

## Original

$$z = (2i + (l - 1)) \frac{L}{2q}$$

Where:  $z = f * \frac{L}{2q}$

## Second Order

$$f = C_0 + C_1(2i + l) + C_2(2i + l)^2$$

$$i = 0 \quad l = 1 \quad f = 0$$

$$f = 0 = C_0 + C_1 + C_2$$

$$i = q \quad l = 1 \quad f = 2q$$

$$f = 2q = C_0 + C_1(2q + 1) + C_2(2q + 1)^2$$

Linearizing this matrix to isolate  $C_2$  leads to:

$$\begin{bmatrix} 1 & 1 \\ 1 & 2q + 1 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \end{bmatrix} = \begin{bmatrix} 0 - C_2 \\ 2q - C_2(2q + 1)^2 \end{bmatrix}$$

Solving in Matlab:

```
syms q C_2
A = [1 1 ;
     1 (2*q+1)]
```

A =

$$\begin{pmatrix} 1 & 1 \\ 1 & 2q + 1 \end{pmatrix}$$

```
B = [0 - C_2 ;
     2*q - C_2*(2*q+1)^2]
```

B =

$$\begin{pmatrix} -C_2 \\ 2q - C_2(2q + 1)^2 \end{pmatrix}$$

```
x = linsolve(A,B)
```

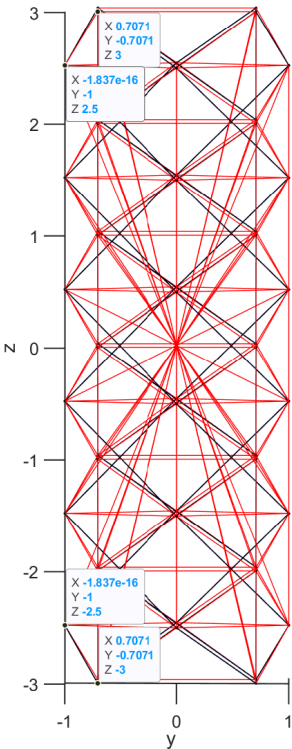
x =

$$\begin{pmatrix} C_2 + 2C_2q - 1 \\ 1 - 2C_2q - 2C_2 \end{pmatrix}$$

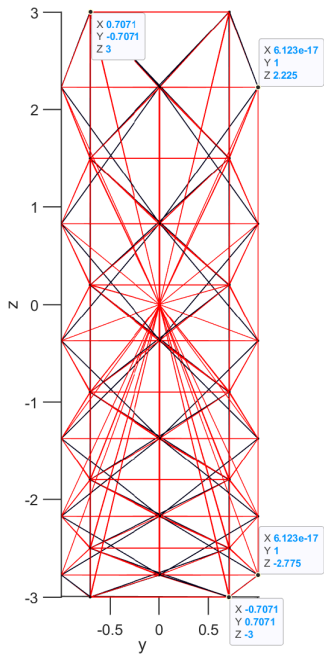
Plug in  $C_0$  and  $C_1$  into  $f$

$$f = C_2 + 2C_2q - 1 + (2i + l)(1 - 2C_2q - 2C_2) + (2i + l)^2C_2$$

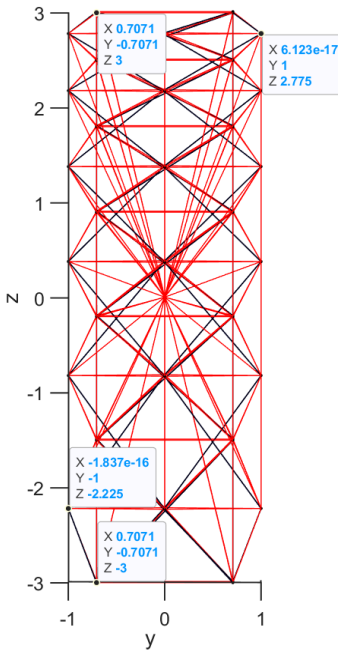
### Output



$$C_2 = 0$$



$$C_2 = 0.05$$



$$C_2 = -0.05$$

## Third Order

$$f = C_0 + C_1(2i + l) + C_2(2i + l)^2 + C_3(2i + l)^3$$

$$i = 0 \quad l = 1 \quad f = 0$$

$$f = 0 = C_0 + C_1 + C_2 + C_3$$

$$i = q \quad l = 1 \quad f = 2q$$

$$f = 2q = C_0 + C_1(2q + 1) + C_2(2q + 1)^2 + C_3(2q + 1)^3$$

Linearizing this matrix to isolate  $C_3$  leads to:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2q + 1 & (2q + 1)^2 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} -C_3 \\ 2q - C_3(2q + 1)^3 \end{bmatrix}$$

For symmetry, we can set  $C_2 = 0$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2q + 1 & 0 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} -C_3 \\ 2q - C_3(2q + 1)^3 \end{bmatrix}$$

Solving in Matlab:

```
clear;
syms C_3 q
A = [1 1 1;
     1 (2*q+1) (2*q+1)^2]
```

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2q + 1 & (2q + 1)^2 \end{pmatrix}$$

$$B = \begin{bmatrix} -C_3; \\ 2q - C_3(2q + 1)^3 \end{bmatrix}$$

$$B = \begin{pmatrix} -C_3 \\ 2q - C_3(2q + 1)^3 \end{pmatrix}$$

$$x = \text{linsolve}(A, B)$$

Warning: Solution is not unique because the system is rank-deficient.

