First-Order Logic

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Outline

Translate the following sentences

- Every gardener likes the sun.

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    - $\forall$ x. gardener(x) $\Rightarrow$ likes(x,Sun)

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  - $\exists$ x. (person(x) $\wedge$ $\forall$ t.(time(t) $\Rightarrow$ can-fool(x,t)))

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- All purple mushrooms are poisonous.

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  - $\forall$ x. (person(x) $\Rightarrow$ $\exists$ t. (time(t) $\wedge$ can-fool(x,t)))
- All purple mushrooms are poisonous.
  - $\forall$ x. (mushroom(x) $\wedge$ purple(x)) $\Rightarrow$ poisonous(x)
- There are exactly two purple mushrooms.
  - ($\exists$ x.$\exists$ y. mushroom(x) $\wedge$ purple(x) $\wedge$ mushroom(y) $\wedge$ purple(y) $\wedge$ -(x=y)) $\wedge$ ($\forall$ z. (mushroom(z) $\wedge$ purple(z)) $\Rightarrow$ ((x=z) $\vee$ (y=z)))

Determine a valid substitution

- parents(x, father(x), mother(Bill)); parents(Bill, father(Bill), y)

Determine a valid substitution

- parents(x, father(x), mother(Bill)); parents(Bill, father(Bill), y)
  - {x/Bill, y/mother(Bill)}

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Determine a valid substitution

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  - {x/Bill, y/mother(Bill)}
- parents(x, father(x), mother(Bill)); parents(Bill, father(y), z)
  - {x/Bill, y/Bill, z/mother(Bill)}
- parents(x, father(x), mother(Jane)); parents(Bill, father(y), mother(y))

Determine a valid substitution

- parents(x, father(x), mother(Bill)); parents(Bill, father(Bill), y)
  - {x/Bill, y/mother(Bill)}
- parents(x, father(x), mother(Bill)); parents(Bill, father(y), z)
  - {x/Bill, y/Bill, z/mother(Bill)}
- parents(x, father(x), mother(Jane)); parents(Bill, father(y), mother(y))
  - Failure

Convert to CNF

$\forall$ x.(P(x) $\Rightarrow$ ($\forall$ y.(P(y) $\Rightarrow$ P(f(x,y))) $\wedge$ -$\forall$ y.(Q(x,y) $\Rightarrow$ P(y))))

Convert to CNF

$\forall$ x.(P(x) $\Rightarrow$ ($\forall$ y.(P(y) $\Rightarrow$ P(f(x,y))) $\wedge$ -$\forall$ y.(Q(x,y) $\Rightarrow$ P(y))))

- Eliminate implication

Convert to CNF

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- Eliminate implication
  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ -$\forall$ y.(-Q(x,y) $\vee$ P(y))))

Convert to CNF

$\forall$ x.(P(x) $\Rightarrow$ ($\forall$ y.(P(y) $\Rightarrow$ P(f(x,y))) $\wedge$ -$\forall$ y.(Q(x,y) $\Rightarrow$ P(y))))

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- Move - inwards

Convert to CNF

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  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ $\exists$ y.(Q(x,y) $\wedge$ -P(y))))

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- Move - inwards
  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ $\exists$ y.(Q(x,y) $\wedge$ -P(y))))
- Standardize Variables

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$\forall$ x.(P(x) $\Rightarrow$ ($\forall$ y.(P(y) $\Rightarrow$ P(f(x,y))) $\wedge$ -$\forall$ y.(Q(x,y) $\Rightarrow$ P(y))))

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  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ $\exists$ z.(Q(x,z) $\wedge$ -P(z))))

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  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ $\exists$ z.(Q(x,z) $\wedge$ -P(z))))
- Skolemize

Convert to CNF

$\forall$ x.(P(x) $\Rightarrow$ ($\forall$ y.(P(y) $\Rightarrow$ P(f(x,y))) $\wedge$ -$\forall$ y.(Q(x,y) $\Rightarrow$ P(y))))

