Propositional Logic

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Outline

1. Translation

2. Valid, Satisfiable, Unsatisfiable

3. Resolution

Translate to Propositional Logic

`p`: You get an A on the final exam
`q`: You do every exercise in the book
`r`: You get an A in this class

Translate to Propositional Logic

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`r`: You get an A in this class

- You get an A in this class, but you do not do every exercise in the book

Translate to Propositional Logic

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- You get an A in this class, but you do not do every exercise in the book
- $r \wedge -q$

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- You get an A in this class, but you do not do every exercise in the book
- $r \wedge -q$
- If you got an A in this class, you must have gotten an A on the final.

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- If you got an A in this class, you must have gotten an A on the final.
- $r \Rightarrow p$
- Getting an A on the final and doing every exercise in the book is sufficient for getting an A in this class

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- Getting an A on the final and doing every exercise in the book is sufficient for getting an A in this class
- $p \wedge q \Rightarrow r$
- You cannot get an A in this class if you do not do every exercise in the book, unless you get an A on the final.

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- You cannot get an A in this class if you do not do every exercise in the book, unless you get an A on the final.
- $-q \Rightarrow (-p \Rightarrow -r)$ or $-q \wedge -p \Rightarrow -r$
- why?

Determine if valid/satisfiable/unsatisfiable

- RockStar $\vee$ -RockStar

Determine if valid/satisfiable/unsatisfiable

- RockStar ∨ -RockStar
- valid, satisfiable

Determine if valid/satisfiable/unsatisfiable

- RockStar $\vee$ -RockStar
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- BigHouse $\wedge$ -BigHouse

Determine if valid/satisfiable/unsatisfiable

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- BigHouse $\wedge$ -BigHouse
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Determine if valid/satisfiable/unsatisfiable

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- Raining $\wedge$ Rainbow

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Determine if valid/satisfiable/unsatisfiable

- RockStar ∨ -RockStar
- valid, satisfiable
- BigHouse ∧ -BigHouse
- unsatisfiable
- Raining ∧ Rainbow
- satisfiable
- $(-P \vee -Q) \Rightarrow ((-P \wedge Q) \vee (P \wedge -Q) \vee (-P \wedge -Q))$

Determine if valid/satisfiable/unsatisfiable

- RockStar $\vee$ -RockStar
- valid, satisfiable
- BigHouse $\wedge$ -BigHouse
- unsatisfiable
- Raining $\wedge$ Rainbow
- satisfiable
- $(-P \vee -Q) \Rightarrow ((-P \wedge Q) \vee (P \wedge -Q) \vee (-P \wedge -Q))$
- valid, satisfiable

Determine if valid/satisfiable/unsatisfiable

- RockStar $\vee$ -RockStar
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- BigHouse $\wedge$ -BigHouse
- unsatisfiable
- Raining $\wedge$ Rainbow
- satisfiable
- $(-P \vee -Q) \Rightarrow ((-P \wedge Q) \vee (P \wedge -Q) \vee (-P \wedge -Q))$
- valid, satisfiable
- $((-P \vee P) \Rightarrow Q) \wedge (Q \Rightarrow (P \wedge -P))$

Determine if valid/satisfiable/unsatisfiable

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- Raining $\wedge$ Rainbow
- satisfiable
- $(-P \vee -Q) \Rightarrow ((-P \wedge Q) \vee (P \wedge -Q) \vee (-P \wedge -Q))$
- valid, satisfiable
- $((-P \vee P) \Rightarrow Q) \wedge (Q \Rightarrow (P \wedge -P))$
- unsatisfiable

Resolution by Contradiction

Sentence:

- Heads I win; tails, you lose

Prove:

- I win

Resolution by Contradiction

Sentence:

- Heads I win; tails, you lose
- Coin must be head or tail but not both.
- Winning and losing are complements for both you and me.
- Exactly one of us will win.

Prove:

- I win

Axioms

- Coin must be head or tail but not both.
- Winning and losing are complements for both you and me.
- Exactly one of us will win.
- Heads I win
- Tails you lose

Axioms

- Coin must be head or tail but not both.
  - $H \Leftrightarrow -T$
- Winning and losing are complements for both you and me.
  - $IW \Leftrightarrow -IL$
  - $YW \Leftrightarrow -YL$
- Exactly one of us will win.
  - $IW \Leftrightarrow -YW$
- Heads I win
  - $H \Rightarrow IW$
- Tails you lose
  - $T \Rightarrow YL$

Axioms

- Coin must be head or tail but not both.
  - $H \Leftrightarrow -T$
- Winning and losing are complements for both you and me.
  - $IW \Leftrightarrow -IL$
  - $YW \Leftrightarrow -YL$
- Exactly one of us will win.
  - $IW \Leftrightarrow -YW$
- Heads I win
  - $H \Rightarrow IW$
- Tails you lose
  - $T \Rightarrow YL$
- Next Step?

KB in Conjunctive Normal Form

Knowledge Base

1. $H \Leftrightarrow -T$	1a. $-H \vee -T$
	1b. $T \vee H$
2. $IW \Leftrightarrow -IL$	2a. $-IW \vee -IL$
	2b. $IL \vee IW$
3. $YW \Leftrightarrow -YL$	3a. $-YW \vee -YL$
	3b. $YL \vee YW$
4. $IW \Leftrightarrow -YW$	4a. $-IW \vee -YW$
	4b. $YW \vee IW$
5. $H \Rightarrow IW$	5. $-H \vee IW$
6. $T \Rightarrow LY$	6. $-T \vee YL$

To prove(α): IW
Add -α to KB

Resolution

1a. $-H \vee -T$
1b. $T \vee H$
2a. $-IW \vee -IL$
2b. $IL \vee IW$
3a. $-YW \vee -YL$
3b. $YL \vee YW$
4a. $-IW \vee -YW$
4b. $YW \vee IW$
5. $-H \vee IW$
6. $-T \vee YL$

7. $T \vee IW$ [1b,5]
8. $H \vee YL$ [1b, 6]
9. $H \vee -YW$ [3a, 8]
10. $H \vee IW$ [4b, 9]
11. $-T \vee IW$ [1a, 10]
12. IW [7,11;11,13;10,18]
13. T [0, 7; 1b,15]
14. YW [0,4b]
15. $-H$ [0,5]
16. $-IW$ [4a,14]
17. YL [15,8]
18. $-YW$ [15,9]
17. *empty* [12, 0; 12,16]