}\ or\ \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A}\ and\ \mathbf{B})$
    $= 0.71 + 0.18 - 0.15$
    $= 0.74$

- **Conclusion:** 74% use Facebook or Twitter.

71% Facebook, 18% Twitter, 15% both

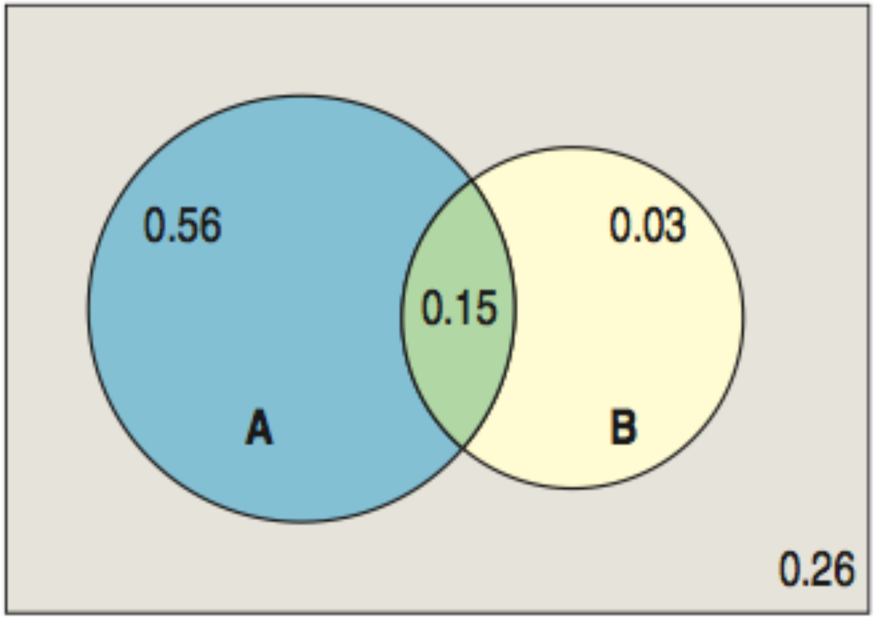
2. *P*(use Facebook or Twitter *but* not both)



- **Mechanics**
  - *P*(**A** *or* **B**) – *P*(**A** *and* **B**) = 0.74 – 0.15 = 0.59
  - *P*((**A** *and* $\mathbf{B}^{\mathbf{C}}$) or ($\mathbf{A}^{\mathbf{C}}$ *and* **B**)) = 0.56 + 0.03 = 0.59

- **Conclusion:** 59% use Facebook or Twitter, but not both.

71% Facebook, 18% Twitter, 15% both



3. *P*(neither Facebook *nor* Twitter)

- **Mechanics**
  - 1 – $P(\boldsymbol{A} \text{ or } \boldsymbol{B})$ = 1 – 0.74 = 0.26
  - $P(\mathbf{A}^C \textit{ and } \mathbf{B}^C) = 0.26$
- **Conclusion:** 26% neither use Facebook nor Twitter.

14.2

Conditional Probability and the General Multiplication Rule

Contingency Table

		Goals			
		Grades	Popular	Sports	Total
Sex	Boy	117	50	60	227
	Girl	130	91	30	251
	Total	247	141	90	478

A table that displays the results of two categorical questions is called a **contingency table**.

- P(girl) = 251/478 = 0.525
- P(girl *and* popular) = 91/478 = 0.190
- P(sports) = 90/478 = 0.188

Conditional Probability

		Goals			
		Grades	Popular	Sports	Total
Sex	Boy	117	50	60	227
	Girl	130	91	30	251
	Total	247	141	90	478

- What if we knew the chosen person was a girl? Would that change the probability that the girl's goal was sports?
- Yes! We write *P*(sports | girl)
- Only look at Girl row: *P*(sports | girl) = 30/251 = 0.120
- Find the probability of selecting a boy given the goal is grades.
- *P*(boy | grades) = 117/247 = 0.474

Conditional Probability Formula

		Goals			
Sex		Grades	Popular	Sports	Total
	Boy	117	50	60	227
	Girl	130	91	30	251
	Total	247	141	90	478

Probability of **B** *Given* **A**:

- $$P(\mathbf{B} \mid \mathbf{A}) = \frac{P(\mathbf{A} \text{ and } \mathbf{B})}{P(\mathbf{A})}$$
- Example:

$$P(\text{girl} \mid \text{popular}) = \frac{P(\text{girl } and \text{ popular})}{P(\text{popular})}$$

$$= \frac{91/478}{141/478}$$

$$= \frac{91}{141} = 0.65$$

College Students: Relationships and Sports

33% in relationships, 25% in sports, 11% in both

- If you see a student athlete, what is the probability that this athlete is in a relationship?
- $P(\mathbf{R}) = 0.33$, $P(\mathbf{S}) = 0.25$, $P(\mathbf{R} \text{ and } \mathbf{S}) = 0.11$
- $P(\mathbf{R} \mid \mathbf{S}) = \dfrac{P(\mathbf{R} \text{ and } \mathbf{S})}{P(\mathbf{S})} = \dfrac{0.11}{0.25} = 0.44$
- There is a 0.44 probability the student involved in sports is in a relationship. Notice this is higher than the probability for the general student (0.33).