- Eliminate implication
  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ -$\forall$ y.(-Q(x,y) $\vee$ P(y))))
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  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ $\exists$ y.(Q(x,y) $\wedge$ -P(y))))
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  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ $\exists$ z.(Q(x,z) $\wedge$ -P(z))))
- Skolemize
  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ (Q(x,g(x)) $\wedge$ -P(g(x)))))

Convert to CNF

$\forall$ x.(P(x) $\Rightarrow$ ($\forall$ y.(P(y) $\Rightarrow$ P(f(x,y))) $\wedge$ -$\forall$ y.(Q(x,y) $\Rightarrow$ P(y))))

- Eliminate implication
  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ -$\forall$ y.(-Q(x,y) $\vee$ P(y))))
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- Skolemize
  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ (Q(x,g(x)) $\wedge$ -P(g(x)))))
- Drop universal

Convert to CNF

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  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ $\exists$ y.(Q(x,y) $\wedge$ -P(y))))
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  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ $\exists$ z.(Q(x,z) $\wedge$ -P(z))))
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  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ (Q(x,g(x)) $\wedge$ -P(g(x)))))
- Drop universal
  - (-P(x) $\vee$ ((-P(y) $\vee$ P(f(x,y))) $\wedge$ (Q(x,g(x)) $\wedge$ -P(g(x)))))

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  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ (Q(x,g(x)) $\wedge$ -P(g(x)))))
- Drop universal
  - (-P(x) $\vee$ ((-P(y) $\vee$ P(f(x,y))) $\wedge$ (Q(x,g(x)) $\wedge$ -P(g(x)))))
- Distribute ‘or’ over ‘and’

Convert to CNF

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  - (-P(x) $\vee$ ((-P(y) $\vee$ P(f(x,y))) $\wedge$ (Q(x,g(x)) $\wedge$ -P(g(x)))))
- Distribute ‘or’ over ‘and’
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  - (-P(x) $\vee$ ((-P(y) $\vee$ P(f(x,y))) $\wedge$ (Q(x,g(x)) $\wedge$ -P(g(x)))))
- Distribute 'or' over 'and'
  - (-P(x) $\vee$ -P(y) $\vee$ P(f(x,y))) $\wedge$ (-P(x) $\vee$ Q(x,g(x))) $\wedge$ (-P(x) v -P(g(x))
- Create separate clauses

Convert to CNF

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- Move - inwards
  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ $\exists$ y.(Q(x,y) $\wedge$ -P(y))))
- Standardize Variables
  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ $\exists$ z.(Q(x,z) $\wedge$ -P(z))))
- Skolemize
  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ (Q(x,g(x)) $\wedge$ -P(g(x)))))
- Drop universal
  - (-P(x) $\vee$ ((-P(y) $\vee$ P(f(x,y))) $\wedge$ (Q(x,g(x)) $\wedge$ -P(g(x)))))
- Distribute 'or' over 'and'
  - (-P(x) $\vee$ -P(y) $\vee$ P(f(x,y))) $\wedge$ (-P(x) $\vee$ Q(x,g(x))) $\wedge$ (-P(x) v -P(g(x))
- Create separate clauses
  - -P(x) $\vee$ -P(y) $\vee$ P(f(x,y))
  - -P(x) $\vee$ Q(x,g(x))
  - -P(x) $\vee$ -P(g(x))

Convert to CNF

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  - $\forall$ x.(-P(x) $\vee$ ($\forall$ y.(-P(y) $\vee$ P(f(x,y))) $\wedge$ (Q(x,g(x)) $\wedge$ -P(g(x)))))
- Drop universal
  - (-P(x) $\vee$ ((-P(y) $\vee$ P(f(x,y))) $\wedge$ (Q(x,g(x)) $\wedge$ -P(g(x)))))
- Distribute 'or' over 'and'
  - (-P(x) $\vee$ -P(y) $\vee$ P(f(x,y))) $\wedge$ (-P(x) $\vee$ Q(x,g(x))) $\wedge$ (-P(x) v -P(g(x))
- Create separate clauses
  - -P(x) $\vee$ -P(y) $\vee$ P(f(x,y))
  - -P(x) $\vee$ Q(x,g(x))
  - -P(x) $\vee$ -P(g(x))
- Standardize variables
  - -P(x) $\vee$ -P(y) $\vee$ P(f(x,y))
  - -P(z) $\vee$ Q(z,g(z))
  - -P(w) $\vee$ -P(g(w))