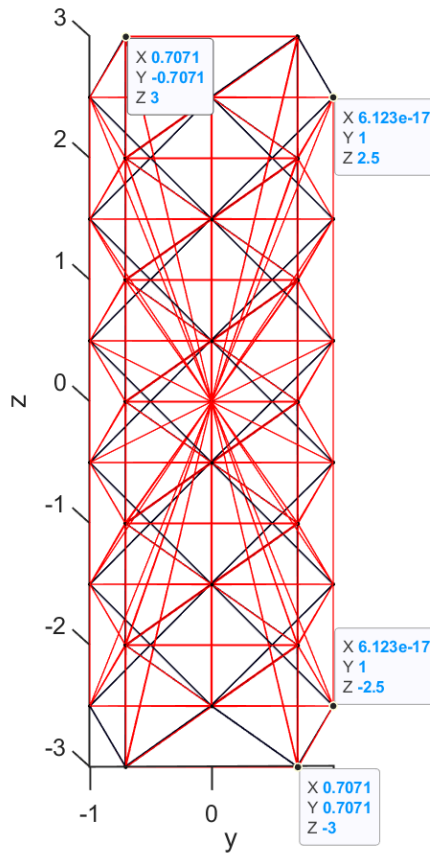
x =

$$\begin{pmatrix} 4 C_3 q^2 + 6 C_3 q + 2 C_3 - 1 \\ -4 C_3 q^2 - 6 C_3 q - 3 C_3 + 1 \\ 0 \end{pmatrix}$$

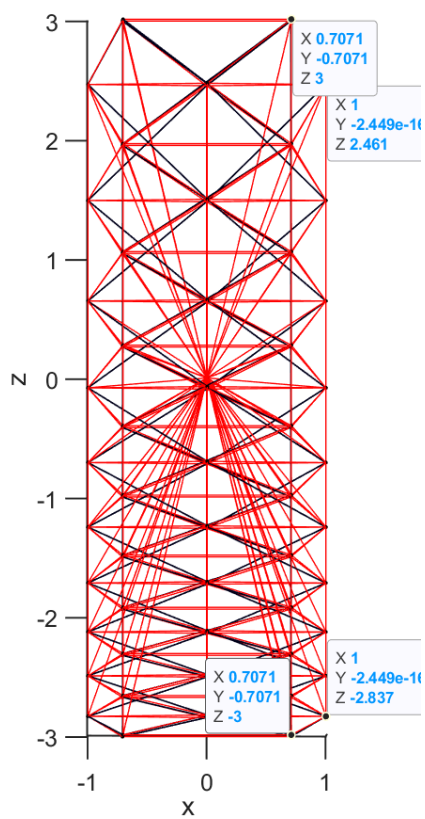
Plug in  $C_0$  and  $C_1$  into  $f$

$$f = C_0 + C_1(2i + l) + C_2(2i + l)^2 + C_3(2i + l)^3$$

$$f = 4C_3q^2 + 6C_3q + 2C_3 - 1 + (-4C_3q^2 - 6C_3q - 3C_3 + 1)(2i + l) + C_3(2i + l)^3$$

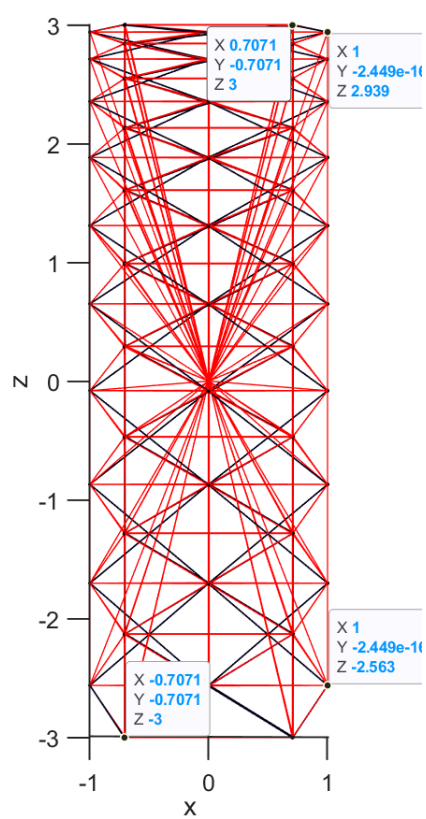


$$C_3 = 0$$



$C_3 = 0.01$

$C_3 = -0.01$



what in the world.