Chap 2 行列式

§2.4 古典伴隨矩陣

Def.

$A: n\times n$，定義 $adj(A) = [d_{ij}]: n\times n$. 其中 $d_{ij} = cof(a_{ji})$

稱 A 之 classical adjoint

Ex. (96雲科)

$A = \begin{bmatrix} 1 & 4 & 2 \\ 0 & 2 & 3 \\ 4 & -1 & 1 \end{bmatrix}$，求 $adj(A)$

sol.

$cof(a_{11}) = + \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} = 5$

$cof(a_{12}) = - \begin{vmatrix} 0 & 3 \\ 4 & 1 \end{vmatrix} = 12$

$\vdots$

$adj(A) = \begin{bmatrix} 5 & -2 & 16 \\ 12 & 9 & -3 \\ -8 & 17 & 2 \end{bmatrix}$

Thm

$A: n \times n$. $A \times adj(A) = adj(A) \times A = det(A) \times I_n$

Note.

$A: n \times n$ 可逆.

$$A \cdot \frac{adj(A)}{det(A)} = I \qquad \therefore A^{-1} = \frac{adj(A)}{det(A)}$$

Lemma:

$A: n \times n$, A 可逆 $\Rightarrow adj(A)$: 可逆 且 $adj(A)^{-1} = \frac{A}{det(A)}$

Ex.

$A: n \times n$ 可逆. 求 (1) $det(adj(A))$ (2) $adj(adj(A))$

sol.

(1) $A \cdot adj(A) = det(A) \cdot I$

$\Rightarrow det(A) \, det(adj(A)) = det(det(A) \cdot I)$

$= det(A)^n$

$\Rightarrow det(adj(A)) = det(A)^{n-1}$

(2) $adj(A) \cdot adj(adj(A)) = det(adj(A)) \cdot I$

$adj(adj(A)) = det(adj(A)) \cdot adj(A)^{-1} I$

$= det(A)^{n-1} \frac{A}{det(A)}$

$= det(A)^{n-2} A$

Thm

$A: n\times n$. A 可逆 $\Longleftrightarrow$ $adj(A)$: 可逆

pf

($\Rightarrow$): ok. Lemma

($\Leftarrow$): $\because$ $adj(A)$: 可逆.

$\cancel{\Rightarrow \det(adj(A)) = \det(A)^{n-1} \neq 0}$

$\cancel{\Rightarrow \det(A) \neq 0 \rightarrow A\text{: 可逆.}}$

設 A 不可逆. 則 $\det(A) = 0$

$\Rightarrow A \cdot adj(A) = \det(A) \cdot I = 0$

$\Rightarrow A = 0 \cdot adj(A)^{-1} = 0$

$\Rightarrow adj(A) = 0$ $\rightarrow\!\leftarrow$

課程5 下. 1:57:27