Resolution by Refutation

Knowledge Base:
Tony, Sam and Ellen belong to the Activity Club. Every member of the Activity Club is either a skier or a mountain climber or both. No mountain climber likes rain, and all skiers like snow. Ellen dislikes whatever Tony likes and likes everything that Tony dislikes. Tony likes rain and snow.
Question:
Which member of the Activity Club is a mountain climber but not a skier?

Translate to FOL Sentences

member(x) means is a member of activity club, S(x) means x is a skier, M(x) means x is a mountain climber, and L(x,y) means x likes y

- Every member of the Activity Club is either a skier or a mountain climber or both.
- No mountain climber likes rain
- All skiers like snow.
- Tony, Sam and Ellen belong to the Activity Club.
- Ellen dislikes whatever Tony likes and likes everything that Tony dislikes.
- Tony likes rain and snow.
- Which member of the Activity Club who is a mountain climber but not a skier?

Translate to FOL Sentences

member(x) means is a member of activity club, S(x) means x is a skier, M(x) means x is a mountain climber, and L(x,y) means x likes y

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  - $\forall$ x. member(x) $\Rightarrow$ (S(x) $\vee$ M(x))
- No mountain climber likes rain
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- No mountain climber likes rain
  - -$\exists$ x. M(x) $\wedge$ L(x, Rain)
- All skiers like snow.
- Tony, Sam and Ellen belong to the Activity Club.
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  - $\forall$ x. member(x) $\Rightarrow$ (S(x) $\vee$ M(x))
- No mountain climber likes rain
  - -$\exists$ x. M(x) $\wedge$ L(x, Rain)
- All skiers like snow.
  - $\forall$ x. S(x) $\Rightarrow$ L(x, Snow)
- Tony, Sam and Ellen belong to the Activity Club.
- Ellen dislikes whatever Tony likes and likes everything that Tony dislikes.
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- Which member of the Activity Club who is a mountain climber but not a skier?

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  - $\forall$ x. S(x) $\Rightarrow$ L(x, Snow)
- Tony, Sam and Ellen belong to the Activity Club.
  - member(Tony)
  - member (Sam)
  - member (Ellen)
- Ellen dislikes whatever Tony likes and likes everything that Tony dislikes.
- Tony likes rain and snow.
- Which member of the Activity Club who is a mountain climber but not a skier?

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- All skiers like snow.
  - $\forall$ x. S(x) $\Rightarrow$ L(x, Snow)
- Tony, Sam and Ellen belong to the Activity Club.
  - member(Tony)
  - member (Sam)
  - member (Ellen)
- Ellen dislikes whatever Tony likes and likes everything that Tony dislikes.
  - $\forall$ y. L(Ellen, y) $\Leftrightarrow$ -L(Tony, y)
- Tony likes rain and snow.

- Which member of the Activity Club who is a mountain climber but not a skier?

Translate to FOL Sentences

member(x) means is a member of activity club, S(x) means x is a skier, M(x) means x is a mountain climber, and L(x,y) means x likes y

- Every member of the Activity Club is either a skier or a mountain climber or both.
  - $\forall$ x. member(x) $\Rightarrow$ (S(x) $\vee$ M(x))
- No mountain climber likes rain
  - -$\exists$ x. M(x) $\wedge$ L(x, Rain)
- All skiers like snow.
  - $\forall$ x. S(x) $\Rightarrow$ L(x, Snow)
- Tony, Sam and Ellen belong to the Activity Club.
  - member(Tony)
  - member (Sam)
  - member (Ellen)
- Ellen dislikes whatever Tony likes and likes everything that Tony dislikes.
  - $\forall$ y. L(Ellen, y) $\Leftrightarrow$ -L(Tony, y)
- Tony likes rain and snow.
  - L(Tony, Rain)
  - L(Tony, Snow)
- Which member of the Activity Club who is a mountain climber but not a skier?

