

$$S = 2 \left(\int_0^{l_{back}} f_{back}(x) dx + \int_0^{l_{front}} f_{front}(x) dx + \int_0^{l_{tip}} f_{tip}(x) dx \right)$$

$$f_{back}(x) = \frac{(w_{tail} - w_{mid})}{2 l_{back}^2} x^2 + \frac{w_{mid}}{2}$$

$$f_{front}(x) = \frac{(w_{tip} - w_{mid})}{2 l_{front}^2} x^2 + \frac{w_{mid}}{2}$$

$$f_{tip}(x) = \frac{w_{tip}}{2 \ln(l_{tip} + 1)} \ln(x + 1)$$

$$\int f_{back}(x) dx = \frac{(w_{tail} - w_{mid})}{6 l_{back}^2} x^3 + \frac{w_{mid}}{2} x$$

$$\int f_{front}(x) dx = \frac{(w_{tip} - w_{mid})}{6 l_{front}^2} x^3 + \frac{w_{mid}}{2} x$$

$$\int f_{tip}(x) dx = \frac{w_{tip}}{2 \ln(l_{tip} + 1)} (x + 1) \ln(x + 1) - \frac{w_{tip}}{2 \ln(l_{tip} + 1)} x$$

$$S = 2 \left(\frac{(w_{tail} - w_{mid})}{6 l_{back}^2} l_{back}^3 + \frac{w_{mid}}{2} l_{back} + \frac{(w_{tip} - w_{mid})}{6 l_{front}^2} l_{front}^3 + \frac{w_{mid}}{2} l_{front} + \frac{w_{tip}}{2 \ln(l_{tip} + 1)} (l_{tip} + 1) \ln(l_{tip} + 1) - \frac{w_{tip}}{2 \ln(l_{tip} + 1)} l_{tip} \right)$$