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1 #include <iostream>
2 #include <vector>
3 #include <limits> // For numeric_limits
4
5 using namespace std;
6
7 // Function to compute the optimal BST cost and structure
8 void optimalBST(const vector<int>& keys, const vector<double>& prob, int n, vector<
9 vector<int> >& root) {
10     vector< vector<double> > cost(n, vector<double>(n, 0));
11
12     // Initialize cost and root for single-key trees
13     for (int i = 0; i < n; i++) {
14         cost[i][i] = prob[i];
15         root[i][i] = i;
16     }
17
18     // Compute optimal costs for larger subtrees
19     for (int len = 2; len <= n; len++) { // Tree size
20         for (int i = 0; i <= n - len; i++) {
21             int j = i + len - 1;
22             cost[i][j] = numeric_limits<double>::max(); // Set to a large value
23
24             // Compute the total probability sum for the range
25             double totalWeight = 0;
26             for (int k = i; k <= j; k++) {
27                 totalWeight += prob[k];
28             }
29
30             // Try each key as a root and find the minimum cost
31             for (int k = i; k <= j; k++) {
32                 double leftCost = (k > i) ? cost[i][k - 1] : 0;
33                 double rightCost = (k < j) ? cost[k + 1][j] : 0;
34                 double totalCost = leftCost + rightCost + totalWeight;
35
36                 // Store the minimum cost and corresponding root
37                 if (totalCost < cost[i][j]) {
38                     cost[i][j] = totalCost;
39                     root[i][j] = k;
40                 }
41             }
42         }
43     }
44
45     cout << "Minimum search cost: " << cost[0][n - 1] << endl;
46 }
47
48 // Function to print the Optimal BST structure
49 void printBST(const vector<int>& keys, const vector< vector<int> >& root, int i, int
50 j, int parent, bool isLeft) {
51     if (i > j) return;
52
53     int r = root[i][j]; // Root of the subtree

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54         cout << "Key " << keys[r] << " is the ROOT" << endl;
55     else if (isLeft)
56         cout << "Key " << keys[r] << " is the LEFT child of " << keys[parent] <<
57     endl;
58     else
59         cout << "Key " << keys[r] << " is the RIGHT child of " << keys[parent] <<
60     endl;
61
62     // Recur for left and right subtrees
63     printBST(keys, root, i, r - 1, r, true);
64     printBST(keys, root, r + 1, j, r, false);
65 }
66
67 int main() {
68     // Sorted keys and corresponding search probabilities
69     int keyArr[] = {10, 20, 30, 40};
70     double probArr[] = {0.4, 0.3, 0.2, 0.1};
71     int n = sizeof(keyArr) / sizeof(keyArr[0]);
72
73     // Convert arrays to vectors (C++98-compatible)
74     vector<int> keys(keyArr, keyArr + n);
75     vector<double> prob(probArr, probArr + n);
76     vector<vector<int>> root(n, vector<int>(n, 0));
77
78     // Compute the optimal BST
79     optimalBST(keys, prob, n, root);
80
81     // Print the structure of the Optimal BST
82     cout << "\nOptimal Binary Search Tree structure:\n";
83     printBST(keys, root, 0, n - 1, -1, false);
84
85     return 0;
86 }

```

OutPut :

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D:\SE Computer\LAB CODES\DSA\DSA8.exe
Minimum search cost: 1.8

Optimal Binary Search Tree structure:
Key 20 is the ROOT
Key 10 is the LEFT child of 20
Key 30 is the RIGHT child of 20
Key 40 is the RIGHT child of 30

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Process exited after 0.2079 seconds with return value 0
Press any key to continue . . .

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