Translate to FOL Sentences

member(x) means is a member of activity club, S(x) means x is a skier, M(x) means x is a mountain climber, and L(x,y) means x likes y

- Every member of the Activity Club is either a skier or a mountain climber or both.
  - $\forall$ x. member(x) $\Rightarrow$ (S(x) $\vee$ M(x))
- No mountain climber likes rain
  - -$\exists$ x. M(x) $\wedge$ L(x, Rain)
- All skiers like snow.
  - $\forall$ x. S(x) $\Rightarrow$ L(x, Snow)
- Tony, Sam and Ellen belong to the Activity Club.
  - member(Tony)
  - member (Sam)
  - member (Ellen)
- Ellen dislikes whatever Tony likes and likes everything that Tony dislikes.
  - $\forall$ y. L(Ellen, y) $\Leftrightarrow$ -L(Tony, y)
- Tony likes rain and snow.
  - L(Tony, Rain)
  - L(Tony, Snow)
- Which member of the Activity Club who is a mountain climber but not a skier?
  - Query ($\exists$ x. member(x) $\wedge$ M(x) $\wedge$ -S(x))
  - Negation of the Query or Answer literal (-$\exists$ x. member(x) $\wedge$ M(x) $\wedge$ -S(x)) $\vee$ Ans(x)

CNF

$\forall$ x. member(x) $\Rightarrow$ S(x) $\vee$ M(x)	1. -member(x1) $\vee$ S(x1) $\vee$ M(x1)
-$\exists$ x. M(x) $\wedge$ L(x, Rain)	2. -M(x2) $\vee$ -L(x2, Rain)
$\forall$ x. S(x) $\Rightarrow$ L(x, Snow)	3. -S(x3) $\vee$ L(x3, Snow)
member(Tony)	4. member(Tony)
member (Sam)	5. member(Sam)
member (Ellen)	6. member(Ellen)
$\forall$ y. L(Ellen, y) $\Leftrightarrow$ -L(Tony, y)	7. -L(Tony, x4) $\vee$ -L(Ellen, x4)
	8. L(Tony, x5) $\vee$ L(Ellen, x5)
L(Tony, Rain)	9. L(Tony, Rain)
L(Tony, Snow)	10. L(Tony, Snow)
(-$\exists$ x. member(x) $\wedge$ M(x) $\wedge$ -S(x)) $\vee$ Ans(x)	11. (-member(x7) $\vee$ -M(x7) $\vee$ S(x7) $\vee$ Ans(x7)

Resolution

1. -member(x1) $\vee$ S(x1) $\vee$ M(x1)
2. -M(x2) $\vee$ -L(x2, Rain)
3. -S(x3) $\vee$ L(x3, Snow)
4. member(Tony)
5. member(Sam)
6. member(Ellen)
7. -L(Tony, x4) $\vee$ -L(Ellen, x4)
8. L(Tony, x5) $\vee$ L(Ellen, x5)
9. L(Tony, Rain)
10. L(Tony, Snow)
11. (-member(x7) $\vee$ -M(x7) $\vee$ S(x7)) $\vee$ Ans(x7)

Resolution

1. -member(x1) $\vee$ S(x1) $\vee$ M(x1)
2. -M(x2) $\vee$ -L(x2, Rain)
3. -S(x3) $\vee$ L(x3, Snow)
4. member(Tony)
5. member(Sam)
6. member(Ellen)
7. -L(Tony, x4) $\vee$ -L(Ellen, x4)
8. L(Tony, x5) $\vee$ L(Ellen, x5)
9. L(Tony, Rain)
10. L(Tony, Snow)
11. (-member(x7) $\vee$ -M(x7) $\vee$ S(x7)) $\vee$ Ans(x7)
12. S(x1) $\vee$ (-member(x1) $\vee$ Ans(x1)) [11,1] {x7/x1}