The General Multiplication Rule

- For **A** and **B** **independent**, we had:

  $P(\mathbf{A}\ \mathit{and}\ \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B})$

- Rearranging the conditional probability equation, we get the **General Multiplication Rule**:

  $P(\mathbf{A}\ \mathit{and}\ \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B} \mid \mathbf{A})$

- Equivalently,

  $P(\mathbf{A}\ \mathit{and}\ \mathbf{B}) = P(\mathbf{B}) \times P(\mathbf{A} \mid \mathbf{B})$

14.3

Independence

Definition of Independence

- Events **A** and **B** are independent if knowing **A** happened does not change the probability of **B**. In symbols:

  A and **B** are independent $\leftrightarrow$ $P(\mathbf{B} \mid \mathbf{A}) = P(\mathbf{B})$

- Equivalent formulas for independence:
  - $P(\mathbf{A} \mid \mathbf{B}) = P(\mathbf{A})$
  - $P(\mathbf{A} \textit{ and } \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B})$

Grades and Girl Independent?

		Goals			
		Grades	Popular	Sports	Total
Sex	Boy	117	50	60	227
	Girl	130	91	30	251
	Total	247	141	90	478

- Determine if the "goal of good grades" and gender are independent.
  - P(grades | girl) = 130/251 ≈ 0.52
  - P(grades) = 247/478 ≈ 0.52
- To two decimal places, they are independent.

- Are the "goal of sports" and gender independent?
  - P(sports | boy) = 60/227 ≈ 0.26
  - P(sports) = 90/478 ≈ 0.19
- No, the goal of sports and gender are dependent.

Relationships, Sports, and Independence

33% in a relationship, 25% involved in sports, 11% both

- Are being in a relationship and being involved in sports independent?
  - *P*(relationship) = 0.33
  - *P*(sports) = 0.25
  - *P*(relationship *and* sports) = 0.11
  - $0.33 \times 0.25 = 0.0825 \neq 0.11$
  - No, they are **dependent**.
- Are they disjoint?
  - *P*(relationship *and* sports) = $0.11 \neq 0$
  - No, they are **not disjoint**.

Independent ≠ Disjoint

Disjoint events cannot be independent.

- Consider the events:
  - Course grade A
  - Course grade B
  - Disjoint: You can't get both.
  - Not independent: $P(\mathbf{A} \mid \mathbf{B}) = 0 \neq P(\mathbf{A})$
  - A and B are disjoint (also called mutually exclusive) but not independent.

Independence and Insurance

An insurance company insures 1000 houses

- If each house is in a different location, then the destruction of each can be treated as independent.
- P(many are destroyed this year) is very small.
- If they are all in the same location, then they can all be destroyed by the same disaster. The events are not independent.
- P(many are destroyed this year) is not as small.
- The insurance company has less risk with the former.

14.4

Picturing Probability:
Tables, Venn Diagrams, and Trees

Marginal and Joint Probabilities

- 71% use Facebook, 18% Twitter, 15% both
- Draw a partial table.
- 0.71 and 0.18 are called marginal probabilities.
- 0.15 is a joint probability.
- How can we complete the table?
- The sum must add up
  - 0.15 + ? = 0.71
  - 0.18 + ? = 1.00

		Use Facebook		
		Yes	No	Total
Use Twitter	Yes	0.15		0.18
	No			
	Total	0.71		1.00

		Use Facebook		
		Yes	No	Total
Use Twitter	Yes	0.15	0.03	0.18
	No	0.56	0.26	0.82
	Total	0.71	0.29	1.00

71% Facebook, 18% Twitter, 15% both

1. Facebook and Twitter mutually exclusive?
2. Facebook and Twitter independent?

- **Plan**
  - **A** = {uses Facebook}
  - **B** = {uses Twitter}
  - $P(\mathbf{A}) = 0.71$, $P(\mathbf{B}) = 0.18$, $P(\mathbf{A} \text{ and } \mathbf{B}) = 0.15$

71% Facebook, 18% Twitter, 15% both

1. Facebook and Twitter mutually exclusive?

- **Mechanics**
  - $P(\mathbf{A} \text{ and } \mathbf{B}) = 0.15 \neq 0$
  - **Conclusion:** Facebook and Twitter are not mutually exclusive.

71% Facebook, 18% Twitter, 15% both

1. Facebook and Twitter independent?

		Use Facebook		
		Yes	No	Total
Use Twitter	Yes	0.15	0.03	0.18
	No	0.56	0.26	0.82
	Total	0.71	0.29	1.00

- **Plan:** Make a table.
- **Mechanics:** $P(\mathbf{T} \mid \mathbf{F}) = 0.15/0.71 \approx 0.21$
  $P(\mathbf{T}) = 0.18 \neq 0.21$
  Not independent
- **Conclusion:** Since the respondents who use Facebook are more likely to tweet, they are not independent.