Resolution

1. -member(x1) $\vee$ S(x1) $\vee$ M(x1)
2. -M(x2) $\vee$ -L(x2, Rain)
3. -S(x3) $\vee$ L(x3, Snow)
4. member(Tony)
5. member(Sam)
6. member(Ellen)
7. -L(Tony, x4) $\vee$ -L(Ellen, x4)
8. L(Tony, x5) $\vee$ L(Ellen, x5)
9. L(Tony, Rain)
10. L(Tony, Snow)
11. (-member(x7) $\vee$ -M(x7) $\vee$ S(x7)) $\vee$ Ans(x7)
12. S(x1) $\vee$ (-member(x1) $\vee$ Ans(x1)) [11,1] {x7/x1}
13. L(x1, Snow) $\vee$ (-member(x1) $\vee$ Ans(x1)) [12,3] {x3/x1}

Resolution

1. -member(x1) $\vee$ S(x1) $\vee$ M(x1)
2. -M(x2) $\vee$ -L(x2, Rain)
3. -S(x3) $\vee$ L(x3, Snow)
4. member(Tony)
5. member(Sam)
6. member(Ellen)
7. -L(Tony, x4) $\vee$ -L(Ellen, x4)
8. L(Tony, x5) $\vee$ L(Ellen, x5)
9. L(Tony, Rain)
10. L(Tony, Snow)
11. (-member(x7) $\vee$ -M(x7) $\vee$ S(x7)) $\vee$ Ans(x7)
12. S(x1) $\vee$ (-member(x1) $\vee$ Ans(x1)) [11,1] {x7/x1}
13. L(x1, Snow) $\vee$ (-member(x1) $\vee$ Ans(x1)) [12,3] {x3/x1}
14. -L(Tony, Snow) $\vee$ (-member(Ellen) $\vee$ Ans(Ellen)) [13,7] {x4/Snow, x1/Ellen}

Resolution

1. -member(x1) ∨ S(x1) ∨ M(x1)
2. -M(x2) ∨ -L(x2, Rain)
3. -S(x3) ∨ L(x3, Snow)
4. member(Tony)
5. member(Sam)
6. member(Ellen)
7. -L(Tony, x4) ∨ -L(Ellen, x4)
8. L(Tony, x5) ∨ L(Ellen, x5)
9. L(Tony, Rain)
10. L(Tony, Snow)
11. (-member(x7) ∨ -M(x7) ∨ S(x7)) ∨ Ans(x7)
12. S(x1) ∨ (-member(x1) ∨ Ans(x1)) [11,1] {x7/x1}
13. L(x1, Snow) ∨ (-member(x1) ∨ Ans(x1)) [12,3] {x3/x1}
14. -L(Tony, Snow) ∨ (-member(Ellen) ∨ Ans(Ellen)) [13,7] {x4/Snow, x1/Ellen}
15. -member(Ellen) v Ans(Ellen) [14,7] {}

Resolution

1. -member(x1) ∨ S(x1) ∨ M(x1)
2. -M(x2) ∨ -L(x2, Rain)
3. -S(x3) ∨ L(x3, Snow)
4. member(Tony)
5. member(Sam)
6. member(Ellen)
7. -L(Tony, x4) ∨ -L(Ellen, x4)
8. L(Tony, x5) ∨ L(Ellen, x5)
9. L(Tony, Rain)
10. L(Tony, Snow)
11. (-member(x7) ∨ -M(x7) ∨ S(x7)) ∨ Ans(x7)
12. S(x1) ∨ (-member(x1) ∨ Ans(x1)) [11,1] {x7/x1}
13. L(x1, Snow) ∨ (-member(x1) ∨ Ans(x1)) [12,3] {x3/x1}
14. -L(Tony, Snow) ∨ (-member(Ellen) ∨ Ans(Ellen)) [13,7] {x4/Snow, x1/Ellen}
15. -member(Ellen) v Ans(Ellen) [14,7] {}
16. Ans(Ellen) [15,6] {}